EXOTIC COMMUTATORS AND IDEAL MIXING

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(Received May 4, 1983)

It is shown that ideality of meson octet-singlet and 15-plet-singlet mixing can be obtained from requirement of vanishing of definite set of exotic commutators. Enlargement of the set does not lead to mass degeneration (as it happens for the octet and 15-plet) but it leaves the ideal mass formulae unchanged.

PACS numbers: 12.70.+q

1. Introduction

To explain the mass spectrum of hadrons one follows either a group theoretic approach based on a broken flavour symmetry or a dynamic one based on the theory of quark interactions (QCD). In the present paper we follow the group theoretic approach. This is the reason why we deal with particle states belonging to some definite representations of the flavour group and it is indifferent to us what is the dynamics gluing quarks into particles.

In such an approach there are perturbative (spurion) methods and nonperturbative ones. The difficulties of the spurion method are well-known. The choice of the simplest set of spurions and their perturbation order (usually the first one) is imposed by requirement to have a calculation scheme with a prediction power. In some cases transformation properties of the simplest spurions can be determined on the basis of the quark model. For example, from the quark mass spectrum $m_u = m_d \neq m_s$ it follows that the simplest spurion breaking SU(3) to $SU(2)_T \otimes U(1)_Y$ is T_3^3 (i.e. the eighth component of an octet). Generally the situation is not so simple (e.g. the well-known alternative $(3,3^*)\oplus(3^*,3)$ or $(8,1)\oplus(1,8)$ in the chiral $SU(3)\otimes SU(3)$). Yet a more complicated situation arises in the case of groups with a spin subgroup (e.g. $SU(6)_S$, $SU(6)_W$; see, for example, discussion in [1]).

The fundamental difficulty within the spurion method is that quantities determining representation mixings remain free. For example, to obtain the Schwinger mass formula for the SU(3) nonet a particular assumption on the octet-singlet transition in the mass matrix must be done (the so called Okubo Ansatz). An enlargement of the internal symmetry

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to the one including the spin subgroup (e.g. SU(3) to $SU(6)_S$ or $SU(6)_W$) is not a way out of troubles. Indeed, the Okubo Ansatz can be justified in $SU(6)_S$ or $SU(6)_W$ but only as result of a special and rather arbitrary choice of spurions.

In the nonperturbative method the mass operator is a sum of an invariant part and noninvariant one but with exactly determined transformation properties [2–4]. The essential point is that such a structure of mass operator is exact, i.e. it is not an approximation based on the assumption that the symmetry breaking is weak. So, we do not in fact expect that noninvariant contribution is smaller than the invariant one. The representation mixing plays a significant role. For example, it is shown [3] that if there is no octet-singlet mixing in SU(3) then all the octet masses are degenerated.

In all nonperturbative methods known to us [2–8] (see discussion in Section 4) restrictions on masses follow from commutators

$$\begin{aligned} [G_a, [m^2, G_b]] &= 0, \\ [G_a, [m^2, [m^2, G_b]]] &= 0, \\ &\cdots\cdots \end{aligned} \tag{1.1}$$

where m^2 is the mass operator squared, $\{G_a\}$ are group generators and (a, b) stand for the exotic combination of indices. Let us remark that restrictions (1.1) can be obtained in the $\vec{p} \to \infty$ limit from the requirement that the exotic commutators of group generators with their time derivatives vanish

$$\begin{aligned} [G_a, \dot{G}_b] &= 0, \\ [G_a, \ddot{G}_b] &= 0, \\ &\cdots\cdots \end{aligned} \tag{1.2}$$

Below, this nonperturbative approach is called the exotic commutator method (ECM).

In the present paper we investigate, as examples, the restrictions on masses following from ECM for meson nonet in SU(3) and 16-plet in SU(4). It is shown that the mass matrix of mixing states is determined almost exactly (there is a freedom in choice of signs of some nondiagonal elements) by commutators with time derivatives only to third order in SU(3) (or SU(4)) case (and only to second order in the case of SU(3)⊗SU(3) (or SU(4)⊗SU(4)) chiral group — see Appendix). The commutators with higher derivatives do not give further restriction. Mass matrix determined in this way corresponds to the ideal mixing. These results are obtained in Sections 2 and 3 (for nonet and 16-plet respectively). In Section 4 some models in which exotic commutators vanish are commented upon. The possible mechanism of ideality violation are discussed in the final section.

2. *SU(3) nonet*

Let $|\eta_8\rangle$ be the isosinglet state from octet. If there exists an SU(3) singlet state $|\eta_0\rangle$ that mixes with $|\eta_8\rangle$ then

$$|\eta_8\rangle = \lambda_1|\eta_1\rangle + \lambda_2|\eta_2\rangle, \tag{2.1}$$

$$|\eta_0\rangle = -\lambda_2|\eta_1\rangle + \lambda_1|\eta_2\rangle, \tag{2.2}$$

where

$$\lambda_1^2+\lambda_2^2 = 1 \tag{2.3}$$

and $|\eta_1\rangle$, $|\eta_2\rangle$ are physical states. Let us introduce the momenta ($k = 1, 2, 3, \ldots$)

$$[\eta_8]_k = \langle\eta_8|\,(m^2)^k\,|\eta_8\rangle, \tag{2.4}$$

where m^2 is the mass operator squared. From (2.4) and (2.1) we obtain the set of equations

$$\lambda_1^2\eta_1+\lambda_2^2\eta_2 = [\eta_8]_1, \tag{2.5}$$

$$\lambda_1^2\eta_1^2+\lambda_2^2\eta_2^2 = [\eta_8]_2, \tag{2.6}$$

$$\lambda_1^2\eta_1^3+\lambda_2^2\eta_2^3 = [\eta_8]_3, \tag{2.7}$$

$$\ldots\ldots\ldots\ldots$$

where

$$\eta_i = \langle\eta_i|m^2|\eta_i\rangle; \qquad (i = 1, 2).$$

Calculating matrix elements of commutators (1.1) in the $\vec{p}\to\infty$ frame between octet states and using as intermediate states one-particle ones from octet we obtain the momenta

$$[\eta_8]_l = \tfrac{1}{3}\, a^l+\tfrac{2}{3}\, b^l \tag{2.8}$$

with $a = \pi$, $b = 2K-\pi$, π (K) is the mass of pion (kaon) squared.

Let us discuss now some properties of the system of equations (2.3), (2.5)–(2.7).

a. If only Eqs. (2.3) and (2.5) are taken into account one obtains

$$\lambda_1^2 = \frac{\eta_2-[\eta_8]_1}{\eta_2-\eta_1}, \qquad \lambda_2^2 = \frac{[\eta_8]_1-\eta_1}{\eta_2-\eta_1}. \tag{2.9}$$

Assuming $\eta_2 > \eta_1$ we have

$$\eta_1 \leqslant [\eta_8]_1 \leqslant \eta_2.$$

b. If Eqs. (2.3), (2.5) and (2.6) are taken into account, we have the consistency condition

$$[\eta_8]_2-(\eta_1+\eta_2)\,[\eta_8]_1+\eta_1\eta_2 = 0. \tag{2.10}$$

This is the Schwinger mass formula.

c. There are two consistency conditions for the system (2.3), (2.5)–(2.7). One is (2.10) and the second one is

$$[\eta_8]_3-(\eta_1+\eta_2)\,[\eta_8]_2+\eta_1\eta_2[\eta_8]_1 = 0. \tag{2.11}$$

From (2.10), (2.11) and (2.8) we obtain

$$\eta_1+\eta_2 = a+b,$$

$$\eta_1\eta_2 = ab.$$

So

$$\eta_1 = \pi, \quad \eta_2 = 2K-\pi \tag{2.12}$$

and

$$\lambda_1^2 = \tfrac{1}{3}, \quad \lambda_2^2 = \tfrac{2}{3}. \tag{2.13}$$

The relative sign of λ_1 and λ_2 remains undetermined; it depends on the sign of the matrix element

$$\alpha = \langle \eta_8 | m^2 | \eta_0 \rangle$$

since

$$\lambda_1 \lambda_2 (\eta_2 - \eta_1) = \alpha = \pm([\eta_8]_2 - ([\eta_8]_1)^2)^{1/2}.$$

If $\alpha < 0$, we have $\lambda_1 = \pm \frac{1}{\sqrt{3}}$, $\lambda_2 = \mp \sqrt{\frac{2}{3}}$. Thus we obtain the ideal mixing (see Appendix). There is no ideality when $\alpha > 0$.

d. It is now obvious that any additional equation

$$\lambda_1^2 \eta_1^k + \lambda_2^2 \eta_2^k = [\eta_8]_k$$

for $k > 3$ is simply an identity.

e. If we apply the above method to the octet only ($\lambda_1 = 1$, $\lambda_2 = 0$) then we obtain

i) from Eq. (2.9) the GMO formula

$$\eta_1 = [\eta_8]_1 = \tfrac{1}{3}(4K-\pi);$$

ii) from Eqs. (2.10) and (2.8) the equality $\pi = K = \eta_1$;

iii) the equations containing third and higher momenta do not give any further restrictions.

So, the representation mixing is necessary in ECM to remove the mass degeneration if momenta higher than first are taken into consideration.

3. *SU(4) 16-plet*

Let $|\eta_8\rangle$ and $|\eta_{15}\rangle$ be the neutral isosinglet states from 15-plet and $|\eta_0\rangle$ be an SU(4) singlet state. Let us define the parameters λ_i, δ_i, κ_i ($i = 1, 2, 3$) as

$$\begin{pmatrix} |\eta_8\rangle \\ |\eta_{15}\rangle \\ |\eta_0\rangle \end{pmatrix} = \begin{pmatrix} \lambda_1 & \lambda_2 & \lambda_3 \\ \delta_1 & \delta_2 & \delta_3 \\ \kappa_1 & \kappa_2 & \kappa_3 \end{pmatrix} \begin{pmatrix} |\eta_1\rangle \\ |\eta_2\rangle \\ |\eta_3\rangle \end{pmatrix} \tag{3.1}$$

where $|\eta_i\rangle$ ($i = 1, 2, 3$) are physical states. The parameter matrix is orthogonal.

Let us introduce the momenta

$$[\eta_8]_k = \langle \eta_8 | (m^2)^k | \eta_8 \rangle, \tag{3.2}$$

$$[\eta_{15}]_k = \langle \eta_{15} | (m^2)^k | \eta_{15} \rangle, \tag{3.3}$$

$$[\alpha]_k = \langle \eta_8 | (m^2)^k | \eta_{15} \rangle = \langle \eta_{15} | (m^2)^k | \eta_8 \rangle. \tag{3.4}$$

According to these three kinds of momenta three systems of equations can be written (instead of the one system (2.3), (2.5)–(2.7) in the case of SU(3))

$$\lambda_1^2\eta_1^k+\lambda_2^2\eta_2^k+\lambda_3^2\eta_3^k = [\eta_8]_k, \tag{3.5}$$

$$\delta_1^2\eta_1^k+\delta_2^2\eta_2^k+\delta_3^2\eta_3^k = [\eta_{15}]_k, \tag{3.6}$$

$$\lambda_1\delta_1\eta_1^k+\lambda_2\delta_2\eta_2^k+\lambda_3\delta_3\eta_3^k = [\alpha]_k, \tag{3.7}$$

$k = 0, 1, 2, 3, \ldots$ and $[\eta_8]_0 = [\eta_{15}]_0 = 1$, $[\alpha]_0 = 0$.

If we consider only first three equations from each system (3.5)–(3.7) we obtain mass formulae that can be regarded as a generalization of the SU(3) Schwinger mass formula to the SU(4) [8].

When we consider first four equations from each system then we obtain the following consistency relations

$$\begin{aligned} A_1[\eta_8]_2-A_2[\eta_8]_1+A_3 &= [\eta_8]_3, \\ A_1[\eta_{15}]_2-A_2[\eta_{15}]_1+A_3 &= [\eta_{15}]_3, \\ A_1[\alpha]_2-A_2[\alpha]_1 &= [\alpha]_3, \end{aligned} \tag{3.8}$$

where A_i ($i = 1, 2, 3$) are invariants of the squared mass operator

$$\begin{aligned} A_1 &= \eta_1+\eta_2+\eta_3, \\ A_2 &= \eta_1\eta_2+\eta_2\eta_3+\eta_3\eta_1, \\ A_3 &= \eta_1\eta_2\eta_3. \end{aligned} \tag{3.9}$$

From exotic commutators in SU(4) we obtain

$$\begin{aligned} [\eta_8]_k &= \tfrac{1}{3}\, a^k+\tfrac{2}{3}\, b^k, \\ [\eta_{15}]_k &= \tfrac{1}{6}\, a^k+\tfrac{1}{12}\, b^k+\tfrac{3}{4}\, c^k, \\ [\alpha]_k &= \frac{1}{3\sqrt{2}}(a^k-b^k), \\ K-\pi &= F-D, \end{aligned} \tag{3.10}$$

where $a = \pi$, $b = 2K-\pi$, $c = 2D-\pi$.

Solving the system (3.8) with the momenta (3.10) we find

$$\begin{aligned} A_1 &= a+b+c, \\ A_2 &= ab+bc+ca, \\ A_3 &= abc. \end{aligned} \tag{3.11}$$

Comparing (3.11) and (3.9) we conclude that

$$\eta_1 = \pi, \quad \eta_2 = 2K-\pi, \quad \eta_3 = 2D-\pi$$

and from (3.5)–(3.7) we obtain

$$\lambda_1^2 = \tfrac{1}{3}, \quad \lambda_2^2 = \tfrac{2}{3}, \quad \lambda_3^2 = 0,$$
$$\delta_1^2 = \tfrac{1}{6}, \quad \delta_2^2 = \tfrac{1}{12}, \quad \delta_3^2 = \tfrac{3}{4},$$
$$\lambda_1\delta_1 = \frac{1}{3\sqrt{2}}, \quad \lambda_2\delta_2 = -\frac{1}{3\sqrt{2}}, \quad \lambda_3\delta_3 = 0.$$

It is now easy to see that equations with higher momenta do not give further restrictions.

Similarly as in the SU(3) there are two essentially different parameter matrices. Indeed, the relative sign of λ_1 and κ_1 remains undetermined

$$\lambda_1\kappa_1(\eta_1-\eta_2) = \langle\eta_8|m^2|\eta_0\rangle = \pm([\eta_8]_2-([\eta_8]_1)^2-([\alpha]_1)^2)^{1/2}.$$

Choosing $\langle\eta_8|m^2|\eta_0\rangle < 0$[1] we obtain the ideal quark contents of physical states

$$|\eta_1\rangle = \left|\frac{1}{\sqrt{2}}(\mathrm{u}\tilde{\mathrm{u}}+\mathrm{d}\tilde{\mathrm{d}})\right\rangle,$$
$$|\eta_2\rangle = |\mathrm{s}\tilde{\mathrm{s}}\rangle,$$
$$|\eta_3\rangle = |\mathrm{c}\tilde{\mathrm{c}}\rangle.$$

The quark contents are not satisfactory if $\langle\eta_8|m^2|\eta_0\rangle$ is positive.

Similarly as in the SU(3) case the representation mixing is necessary to avoid the degeneracy in the 15-plet if the second and higher momenta are taken into consideration.

4. *Some remarks on models*

The absence of exotic terms on the right-hand side of Eqs. (1.1) can be meant either as a particular dynamic assumption, or as a property of some models. We would like to make some remarks on two models based on the SU(3) (or SU(4)) symmetry.

a. The free quark model

The relations (1.1) can be proved [7] if the symmetry breaking is caused by the noninvariant piece of quark mass operator in the Lagrangian

$$L = \bar{q}(i\gamma_\mu\partial^\mu+M)q,$$

i.e.

$$M = M_{\mathrm{inv}}+\Delta M\lambda_8 \quad \text{in SU(3)};$$
$$M = M_{\mathrm{inv}}+\Delta M_1\lambda_8+\Delta M_2\lambda_{15} \quad \text{in SU(4)};$$

where $\{\tfrac{1}{2}\lambda_i\}$ are generators of quark representation in SU(3) (or SU(4)). Then

$$\left[T_a(t), \frac{d^n}{dt^n}T_b(t)\right] = \frac{i^n}{4}\int d\vec{x}q^\dagger(\vec{x},t)(\gamma^0)^n[\lambda_a, \underbrace{[M, \dots [M}_{n}, \lambda_b]\dots]q(\vec{x},t). \qquad (4.1)$$

So, there is no exotic term on the right-side of Eq. (1.2).

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[1] This sign has been obtained in the broken $SU(8)_W$ symmetry [1].

Let us consider the matrix elements of commutators (1.2) between one-particle states in the $\vec{p} \to \infty$ limit. Selecting one-particle intermediate states and using the standard method (e.g. see [9]) we obtain

$$\langle \alpha | [T_a, [m^2, T_b]] | \beta \rangle_{1\text{-particle}} = \int \frac{d\nu}{\nu} \varrho_{ab}(\nu), \tag{4.2}$$

$$\langle \alpha | [T_a, [m^2, [m^2, T_b]]] | \beta \rangle_{1\text{-particle}} = \int d\nu \varrho_{ab}(\nu), \tag{4.3}$$

$$\langle \alpha | [T_a, [m^2, [m^2, [m^2, T_b]]]] | \beta \rangle_{1\text{-particle}} = \int d\nu \nu \varrho_{ab}(\nu), \quad \text{etc.,} \tag{4.4}$$

where the integrals represent many-particle contributions. The spectral functions $\varrho_{ab}(\nu)$ are calculated in the $\vec{p} \to \infty$ limit and they are sums over many-particle intermediate states $|n\rangle$

$$\varrho_{ab}(\nu) = (2\pi)^3 \sum_n \delta^4(p+q-p_n) \{\langle \alpha, \vec{p} | D_a(0) | n \rangle \langle n | D_b(0) | \beta, \vec{p} \rangle$$
$$- \langle \alpha, \vec{p} | D_b(0) | n \rangle \langle n | D_a(0) | \beta, \vec{p} \rangle\}_{\vec{q}=0},$$

where $\nu = p_0 q_0$ and $D_a = \partial_\mu V_a^\mu$ (4-divergence of vector current V_a^μ). The Regge-pole model for the asymptotic behaviour of $\varrho(\nu)$ gives [10]

$$\varrho(\nu) \sim \nu^{\alpha_e(0)},$$

where $\alpha_e(0)$ is the intercept of the trajectory with exotic quantum numbers. We see that convergence of the many-particle integrals requires $\alpha_e(0) < 0$, $\alpha_e(0) < -1$, and $\alpha_e(0) < -2$ for (4.2), (4.3) and (4.4) respectively.

b. Nonlinear realization of SU(3) [3] and SU(4) [4]

There are Goldstone particles in such theories. Restrictions on the mass operator are a consequence of the assumption about the Regge asymptotic behaviour of Goldstone-hadron scattering amplitude approximated by tree graphs. If $\alpha_e(0) < 0$, then [see [3, 4]]

$$\langle \alpha | [T_a, [m^2, T_b]] | \beta \rangle_{1\text{-particle}} = 0;$$

if $\alpha_e(0) < -1$, then (see [3])

$$\langle \alpha | [T_a, [m^2, [m^2, T_b]]] | \beta \rangle_{1\text{-particle}} = 0;$$

if $\alpha_e(0) < -2$, then

$$\langle \alpha | [T_a, [m^2, [m^2, [m^2, T_b]]]] | \beta \rangle_{1\text{-particle}} = 0;$$

where combinations of indices (a, b) are exotic.

To end the remarks on models we note that the experimental situation about exotic trajectories and especially their intercepts is far from being clear. In the SU(3) case the requirement $\alpha_e(0) < -1$ does not contradict the experimental data [11]. However it is not known to us if $\alpha_e(0) < -2$ does not.

5. *Final remarks*

Generally, the mass formulae corresponding to the ideal mixing are not well obeyed by the experimental values of masses. The most spectacular disagreement happens in the case of the 0^- nonet in SU(3). On the other hand the 1^- nonet in SU(3) is often quoted as an example of a good agreement. But in SU(4) the 1^- mesons do not obey the mass formulae for ideal mixing (e.g. from $\psi = 2D^* - \varrho$ one obtains $m_{\psi/J} = 2.75$ GeV).

Different mechanisms might be responsible for the violation of ideality of mixing. We do not discuss that ones which are connected with QCD (as, for example, the U(1) problem) but mention some of those related to the broken flavour symmetry.

a. Contribution of many-particle states

The crucial role for applicability of ECM plays the value of intercepts $\alpha_e(0)$. This problem has been discussed in Section 4. If the multiparticle contributions appear to be finite then the mass sum rules should be modified. However such a modification would be strongly model dependent.

b. Mixing of the larger number of one-particle states

There is a possibility in the framework of ECM to admix additional flavour singlets (e.g. gluonium) and/or full multiplets (e.g. radially excited states [12, 13]). There are some arguments in favour of taking into account the radially excited states:

— the masses of SU(4) 16-plet cover the mass region of radially excited nonets of SU(3);

— the standard mixing (octet + singlet) in the 0^- case disagrees not only with the mass spectrum but also with the Alexander–Lipkin sum rules [12].

This mechanism of ideality violation is now under consideration. The results will be given elsewhere.

c. Additional mixing of ideal states via gluons

This additional mixing seems to be necessary. However definite results can be expected only for heavy multiplets (when $\alpha_s \ll 1$). A situation is similar to that in QCD when meson spectrum is calculated: good results are obtained only for heavy mesons ($\psi(c\bar{c})$, $\Upsilon(b\bar{b})$) [14].

The authors are very grateful to Professor V. Ogievetsky and Drs P. Kosiński and J. Rembieliński for illuminating discussions.

APPENDIX

To obtain the mass formulae corresponding to ideal mixing in the chiral SU(3)⊗SU(3) symmetry we apply a little different formalism.

The mass matrix for the octet isosinglet $|\eta_8\rangle$ and the SU(3) singlet $|\eta_0\rangle$ is of the form

$$\begin{pmatrix} \eta_8 & \alpha \\ \alpha & \eta_0 \end{pmatrix}, \tag{A1}$$

where $\eta_8 = \langle\eta_8|m^2|\eta_8\rangle$, $\eta_0 = \langle\eta_0|m^2|\eta_0\rangle$ and $\alpha = \langle\eta_8|m^2|\eta_0\rangle = \langle\eta_0|m^2|\eta_8\rangle$. In the SU(3) symmetry we can calculate η_8, η_0 and α^2 from the momenta $[\eta_8]_i$ ($i = 1, 2, 3$) using the following relations

$$[\eta_8]_1 = \eta_8, \tag{A2}$$

$$[\eta_8]_2 = \eta_8^2+\alpha^2, \tag{A3}$$

$$[\eta_8]_3 = \eta_8^3+(2\eta_8+\eta_0)\alpha^2. \tag{A4}$$

The values of η_8, η_0 and α^2 can be calculated as well in the chiral SU(3)⊗SU(3) symmetry from following exotic commutators

$$[T_a, \dot{T}_b] = 0, \tag{A5}$$

$$[T_a, \ddot{T}_b] = 0, \tag{A6}$$

$$[A_a, T_b] = 0, \tag{A7}$$

$$[A_a, \dot{T}_b] = 0, \tag{A8}$$

$$[A_a, \ddot{T}_b] = 0, \tag{A9}$$

where $T_a \pm A_a$ are the generators of SU(3)⊗SU(3).

From Eqs. (A5)–(A9) we obtain[2] $[\eta_8]_1$, $[\eta_8]_2$ and $[\alpha]_2 \equiv \langle\eta_8|m^4|\eta_0\rangle = 2\alpha K$. Using the relation

$$[\alpha]_2 = \alpha(\eta_8+\eta_0) \tag{A10}$$

we obtain η_0 independently of the α sign.

In both cases (SU(3) and SU(3)⊗SU(3)) we get

$$\eta_8 = \tfrac{1}{3}(4K-\pi),$$

$$\eta_0 = \tfrac{1}{3}(2K+\pi),$$

$$\alpha^2 = \tfrac{8}{9}(K-\pi)^2.$$

The eigenvalues of the matrix (A1) are

$$\eta_1 = \pi, \qquad \eta_2 = 2K-\pi$$

independently of the sign of α.

Because of

$$\begin{pmatrix} |\eta_8\rangle \\ |\eta_0\rangle \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{3}} & -\frac{2}{\sqrt{6}} \\ \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix} \begin{pmatrix} |\eta_u\rangle \\ |\eta_s\rangle \end{pmatrix}$$

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[2] For the details of the calculations see Ref. [5, 6] where the commutators (A5), (A7) and (A8) only have been discussed.

where $|\eta_u\rangle = \left|\frac{1}{\sqrt{2}}(u\tilde{u}+d\tilde{d})\right\rangle$, $|\eta_s\rangle = |s\tilde{s}\rangle$ and choosing

$$\alpha = -\frac{2\sqrt{2}}{3}(K-\pi)$$

we have

$$|\eta_1\rangle = |\eta_u\rangle, \quad |\eta_2\rangle = |\eta_s\rangle$$

i.e. ideal quark contents corresponding to the ideal octet-singlet mixing with $\mathrm{tg}\,\theta = -\sqrt{2}$. The opposite sign of α leads to nonsatisfactory quark contents.

Let us make some remarks on the chiral case.

a. Commutators containing derivatives higher than second order do not give further restrictions on masses.

b. The commutators (A5), (A7) and (A8) without additional assumptions do not give information on octet-singlet mixing [5].

c. If we use only the commutators (A7)–(A9) then we get $\eta_1 = \pi$ (η_8 and α being free parameters). So, the mass equality $\eta = \pi$ (typical for ideal mixing) can be obtained as well under weaker assumptions.

d. All the SU(3)⊗SU(3) results can be copied in the SU(4)⊗SU(4) symmetry.

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