ON THE NUMERICAL COMPUTATION OF QCD MASS SPECTRUM: AN INTRODUCTION*

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Technical details of the structure of numerical computations of mass spectra are presented and discussed.

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1. On a finite lattice

At present Lattice Gauge Theories [1] are showing to be a very useful tool: the possibility of obtaining non-perturbative results, concerning the "supposed to be" theory of strong interactions, is enormously appealing, and it is beginning to be exploited nowadays (for a general review about pure gauge lattice gauge theory, non perturbative methods, and related references, see the lectures of Peter Hasenfratz at this school [2]).

At the same time a complete turn over of the role of computers is happening in theoretical physics. From being simple (and somehow superfluous) instruments, they are going to play an essential role in a remarkable part of the theoretical research: they are going to be built by implementing the fundamental features of the problem that has to been solved at an *hardware* level [3, 4], from the physicists in first person. A general spreading of a precise "weltanschaung" about numerical computing in theoretical physics seems to be happening.

We should enter a little bit deeper in our problem, now. The crucial step that leads to the possibility of using a wide class of non perturbative methods is the rotation of time

$$t \to it \tag{1.1}$$

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leading from the Minkowskyan formulation in $(d-1,1)$ dimensions to the Euclidean one, in d dimensions. Now the factor

$$e^{-S}, \tag{1.2}$$

where S is the action of the interesting theory, is a probability measure (when properly normalized). It will be the Boltzmann factor of a Statistical Mechanics in d dimensions ($3+1=4$ in the real word). Now the main goal is reached: lot of knowledge has been accumulated in Statistical Mechanics, and we can use it to study our (Lattice) Gauge Theory.

Consider the correlation length ξ of such a theory (expressed in units of lattice spacings). We are on a lattice of length L, and we would like the conditions

$$1 \ll \xi, \tag{1.3a}$$

$$\xi \ll L \tag{1.3b}$$

to be satisfied. Condition (1.3a) means that we are close enough to the continuum limit: the coarse graining of our lattice should be forgotten. If we define $\beta = \frac{2N}{g^2}$, where g^2 is the coupling constant of the non abelian gauge theory we are considering, and N is the number of colours (SU(N), $N = 3$ in principle, is the gauge group we care about), (1.3a) will be satisfied in the limit in which $\beta \to \infty$. Condition (1.3b) means that the lattice is big enough, and that finite size effects are negligible.

Now we realize that, in view of a numerical simulation, the situation is not so easy : if we want to simulate, for example, a pure gauge SU(3) theory on a lattice containing 10^4 sites, we will need to store in our computer of the order of 700,000 link variables ($4 \cdot 10^4$ links, times 18 variables for link). The lattice sounds small, the number of variables enormous.

The success of Monte Carlo methods for studying pure gauge lattice theories is based on a crucial result that makes this difficulty avoidable : for the operators used to compute, for example the string tension [5, 21] or the mass of the 0^{++} gluonic excitation, the continuum behaviour given by the Renormalization Group (see 2) sets in when $\xi_G \sim 1$ (where by ξ_G we indicate the typical correlation length in the gluonic sector). This will happen, for example for SU(3), for $g^2 \sim 1$.

Exploiting MC techniques for analyzing a lattice gauge theory coupled to fermionic matter fields is a more difficult task. We will just quote here the 3 main difficulties : first the anticommuting character of the fermionic fields, implying a strong non locality of the effective action, obtained by integrating out the fermionic fields. The second point is that the bare quark mass is not allowed, on a finite lattice, to be arbitrarily small : it is controlling the correlation length of the fermionic sector of the theory. The possible way out will consist in working with unphysically large quark masses, and eventually trying to extrapolate. Last but not least we will have to face the so called doubling problem : the discretization of the continuum theory will make a number of unwanted fermionic species appearing, and/or will explicitly break the chiral invariance of the theory.

2. *About the action : mainly the fermionic part*

The action governing our lattice gauge theory reads

$$S = S_G + S_F, \tag{2.1}$$

where S_G is the pure gauge term and S_F is the part including the contribution of the matter fields. A possible choice for S_G is the Wilson original one [1]

$$S_G = \beta \operatorname{Re} \sum_{\square} \operatorname{Tr} U_{\square}, \tag{2.2}$$

where $\beta = \dfrac{2N}{g^2}$, the sum runs over the plaquettes (elementary squares of the lattice), and

$$U_{\square} = U_\mu(n) U_\nu(n+\hat{\mu}) U_\mu^+(n+\hat{\nu}) U_\nu^+(n). \tag{2.3}$$

The gauge variable $U_\mu(n)$ lies on the link joining the site n and the site $n+\hat{\mu}$. The fermionic contribution can be formally written down as

$$S_F = \sum_n \bar{\psi}(n) [(-\mathscr{D} + m)\psi] (n), \tag{2.4}$$

where the ψ are anticommuting variables defined on sites, and $\mathscr{D}$ should be defined in a suitable way. We mean we will have to face here the problem of "species doubling", that is connected to the problem of the chiral anomaly.

For sake of clearness we will start by illustrating the problem in $d = 1+1$ dimensions [7, 8]. Consider the Dirac equation in the continuum : our spinors will have the form

$$\psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix}, \tag{2.5}$$

and we will use the conventions

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \sigma_3, \quad \gamma_5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_1. \tag{2.6}$$

The Dirac equation will be

$$i\dot{\psi} = -i\gamma_5 \partial_z \psi. \tag{2.7}$$

If we consider the eigenstates of γ_5, such that

$$\dot{\psi}_\pm = \mp \partial_z \psi_\pm \tag{2.8}$$

for planar waves

$$\psi_\pm \sim e^{i(-Kz+Et)} X_\pm. \tag{2.9}$$

We will get

$$iEX_\pm = \mp(-ik) X_\pm \tag{2.10}$$

and the dispersion relation

$$E = \pm k, \quad -\infty < k < \infty \tag{2.11}$$

(see Fig. 2.1).

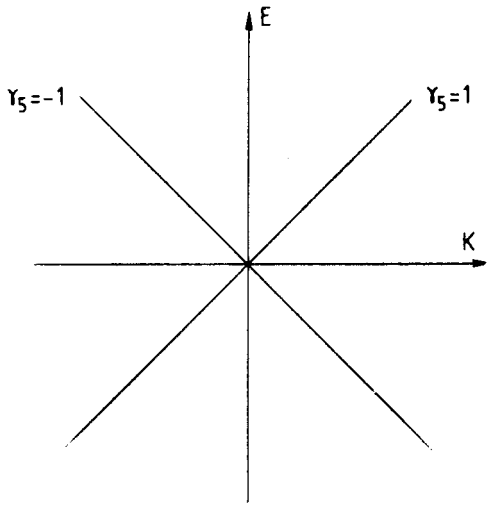

Fig. 2.1. Dispersion relation in the continuum

We try now to discretize the spatial part of our Dirac equation, being as naive as we can. So we define

$$a\partial_z\psi(n) \equiv \frac{\psi((n+1)a)-\psi((n-1)a)}{2}, \tag{2.12}$$

and we consider the plane waves

$$\psi_\pm = e^{i(-Kna+Et)}X_\pm. \tag{2.13}$$

Now we get

$$iEX_\pm = \pm \frac{e^{iKa}-e^{-iKa}}{2a} X_\pm \tag{2.14}$$

and the lattice dispersion relation

$$E = \pm \frac{\sin ka}{a}. \tag{2.15}$$

Now we will get a low energy excitation not only for

$$ka \ll 1, \quad E = \pm k + O(k^3a^2), \tag{2.16}$$

but also for

$$ka = \pi - k'a, \quad k'a \ll 1, \quad E \simeq \mp k' + O(k'^2a^3). \tag{2.17}$$

The phenomenon of species doubling is here. Fig. 2.2 illustrates that in the continuum limit two Dirac spinors are present.

The generality of this phenomenon is impressing (and deeper, perhaps, of what is proved nowadays). To make it clear we will give here the content of the Nielsen-Ninomiya theorem [9]: consider the class of lattice fermionic theories for which the bilinear part of the action reads

$$S = -i \int dt \sum_x \bar{\psi}(x)\dot{\psi}(x) - \int dt \sum_{x,y} \bar{\psi}(x) H(x-y)\psi(y). \tag{2.18}$$

Assume now locality of the interaction, translational invariance on the lattice and hermiticity of H. Assume also exact conservation of the charge Q (lepton number), and that it is locally defined, quantized and bilinear in $\psi(x)$. The statement is that under these assumptions the appearance of equally many right- and left-handed species of Weyl particles is unavoidable.

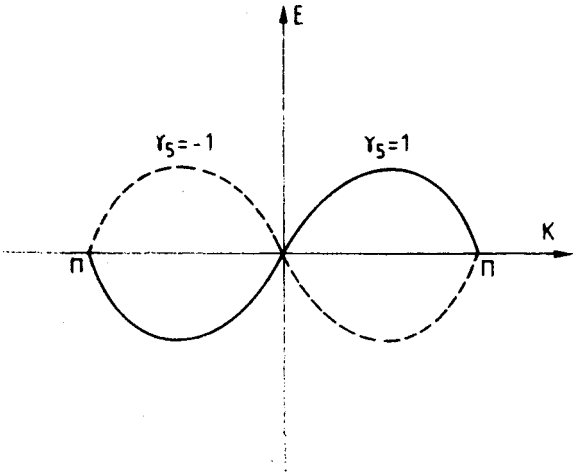

Fig. 2.2. And on the lattice ("naive" discretization)

Let us resume what is happening : we start from a 1 flavor theory, and we discretize it in a "naive" way (defining the lattice derivative as in (2.12)). Now we take the continuum limit and we get a theory with 2^d fermionic species. We know, moreover, that it will be very difficult to eliminate this disease. We will have to pay a price, that will usually consist in giving up all or part of the chiral invariance on the lattice. We will describe now two possible approaches to the problem.

2a) A la Wilson

Define

$$[\mathscr{D}\psi](n) = \tfrac{1}{2} \sum_\mu [(\gamma_\mu - r\mathbf{1})U_\mu(n)\psi(n+\hat{\mu}) - (\gamma_\mu + r\mathbf{1})U_\mu^+(n-\hat{\mu})\psi(n-\hat{\mu})] \tag{2.19}$$

with $0 < r \leqslant 1$. We have added here an irrelevant operator to the action (the kinetic energy of a boson multiplied times a) : the effect of this term will be of removing the unwanted modes, but also of breaking *all* the γ_5 invariances of the theory. Nothing will

prevent the interactions from inducing a mass for the fermions. We rewrite S_F as

$$\sum_n \bar{\psi}(n) \{K \sum_\mu [(\gamma_\mu - r\mathbf{1}) U_\mu(n) \psi(n+\hat{\mu}) - (\gamma_\mu + r\mathbf{1}) U^+(n-\hat{\mu}) \psi(n-\hat{\mu})] + \psi(n)\}. \tag{2.20}$$

The bare quark mass can be identified with

$$m_q^B = m^B - m_c = \frac{1}{2K} - \frac{1}{2K_c}, \tag{2.21}$$

where m_c is the parameter needed for removing the explicit chiral symmetry breaking. In this formalism the problem we were discussing at the end of the first section (going to the critical point of the fermionic sector of the theory) is the problem of approaching K_c. We will be forced from the finiteness of our lattice to work at some $(K_c - K) \neq 0$, and to extrapolate at $K = K_c$. But since here the fundamental symmetry of the theory (chiral symmetry) is explicitly and completely broken at $K \neq K_c$, the safety of the extrapolation will be very unclear : PCAC results cannot be trusted a priori, and essential changes of behaviour of the forces of the theory in the limit $K \to K_c$ cannot be excluded. Moreover K_c is analytically known just in the two limits

$$\beta = \infty \text{ (free fermions)}, \quad K_c = \tfrac{1}{8}; \tag{2.22a}$$

$$\beta = 0 \quad , \quad K_c = \tfrac{1}{4}. \tag{2.22b}$$

At finite non zero β K_c has to be found numerically : that introduces a new problem that will complicate the numerical experiment.

2b) A la Kogut-Susskind [8, 11, 12, 13]

We assume we are working in $d = 4$ euclidean dimensions. The naive discretization of a 1 flavor theory leads to a 16 flavors one : the mechanism we are going to describe here will lead from a 1 flavor theory, via the lattice, to a 4 flavours continuum theory. We state the final result. By defining on every site of the lattice a *one* component anticommuting variable we get the fermionic lattice action

$$S_F^{(KS)} = \sum_n \bar{\chi}(n) \{[D_x \chi](n) + (-)^{n_x} [D_y \chi](n)$$

$$+ (-)^{n_x + n_y} [D_z \chi](n) + (-)^{n_x + n_y + n_z} [D_t \chi](n) + m\chi(n)\}, \tag{2.22}$$

where D_μ is the covariant version of the central derivative

$$[\partial_\mu \psi](n) = \frac{1}{2a} \{\psi(n+\hat{\mu}) - \psi(n-\hat{\mu})\}. \tag{2.23}$$

Which is the procedure for building the χ variables? We start from the original Dirac spinors

$$\psi_a^i(n),$$

where i is a colour and a is a Dirac index. Now we define the new 4 component spinors $\eta^i_a(n)$, such that [11]

$$\psi^i_a(n) = (\gamma^1)^{n_1}_{ab}(\gamma^2)^{n_2}_{bc}(\gamma^3)^{n_3}_{cd}(\gamma^4)^{n_4}_{de}\eta^i_e(n) \equiv T_{ae}\eta^i_e(n). \tag{2.24}$$

The transformation T is unitary, and is such that, when we express S_F in terms of the η fields, we find that the 4 Dirac components decouple. The left over of the γ matrices structure is a collection of minus signs, that with the choice done in (2.24) are represented by the matrices

$$\Delta_\mu(n) = -\alpha_\mu(n)\mathbf{1}, \qquad \alpha_\mu(n) = (-1)^{n_1+n_2+\ldots+n_{\mu-1}}. \tag{2.25}$$

As it has been remarked in Ref. [14] it is necessary for the Δ_μ matrices to satisfy the condition

$$\Delta_\mu(n)\Delta_\nu(n+\hat{\mu})\Delta^+_\mu(n+\hat{\nu})\Delta^+_\nu(n) = -\mathbf{1}. \tag{2.26}$$

Conversely it can be shown [12] that any set of Δ matrices satisfying on the lattice the condition (2.26) will lead to the correct γ matrices structure for the Dirac spinors. Since the action decouples we can just discard 3 of the Dirac components, by setting

$$\chi^i(n) \equiv \delta_{1a}\eta^i_a, \tag{2.27}$$

for example. We just remark (and we will go back to this point) that the same mechanism will apply to the fermionic Green functions.

In this approach we have a discrete γ_5 invariance, preventing the presence of mass counterterms. A residual doubling ($2^{d/2} = 4$ fermionic species in $d = 4$) is present in this formulation.

3. *Various masses and chiral symmetry breaking*

In doing a numerical simulation of a gauge theory coupled to fermionic fields one is as first trying to answer the two questions:

a) What is the mass spectrum of the theory?

b) Which is the realization of chiral symmetry?

These are obviously just two of the many problems asking for a solution, the ones we will discuss here. One would also like to learn, for example, what is the effect of the fermions on the deconfining transition at finite temperature. Measurements have been done for the anomalous magnetic moments μ_p and μ_n [15], and the computation of the structure functions of the proton seems doable. At the moment, anyhow, the most important point to be investigated is by far more preliminar : what are the size, the origins of the systematical and statistical errors? Is it possible to keep them under control with the present disponibility of computer memory and time?

We want to compute the pattern of the hadronic spectrum for the continuum limit of our lattice theory: we will look at the asymptotic behaviour ($n_t \to \infty$) of the fermionic Green functions. When we use Wilson fermions the identification of quantum numbers will be straightforward: with Kogut-Susskind fermions, on the contrary, it will be by far

more delicate. The second point we will discuss will be the spontaneous breaking of chira symmetry. Here we really want to avoid an explicit breaking, so we will have to think in terms of Kogut-Susskind fermions: we will look at the order parameter

$$\langle\bar{\psi}(0)\psi(0)\rangle = \int d\mu \ \mathrm{Tr}\ \{G(0,0)\}. \tag{3.1}$$

Here G is the quark propagator, solution of

$$(-\not{D}+m)G(n,0) = \delta_{n,0}, \tag{3.2}$$

and $d\mu$ is the appropriate integration measure. We are integrating over all possible configurations of the gauge fields.

In the mesonic sector of the mass spectrum we will have to look at operators like (for the pion, for example)

$$G_\pi(n_t) \equiv \sum_{\vec{n}} \langle\pi(n)\pi(0)\rangle, \tag{3.3}$$

where

$$\pi(n) \equiv \bar{\psi}(n)\gamma_5\psi(n), \tag{3.4}$$

and G_π turns out to be

$$G_\pi(n_t) = \sum_{\vec{n}} \int d\mu \ \mathrm{Tr}\ \{G(n,0)G^+(n,0)\}. \tag{3.5}$$

For Wilson fermions we will just write in a straightforward way the lattice transcription of the continuum operators (see [16] for a complete analysis). For mesons

$$M(x) = \sum_{f_1,f_2} C_{f_1f_2}\bar{\psi}^{f_1,A}(x)\Gamma\psi^{f_2,A}(x), \tag{3.6}$$

where the sum is over flavour indices, $C_{f_1f_2}$ are the appropriate Clebsch-Gordon coefficients. A is a colour index, and Γ is the appropriate combination of γ matrices. For example

$$\varrho_\mu^+(x) = \bar{u}^A(x)\gamma_\mu d^A(x). \tag{3.7}$$

For baryons, indicating by $\boldsymbol{C}$ the charge conjugation matrix,

$$B_\delta(x) = \sum_{f_1,f_2,f_3} C_{f_1f_2f_3}[\psi^{f_1,A}(x)\boldsymbol{C}\gamma_5\psi^{f_2,B}(x)]\psi_\delta^{f_3,C}(x)\varepsilon_{ABC}. \tag{3.8}$$

The proton operator, for example, is

$$P_\delta(x) = [u^A(x)\boldsymbol{C}\gamma_5 d^B(x)]u_\delta^C(x)\varepsilon_{ABC}. \tag{3.9}$$

For a meson we will compute

$$G_\mu(x) = \langle M(x)M(0)^+\rangle$$

$$= \frac{1}{Z}\sum_{f_1,\ldots,f_4} C_{f_1f_2}C^*_{f_3f_4}\int d[U]d[\bar{\psi}]d[\psi]e^{-S}\bar{\psi}^{f_1,A}(x)\Gamma\psi^{f_2A}(x)\bar{\psi}^{f_4B}(0)\Gamma^+\psi^{f_3B}(0), \tag{3.10}$$

and we will go to zero momentum by integrating over the spatial hyperplane.

In the Kogut–Susskind formalism we will have to express the operators of interest as functions of the one variable fermionic varibles $\chi(n)$, defined in (2.27), combined with some suitable minus signs. We will use here the most simple approach: we will assume we are close enough to the continuum limit to have an unbroken flavour symmetry, and we will identify the wanted operators with their continuum expressions. For a more complete lattice analysis see [18]. We indicate now by $\mathscr{D}^{KS}$ the covariant form of the K–S derivative

$$\mathscr{D}^{KS}\psi(n) = \sum_{\mu=1}^{d} (\partial_\mu \psi(n))\,(-)^{\sum_{\nu=1}^{\mu-1} n_\nu}, \tag{3.11}$$

where ∂_μ is the usual central derivative, $n = (n_1, \ldots, n_d)$, (3.11) is the expression we get by applying the transformation (2.24), and we define $g(n)$ by

$$(-\mathscr{D}^{KS} + m)g(n) = \delta_{0,n}. \tag{3.12}$$

If we compute numerically

$$P(n) \equiv \int d\mu \,\mathrm{Tr}\,\{g(n)g^*(n)\} \tag{3.13}$$

we can eventually get information about the zero momentum operators ($\vec{n} \equiv (n_1, n_2, n_3)$)

$$\begin{aligned}
G_\pi(n_4) &= \sum_{\vec{n}} P(n), \\
G_\varrho(n_4) &= \sum_{\vec{n}} P(n)\,\{(-)^{n_1} + (-)^{n_2} + (-)^{n_3}\}, \\
G_{A_1}(n_4) &= \sum_{\vec{n}} P(n)\,\{(-)^{n_1+n_2} + (-)^{n_1+n_3} + (-)^{n_2+n_3}\}, \\
G_\delta(n_4) &= \sum_{\vec{n}} P(n)\,(-)^{n_1+n_2+n_3}.
\end{aligned} \tag{3.14}$$

By applying the transformation (2.24) to the operators having in the continuum theory the quantum numbers of the π, ϱ, A_1, ϱ and B (tensor) if the contributions to the K–S action breaking part of the chiral symmetry [12] (terms of order a) are small enough [18], we get that in the limit $n_t \to \infty$

$$\begin{aligned}
G_\pi &\sim e^{-m_\pi n_t}, \\
G_\varrho &\sim e^{-m_\varrho n_t} + (-)^{n_t} e^{-m_B n_t}, \\
G_{A_1} &\sim e^{-m_\varrho n_t} + (-)^{n_t} e^{-m_{A_1} n_t}, \\
G_\delta &\sim e^{-m_\pi n_t} + (-)^{n_t} e^{-m_\delta n_t}.
\end{aligned} \tag{3.15}$$

4. *The numerical computation*

4a) A general scheme

We are able now to describe, in a very schematic way, all the steps we need to go through our numerical computation.

Point 1: We have to generate, by some standard (or non standard [28]) algorithm, a number (big enough, in the sense of giving rise to a small enough statistical error) of configurations of gauge fields. In principle we should thermalize our gauge fields according to the measure

$$e^{-(S_G+S_F)}, \tag{4.1}$$

where a possible S_G (Wilson) is defined in (2.2), and the formal definition of S_F is given in (2.4).

The fermionic contribution to the action is quite complicated to be taken into account: the Grassmanian character of the fermionic variables implies that S_F has a non local character. And, on the contrary, probabilistic algorithms work well for actions as S_G just thanks to their locality. So using as weight (4.1) is still an open problem: some efficient methods begin to be implemented (see for example Ref. [22], where results for $\langle\bar{\psi}\psi\rangle$ in presence of the full weight (4.1), for $d = 4$ lattice QCD are obtained).

The solution used up to the moment is to implement what is called (from a solid state terminology) the "*quenched approximation*" [23, 17, 24, 19, 25]: we will give some details in Sec. 4c. Here we just give its definition: it just means that the gauge fields are equilibrated according to the measure

$$e^{-S_G}. \tag{4.2}$$

The feedback of fermions on the gauge fields is neglected. Let us remark that it seems plausible nowadays that a computation done by using (4.1) takes less than a factor 10 more than a quenched one [22]: moreover the only difference between a quenched computation and a "full" one is in the choice of the measure. All the other steps (points 2 and 3 of this section) are identical.

It should be noticed that the configurations we are talking about should be statistically independent: since the computation of the observables (quark Green functions, in point 2), takes the main part of the time of the computation we like the terms that will contribute to the statistical sum to be completely independent one from each other.

Point 2: We have to compute the quarks Green function. Since we are interested in the long time behaviour (see for example 3.15) we will have to compute very small numbers. We will use a relaxation procedure (see Sec. 4b).

Point 3: Given our Green functions we will form the propagators of the interesting particles, and extract masses (with suitable fits).

4b) Large time behaviour of the fermionic Green functions

We already said that, since we are interested in the large time behaviour, we will better not use a statistical procedure. We want to solve

$$(-\mathscr{D}+m)_{ij}G_{jk}(0, n) = \delta_{0,n}\delta^{i,k}. \tag{4.3}$$

A possible numerical way of solving this equation consists in introducing a computer time τ, and writing down

$$\frac{dG_\tau(0, n)}{d\tau} = -(\mathscr{D}+m)G_\tau(0, n)+\delta_{0,n} \tag{4.4}$$

(where we omit the colour indices). If the limit exists then

$$\lim_{\tau\to\infty} G_\tau(0, n) = G(0, n). \tag{4.5}$$

With a discretized time

$$G_{\tau+1} = (1-\varepsilon m)G_\tau+\varepsilon\mathscr{D}G_\tau+\varepsilon\delta_{n,0}. \tag{4.6}$$

If we set $\varepsilon = \frac{1}{m}$ in (4.6) we get the Gauss-Seidel scheme

$$G_{\tau+1} = \frac{1}{m}\mathscr{D}G_\tau+\frac{1}{m}\delta_{n,0}. \tag{4.7}$$

For Wilson fermions the procedure (4.6) is quite effective, and the optimal ε is found to be quite close to $\frac{1}{m}$ (see Ref. [25]): for Kogut-Susskind fermions the diagonal part of the operator we want to inverse is smaller than in the Wilson case, and a too small ε would be needed to make (4.6) converge. So the use of a second order algorithm is suitable. References to other inversion schemes can be found in the literature (see for example Ref. [13] for the conjugate gradient method).

Consider (4.6): when $m \to m_c$ (Wilson fermions) the ε which guarantees the convergence becomes very small, and the convergence prohibitively slow. Moreover in a finite system the operator $-\mathscr{D}+m$ may have a negative eigenvalue already for $m > m_c$.

We said up to now that statistical methods are not useful to compute the long time behaviour of the Green functions: as far as $\langle\bar{\psi}\psi\rangle$ is concerned, anyhow, we are dealing with a number of order 1. By means, for example, of a Langevin equation, we can get a very precise estimate in a reasonable computer time. Let us consider the two complex fields φ_1 and φ_2, and the two equations

$$\begin{aligned}\frac{d}{d\tau}\varphi_1(x, \tau) &= (\mathscr{D}+m)\varphi_1(x, \tau)+\eta(x, \tau),\\ \frac{d}{d\tau}\varphi_2(x, \tau) &= (-\mathscr{D}+m)\varphi_2(x, \tau)+\eta(x, \tau),\end{aligned} \tag{4.8}$$

where τ is here the Langevin time, and the noise η is the same in the two equations, with

$$\langle\eta(x, \tau)\rangle = 0, \quad \langle\eta(x, \tau)\eta(y, \tau')\rangle = 2\delta(\tau-\tau')\delta^{(4)}(x-y). \tag{4.9}$$

Now it is easy to check [22] that

$$\lim_{\tau\to\infty}\langle\varphi_1(x, \tau)\varphi_2^*(y, \tau)\rangle = G(x, y). \tag{4.10}$$

4c) Quenching

Which is the sense of the quenched approximation? The action of our original theory has the form

$$S = S_G[U] + \sum_{i,j} \bar{\psi}_i \Delta_{ij}[U] \psi_j. \tag{4.11}$$

By defining the Grassmanian integration in the usual way

$$\int d[\psi] d[\bar{\psi}] e^{-S} = \det \{\Delta[U]\} e^{-S_G[U]} \tag{4.12}$$

we can define an effective action

$$S_{\text{eff}}[U] = S_G[U] - \text{Tr} \ln \{\Delta[U]\}. \tag{4.13}$$

We want to compute $G(0, n)$: the situation is shown in Fig. 4.1. The two sources are in 0 and n, the dashed line is the external fermionic line, integer lines are internal fermionic lines and wavy lines are the gluonic contributions. The gluonic contributions are given by the measure $\exp\{-S_G\}$, the internal fermionic lines by the det (Δ). So a quenched

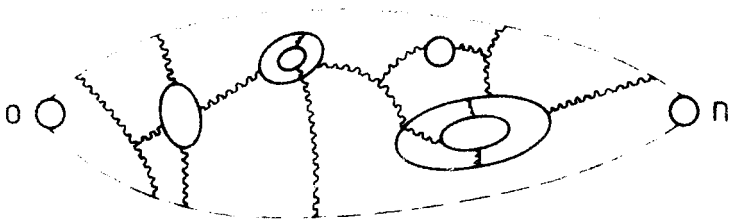

Fig. 4.1. Typical graph contributing to $G(0, n)$

approximation turns out to be the equivalent of neglecting the internal quark loops. If we consider a theory with n_f flavours we will get

$$S_{\text{eff}} = S_G - n_f \ln \{\det(-\not{\partial} + m)\}. \tag{4.14}$$

So our approximation is an $n_f = 0$ approximation. In second it is equivalent to implement the Zweigg rule: is very plausible [24] (and well confirmed by numerical experiments on the Schwinger model [23]) that his main result is the readjustment of the unphysical length scale of the lattice theory. Lastly in the limit of the infinite number of colours internal fermionic loops are uneffective.

5. *Conclusions (preliminary)*

We have given here some technical details about the structure of a numerical mass spectrum computation. It should be said that the situation of MC simulations is at a turning point: lattices are becoming closer to be "big enough" [29] ($\sim$ 1 Fermi in Ref. [19]), making small the correlations between propagators computed in different configurations

(see Fig. 1 in Ref. [19]). Astonishingly enough, one of the most important outcomes of quenched computations is going to become the determination of the string tension, a pure gauge quantity. The output of Ref. [19] is that, due to the too small evaluated Wilson loops, the result of Ref. [27] (and the first Creutz result, Ref. [5]), can be off of a factor 2. This trend is confirmed by Ref. [20], and by the analysis of Ref. [21]. The crucial point is that the analogous of Eq. (4.6) for Wilson loops does not exist: they have to be evaluated by a statistical procedure, and too big Wilson loops (the ones giving the wanted asymptotic value) have a too small value to be observed in a reasonable computer time.

The structure of Kogut–Susskind fermions is going to be understood better and better: analytical (strong coupling) work [12] is making transparent the role of the symmetries (mainly the discrete lattice chiral symmetry), and numerical simulations for SU(2) with a high statistics (50 configurations) on a fairly big lattice ($8^3 \cdot 16$ in the time direction) are showing that mesons have a quite stable and credible pattern [18]. Masses of ϱ, A_1 are coming out very stable and in a good agreement with "Rosenfeld tables".

The pattern of the explicit restoration of chiral symmetry for $K \to K_c$ in Wilson fermions (and the supposed to be spontaneous breaking) has to be clarified. Also for Wilson fermions information about the spectrum is starting to be more reliable [19]: statistical errors seem to be now under control. Dedicated machines should provide, in the next period, the way of getting real checks about the consistency between Q.C.D. and nature. Lot of efforts are needed: but they seem to be worthwhile!

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REFERENCES

[1] K. G. Wilson, *Phys. Rev.* **D10**, 2445 (1974); R. Balian, J. M. Drouffe, C. Itzykson, *Phys. Rev.* **D10**, 3376 (1974); **D11**, 2098, 2104 (1975).

[2] P. Hasenfratz, lectures given at this school.

[3] N. H. Christ, A. E. Terrano, Columbia Univ. preprint, CU-TP-261.

[4] E. Brooks, G. Fox, R. Gupta, O. Martin, S. Otto, E. De Benedictis, Caltech preprint CALT-68--867 (September 1981).

[5] M. Creutz, *Phys. Rev. Lett.* **43**, 553 (1979); *Phys. Rev.* **D21**, 2308 (1980).

[6] M. Falcioni, E. Marinari, M. L. Paciello, G. Parisi, F. Rapuano, B. Taglienti, Zhang Yi-Cheng, *Phys. Lett.* **110B**, 295 (1982); B. Berg, A. Billoire, Saclay preprint SPh.T/42 (March 1983), and references therein.

[7] T. Banks, S. Raby, L. Susskind, J. Kogut, D. R. T. Jones, P. N. Scherbach, D. K. Sinclair, *Phys. Rev.* **D15**, 1111 (1977); L. Susskind, *Phys. Rev.* **D16**, 3031 (1977).

[8] For further details see J. Kogut, lectures given at the Les Houches Summer School (July 1982).

[9] H. B. Nielsen, N. Ninomiya, *Nucl. Phys.* **B185**, 20 (1981); *Phys. Lett.* **105B**, 219 (1981); *Nucl. Phys.* **B193**, 173 (1981).

[10] K. G. Wilson, *New phenomena in subnuclear physics*, Erice 1975, ed. A. Zichichi, Plenum Press, New York 1977.

[11] K. Kawamoto, J. Smit, *Nucl. Phys.* **B192**, 100 (1981).

[12] H. Kluberg-Stern, A. Morel, O. Napoly, B. Peterson, *Nucl. Phys.* **B190** [FS3], 504 (1981); *Flavours of Lagrangian Susskind Fermions*, to appear in *Nucl. Phys.* B; H. Kluberg-Stern, A. Morel, B. Peterson, *Phys. Lett.* **114B**, 152 (1982); *Nucl. Phys.* **B215** [FS7], 527 (1983).
[13] J. Kogut, M. Stone, H. W. Wyld, S. H. Shenker, J. Shigemitsu, D. K. Sinclair, preprint DOE/ER/01545-335.
[14] T. Banks, P. Windey, *Nucl. Phys.* **B198**, 226 (1982).
[15] G. Martinelli, G. Parisi, R. Petronzio, F. Rapuano, *Phys. Lett.* **B116**, 434 (1982); C. Bernard, T. Draper, K. Olynyk, M. Rushton, *Phys. Rev. Lett.* **49**, 1076 (1982).
[16] I. Ioffe, *Nucl. Phys.* **B188**, 317 (1981).
[17] E. Marinari, G. Parisi, C. Rebbi, *Phys. Rev. Lett.* **47**, 1795 (1981).
[18] A. Billoire, R. Lacaze, E. Marinari, A. Morel, *Phys. Lett.* **B**, to be published.
[19] H. Lipps, G. Martinelli, R. Petronzio, F. Rapuano, *Phys. Lett.* **126B**, 250 (1983).
[20] F. Gutbrod, P. Hansenfratz, Z. Kunszt, J. Montvay, CERN preprint TH. 3591 (May 1983).
[21] G. Parisi, R. Petronzio, F. Rapuano, CERN preprint TH. 3596 (May 1983).
[22] H. Hamber, E. Marinari, G. Parisi, C. Rebbi, *Phys. Lett.* **B124**, 99 (1983); H. Hamber, E. Marinari, G. Parisi, C. Rebbi, Saclay preprint SPh.T/83/54 (April 1983).
[23] E. Marinari, G. Parisi, C. Rebbi, *Nucl. Phys.* **B190** [FS3], 734 (1981).
[24] D. Weingarten, *Phys. Lett.* **109B**, 57 (1982); Indiana University preprint IUHET-82 (1982).
[25] K. Bowler, E. Marinari, S. Pawley, F. Rapuano, D. Wallace, *Nucl. Phys.* **B220** [FS8], 137 (1983).
[26] F. A. Berezin, *The Method of Second Quantization*, Academic Press, 1966.
[27] C. Pietarinen, *Nucl. Phys.* **B190** [FS3], 349 (1981); M. Creutz, K. J. M. Moriarty, *Phys. Rev.* **D26**, 2166 (1982); R. W. B. Ardille, M. Creutz, K. J. M. Moriarty, BNL preprint 32377 (1982).
[28] N. Cabibbo, E. Marinari, *Phys. Lett.* **119B**, 387 (1982).
[29] P. Hasenfratz, I. Montvay, *Phys. Rev. Lett.* **50**, 309 (1983).