BINDING OF MATTER TO A MAGNETIC MONOPOLE*

BY TH. W. RUIJGROK

Instituut voor Theoretische Fysica, Utrecht**

(*Received October 4, 1983*)

After Dirac had firmly planted the concept of a magnetic monopole in the minds of experimental and theoretical physicists the search for this particle has not stopped. Since it can manifest itself only through the interaction with ordinary matter many matter-monopole systems have been investigated. The purpose of the present paper is to give a short and therefore incomplete review of the literature concerning the question whether an electron, a proton or an atom can be bound to a magnetic monopole.

PACS numbers: 14.80.Hv

1. *Introduction*

In 1931 Dirac [1] showed that the formalism of quantum mechanics inevitably leads to wave functions whose only physical interpretation is the motion of an electron in the field of a single magnetic pole. He then remarks that "Under these circumstances one would be surprised if Nature had made no use of it". Dirac's proof of the quantisation of the magnetic charge g, i.e., of the statement that the quantity $2q \equiv 2eg$ must be an integer, was not transparent enough to the taste of Fierz [2]. He subsequently produced a proof based on the requirement that the solution of the Schrödinger equation must form part of a representation of the rotation group.

The bibliographies of Stevens [3] and of Strazhev and Tomol'chik [4] show that the curiosity generated by this hypothetical particle has never disappeared, although waves of interest can be seen. One such wave started when 't Hooft [5] and Polyakov [6] found a magnetic monopole solution to a classical non-abelian gauge theory. The period between 1973 and 1976 is described in the bibliography of Carrigan [7]. Recently the interest in monopoles has been intensified for two reasons: Cabrera's experimental observation [8] of a candidate for the monopole and Rubakov's theoretical analysis [9] of monopole catalysis of proton decay.

* Lecture presented at the XXIII Cracow School of Theoretical Physics, Zakopane, May 29-June 12, 1983.

** Address: Institute of Theoretical Physics, Rijksuniversiteit, Princetonplein 5, P. O. Box 80.006, 3508 TA Utrecht, The Netherlands.

In order to have additional methods for detecting magnetic monopoles it is natural to investigate the possibility of binding such a monopole to an atomic nucleus or to a molecule. Such an investigation, both theoretically and experimentally, was first performed by Malkus [10], who suggested that chemical binding indeed is possible. He obtained his results by interpolating between two extreme cases:

(*i*) the electron moving in the combined field of a monopole and an atomic nucleus both situated at the same position and

(*ii*) calculation of the diamagnetic and paramagnetic energy for a large separation of the monopole and the nucleus.

The result of such a calculation may, however, be very misleading, especially since it has been shown [11] that the singular magnetic field requires special methods for the solution of the Dirac equation. The simplest molecule contains one monopole, one proton and one electron. This molecule is of atomic size ($\sim$ 1 Å) and has atomic binding energy ($\lesssim$ 1eV). A detailed analysis of this molecule was presented in a recent paper [12] and will be discussed in Section 3. While this molecule is of interest by itself, there are several additional motivations for its investigation. First it is the prototype of a new class of molecules. Many other similar molecules can be easily envisaged, including for example the binding of a monopole to organic molecules or to ferromagnetic materials [13–14].

Secondly, if the Rubakov-effect does exist, it is a priori not excluded that such monopole molecules will speed up the decay of nucleons, possibly leading to spectacular events.

In the next section a short review is given of papers dealing with the binding of one charged particle by a magnetic monopole. In the third section the existence of monopole-molecule bound states is discussed.

2. *Can a monopole bind an electron, a proton or an atomic nucleus?*

The classical dynamics of a particle with charge e moving in the magnetic field of a fixed monopole of strength g has been reviewed by Carter and Cohen [15] and only the most essential properties of this system will be mentioned here.

(*i*) The charge e moves with constant velocity on the surface of a right circular cone, which has its top at the position of the monopole and its axis in the direction of the vector

$$\vec{J} = \vec{L}+\vec{s},$$

where $\vec{L}$ is the orbital angular momentum and $\vec{s} = -q\vec{e} = -eg\frac{\vec{r}}{r}$ is a vector of constant length in the direction from monopole to charged particle.

(*ii*) When the cone is cut open and spread on a plane the orbit is a straight line. This shows that in the classical case no bound states are possible.

(*iii*) The angular momentum contained in the electromagnetic field and defined by $\frac{1}{4\pi}\int \vec{r}\times(\vec{E}\times\vec{B})d\vec{r}$ turns out to be equal to $\vec{s}$.

(*iv*) The vector $\vec{J}$ is conserved and is interpreted as the total angular momentum.

(*v*) Goldhaber [16] has shown that Newton's equation of motion can also be derived from a Hamiltonian $H = \frac{|\vec{p}|^2}{2m} + \frac{\vec{L}\cdot\vec{s}}{mr^2}$, with $\vec{s}$ considered as an independent vector-variable with Poisson-brackets $(s_i, s_j) = -\varepsilon_{ijk}s_k$, provided $\vec{s} = -q\vec{e}$ is satisfied at some initial time. It can be said, therefore, that because of the presence of the magnetic monopole the particle acquires an additional intrinsic spin, which in the quantum theory has the value $q = \frac{1}{2}$.

(*vi*) When the charged particle passes straight through the monopole $\vec{L} = 0$, but $\vec{s}$ changes sign suddenly, so that the total angular momentum is not conserved. This is the first indication that trouble will arise if the monopole is considered as a structureless particle.

As to the quantum case, already in his original paper [1] Dirac showed that an electron without spin and described by the non-relativistic Schrödinger equation, cannot be bound to a magnetic monopole. For an *s*-state this equation takes the form

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} - \frac{\lambda_n}{r^2}\right]f(r) = -\frac{2mW}{\hbar^2}f(r),$$

in which $\lambda_n = n^2+2n+\frac{1}{2}$ is a dimensionless number. This shows that there is no natural unit a_0 with the dimension of a length $\left(\text{like the Bohr-radius } a_0 = \frac{\hbar}{\alpha mc}\right)$ from which an energy $E_H = \frac{e^2}{2a_0}$ can be formed.

This result was confirmed by Tamm [17] in a paper in which he introduced generalized spherical harmonics. These have additional lines of zeroes as compared to the usual spherical harmonics and this created the idea that a quantum mechanical description of a charged particle moving in the field of a magnetic monopole is possible only when the monopole has attached to it a string of magnetic dipoles, extending to infinity. This picture was corrected by Wu and Yang [11] who showed that the Dirac string disappeared when instead of working with a single wave function the concept of section was used. Roughly speaking this amounts to having two wave functions, one for the northern and one for the southern hemisphere, which are connected by a gauge transformation in a region on both sides of the equator. Their method solves the problem of how to avoid the singularities which necessarily arise if one tries to describe the nonsingular magnetic field of a monopole by a single vector potential. The same problem occurs if one tries to use one network of coordinates for the non-singular geometry of a sphere. Banderet [18], using the Dirac-equation for a charged particle with spin $\frac{1}{2}$ and anomalous magnetic moment M, showed that for $M \neq 0$ the wave function was unacceptably singular in the origin, where the monopole is sitting. For $M = 0$, i.e., for the Dirac-electron, the solutions are well defined, but it turned out that also in this case the electron could not be bound to the monopole. Teller [19] has remarked that this result is an immediate consequence of the form of the Dirac-equation for a particle moving in an arbitrary magnetic field describable by a vector potential, but in the absence of an electric field.

The singular behaviour of the wave function of the Dirac-equation was partly avoided by Malkus [10], who considered the Pauli-equation for a charged particle with arbitrary magnetic moment. Below a certain critical value he finds that there are no bound states. Above this critical magnetic moment states with an arbitrarily low value of the energy are found. This is an example of the well known difficulties [20] arising in quantum mechanics when the potential is highly singular. Therefore, in order to give a correct treatment of the wave function close to the monopole one should either take the structure of the monopole into account., e.g. by considering its non-abelian character, or prevent the particle to come too close to the monopole. The latter can be done in two ways.

The first is to give the monopoles a hard core. This was done by Bracci and Fiorentini [21], who considered the Pauli-equation for a charged particle with an anomalous magnetic moment. They find that the proton with an anomalous magnetic moment $\kappa = 1.79$ can bind in a $j = 0$ state to the magnetic monopole. The binding energy is 15 keV and for the linear dimension of the wave function they obtain a value of 32 fm. These results, however, are in disagreement with the calculations of Ref. [22], which are based on the Dirac--equation. It is also suggested [21] that once a bound state of a monopole with an atom has been formed, it will soon contract to a system in which the atomic nucleus is bound directly to the monopole. In the last section, however, evidence will be given to show that, contrary to this suggestion, an atomic bound state will have a very long life time and therefore will not make a rapid transition to the nuclear bound state.

A second way of preventing the charged particle to pass through the monopole was given by Kazama, Yang and Goldhaber [23]. They observed that the Hamiltonian derived from the Dirac-equation was not well defined when the total angular momentum was equal to $j = |q|-\frac{1}{2} \geqslant 0$, where $q = Zeg$, which is the product of the electric charge of the particle and the magnetic charge of the monopole. According to Dirac [1] the value of $2q$ should be an integer. The Hamiltonian became well defined after adding a small amount to the magnetic moment, which thereby became $\frac{Ze}{2M}(1+\kappa)$. The value of κ, which could be positive or negative, eventually was approaching zero. In this way the singularity in the origin was avoided for $\kappa \neq 0$. It appears of course for $\kappa = 0$, but the authors of Ref. [23] were able to show that by changing the boundary condition in the origin the problem remained well defined for $\kappa \to 0$. For the case of electron-monopole scattering ($E > M$) this was done as follows. After a simple transformation the Dirac-equation for the large and small components $F(r)$ and $G(r)$ took the form (here shown only for $\kappa > 0$ and $q = \frac{1}{2}$)

$$\frac{dF}{dr} = \left[M+E-\frac{\kappa}{4Mr^2}\right]G(r), \quad \frac{dG}{dr} = \left[M-E-\frac{\kappa}{4Mr^2}\right]F(r).$$

The κ-terms can be neglected for $r \gg R = \frac{\sqrt{|\kappa|}}{M}$. In this case it follows from the above equations that $\frac{d}{dr}[(E+M)G^2+(E-M)F^2)] = 0$. This means that in the $F-G$ plane the point representing the two amplitudes moves on an ellipse with varying r. The size of the ellips, however, shrinks to zero for $|\kappa| \to 0$. On the other hand, for $r \to 0$, the κ-terms are the only important terms on the right hand side of the differential equations. The solution is given by $F(r) = -G(r) = e^{-\kappa/4Mr}$. In the $F-G$ plane, when $r \to 0$, the point approaches the origin along a straight line, which corresponds to the correct boundary condition. From the above construction it is seen that if now the limit $\kappa \to 0$ is taken the condition $F(0) = G(0) = 0$ is replaced by $\lim_{r \to 0} \frac{F(r)}{G(r)} = -1$. This Kazama–Yang–Goldhaber (KYG) boundary condition gives a self-adjoint Hamiltonian, which is free of the singularities with which previous formulations were plagued. This method of letting κ go to zero either from the positive or from the negative side is a way to describe the short distance properties of the monopole. This is certainly not unique as was recently shown by Wu [24]. He constructed not just two ($\kappa \to 0^+$ and $\kappa \to 0^-$), but a continuous set of limiting cases, described by a parameter ω ranging from $-\pi$ to $+\pi$ and consistent with the requirement of relativistic invariance.

With these KYG-boundary conditions and using the method of sections of Ref. [11], Kazama and Yang [25] then considered the question whether a Dirac particle, with an extra magnetic moment given by κ, can be bound to an infinitely heavy monopole. They find that for $\kappa > \frac{1}{4|q|}$ there are infinitely many bound states with $j = |q| - \frac{1}{2}$. A more detailed investigation of this case has been given by the authors of Ref. [22]. They find for instance a proton-monopole bound state with a binding energy of 263 keV and a radius $a_0 = 9.3$ fm. These bound states disappear when $\kappa \to 0$. For $\kappa > 0$ Kazama and Yang find an additional bound state with energy $E = 0$, i.e., with a binding energy equal to the rest mass of the Dirac particle. The analogous state in the work of Wu [24] has an energy between $-M$ and $+M$, depending on the parameter ω. Although the electron has a non-vanishing anomalous magnetic moment, it is not certain that this $E = 0$ state (or its Wu analog) really exists for the electron-monopole system, since the anomalous moment is due to radiative corrections and is therefore not completely pointlike. Its existence will also depend on the structure of the monopole as is shown by the example of the SU(5) grand unified monopole. For this case Walsh, Weisz and Wu [26] showed that there is no zero-energy bound state.

3. Can a monopole bind an atom?

The answer to this question may be of importance for the possible monopole catalysis of fusion [27] and fission [28], but not much work has been done on this subject. There is of course always the possibility to add one or more electrons to the deep lying states of a monopole bound to an atomic nucleus, as discussed in the previous section. Here, however, the attention is restricted to monopoles which are bound chemically, i.e., with energies of the order of 1 eV.

Malkus [10] has considered the Pauli-equation for an electron in the magnetic field of a monopole together with the electric field of a charge $Z|e|$, both fixed in the same position. He finds that this system gives the same binding as when the monopole and the free atom are separated by an infinite distance. Since the combined effect of diamagnetic and paramagnetic energy is to give more binding when the monopole and the atom come closer together, there will be a minimum in the energy of the ground state as a function of their separation. For the electron-proton-monopole system he estimates the lowest energy to occur at a proton-monopole separation $R = 0.56\ a_0$ (a_0 is the Bohr radius) and to lie about 7 eV below the ground state energy of -13.6 eV of the H-atom. The monopole-atom potential calculated in this way, i.e., using the Born-Oppenheimer approximation [29], is certainly deep enough to give bound states. Actually, since at large distances the Zeeman-term, which is linear in the magnetic field and therefore proportional to R^{-2}, dominates the repulsive diamagnetic term $\sim B^2 \sim R^{-4}$, the potential will support an infinite number of bound states.

From these rather crude estimates, and similar ones given by Bracci and Fiorentini [21], one can conclude that if the (neutral) magnetic monopole exists, it can certainly be bound to atoms, molecules and other chemical compounds with a nonvanishing magnetic moment. In order, however, to find the vibrational spectrum and to determine the stability of these bound states against transitions to the states in which the monopole is bound directly to the atomic nucleus, it is necessary to perform a more accurate calculation and to find the form of the monopole-atom potential.

Such a calculation was presented in Ref. [12] and the results will now be described briefly. The system considered consisted of one electron with mass M and magnetic moment $\frac{e}{2M}(1+\kappa)$, of a neutral magnetic monopole which was fixed in the origin of the coordinate system and of a proton, without magnetic moment, which was also fixed, but at a distance R from the monopole. The Dirac-equation for the four-component electron wave function was written as $H\psi = E\psi$ with the Hamiltonian

$$H = \alpha\cdot(-i\vec{\nabla}-e\vec{A})+\beta M-\frac{\kappa q\beta\vec{\sigma}\cdot\vec{r}}{2Mr^3}-\frac{e^2}{|\vec{r}-\vec{R}|}\,.$$

The number $q = eg$ has the Dirac-value $q = \frac{1}{2}$, while $\vec{A}$ is the vector potential for the magnetic field of the monopole. To avoid singularities in this vector potential the method of Kazama and Yang [25] was followed and the wave function ψ was treated as a section [11]. Without the Coulomb-interaction between electron and proton the total angular momentum

$$\vec{J} = \vec{r}\times(\vec{p}-e\vec{A})-q\frac{\vec{r}}{r}+\tfrac{1}{2}\vec{\sigma}$$

would be a conserved quantity with eigenvalues $j(j+1)$ and m_z for J^2 and J_z (j and m_z integers). With the proton situated on the positive z-axis m_z is still a good quantum number, but the total wave function now contains all j-values. Only $m_z = 0$ is considered. For $j = 0$ there are two radial wave functions denoted by $v_1(r)$ and $v_2(r)$, whereas for each $j > 0$

there are four functions, collectively denoted by $v_n(r)$ with $n = 3, 4, \ldots$. In order to find bound states the usual boundary conditions must be imposed on the functions $v_3, v_4, \ldots$, viz.

$$\lim_{r\to 0} v_n(r) = \lim_{r\to\infty} v_n(r) = 0 \quad \text{for} \quad n \geqslant 3.$$

The same boundary conditions apply to $v_1(r)$ and $v_2(r)$ as long as $\kappa \neq 0$. For the limiting case $\kappa \to 0^{\pm}$ the boundary conditions for $r \to 0$ must, however, be replaced by those derived by Kazama, Yang and Goldhaber [23], i.e., by

$$\lim_{r\to 0} \frac{v_1(r)}{v_2(r)} = -\frac{\kappa}{|\kappa|}.$$

Before discussing the results of the numerical solution of the eigenvalue problem the case is considered where the functions $v_n(r)$ for $n \geqslant 3$ can be neglected. This will happen when R becomes small compared to the Bohr radius a_0. The equations for $v_1(r)$ and $v_2(r)$ then become

$$\frac{dv_1}{dr} = (M+E+V_0)v_2(r), \quad \frac{dv_2}{dr} = (M-E-V_0)v_1(r),$$

with

$$V_0(r) = \begin{cases} \dfrac{e^2}{R} \text{ for } r \leqslant R; \\[2ex] \dfrac{e^2}{r} \text{ for } r \geqslant R. \end{cases}$$

These equations are exact if the proton is replaced by a uniformly charged spherical shell of radius R, whose center coincides with the monopole. Without Coulomb-interaction it is seen that for $\kappa \to 0^+$ a solution is given by $E = 0$ and $v_1(r) = -v_2(r) = e^{-Mr}$. This is the deeply bound state found already by Kazama and Yang [25]. Because of the addition-

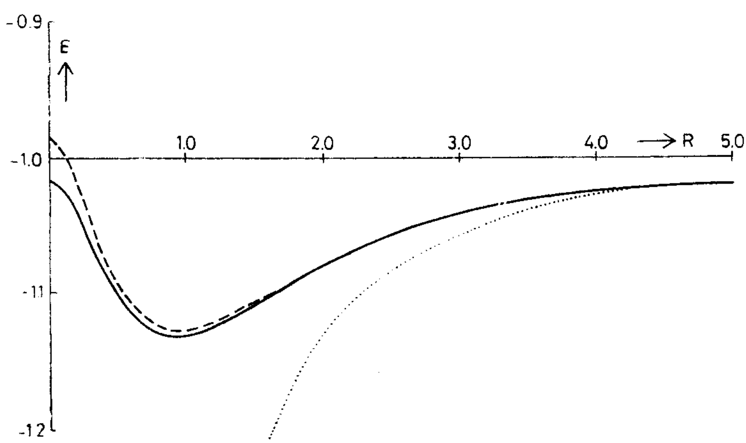

Fig. 1. Effective monopole-atom potential for $\kappa \to 0^-$ (full line) and for $\kappa \to 0^+$ (dashed line). The dotted line gives the dia- and paramagnetic energy

al Coulomb attraction this state will persist and even have a negative energy, which means that the binding energy is larger than the rest mass of the electron. It is questionable whether under such circumstances the Dirac-equation is still applicable. The attention is therefore restricted to the states with binding energies of the order of 1 eV or less. The result of a numerical solution of the full eigenvalue problem, both for $\kappa \to 0^+$ and for $\kappa \to 0^-$, is shown in Fig. 1. Note that this effective potential between atom and monopole is much less attrac-

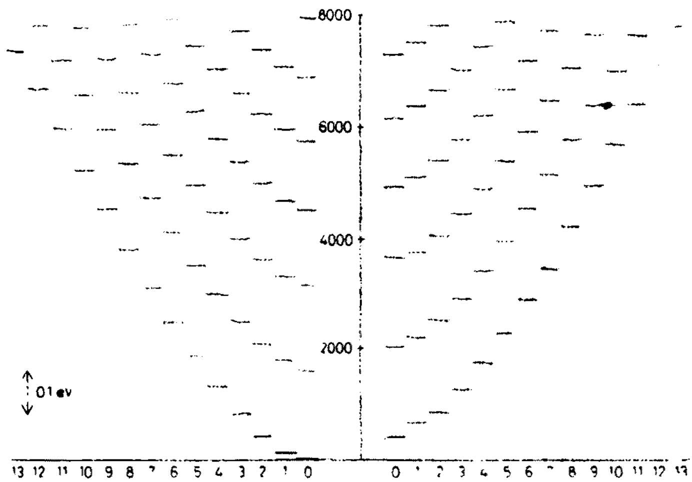

Fig. 2. Level scheme of monopole-atom bound states for $\kappa \to 0^-$ (on the left) and $\kappa \to 0^+$ (on the right). The energy unit is cm^{-1}. The values of the angular momentum are shown along the horizontal axis, which lies 1.646 eV below the energy of the free atom

tive than the rough estimate of Malkus [10]. In the present case the potential has a minimum at $R = 0.9\ a_0$ with a value $-1.13\,E_H$, in which $E_H = \dfrac{e^2}{2a_0} = 13.6\,\text{eV}$. Also shown in Fig. 1 is the para- and diamagnetic energy of a H-atom in the field of the monopole at a distance R. With this effective potential the Schrödinger equation was used for the calculation of the vibrational and rotational levels of the monopole-atom system. The results are shown in Fig. 2. It is seen that there are differences depending on whether the limit $\kappa \to 0^+$ or $\kappa \to 0^-$ is taken. A continuum of results would be obtained, probably lying within these two extremes, when the boundary conditions as derived by Wu [24] would have been used. It should therefore be stressed that the structure of the monopole, even in the limit of a point particle, is important for the atomic properties of the bound state. The correct choice for this structure should be made on the basis of a more comprehensive theory.

4. *Conclusions*

In the previous section it was shown that the potential which can bind an atom to a magnetic monopole is not as deep as was estimated by Malkus [10]. Nevertheless, once such a bound state is formed it is separated from the deep lying nucleus-monopole states mentioned in Section 2 and from the Kazama–Yang state with $E = 0$ by a barrier which still has the sizable height of about 1.5 eV (Fig. 1). The atomic states will therefore be depopulated and make transitions to the tightly bound nuclear states, only when the temperature of the environment is about 20000 K or higher. Then also dissociation into a monopole and a free atom will be possible. In principle a transition to a nuclear bound state can also be made by tunneling through this barrier of height 1.5 eV and width of about 1 Å. The average time necessary for this process is of the order of $\tau = \frac{4\pi}{\omega} e^{2\kappa}$. In this formula ω is a characteristic frequency of the bound state, so $\hbar\omega \simeq 0.3$ eV. The factor in the exponent is given by

$$K = \frac{2\pi}{\hbar} \int_{r_1}^{r_2} |p| dr \simeq \frac{2\pi a_0}{\hbar} \sqrt{2M_A B},$$

where B is the barrier height $B \simeq 1.5$ eV. Substituting these values gives a lifetime against tunneling which is much larger than the age of the universe.

From this it is clear that once a monopole has lost its kinetic energy by atomic or nuclear collisions in the interior of the earth it can easily form absolutely stable chemical bonds with molecules of the surroundings. The question how a slowly moving monopole loses its energy in matter has been answered in a recent paper by Drell, Kroll, Mueller, Parke and Ruderman [30]. A typical value of $\frac{1}{\varrho}\frac{dE}{dx}$, where ϱ is the density of matter, is 1 MeV $\cdot$ cm^2/g for a velocity $\beta = \frac{v}{c} = 10^{-4}$. This means that a monopole with mass 10^{16} GeV/c^2 and with this velocity can completely lose its kinetic energy of 5.10^7 GeV when passing through the center of the earth. Once it is stopped it can easily form a monopole molecule with the surrounding material. It will not fall to the center of the earth, because, when a monopole with this mass is displaced over a distance of 1 Å in a gravitational field of 10 m/sec^2 it only loses 0.1 eV in potential energy and this is small compared with the energy difference of 1.8 eV between the H-monopole bound state and the dissociated H-monopole system.

An opposite effect is given by the earth magnetic field [10]. A monopole of strength $g = \frac{\hbar c}{2e} = \frac{e}{2\alpha}$ gains $\frac{300}{2\alpha} H$ eV/cm when moving in the direction of a magnetic field of H Gauss. For a field of one Gauss this amounts to only 10^{-4} eV/Å. There is therefore no hope that under the influence of the earth magnetic field the monopole molecules would diffuse to the surface of the earth, so that they could simply be fished up from the Hudson Bay.

Also the Rubakov effect is not a promising way for detecting monopole molecules, because the field of the monopole at the position of the proton 1 Å away is not strong enough to appreciably accelerate the proton decay.

Nevertheless, if monopoles really exist the work of Drell c.s. [30] shows that since the formation of the earth quite a few could have been captured by molecules. Extensive searches in deep ocean deposits and lunar rocks [31], however, have not resulted in the discovery of a single monopole and have only set limits on the production cross section and on the occurrence in cosmic radiation.

Since the vibrational spectrum of a monopole molecule, as shown in Fig. 2, is different from that of an ordinary molecule, it may pay to have the IRAS satellite look at the far infrared spectrum of a celestial object which is suspected to have an abundance of monopoles. The author, however, will neither look nor pay.

It was a pleasure to cooperate with Professor J. A. Tjon and Professor T. T. Wu.

REFERENCES

[1] P. A. M. Dirac, *Proc. Roy. Soc. London* **A133**, 60 (1931).
[2] M. Fierz, *Helv. Phys. Acta* **17**, 27 (1944); For a formulation in modern language the reader is referred to C. A. Hurst, *Ann. Phys.* (USA) **50**, 51 (1968).
[3] D. M. Stevens, VPI-EPP-73-5 (Oct. 15, 1973). Virginia Polytechnic Institute.
[4] V. I. Strazhev, L. M. Tomil'chik, *Fiz. El. Chast. Atom. Yad.* **4**, 187 (1973); *Sov. J. Part. Nucl.* **4**, 78 (1973).
[5] G.'t Hooft, *Nucl. Phys.* **B79**, 276 (1974).
[6] A. M. Polyakov, *Pis'ma v Zh. Eksp. Teor. Fiz.* **20**, 430 (1974); *JEPT Lett.* **20**, 194 (1974).
[7] R. A. Carrigan, Jr., *Magnetic Monopole Bibliography 1973–1976*, FERMILAB-77/42, January 1977.
[8] B. Cabrera, *Phys. Rev. Lett.* **48**, 1378 (1982).
[9] V. A. Rubakov, *JETP Lett.* **33**, 644 (1981).
[10] W. V. Malkus, *Phys. Rev.* **83**, 899 (1951).
[11] T. T. Wu, C.N. Yang, *Nucl. Phys.* **B107**, 365 (1976).
[12] Th. W. Ruijgrok, J. A. Tjon, T. T. Wu, *Phys. Lett.* **129B**, 209 (1983).
[13] P. H. Eberhard, R. R. Ross, J. D. Taylor, L. W. Alvarez, *Phys. Rev.* **D11**, 3099 (1975).
[14] C. Kittel, A. Manoliu, *Phys. Rev.* **B15**, 333 (1977).
[15] E. F. Carter, H. A. Cohen, *Am. J. Phys.* **41**, 994 (1973).
[16] A. S. Goldhaber, *Phys. Rev.* **B140**, 1407 (1965).
[17] Ig. Tamm, *Z. Phys.* **71**, 141 (1931).
[18] P. P. Banderet, *Helv. Phys. Acta* **19**, 503 (1946).
[19] E. Teller, see footnote 2 in Ref. [10].
[20] K. M. Case, *Phys. Rev.* **80**, 797 (1950).
[21] L. Bracci, G. Fiorentini, *Phys. Lett.* **124B**, 29, 493 (1983) and preprint IFUP TH-83/2, Pisa 1983.
[22] K. Olaussen, H. A. Olsen, P. Osland, I. Øverbø, *Kazama-Yang Monopole-Fermion Bound States*, DESY 83-041, June 1983.
[23] Y. Kazama, C. N. Yang, A. S. Goldhaber, *Phys. Rev.* **D15**, 2287 (1977).
[24] T. T. Wu, *Interaction of a Fermion with a Monopole*, DESY 83-021, March 1983.
[25] Y. Kazama, C. N. Yang, *Phys. Rev.* **D15**, 2300 (1977).
[26] T. F. Walsh, P. Weisz, T. T. Wu, DESY 83-022, March 1983.
[27] J. S. Trefil, H. P. Kelly, R. T. Rood, *Nature* **302**, 111 (1983).
[28] S. N. Anderson, J. J. Lord, S. C. Strausz, R. J. Wilkes, *Phys. Rev.* **D28**, 2308 (1983).
[29] M. Born, R. J. Oppenheimer, *Ann. Phys.* (Germany) **84**, 457 (1927).
[30] S. D. Drell, N. M. Kroll, M. T. Mueller, S. J. Parke, M. A. Ruderman, *Phys. Rev. Lett.* **50**, 644 (1983).
[31] R. L. Fleischer, H. R. Hart, Jr., I. S. Jacobs, P. B. Price, W. M. Schwarz, R. T. Woods, *J. Appl. Phys.* **41**, 958 (1970).