ON THE PHYSICAL INTERPRETATION OF A SOLUTION OF A NONSYMMETRIC UNIFIED FIELD THEORY

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We re-examine the physical interpretation of a spherically symmetric static solution of the Einstein-Straus-Klotz non-symmetric unified field theory.

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1. *Introduction*

In this paper we examine the background (pseudo-Riemannian) space-time generated by a static spherically symmetric solution of the Einstein-Straus-Klotz nonsymmetric unified field theory (GFT). On the basis of this analysis a new physical interpretation of the space-time will be proposed. This interpretation will be compared and contrasted with the previously proposed one (Refs. [1–5]).

The original interpretation asserted that the space-time under consideration corresponds to that of an expanding universe (somewhat similar to the de Sitter one, except non-empty). Attempts have been made (Refs. [1–5]) to identify the general features of this model with those of our universe. We shall argue, however, that on the contrary the background space-time represents the exterior geometry of a static (Schwarzschild) black hole in a static (Einstein) universe. Thus this space-time cannot represent a cosmological model in the accepted meaning of the term. We note, however, that Ellis (Ref. [6]) has shown, in the context of General Relativity, that a class of static spherically symmetric space-times with two centres can account for many of the observed features of our universe. Generalization of Ellis' work to GFT may make it possible to retain the cosmological interpretation (although radically different one from that advanced in (Ref. [1–5]). We shall return to this point later.

Before writing down the actual line element of the background geometry we give a brief account of the underlying theory. The GFT, which consists of the weak field equa-

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tions of Einstein and Straus (Ref. [7]) together with the metric hypothesis introduced by Klotz (Ref. [5]), aims to unify gravitation and electromagnetism on the macroscopic level. The solutions (under various symmetry restrictions) of the weak field equations are given in terms of the fundamental nonsymmetric field $g_{\mu\nu}$ which induces, via the metric hypothesis, a pseudo-Riemannian metric tensor field $a_{\mu\nu}$ in the underlying manifold. For a comprehensive description of this theory see the review article (Ref. [8]) or the book (Ref. [5]) devoted to this subject. The metric hypothesis is discussed, in some detail, by Klotz and McInnes (Ref. [9]).

2. *The background metric*

The Einstein-Straus weak field equations have been solved, in the case of a static spherically symmetric field $g_{\mu\nu}$, by Vanstone (Ref. [10]). This solution is given in terms of an arbitrary function of the radial coordinate, thus there are effectively infinitely many static spherically symmetric solutions. However, the requirement of GFT that the Vanstone solution be compatible with the metric hypothesis reduces the Vanstone class to two possible solutions (Ref. [5]). One of these solutions is identified with static, spherically symmetric, electric charge. The cosmological model in question is based on the background metric of this solution, which is given by

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2+\frac{dr^2}{(1-2m/r)(1+r^2/r_0^2)^2} + \frac{r_0^2 r^2}{r^2+r_0^2}(d\theta^2+\sin^2\theta d\psi^2), \quad 2m < r < \infty, \tag{1}$$

where m and r_0 are parameters. In order to analyse this geometry we examine the obvious limiting cases of line element (1) i.e. (*i*) $r_0 \to \infty, m \to 0$, (*ii*) $r_0 \to \infty, m$ fixed (> 0), and (*iii*) $m \to 0, r_0$ fixed ($> 2m$).

It is clear that in the first case (1) reduces to the Minkowski line element (also $g_{\mu\nu}$ becomes symmetric). In the second case (Ref. [4]) the line element (1) reduces to

$$ds^2 = -\left(1-\frac{2m}{r}\right)dt^2+\left(1-\frac{2m}{r}\right)^{-1} dr^2+r^2(d\theta^2+\sin^2\theta d\psi^2), \tag{2}$$

and $g_{\mu\nu}$ becomes symmetric. This is the familiar line element describing the exterior geometry of the static black hole solution in General Relativity. Finally, in the third case (1) reduces to

$$ds^2 = -dt^2+\frac{1}{\left(1+\frac{r^2}{r_0^2}\right)^2}dr^2+\frac{r_0^2 r^2}{r_0^2+r^2}(d\theta^2+\sin^2\theta d\psi^2), \tag{3}$$

while $g_{\mu\nu}$ remains nonsymmetric. This is the line element of the static Einstein universe in General Relativity.

The limiting line elements (2) and (3) suggest that the space-time represented by (1) corresponds to that of a static black hole in a static universe. Furthermore, the two parameters m and r_0 can be identified with the mass of the black hole and the radius of the surrounding universe, respectively. It is also evident that the origin of the coordinate system (t, r, θ, ψ) coincides with the black hole's essential singularity.

3. *Singularities*

The line element (1) is singular at $r = 0$ and $r = 2m$. An important qustion which now arises is whether these singularities are essential or removable (coordinate system dependent). It has been claimed (Ref. [5] and references given therein) that $r = 2m$ corresponds to an essential singularity and that, as a consequence of this, the region $r \leqslant 2m$ should be identified with the 'primeval atom'. We can show, however, that the physical components of the Riemann tensor are regular on the surface $r = 2m$, which implies that the $r = 2m$ singularity is removable.

Consider the line element (1). The physical components of the Riemann tensor, i.e. components with respect to the frame carried by an observer in free fall can be calculated along the lines of (Ref. [11]). If the observer carries an orthonormal frame $w^{\hat{\tau}}$, $w^{\hat{\varrho}}$, $w^{\hat{\theta}}$, $w^{\hat{\psi}}$ then, with respect to this frame, the only non-zero components of the Riemann tensor are

$$R_{\hat{\tau}\hat{\varrho}\hat{\tau}\hat{\varrho}} = -\frac{2m}{r^3}\left(1+\frac{r^2}{r_0^2}\right), \quad R_{\hat{\tau}\hat{\theta}\hat{\tau}\hat{\theta}} = R_{\hat{\tau}\hat{\psi}\hat{\tau}\hat{\psi}} = \frac{m}{r^3}\left(1+\frac{r^2}{r_0^2}\right),$$

$$R_{\hat{\theta}\hat{\psi}\hat{\theta}\hat{\psi}} = \frac{2m}{r^3} + \frac{1}{r_0^2}, \quad R_{\hat{\varrho}\hat{\theta}\hat{\varrho}\hat{\theta}} = R_{\hat{\varrho}\hat{\psi}\hat{\varrho}\hat{\psi}} = -\frac{m}{r^3}\left(1+\frac{r^2}{r_0^2}\right) + \frac{1}{r_0^2}\left(1-\frac{2m}{r}\right), \tag{4}$$

and those obtained from the above via the symmetries of the Riemann tensor.

We observe that none of these physical components is singular on the surface $r = 2m$. Thus the tidal forces experienced by an infalling observed are finite on $r = 2m$ and so we conclude that the singularity of the line element (1), on this surface, is removable.This implies that there exists a coordinate system in terms of which the line element (1) becomes regular on $r = 2m$. Such a coordinate system (generalization of the Kruskal-Szekeres one) has in fact been found (Ref. [12]). On the other hand the singularity of the line element (1) at $r = 0$ is irremovable as can be seen on examination of the curvature invariant

$$R \equiv R_{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}}R^{\hat{\alpha}\hat{\beta}\hat{\gamma}\hat{\delta}} = \frac{48m^2}{r^6} + \frac{96m^2}{r^4 r_0^2} + \frac{96m^2}{r^2 r_0^4} - \frac{48m}{r r_0^4} + \frac{12}{r_0^4}. \tag{5}$$

This expression, which has the same value in every local Lorentz frame, is clearly singular at $r = 0$.

We can therefore conclude that there is no evidence for the "primeval atom" interpretation. In fact we are dealing with a static black hole which has event horizon at $r = 2m$. Note, however, that the radial coordinate r in (1) is not the usual Schwarzschil radial coordinate, but approaches it as $r_0 \to \infty$.

4. *The Killing vector field*

Astronomical observations reveal that our universe is isotropic about our position. Since we have no reason to believe that we occupy a privileged position in the cosmos it is conjectured that the universe is isotropic about every one of its points. Furthermore, it is assumed that on a large scale the universe is spatially homogeneous. These assumptions are incorporated into the standard cosmological models. It has been shown (Ref. [13]) that a space-time which is isotropic about each of its points is also spatially homogeneous and admits a six-parameter group of isometries whose surfaces of transitivity are space--like three-surfaces of constant curvature.

The isometries on a given space-time are generated by the Killing vector fields which satisfy

$$\zeta_{\mu;\nu}+\zeta_{\nu;\mu} = 0, \tag{6}$$

or

$$a_{\mu\nu,\lambda}\zeta^{\lambda}+a_{\mu\lambda}\zeta^{\lambda}{}_{,\nu}+a_{\lambda\nu}\zeta^{\lambda}{}_{,\mu} = 0. \tag{7}$$

In order to be isotropic about each of its points the space-time defined by (1) would have to admit six linearly independent Killing vector fields. The line element (1) is independent of t and ψ thus we would expect that this spacetime admits, at least, a two-parameter Killing vector field. In fact, it is not difficult to show that in the case of the geometry (1), equation (7) can be solved to yield the following four-parameter Killing vector field:

$$\zeta = \alpha\partial_t+(\beta \sin\psi+\gamma \cos\psi)\partial_\theta+\{(\beta \cos\psi-\gamma \sin\psi)+\delta\}\partial_\psi, \tag{8}$$

(which is identical to that admitted by the exterior Schwarzschild geometry) where ∂_t, ∂_θ and ∂_ψ are the coordinate basis vector fields. Thus the background space-time (1) admits a four-parameter group of isometries, which implies that there are at most two points from which the spacetime looks spherically symmetric. From this we immediately deduce that the line element (1) cannot be transformed into the Robertson-Walker line element (this has also been proved by direct calculation (Ref. [14])).

We conclude that line element (1) represents static spherically symmetric space-time with two centres, one of which is occupied by a static black hole. Consequently, in order to use this space-time as a model for the physical universe one would have to assume that the solar system is situated near one of the two centres while the other is occupied by a static black hole of large enough mass to account for recession of galaxies.

5. *Expansion?*

At this point we consider the assertion (Ref. [4]) that the space-time represented by the line element (1) expands, with a Hubble-like law of expansion. It will prove helpful to summarize the arguments used in [4] to deduce expansion.

The method employed in [4] is based on the analysis of radial geodesics (along the same lines as in the de Sitter universe). Starting from the radial part of the line element (1), with a change of signature, it is shown that one obtains a radial geodesic equation of the form

$$\ddot{r}+\frac{m}{r^2}\left(1+\frac{r^2}{r_0^2}\right)^2-\frac{2r}{r_0^2}\left(k+\frac{2m}{r}\right)\left(1+\frac{r^2}{r_0^2}\right)=0, \tag{9}$$

where k is a constant. This is equation (31) of [4] with ϱ replaced by r. It is then argued that for r in the range $2m \ll r \ll r_0$ equation (9) reduces to

$$\ddot{r} \approx \frac{2kr}{r_0^2}, \tag{10}$$

which corresponds (provided that k is positive) to a Hubble-like law of expansion. In order to satisfy the $k > 0$ requirement r_0 is identified with the "distance" from the observer to the Hubble horizon, and the condition

$$\frac{dr}{dt}=1 \quad \text{when} \quad r=r_0 \gg 2m, \tag{11}$$

is used.

The conclusions reached in [4] using the above described analysis are questionable. Firstly, as already noted in [4], for values of r in the range $2m < r \ll r_0$ equation (9) reduces to

$$\ddot{r} \approx -\frac{m}{r_0^2}, \tag{12}$$

which implies that for r in this range a test particle moving along a radial geodesic experiences an attractive force directed towards $r = 0$. Thus if we accept the analysis of [4] then we have a model with "short" range attraction (towards the observer, since the coordinate system is taken as observer based in [4]) and long range repulsion! At this point we note that the (t, r, θ, φ) coordinate system cannot be observer based because the space--time described by (1) does not admit a six-parameter isometry group (it does not satisfy the standard Cosmological Principle). Thus we cannot move the origin of this coordinate system from point to point as we do when dealing with Friedman-Robertson-Walker cosmologies. The origin of our coordinate system is located at the essentially singular point of the space-time (1). Secondly, one must be very careful when interpreting equation (9), for consider the radial part of the line element (1), with a change of signature, in terms of another set of coordinates, i.e.

$$ds^2=+\left(1-\frac{2m}{r_0}\cot\frac{z}{r_0}\right)dt^2-\left(1-\frac{2m}{r_0}\cot\frac{z}{r_0}\right)^{-1}dz^2, \tag{13}$$

where

$${}_0\tan^{-1}\left(\frac{2m}{r_0}\right) < z < r_0. \tag{14}$$

Applying the analysis of [4] to line element (13) we obtain

$$\ddot{z} = -\frac{m}{r_0^2}\operatorname{cosec}^2\frac{z}{r_0}. \tag{15}$$

Now with z in the range (14) $\operatorname{cosec}^2 \dfrac{z}{r_0}$ is a positive monotonically decreasing function. Thus in terms of this new coordinate system we would seem to have attractive force (directed towards $z = 0$) at all points of our space-time. This implies that the results of [4] are in some way dependent on the coordinate system used there.

The above analysis suggests that we should study the problem of expansion in terms of a co-ordinate system which can be interpreted in terms of some physical phenomenon. Since we are interested in the interpretation of (1) in terms of cosmology we should select a co-moving (with galaxies taken as test particles) coordinate system. The most suitable one should be the generalization of Novikov's [11] system because in addition to being co-moving it eliminates the coordinate singularity at $r = 2m$. This approach will not be considered in this paper, instead we shall show that the line element (1) is equivalent to the General Relativistic line element which describes a Schwarzschild black hole in the background of Einstein's universe (SE)!

In General Relativity the SE line element can be obtained as a special case of the line element of the Kerr black hole in the background of Einstein's universe which was derived by Vaidya [15]. The result is

$$\begin{aligned} ds^2 &= \left(1-\frac{2m}{R}\cot\frac{w}{R}\right)dy^2 + \frac{4m}{R}\cot\frac{w}{R}\,dydw \\ &- \left(1+\frac{2m}{R}\cot\frac{w}{R}\right)dw^2 - R^2\sin^2\frac{w}{R}\,(d\theta^2+\sin^2\theta d\psi^2), \end{aligned} \tag{16}$$

where m and R are parameters. Under the coordinate transformation

$$y = \tau + f(\xi), \qquad w = \xi,$$

where

$$\frac{df(\xi)}{d\xi} = -\frac{2m}{R}\cot\frac{\xi}{R}\left(1-\frac{2m}{R}\cot\frac{\xi}{R}\right)^{-1} \tag{17}$$

the line element (16) becomes

$$\begin{aligned} ds^2 &= \left(1-\frac{2m}{R}\cot\frac{\xi}{R}\right)d\tau^2 - \left(1-\frac{2m}{R}\cot\frac{\xi}{R}\right)^{-1}d\xi^2 \\ &- R^2\sin^2\frac{\xi}{R}\,(d\theta^2+\sin^2\theta d\psi^2). \end{aligned} \tag{18}$$

Further transformation of (18) under

$$\xi = R \tan^{-1} \frac{r}{R}, \quad \tau = t, \quad R = r_0, \tag{19}$$

yields

$$ds^2 = \left(1 - \frac{2m}{r}\right) dt^2 - \frac{dr^2}{\left(1 - \frac{2m}{r}\right)\left(1 + \frac{r^2}{r_0^2}\right)^2} - \frac{r_0^2 r^2}{r_0^2 + r^2} (d\theta^2 + \sin^2\theta d\psi^2). \tag{20}$$

But this is just the line element (1) with a change of signature!

6. *Conclusion*

We have shown that the background space-time corresponding to a GFT solution is equivalent to the SE solution of General Relativity. Thus we must reject the interpretation of (1) which asserts that (1) represents an expanding universe. Clearly (1) should be interpreted as representing the exterior geometry of a static black hole (as opposed to primeval atom) in the background of a static universe. It is possible, however, to retain (1) as a cosmological model. As we pointed out in the introduction such spherically symmetric static models have been studied in General Relativity by Ellis. In these General Relativistic models there is some difficulty with the fit of the (m, z) observations (Ref. [6]), however, it may be possible to overcome this problem in GFT (Ref. [16]).

We note that such two-centered models imply a privileged position in space. This may or may not be appealing, depending on one's philosophical viewpoint. Furthermore these models may prove to be unstable under various perturbations (eg. density perturbations etc.). Thus it seems reasonable that, in order to resolve the problem of cosmology in terms of GFT, we should also examine solutions (of the GFT equations) with stationary metrics. Unfortunately no such solutions are available at the moment.

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