HYDRODYNAMICAL SPACE-TIME EVOLUTION OF THE CENTRAL REGION OF RAPIDITIES IN ULTRA-RELATIVISTIC HEAVY ION COLLISIONS†,*

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The solutions of hydrodynamic equations describing a longitudinally expanding quark-gluon plasma produced in central collisions of two large nuclei are investigated. It is argued that a fair description of the transverse flow of plasma is given — irrespective of the initial conditions — by the relativistic simple wave of Riemann, modulated by a factor resulting from cooling due to the plasma longitudinal expansion. The role of fluctuations in the flow of plasma is also briefly discussed.

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1. Introduction

Recently there have been some new steps taken in applications of the Landau hydrodynamical model of particle production [1] in ultra-relativistic heavy ion collisions. Firstly, it was proposed by Bjorken [2] that the existence of a plateau in the rapidity distributions of produced particles is a consequence of the longitudinal expansion of this part of excited hadronic matter (presumably a quark-gluon plasma) which, after a phase transition, populates the central region of rapidities: He gave in fact an explicit solution of the hydrodynamical equations which satisfies this requirement. Secondly, the one dimensional description of Bjorken was generalized to a three dimensional expansion of cylindrically symmetric systems [3]. That is to say a system of equations was constructed which couple

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the longitudinal expansion with a cylindrical transverse expansion and reduce to the Bjorken equations when there is no transverse radial motion[1].

Fig. 1 shows a schematic view of a central collision of two large Lorentz-contracted nuclei in their c.m. system. The region of the transverse expansion coupled to — and driven by the longitudinal Bjorken expansion is marked *C*.

Ref. [3] discusses the space-time evolution of hadronic fluid which does not undergo a phase transition. In other words the hydrodynamic equation of motion with a given equation of state are assumed to be valid in all space-time and for all temperatures, and

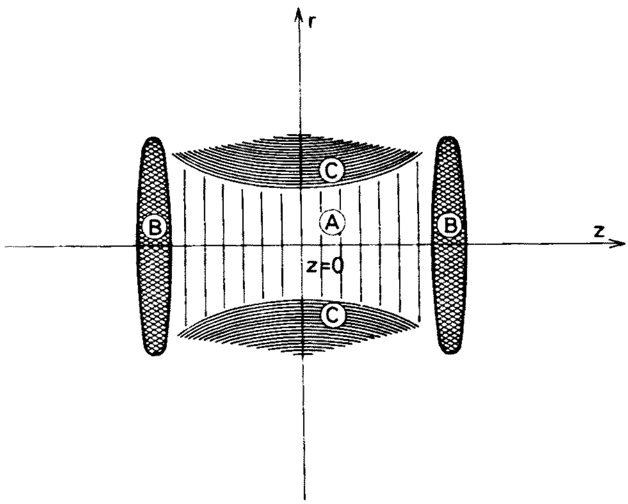

Fig. 1. Schematic view of a central nucleus-nucleus collision at ultra-relativistic energies; *A* — the region of purely longitudinal expansion described by the scaling solution of Ref. [2]; *B* — Lorentz contracted fragmentation regions of the two nuclei (not discussed in this paper); *C* — the region of both longitudinal and transverse motions. The longitudinal motion is as in *A*, the transverse one is a solution of Eqs (2.3) (see the text)

the dynamical role of a phase transition in which quark-gluon plasma is converted into physical particles is neglected.

In the present paper we go a little further towards including a phase transition and introduce a surface of the phase transition (critical surface) within which plasma stays above the critical temperature and is governed by the relativistic hydrodynamics. Outside of this surface we assume the matter to be just physical particles which are being created as the quark-gluon plasma leaks through the critical surface where the temperature stays critical at all times and where the phase transition takes place. As we have already stated the relativistic hydrodynamics is valid only inside the critical surface. This condition influences the evolution of the region filled with plasma and defines the motion of the critical

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[1] In Ref. [3] only the radial motion of the fluid is allowed. It is possible to generalize these equations to include a vortex motion of the fluid (which preserves cylindrical symmetry) [4].

surface. Since temperatures decrease with time the plasma bubble shrinks with time and eventually disappears.

The model of the phase transition described above is very incomplete: it skips any detailed discussion of the phase transition and its main physical content is the very existence of the critical surface. Yet it leads to an evolution of the plasma somewhat different from the one discussed in [3], and enables one to introduce arbitrary initial conditions without relating hydrodynamically the regions containing plasma with the regions with temperatures below the critical temperature.

Understanding of plasma cooling is incomplete without description of emission of electromagnetic radiation and, possibly, also of radiation of pions from a hot plasma [5–7]. These processes might play an important role in the cooling of plasma. It is however a separate complex problem and we shall not discuss it here.

In Section 2 we construct solutions which have no causal relations (through hydrodynamical equations) with these parts of the system which are outside the critical surface. In Section 3 we discuss an analytic approximation to the quasi-Riemann solutions in cylindrically symmetric plasma. In Section 4 we present the various computer generated numerical solutions with smooth initial conditions and compare them with the analytic solution of Section 3 and with each other. In Section 5 we discuss solutions determined by non--smooth initial conditions. Section 6 contains the summary and conclusions.

2. *Space-time evolution of the quark-gluon plasma within boundaries of the critical temperature isotherm*

The starting point of our discussion is the equation [1,8]

$$\frac{\partial T^{\mu\nu}}{\partial x_\mu} = 0, \tag{2.1}$$

with the conventional energy-momentum tensor (without dissipation)

$$T^{\mu\nu} = (\varepsilon + p)u^\mu u^\nu - g^{\mu\nu} p, \tag{2.2}$$

where ε is the energy density, p — pressure, $g^{\mu\nu}$ — metric tensor $(+ - - -)$, $u^\mu (= \gamma(1, \boldsymbol{v})$, $\gamma = (1-\boldsymbol{v}^2)^{-1/2})$ — fluid four velocity. Eq. (2.1) is then specialized for solutions which are invariant under a boost along the longitudinal direction (z-axis) and have cylindrical symmetry [3]. As in [3] we assume the longitudinal expansion to be given in the form of the scaling solution [2] and construct from (2.1) a system of equations which have the boost invariance built into them. Such equations couple the transverse motion to the longitudinal one in an asymmetric manner: the transverse motion is to be found from the initial conditions and the equations, the longitudinal motion however is completely specified. We believe that the treatment of the longitudinal motion as a "driving mechanism" is a realistic physical Ansatz. The quark-gluon plasma we are dealing with is assumed to have the baryon chemical potential equal zero and is a one parameter thermodynamic system. Hence e.g. the temperature T may be taken as the only independent thermodynamic variable.

After employing a few thermodynamic relations and some algebra, the problem is reduced to solving the following characteristic equations for the temperature, T, and the transversal rapidity, α, valid in the $z = 0$ plane, for the plasma of the central region of rapidities, in the c.m. system of two identical nuclei colliding centrally [3]:

$$\left(\frac{\partial}{\partial t}+\mathrm{v}_{+}\frac{\partial}{\partial r}\right)\left(\frac{1}{\mathrm{v_s}}\ln T+\alpha\right) = -\frac{\mathrm{v_s}\left(\frac{1}{r}v_r+\frac{1}{t}\right)}{1+\mathrm{v_s}v_r} = -f_{+}, \tag{2.3a}$$

$$\left(\frac{\partial}{\partial t}+\mathrm{v}_{-}\frac{\partial}{\partial r}\right)\left(\frac{1}{\mathrm{v_s}}\ln T-\alpha\right) = -\frac{\mathrm{v_s}\left(\frac{1}{r}v_r+\frac{1}{t}\right)}{1-\mathrm{v_s}v_r} = -f_{-}, \tag{2.3b}$$

$$\mathrm{v}_{\pm} = \frac{v_r \pm \mathrm{v_s}}{1 \pm v_r \mathrm{v_s}}. \tag{2.4}$$

Here $v_r = \tanh\alpha$ is the transverse radial velocity, $\mathrm{v_s}$ — velocity of sound $\left(\mathrm{v_s^2} = \frac{dp}{d\varepsilon}\right)$. $\alpha(t, r, 0)$ and $T(t, r, 0)$ depend only on the time, t, and the transverse distance, r, from the axis of cylindrical symmetry. In order to get the motion for an arbitrary z, one has to boost v_r along the z-axis to a system in which the longitudinal velocities become zero:

$$v_r(t, r, z) = v_r(t, r, 0)\frac{\tau}{t},$$

$$T(t, r, z) = T(\tau, r, 0),$$

$$\tau = \sqrt{t^2-z^2}. \tag{2.5}$$

Eqs. (2.3) and (2.4) are supplemented by an equation of state whose role is reflected in the presence of the velocity of sound, $\mathrm{v_s}$, which we approximate as a constant. This system of equations was extensively discussed in Ref. [3] under the assumption that it is valid for all space and time. In this paper we restrict its validity to these regions of space--time where the condition

$$T(\tau, r) \geqslant T_c \tag{2.6}$$

is satisfied. Here T_c is the critical temperature above which we are dealing with the quark--gluon plasma and below — with hadrons.

In order to have a complete model of the role of a phase transition in the transverse motion of the plasma, one should also introduce some boundary conditions for velocities at the critical surface. This would force us to decide what happens outside of the plasma bubble and therefore we shall refrain from doing it here.

The process of solving (2.3a, b) goes as follows. We start with the initial conditions that in a certain volume a quark-gluon plasma is created with a given spatial distributions of T and α. The surface of this bubble of plasma stays all the time (until plasma disappears) at the critical temperature T_c. This is a supplementary boundary condition which our hydrodynamical solutions governed by (2.3) are assumed to satisfy. Except for few limiting cases Eqs. (2.3) have to be solved numerically. As the computer builds $T(t, r)$ along the characteristics step by step from the initial conditions $T(t_0, r)$ and $\alpha(t_0, r)$ given for $t = t_0$, and reaches T_c, it faces three possibilities when it makes the next step: (*i*) T stays equal T_c, (*ii*) T goes below T_c, (*iii*) T goes above T_c. In (*i*) the critical surface stays put, in (*ii*) it is moved along r toward higher temperatures, in (*iii*) it is moved along r toward lower temperatures. Some details of this procedure are given in the Appendix.

Thus we get a space-time evolution of the quark-gluon bubble. The simplest mechanism of production of hadrons in this model is to assume that they leak through the critical surface with rapidities which are identified with rapidities of the quark-gluon plasma. For the purpose of crude estimates we shall follow this prescription.

3. *The quasi-Riemann solutions*

The initial conditions at $t = t_0$ (at t_0 is the beginning of the hydrodynamical expansion) introduce considerable uncertainties. Before discussing the numerical solutions of (2.3a, b) for arbitrary initial conditions we give the approximate analytic solutions which are as close as possible to the Riemann one dimensional simple relativistic wave [9]. Let us first recall the Riemann solution of (2.3a, b) with $f_+ = f_- = 0$ in one dimension and the quasi-Riemann approximate solutions for the transverse expansion of cylindrically symmetric matter ($f_\pm \neq 0$). Following [3] we define

$$\zeta_\pm(t, r) = \int_{t_0}^{t} dt' f_\pm(r_\pm(t'), t'), \tag{3.1}$$

where the integration is taken along the characteristics (+) or (−), hence along $r = r_\pm(t)$. Then Eqs. (2.3a, b) become

$$\left(\frac{\partial}{\partial t} + v_+ \frac{\partial}{\partial r}\right)\left(\frac{1}{v_s} \ln T + \alpha + \zeta_+\right) = 0, \tag{3.2a}$$

$$\left(\frac{\partial}{\partial t} + v_- \frac{\partial}{\partial r}\right)\left(\frac{1}{v_s} \ln T - \alpha + \zeta_-\right) = 0. \tag{3.2b}$$

The quasi-Riemannian wave is defined by assuming the condition

$$\frac{1}{v_s} \ln T + \alpha + \zeta_+ = \text{const}, \tag{3.3}$$

to be a universal relation for all t, r. Using (3.3) we get for the space-time evolution the equation

$$\left(\frac{\partial}{\partial t} + v_- \frac{\partial}{\partial r}\right)\left(\frac{2}{v_s} \ln T + \zeta_- + \zeta_+\right) = 0. \tag{3.4}$$

When $\zeta_+ = \zeta_- = 0$, Eqs (3.3) and (3.4) become the one dimensional equations of Riemann and the well known rarefaction wave [9] is their solution. In this case $v_- = (v_r - v_s)/(1 - v_r v_s)$ is constant along the characteristics (and so is v_r and T). As it was pointed out in [3] one gets an approximate solution of (2.3a, b) with the initial conditions of a Riemann rarefaction wave when one builds it up around the one-dimensional relativistic Riemann solution. Indeed, one may then approximate

$$\zeta_-(t, r) = v_s \int_{t_0}^{t} dt' \frac{\left(\frac{1}{r_-(t')} v_r + \frac{1}{t'}\right)}{1 - v_s v_r} \approx \frac{v_s}{1 - v_R v_s} \ln\left(\frac{t}{t_0}\right), \tag{3.5}$$

where v_R is taken from the Riemann ($\zeta_+ = \zeta_- = 0$) solution of Eq (3.4) (a constant on (−) characteristics), and

$$\zeta_+(t, r) = v_s \int_{t_0}^{t} dt' \frac{\left(\frac{1}{r_+(t')} v_r + \frac{1}{t'}\right)}{1 + v_s v_r} \approx v_s \ln\left(\frac{t}{t_0}\right). \tag{3.6}$$

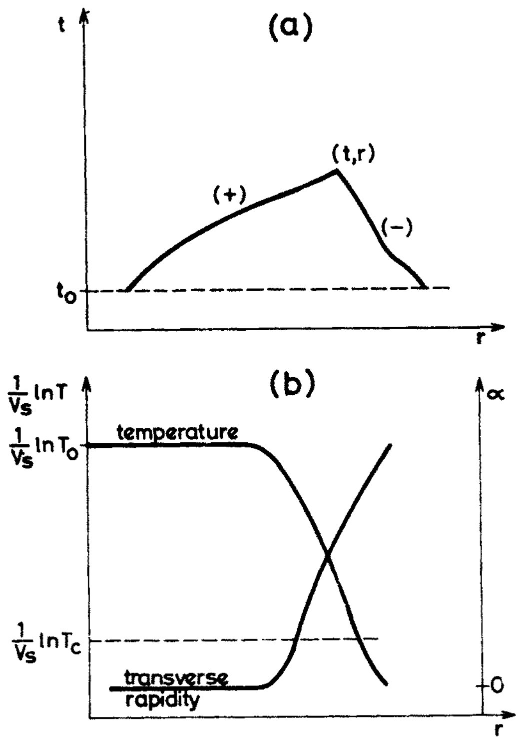

Fig. 2. (a) Two characteristics leading from t_0 to a given point. (b) A typical temperature and rapidity initial profiles leading to a quasi-Riemann solution: $\alpha + \frac{1}{v_s} \ln T = const$, for $t = t_0$

The asymmetry in approximations (3.5) and (3.6) follows from the properties of the characteristics of the quasi-Riemann solutions of (3.2a, b). First of all $\mathrm{v}_+ > \mathrm{v}_-$. This implies that (see Fig. 2a) from the initial moment t_0, to a given point t, r, the (+) characteristic passes through a long stretch of small r's while the (−) one passes only through r's neighbouring the end point r. On the other hand the temperature and velocity profiles of the quasi-Riemann solutions are such that small r's are at high temperatures and small v_r's, and large r's are at low temperatures and large v_r's (Fig. 2b). Therefore, in the first approximation, v_r can be neglected in (3.6) but should be kept in (3.5). This is done by approximating $v_r \approx v_R$ in (3.5) and neglecting it in (3.6).

With the approximate expressions (3.5), (3.6) and

$$\mathrm{v}_- \cong \frac{v_R - \mathrm{v}_s}{1 - v_R \mathrm{v}_s}, \tag{3.7}$$

we can solve (3.3) and (3.4), since (3.4) becomes then one dimensional Riemann equation:

$$\alpha(t, r) = -\frac{1}{\mathrm{v}_s} \ln T - \mathrm{v}_s \ln\left(\frac{t}{t_0}\right) + \text{const}, \tag{3.8}$$

$$\frac{2}{\mathrm{v}_s} \ln T + \frac{\mathrm{v}_s}{1 - v_R \mathrm{v}_s} \ln\left(\frac{t}{t_0}\right) + \mathrm{v}_s \ln\left(\frac{t}{t_0}\right) = \frac{2}{\mathrm{v}_s} \ln T_R, \tag{3.9}$$

where $T_R(t, r)$ is the well known relativistic Riemann solution [9] of the one dimensional problem. Solving (3.8) and (3.9) for T and α with the proper initial conditions we get

$$T(t, r) = T_R(t, r) \left(\frac{t_0}{t}\right)^{\frac{1}{2} \mathrm{v}_s^2 \left(1 + \frac{1}{1 - v_R \mathrm{v}_s}\right)}, \tag{3.10}$$

$$\alpha(t, r) = \alpha_R(t, r) + \tfrac{1}{2} \mathrm{v}_s^2 \frac{v_R}{1 - v_R \mathrm{v}_s} \ln\left(\frac{t}{t_0}\right), \tag{3.11}$$

where

$$\alpha_R(t, r) = -\frac{1}{\mathrm{v}_s} \ln \frac{T_R(t, r)}{T_0}. \tag{3.12}$$

$T_R(t, r)$ one obtains (see e.g. [9]) from the trajectory of a point of the temperature profile, $r(T_R, t)$, at constant temperature T_R

$$r(T_R, t) = \frac{\tanh\left[-\frac{1}{\mathrm{v}_s} \ln \frac{T_R}{T_0}\right] - \mathrm{v}_s}{1 - \mathrm{v}_s \tanh\left[-\frac{1}{\mathrm{v}_s} \ln \frac{T_R}{T_0}\right]} (t - t_0) + r(T_R, t_0), \tag{3.13}$$

by solving it for T_R. Note that $r(T_R, t_0)$ specifies the initial conditions (the initial profile of temperatures). The relations (3.10)–(3.13) give a complete approximate solution of our basic equations (2.3a, b) with the initial conditions at $t = t_0$ identical to the ones of the Riemann one dimensional simple relativistic wave. The physical significance of these solutions is discussed in the following section.

4. Numerical solutions

In this and the following sections we present some of the numerical solutions of the equations (2.3a, b) with particular emphasis on their dependence on initial conditions. We have considered three kinds of initial conditions:

(*a*) The Riemann simple wave initial conditions, Fig. 2, given by (3.12) with $t = t_0$ [8]. Thus by giving an initial profile of $T(t_0, r)$ one determines the rapidities through (3.12). These solutions are very much like the ones discussed in Ref. [3] and are well approximated by (3.10)–(3.13).

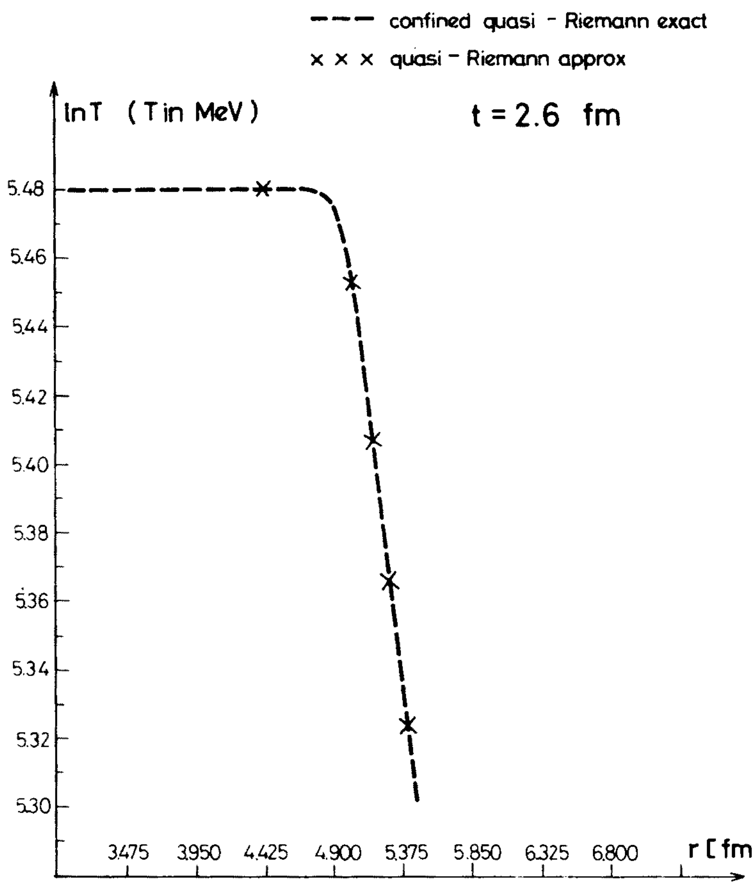

Fig. 3. Comparison of the approximation (3.10) with the exact solution of the equations (3.2) at $t \simeq 2.6$ fm with the Riemann initial conditions at $t = t_0 = 1$ fm. One sees that the curves are undistinguishable on our plot even after the relatively long time after the beginning of hydrodynamical evolution ($t_0 = 1$ fm) (compare also Fig. 4). We always stop the plots at the critical temperature $T_c = 200$ MeV ($\ln T_c = 5.3$)

(*b*) The static initial conditions in which $\alpha(t_0, r) = 0$ for all r, i.e. there is no radial flow of the quark-gluon plasma at the beginning. Here, introduction of a confinement of plasma to within the critical surface does modify its evolution relative to a unrestricted hydrodynamical expansion. Nevertheless, after a relative short time, the quasi-Riemann relations are restored and further evolution goes as in (*a*).

(*c*) The perturbed static initial conditions in which perturbations are introduced into the $\alpha(t_0, r) = 0$ condition, i.e. rapidities locally different from zero. Again, the quasi--Riemann wave gives — *in the average* — a reasonably good description of the plasma evolution. These results are discussed in the next section.

Now let us supplement (*a*) and (*b*) with some details. In the case (*a*) the space-time

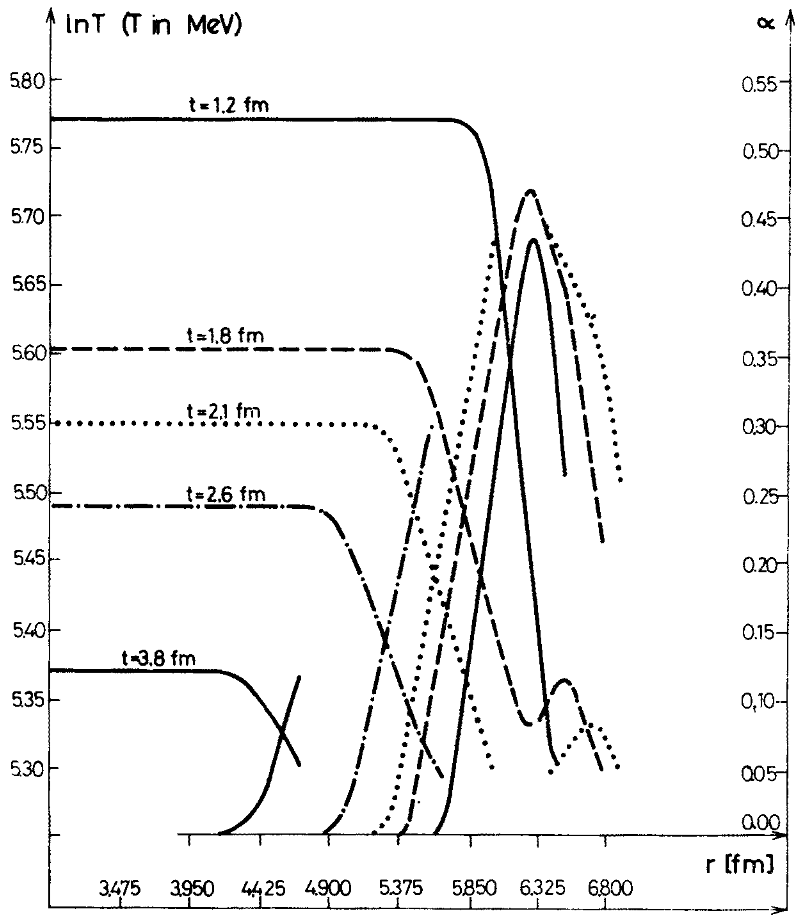

Fig. 4. Time-evolution of the temperature profile of the plasma confined within the region $T \geqslant T_c$, and of the corresponding profile of the transversal rapidity α. One can observe a ring of the plasma separating from the main body of matter and disappearing below $T = T_c$. Note that the evolution of the ring is determined by its internal hydrodynamics, hence it moves independently from the central part of plasma

evolution of the system is such that the condition of restriction of hydrodynamic expansion to within the region of space-time determined by the critical surface, $T_c = T(t, r)$, — we call it confinement of plasma — is trivially satisfied. Indeed, the space-time evolution given by (3.3) and (3.4) goes along the characteristic (−) whose slope increases with increasing r and decreasing T. Thus the critical surface, which has lower $T(= T_c)$ than any point inside, moves faster (with increasing r) or slower (with decreasing r) than any point on a characteristic which starts inside. Therefore the characteristics can never catch up with the critical surface and there is no "communication" through the differential equation (3.4) of the inside with the outside (or vice versa).

Fig. 3 shows a confined quasi-Riemann solution generated from the Riemann initial condition compared with the approximate solution (3.10). They are undistinguishable

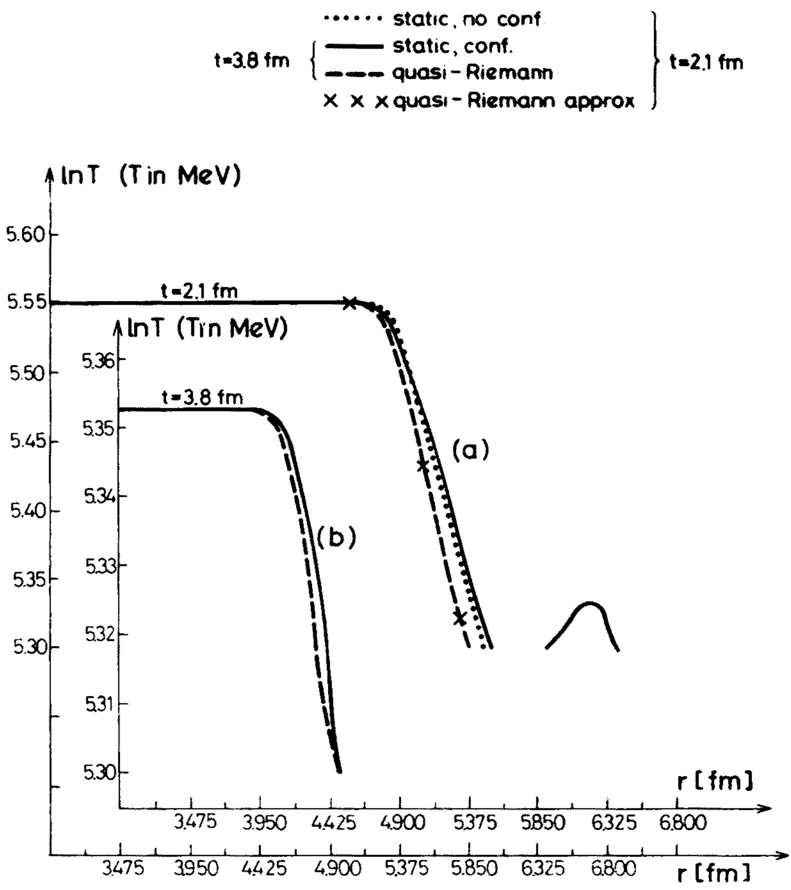

Fig. 5. (a) Comparison of the temperature profiles of the confined (solid line) and unconfined (dotted line) plasma at $t = 2$ fm, with the static initial conditions. The quasi-Riemann solution (dashed line) is also plotted to show that it describes well the main body of matter even at the time when the transient (the ring of plasma) is still around. (b) Long time after the system gets rid of the ring ($t = 3.8$ fm) the confined solution is yet still better represented by the quasi-Riemann solution (hence also by the approximation (3.10))

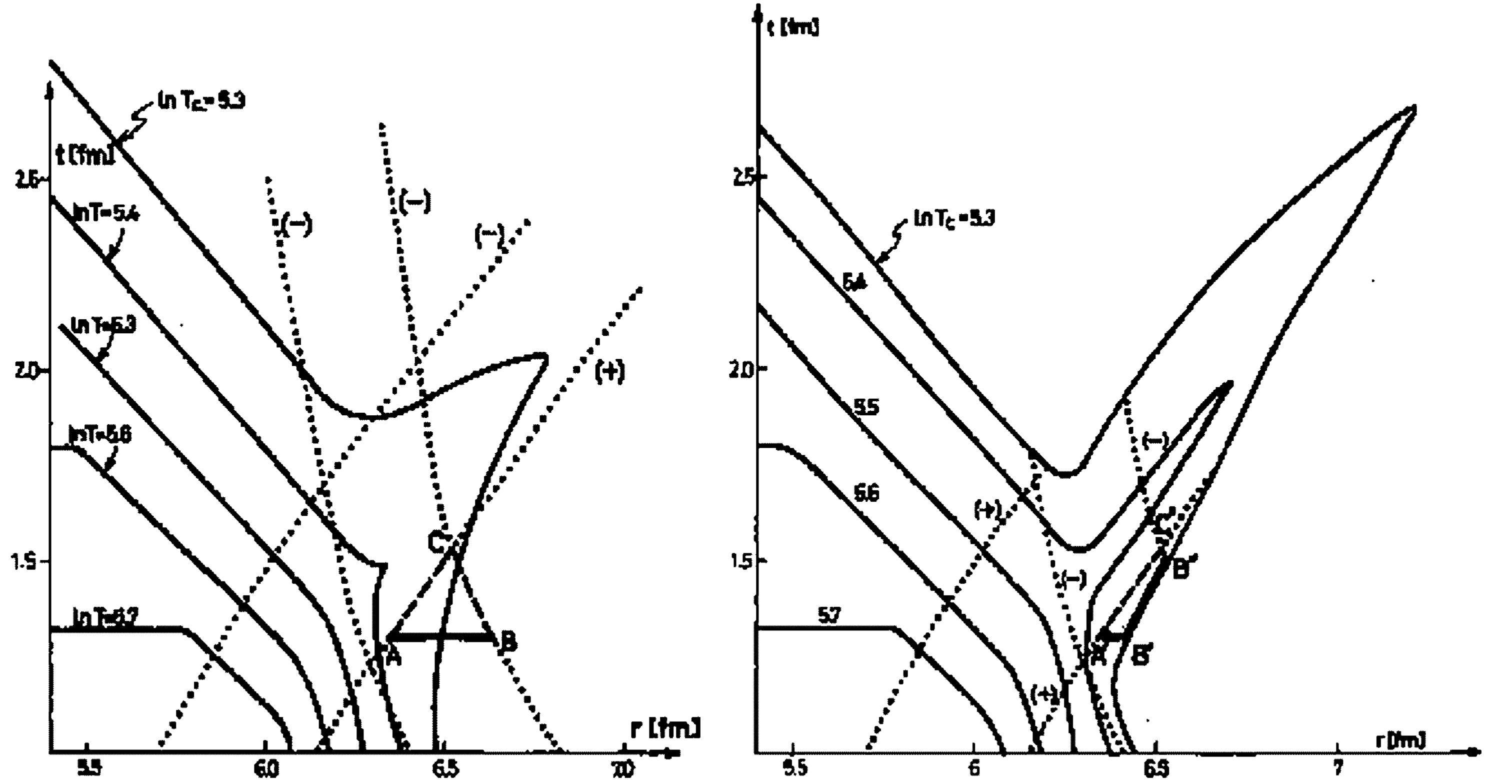

Fig. 6. (a) The critical surface, $T(t, r) = T_c$, for the unconfined plasma with the static initial conditions ($v = 0$). Few characteristics are also plotted to show that the information from the region $T < T_c$ does influence the solutions for $T > T_c$ at some point C. (b) The same for the confined plasma. In this case the information at the point C' from the region $T < T_c$ is cut out along the critical surface

on the logarithmic plots. The computer generated solutions obtained with and without confinement give identical results.

Let us turn now to the static initial conditions, (*b*), which are as follows: we take the temperature profile as in Fig. 2(b) but set $\alpha = 0$ everywhere at $t = t_0$. (Indeed, we are not aware of any convincing arguments that the newly created quark-gluon plasma should satisfy (3.12), hence the initial conditions depicted in Fig. 2(b).)

Fig. 4 shows the space-time evolution of the plasma confined to the region with $T \geqslant T_c$. One can observe a ring of plasma separating from the shrinking main body of matter and disappearing below $T = T_c$.

In Fig. 5 (a) the temperature profile of the confined plasma at $t = 2$ fm is compared with (*i*) the solution for unconfined plasma (also with the static initial conditions) and (*ii*) the solution with the Riemann initial conditions. One sees that the quasi-Riemann wave is quite close to the confined solution for the main body of matter even though the system has just barely got through the process of expelling the ring of plasma. From Fig. 5(b), at a later time $t = 3.8$ fm when the transient ring has long since disappeared, one sees that these two solutions are even closer to each other. So, although it does make a difference whether the plasma is or is not confined, the main difference is just a transient effect.

Fig. 6 shows some isotherms and characteristics for the non confined and confined plasma. We see that lack of confinement does lead to communications between regions separated by the critical isotherm. Introduction of confinement removes this effect.

Before closing this section let us summarize: From Figs. 4 and 5 one can infer that the system — in spite of the fact that its initial conditions are not the ones depicted in Fig. 2(b) — becomes quickly a quasi-Riemann wave. In fact the transient effect which takes shape of a ring of plasma separating from the main body of the plasma and then disappearing, lasts only $\sim 10\%$ of the lifetime of the plasma. The remaining process looks just like the one started with the initial conditions of Fig. 2(b). The approximate solution (3.10) stays close to the machine generated one with the static (non Riemann) initial conditions.

5. *Non-smooth initial conditions*

One may also wonder what are the consequences of possible local disturbances introduced into the initial conditions at $t = t_0$. In Figs. 7 and 8 we show some examples of evolutions of plasma with disturbed (relative to the smooth initial conditions we have considered so far) initial conditions. We can see that, although disturbances last as long as plasma lives, they do not grow with time but "ride on the back" of a quasi-Riemann wave (Fig. 7). This lack of damping presumably results from the lack of dissipative effects in our equations: the energy, which is conserved, gets transferred from the temperature to the flow of plasma and back. A disturbance introduced into e.g. temperature profiles is promptly reflected in the velocity distributions and vice versa.

The parameter which is important in determining the amplitudes of these oscillations is the sound velocity v_s. This follows from the relation (3.8) which is an approximate integral of motion. Thus

$$\Delta\alpha \approx \Delta\left(\frac{1}{\mathrm{v_s}}\ln T\right) - \Delta(\mathrm{v_s}\ln t)$$

$$\approx \frac{1}{\mathrm{v_s}}\,\frac{\Delta T}{T} - \mathrm{v_s}\frac{\Delta t}{t}, \tag{5.1}$$

and, for a given $\Delta\alpha$, small $\mathrm{v_s}$ implies small $\frac{\Delta T}{T}$. Fig. 8 confirms this observation.

The possibility of a prompt transfer of energy between α and T degrees of freedom might drastically change the picture of plasma evolution. For instance, a fluctuation of

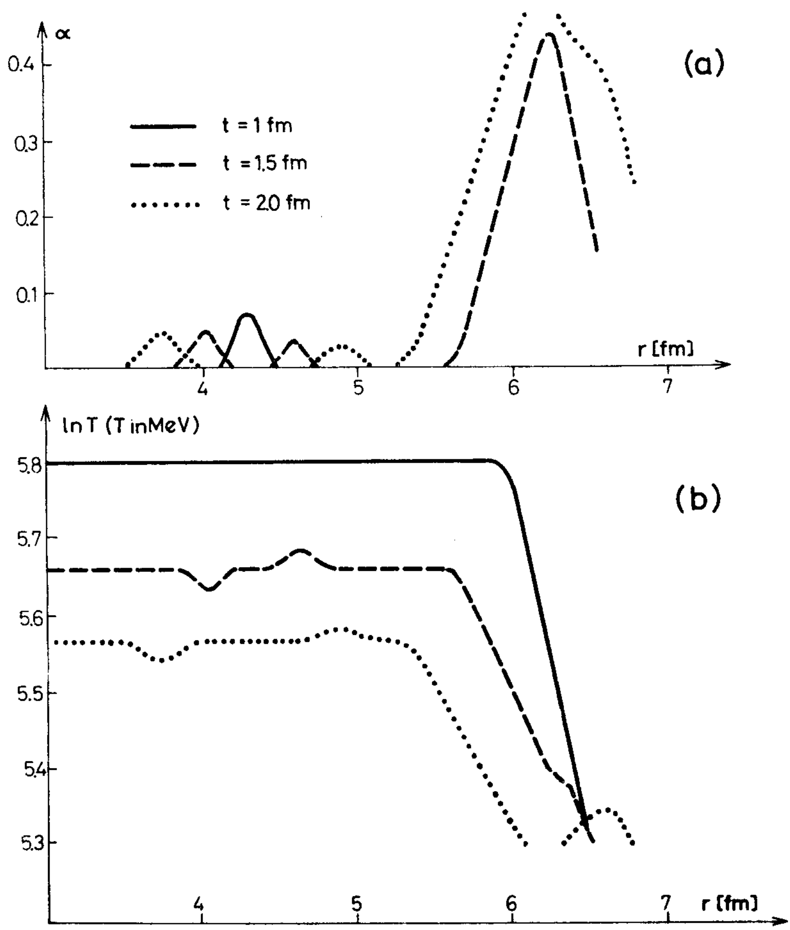

Fig. 7. An example of the non-smooth initial conditions with a small disturbance of $v_r \neq 0$ introduced at $t = 1$ fm in the middle of the plasma at rest (a). We can see a prompt response of the temperature distribution (b). The amplitudes of the disturbances remain small for as long as the plasma exists. The bulk of high velocities at the edge of the plasma is the re-adjustment of the flow to comply with the quasi-Riemann propagation

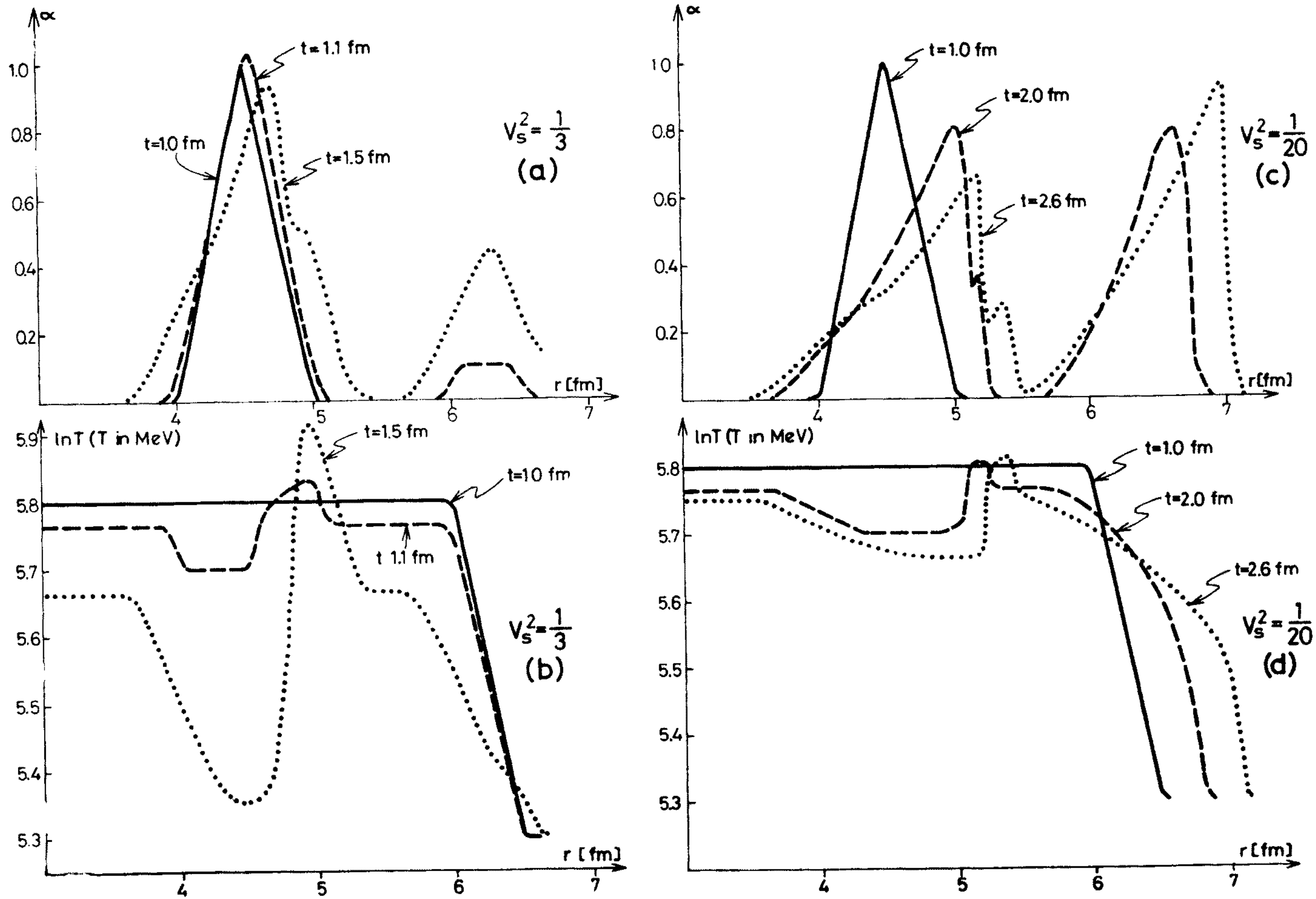

Fig. 8. A large disturbance in velocities (a) is promptly reflected in a considerable local cooling of plasma to $T = T_c$, (b), when $v_s^2 = \frac{1}{3}$. However, for $v_s^2 = \frac{1}{20}$, the amplitude of the response of the temperature to an identical as in (a) initial disturbance shown in (c) is much weaker than in (b) — as (d) illustrates. This is all in accordance with Eqs. (5.1)

velocity of the order of $\Delta v \sim 0.6$ leads to a local cooling close to $T_c \approx 200$ MeV within $\Delta t \sim 0.5$ fm, when the initial temperature $T_0 \approx 330$ MeV. This is illustrated also in Fig. 8.

Clearly, our model without any dissipative effects built in is too crude to be very realistic. Nevertheless, the proximity of T_0 and T_c may lead to a comparatively easy break-ups of the plasma into a few tubes separated by a matter below the critical temperature. Whatever that matter might be, it would probably interact with the plasma leading the two phases back to the plasma phase, provided the average temperature were high enough and lasted long enough.

6. Summary and conclusions

We have investigated solutions of the hydrodynamical equations describing space-time evolution of the quark-gluon plasma with cylindrical symmetry along the collision axis. In our analysis we have tried to take into account (although in a rather limited sense) the effects of a phase transition from plasma to hadronic matter. Since the relevant equations are non-linear, the character of the solutions depends, in general, in a non-trivial way on the assumed initial conditions. We have investigated three types of initial conditions: (*a*) the simple wave (Riemann) initial conditions, (*b*) the static initial conditions, and (*c*) some locally perturbed static initial conditions. Our main conclusion is that in all these three cases the behaviour of plasma can be — on the average — described very well by the approximate quasi-Riemann solutions (Eqs (3.10) and (3.11)). First, it turns out that after relatively short time (compared to a total life-time of the plasma) the Riemann rarefaction wave provides a good description of the quark-gluon plasma. Second, the local disturbances do not increase with time but fluctuate around the Riemann wave.

This situation indicates that by smoothing out the oscillations and transients — a procedure which, hopefully, does away with the non-essential but does not disturb the essential features of the plasma evolution — we can determine quite reliably the lifetime of the plasma and its transverse cross section when it reaches the critical temperature.

It turns out (see Figs. 4 and 5) that, for large nuclei of radius ~ 7 fm, the transverse rarefaction wave is some distance from the center of the cylinder when the critical temperature is reached. This means that some sizable fraction of plasma whose transverse flow is still zero goes through the phase transition. Thus the transverse rapidities of hadrons emerging from this part of plasma are approximately determined by T_c since the transverse flow is nil. We can also see that, due to the Bjorken cooling, the transverse rapidities in the region of the rarefaction wave are also small. So, the transverse motion of our model results altogether in small transverse momenta.

Finally, we would like to comment on the significance of the fluctuations inside the plasma. According to our calculations the fluctuations in velocity can induce some fluctuations in temperature which are so large that they can bring the plasma close to the critical point $T = T_c$. Were this the case, the fluctuations could play an important role in determining the lifetime and the evolution of the plasma. However, one should bear in mind that our discussion does not include any dissipative effects which might smooth and damp out these fluctuations. In fact also the fast re-adjustment of the solutions with the static initial conditions to the quasi-Riemann evolution (Figs. 4 and 5) is possible because of the prompt response of the velocity distribution to the initial temperature distribution. This, again, may look different when a dissipation is important. Thus an analysis of such effects for the quark-gluon plasma is of importance to give credibility to the hydrodynamic equations (2.1) and (2.2).

APPENDIX

We construct the confined solutions using the following procedure. First, we replace the original profile of temperatures ACB by ACB′ (Fig. 9): the section of the original profile which is below T_c is replaced by a constant $T = T_c$. Then, we solve the hydrodynamic equations with these new initial conditions in the whole space. Two processes regulate

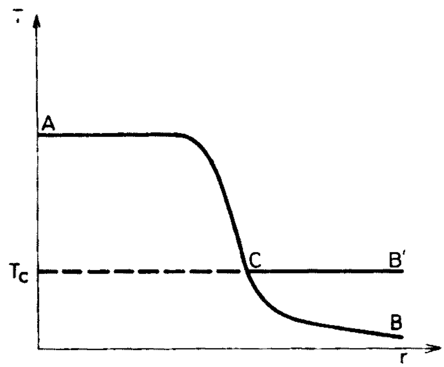

Fig. 9. Modification of the initial temperature profile (ACB → ACB′) which leads to confinement

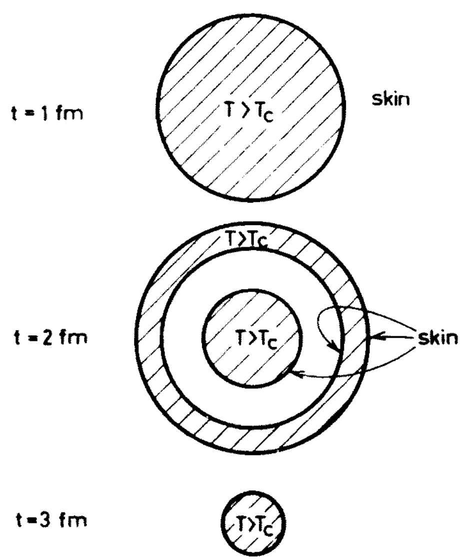

Fig. 10. Schematic view of a transverse cross section through a confined quark-gluon plasma whose temperature and transverse rapidity profiles are given in Fig. 4

the motion of C (critical surface): (*a*) a decay of the high temperature region due to the transverse flow of matter, (*b*) the overall Bjorken cooling due to the longitudinal expansion. In the neighbourhood of C the temperature may either go up (process (*a*) dominates) or down (process (*b*) dominates). When T goes up (above T_c), C shifts to larger r's up to the point at which T falls below T_c (because of (*b*)). When T goes down (below T_c), C shifts to smaller r's until T becomes larger than T_c. After each iteration, in all regions where $T < T_c$, the computer sets it equal to T_c.

One should stress that the above described procedure is equivalent to introducing in the neighbourhood of C just a skin of 2 lattice steps deep kept at all times at $T = T_c$. What happens beyond this skin is irrelevant. This means that in effect the hydrodynamical equations are being solved inside the space-time region with temperatures $T > T_c$. It also means that when the original region occupied by plasma splits into e.g. two (see Fig. 10) regions with $T > T_c$ they evolve independently from each other.

To summarize: The computer program works as follows. After each iteration the newly obtained temperatures are checked at each point of the space lattice starting from the center. Where $T > T_c$ — the temperature is accepted, where $T \leqslant T_c$ — the temperature is set equal T_c. The boundaries of the critical surfaces are determined by the coordinates of the first points at which T goes through T_c (either going up or down). That is to say: the coordinate r of the first point at which T falls below T_c determines the internal boundary (see Fig. 10). The coordinate of the next (going towards larger r) boundary determines the point at which T goes above T_c (Fig. 10).

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