# STUDIES OF CHIRAL SYMMETRY BREAKING IN LATTICE GAUGE THEORY*

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We study chiral symmetry breaking in SU(2) and SU(3) lattice gauge theory using the quenched approximation. For the SU(2) theory we consider quarks in the $l = 1/2$, $l = 1$, $l = 3/2$ and $l = 2$ representations and find evidence for a hierarchy of relevant energy scales. In the SU(3) theory with fundamental quarks both the deconfinement and the chiral symmetry restoration transition occurs at the same temperature. In the absence of dynamical quark loops both transitions are first order.

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## 1. Introduction

The realizations of chiral symmetries in systems involving fermions coupled to gauge fields remains a topic of great importance in particle physics [1]. Even before the advent of non-Abelian gauge theories it was suggested that any theory of the strong interactions should exhibit approximate chiral symmetries. Furthermore these symmetries must be broken spontaneously giving rise to massive baryons and massless pions. It is now generally believed that QCD as the theory of hadronic matter does indeed break chiral symmetry dynamically. Evidence for this comes from studies of instanton physics [2], anomaly constraints [3], large N analysis [4] and most recently the hadron spectrum calculations in lattice gauge theory [5]. In spite of these developments our understanding of the basic mechanism responsible for chiral symmetry breaking ($\chi$SB) is still very limited. The need for a deeper understanding becomes urgent when one tries to go beyond the standard $SU(3)_c \otimes SU(2) \otimes U(1)$ model to more general and more ambitious models that attempt to derive the quark and lepton masses from the theory rather than regard them as input parameters. Many of these dynamical theories of fermion mass matrices require that chiral symmetry be realized in ways different from in QCD. Whether some type of non-Abelian

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gauge theory can account for the proper dynamics and whether we will be able to develop methods powerful enough to investigate complicated dynamical issues is still an important open question.

In today's talk I would like to present a progress report on studies of $\chi$SB in situations that go beyond conventional QCD. Using lattice techniques we are investigating $\chi$SB in theories with fermions in higher representations of the gauge group and also $\chi$SB at finite temperatures [6]. Most of the models that we discuss (except for QCD at finite temperatures) are not meant to represent realistic theories of elementary particles. Rather, we are studying them in the hope that they will provide useful information on the dynamics of $\chi$SB. We have concentrated on the following questions.

1. What is the connection between $\chi$SB and confinement?
2. Is it possible to introduce disparate length scales into a theory with a single gauge group? Can the "tumbling" mechanism [7] be made to work?
3. What is the nature of the chiral symmetry restoration transition at finite temperatures?

Before describing our methods for investigating the questions let us make a few comments. Question 1 asks whether confining forces are necessary in order to trigger $\chi$SB. According to the intuitive picture one now has of $\chi$SB in QCD, the vacuum is believed to be unstable with respect to the formation of a fermion-antifermion pair condensate [8]. Once such a condensate has formed, chirality is no longer a good quantum number and chirally noninvariant quantities such as $\bar{\psi}\psi$ can develop vacuum expectation values, $\langle\bar{\psi}\psi\rangle \neq 0$. This picture of $\chi$SB tells us that at the very least attractive binding forces must exist. Although these forces may have to be fairly strong in order for pairs to condense, it is not at all clear whether confining forces are required (by "confining forces" we mean forces that prohibit the appearance of asymptotic states with quark quantum numbers). We have tried to settle this issue by considering $\chi$SB in theories in which fermions do not experience confining forces at large distance. For instance we have looked at quarks in $l$ = integer representations of SU(2). The color charge of these quarks can be screened by the gauge degrees of freedom. A fermion need not pair up with an antifermion to bind into a color singlet state. It can also form fermion-gluon color singlet fermionic states. We find however that $\chi$SB still occurs in these theories. This leads to the conclusion that $\chi$SB can take place even in the absence of long distance confining forces.

Question 2 looks for evidence that the tumbling picture of Dimopoulos, Raby and Susskind [7] is a viable scenario. These authors have suggested that one can introduce a hierarchy of length scales into a non-Abelian gauge theory by allowing different types of condensates to form sequentially. Of course, this idea assumes that confinement and $\chi$SB are independent phenomena, since otherwise there should be only one scale, the confinement scale (the equivalent of $\Lambda_{QCD}$), that characterizes the theory. Suppose $\chi$SB occurs when the "effective coupling"

$$e_{eff}^2 \equiv g^2 C_2(l) \tag{1.1}$$

takes on a critical value $e_c^2$, where $C_2(l)$ is the quadratic Casimir for representation $l$. The coupling $e_{eff}^2$ appears in the attractive Coulomb potential between a quark and an antiquark in the single gluon exchange approximation (or more generally the ladder approximation). It also is the combination that determines the stability of the chirally symmetric vacuum in the effective potential formalism [9].

In a renormalizable field theory the $g^2$ in Eq. (1.1) should be replaced by the running coupling constant $g^2(\mu)$. So if one compares two different representations $l_1$ and $l_2$ with $C_2(l_1) < C_2(l_2)$, then the two types of pairs should condense at different scales $\mu_1$ and $\mu_2$. Namely one must have,

$$g^2(\mu_1)C_2(l_1) = g^2(\mu_2)C_2(l_2) = e_c^2. \tag{1.2}$$

If Eq. (1.2) can be satisfied with $g^2(\mu_{1,2})$ sufficiently small, one can use the one loop expression for the renormalization group $\beta$-function to obtain ($\beta = -b_0 g^3 \ldots$),

$$\frac{\mu_2}{\mu_1} = \exp\left[\frac{1}{2b_0 e_c^2}(C_2(l_2)-C_2(l_1))\right]. \tag{1.3}$$

One sees that relatively small changes in the Casimir lead to huge ratios in energy scales. We have used our studies of SU(2) gauge theory with higher representation quarks to show that indeed several scales emerge from the dynamics and also to estimate the value for $e_c^2$. We find

$$e_c^2 \approx 4. \tag{1.4}$$

This value is smaller than previous estimates of $e_c^2$ [1–9]. So it may be easier to find representations that satisfy Eq. (1.2), while on the one hand keeping $g^2$ sufficiently small so that Eq. (1.3) is applicable and at the same time remaining asymptotically free [10].

The last question we have asked, question 3, concerns the fate of $\chi$SB at finite temperatures. In analogy with spontaneous symmetry breaking in condensed matter systems, one expects all symmetries to eventually become restored as the physical temperature is raised. This should also be true of $\chi$SB in QCD. We have studied $\chi$SB for SU(3) gauge theory with quarks in the fundamental representation. We estimate the chiral symmetry restoration temperature $T_{\chi\mathrm{SB}}$ in QCD to be

$$T_{\chi\mathrm{SB}} \approx 200\ \mathrm{MeV}. \tag{1.5}$$

We have been interested in $\chi$SB at finite temperature also for purely technical reasons. Even to investigate questions 1 and 2 we have found it convenient to go to finite temperature. The chiral symmetry restoration temperature $T_l$ for different representations $l$ provides a precise and very physical definition of what is meant by the "relevant scale of $\chi$SB".

## 2. *The method*

In order to investigate $\chi$SB one must employ nonperturbative methods for analyzing non-Abelian gauge theories. We have taken the lattice approach and have carried out Monte Carlo calculations of the order parameter $\langle\bar{\psi}\psi\rangle$. We work on a four dimensional hypercubic Euclidean lattice with sites labelled by a set of integers $n \equiv (n_0, n_1, n_2, n_3)$. The effect of nonzero temperature can be incorporated by using asymmetric lattices, lattice size $= N_t N_s^3$ with $N_t \ll N_s$. The connection between $N_t$, the number of sites in the timelike direction, and the physical temperature $T$ is given by

$$T = \frac{1}{aN_t}, \tag{2.1}$$

where $a$ is the lattice spacing.

The gauge degrees of freedom are unitary matrices $U_\mu(n)$ that reside on the links between neighboring sites $n, n+\mu$. The $U$-matrices can be viewed as the lattice analogues of the path ordered non-Abelian phase

$$\mathrm{P}\cdot\exp\,[ig \int_n^{n+\mu} A_\mu dl] \to e^{iagA_\mu} \sim U_\mu(n). \tag{2.2}$$

The matter degrees of freedom live on lattice sites. One defines, for instance, four component spinors $\psi(n)$ and $\bar\psi(n)$ at each site $n$. The lattice action splits up into two parts,

$$S = S_G + S_F. \tag{2.3}$$

The pure gauge part $S_G$ is the Wilson action

$$S_G = \frac{1}{g^2}\sum_p \mathrm{tr}\,\{2-[U_p+U_p^\dagger]\}. \tag{2.4}$$

Where $\sum_p$ is a sum over unoriented plaquettes (squares) and $U_p$ is the product of the four $U$'s on links bordering the plaquette. For the fermionic action $S_F$ several different candidates exist, none of them completely satisfactory. The most straight forward discretization of the continuum Dirac operator leads to the so called "naive" lattice fermion action.

$$S_F^{\text{naive}} = \sum_n \bar\psi(n)\,\{\tfrac{1}{2}\sum_\mu \gamma_\mu(U_\mu(n)\psi(n+\mu) - U_\mu^\dagger(n-\mu)\psi(n-\mu)) + m\psi_{(n)}\}. \tag{2.5}$$

The action (2.5) describes 16 species (flavors) of Dirac fermions. One can reduce the number of species from 16 to 4 by working with single component fermionic variables $\varphi(n)$ and $\bar\varphi(n)$ at each site. This leads to the Kogut-Susskind or staggered fermion method

$$S_F = \sum \bar\varphi(n)\,\{\tfrac{1}{2}\sum \eta_\mu(n)\,(U_\mu(n)\varphi(n+\mu) - U_\mu^\dagger(n-\mu)\varphi(n-\mu)) + m\varphi(n)\}, \tag{2.6}$$

with

$$\eta_0(n) = 1, \quad \eta_1(n) = (-1)^{n_0}, \quad \eta_2(n) = (-1)^{n_0+n_1}, \quad \eta_3(n) = (-1)^{n_0+n_1+n_2}. \tag{2.7}$$

The connection between the four component $\psi$'s and the $\varphi$'s is given by (let $\xi$ be a constant spinor normalized to one)

$$\begin{aligned} \bar\psi(n) &= \gamma_0^{n_0}\gamma_1^{n_1}\gamma_2^{n_2}\gamma_3^{n_3}\xi\varphi(n) \equiv T(n)\xi\varphi(n), \\ \bar\psi(n) &= \bar\varphi(n)\xi^\dagger T^\dagger(n). \end{aligned} \tag{2.8}$$

Eq. (2.6) is the fermionic action we have used for all our numerical simulations. Since it describes 4 flavors one expects in the continuum limit and for $m = 0$ to have a theory with an SU(4)⊗SU(4)⊗U(1) global flavor symmetry, which is then spontaneously broken down to $[SU(4)]_{vector}\otimes U(1)$ if $\langle\bar{\psi}\psi\rangle \neq 0$. For finite lattice spacing the action (2.6) has only a subset of the flavor symmetry. This subset includes one continuous flavor nonsinglet axial symmetry and one continuous vectorlike symmetry (fermion number). In addition there are 15 discrete axial and 15 discrete vectorlike symmetries. These remnants of the full flavor symmetry ensure that no fermion bilinear counter terms are produced as one approaches the continuum limit. So $\langle\bar{\varphi}(n)\varphi(n)\rangle \neq 0$ (which one can show implies $\langle\bar{\psi}\psi\rangle \neq 0$) is a true signal for χSB.)

To evaluate $\langle\bar{\varphi}\varphi\rangle$ one uses the formula,

$$\langle\bar{\varphi}\varphi\rangle = \lim_{m\to 0}\frac{1}{\mathscr{Z}}\int dU \det(G^{-1})\,[\mathrm{tr}\, G(n=0, m, U)]e^{-S_G},$$

$$\mathscr{Z} = \int dU \det(G^{-1})e^{-S_G}, \tag{2.9}$$

where $G(n, m, U)$ is the fermion propagator in a background gauge field configuration $\{U\}$. The quenched approximation which we will be using throughout the present work consists of setting the determinant $\det(G^{-1}) = 1$. There is then no feedback from the fermions onto the gauge field dynamics. In calculations of $\langle\bar{\varphi}\varphi\rangle$ one first performs a pure gauge Monte Carlo to set up typical gauge field configurations at fixed values of the coupling constant. For each gauge field configuration one inverts the fermionic action to obtain the propagator $G(n, m, U)$ and in particular $G(n = 0, m, U)$ for several masses $m$. One must then average over as many different gauge field configurations as possible and then try to extrapolate to the limit $m \to 0$.

Let us now go on to the numerical data and explain how one reaches the conclusions discussed in the Introduction.

## 3. *Results for higher representations of SU(2)*

Fig. 1 shows the raw data for $\langle\bar{\psi}\psi\rangle_{m\neq 0}$ vs $m$ for the gauge group SU(2) and $l = 1$ adjoint quarks on lattices with $N_t = 4$. The error bars reflect statistical errors only resulting from an averaging over 48~60 inversions of the lattice Dirac operator for fixed $\beta \equiv 4/g^2$ and $m$. Although we are interested in the limit $m \to 0$ one sees that there is considerable freedom in how this extrapolation is performed. On the other hand it is well known that one cannot have spontaneous symmetry breaking in a finite system. So as long as $N_s$, the spatial lattice size, is finite $\langle\bar{\psi}\psi\rangle_{m\neq 0}$ will eventually vanish as $m$ decreases below a certain $m_{min}$. If one is trying to extract infinite volume physics one must be careful not to rely on data for $m < m_{min}$ in making the $m \to 0$ extrapolations. One can obtain a rough estimate of $m_{min}$ by using the relationship between $\langle\bar{\psi}\psi\rangle$ and the density of eigenvalues $\varrho(\lambda)$ of the Dirac operator near $\lambda \approx 0$

$$\langle\bar{\psi}\psi\rangle = \frac{\pi}{V}\,\varrho(0), \tag{3.1}$$

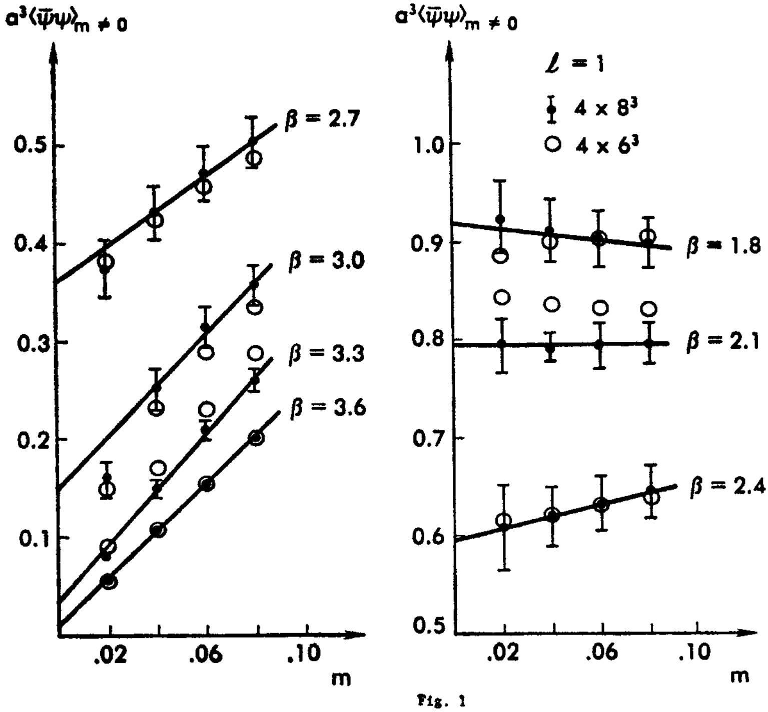


Fig. 1. The raw data $\langle\bar\psi\psi\rangle_{m\neq0}$ vs $m$ on an $N_t = 4$ lattice for $l = 1$ quarks

where $V$ is the number of lattice sites. In order for a finite system to mimic infinite volume physics one should ensure that

$$m > \text{mean separation between eigenvalues near } \lambda \approx 0 \tag{3.2}$$

or

$$m_{\min} > \frac{1}{\varrho(0)} = \frac{\pi}{V}\frac{1}{\langle\bar\psi\psi\rangle}. \tag{3.3}$$

Keeping Eq. (3.3) in mind, we have extrapolated to $m \to 0$ along the full lines drawn through the data in Fig. 1. The extrapolated values are then plotted vs $\beta$ in Fig. 2. From Fig. 2 one estimates a critical value $\beta_c$ at which $\langle\bar\psi\psi\rangle$ vanishes for $N_t = 4$. Similar plots are shown in Fig. 3 for $N_t = 2$ and $N_t = 6$. One reads off,

$$\beta_c = \begin{cases} 2.80 \pm 0.20, & N_t = 2, \\ 3.15 \pm 0.15, & N_t = 4, \\ 3.30 \pm 0.20, & N_t = 6. \end{cases} \tag{3.4}$$

In the next step we must convert the results of Eq. (3.4) into a physical quantity, the chiral symmetry restoration temperature $T_A$ for adjoint quarks. In the weak coupling ($\beta \to \infty$)

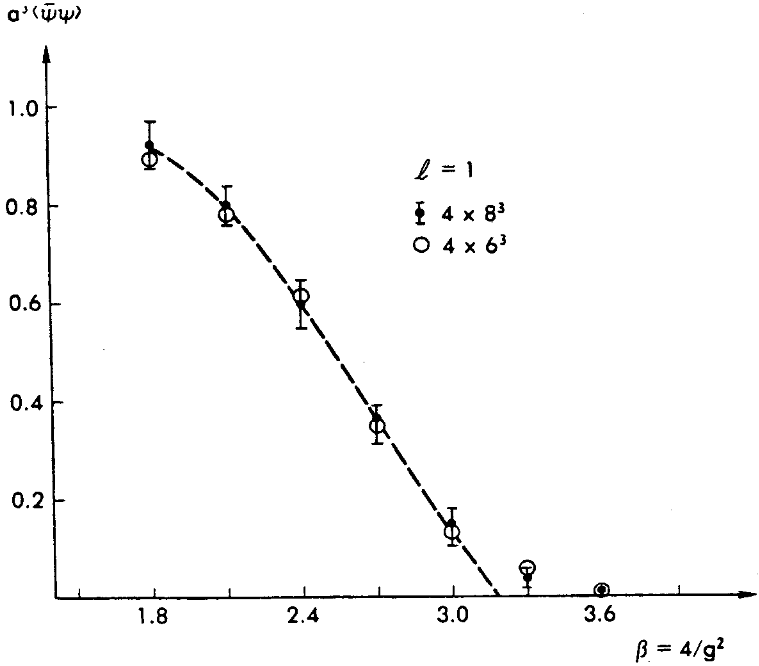


Fig. 2. $\langle\overline{\psi}\psi\rangle$ vs $\beta$ for $l = 1$ quarks on an $N_t = 4$ lattice

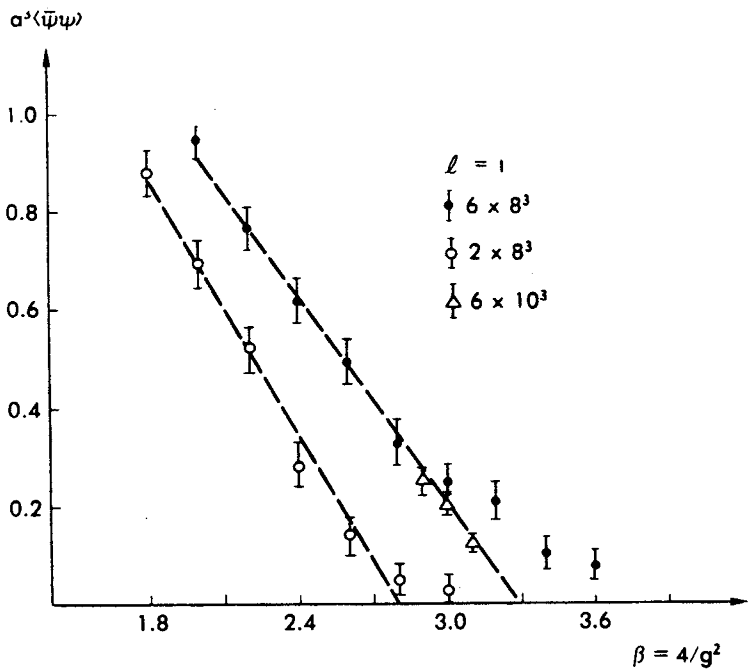


Fig. 3. Same as Fig. 2 for $N_t = 2$ and $N_t = 6$

region $T_A$, being a renormalization group invariant, must depend on the coupling and the lattice spacing according to,

$$aT_A \propto \beta^{\frac{51}{121}} e^{\frac{-3\pi^2\beta}{11}} . \tag{3.5}$$

In Fig. 4 we plot $aT_A$ and $aT_F$, the corresponding critical temperature for $l = 1/2$ fundamental quarks. The data for $N_t = 4$ and $N_t = 6$ are consistent with scaling (Eq. (3.5)). In addition the data already shows that $\chi$SB occurs in $l =$ integer (screenable) representations and also that different critical temperatures characterize different representations.

We have also accumulated data for $l = 3/2$ and $l = 2$ quarks. All our SU(2) results are summarised in Fig. 5. We also show the deconfinement temperature $T_{dec}$ for the pure SU(2) gauge theory. From Fig. 5 one can read off various ratios of scales.

$$1.0 \leqslant T_F/T_{dec} \leqslant 1.3, \tag{3.6}$$

$$T_A/T_F \equiv T_{l=1}/T_{l=1/2} \approx 8.6 \pm 4.5, \tag{3.7}$$

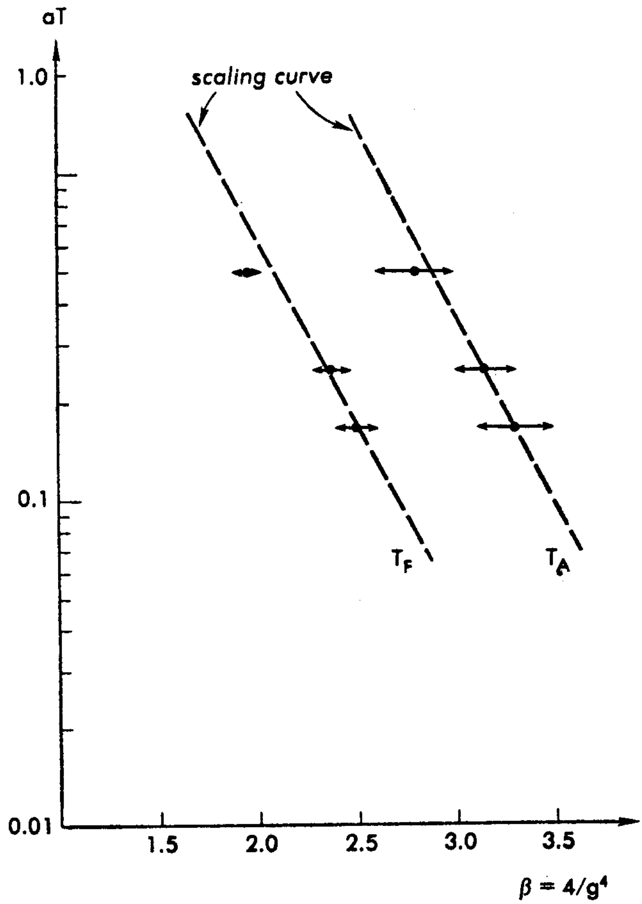


Fig. 4. The chiral symmetry restoration temperature for $l = 1$ adjoint ($T_A$) and $l = 1/2$ fundamental ($T_f$) quarks

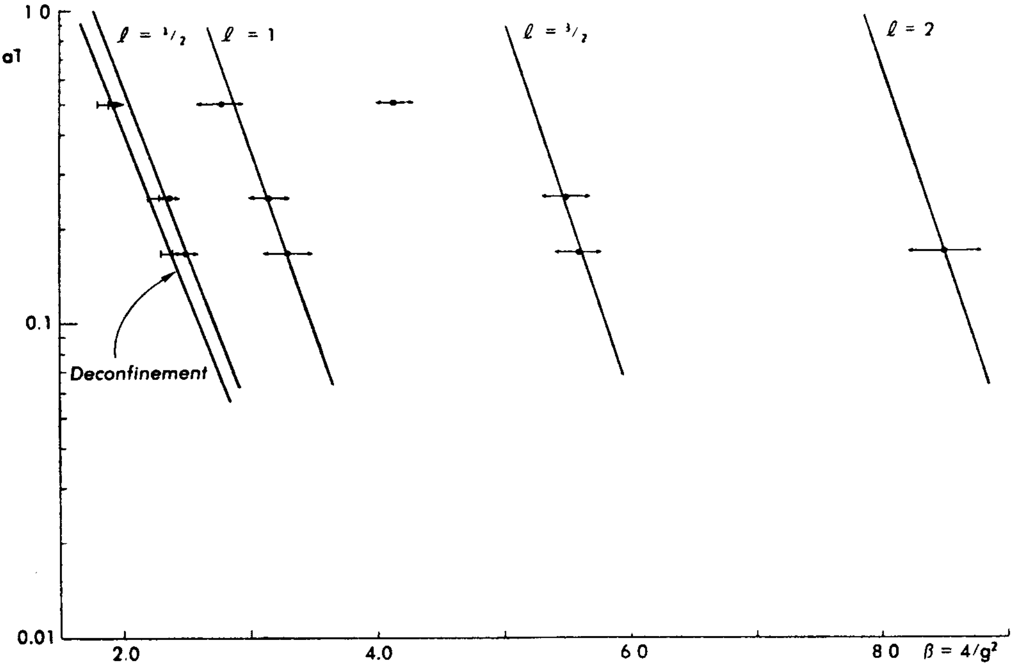


Fig. 5. The deconfinement and chiral symmetry restoration temperatures in SU(2) lattice gauge theory

$$T_{l=3/2}/T_{l=1} \approx 10^{2.7\pm0.3}, \tag{3.8}$$

$$T_{l=2}/T_{l=3/2} \approx 10^{3.4\pm0.4}. \tag{3.9}$$

One sees that condensates for the higher representations $l = 3/2$ and $l = 2$ introduce a large hierarchy in scales. It is interesting to use (3.8) and (3.9) to estimate the critical coupling $e_c^2$ of Eq. (1.2). We restrict ourselves to the higher representations since only they involve $g_c^2$'s that are sufficiently small so as to make the Casimir scaling hypothesis applicable. According to (1.2) one has,

$$e_c^2 = g_c^2 C_2(l). \tag{3.10}$$

As $g_c$ we will take the running coupling constant at the scale given by $T_l$. In terms of the "momentum space" definition of the coupling constant,

$$g_c^2 \to g_{\text{mom}}^2(T_l) = \frac{1}{2b_0 \ln T_l/\Lambda_{\text{mom}}} = \frac{1}{2b_0[\ln T_l/\Lambda_{\text{L}} - \ln \Lambda_{\text{mom}}/\Lambda_{\text{L}}]}. \tag{3.11}$$

$T_l/\Lambda_{\text{L}}$ can be read off from Fig. 5 and from Ref. [11] one has $\Lambda_{\text{mom}}/\Lambda_{\text{L}} = 57.5$ for SU(2), so, one finds,

$$e_c^2 = \begin{cases} 4.3 \pm 0.2, & l = \frac{3}{2}; \\ 3.8 \pm 0.2, & l = 2. \end{cases} \tag{3.12}$$

The two values for $l = 3/2$ and $l = 2$ are close enough to each other, so that our results are consistent with Casimir scaling with $e_c^2 \approx 4$.

### 4. *QCD at finite temperature*

Up to this point we have been using finite temperature mainly as a technical device for getting at the relevant length scales in $\chi$SB. On the other hand studies of QCD and the electroweak interactions at finite temperature has recently been attracting more and more attention [12]. The main interest has been in understanding physics in the hot early universe soon after the initial Big Bang. One now believes that the universe underwent several phase transitions as it cooled down. It is important to estimate consequences of these transitions and look for consistency checks between our current theories of elementary particle interactions, cosmology and present day astrophysical observations. In addition there exists the exciting possibility that unusual phases of matter can be created and studied experimentally in heavy ion collisions.

There are two phase transitions that are associated with QCD. One is the chiral symmetry restoration transition that we have been discussing a lot already. It is determined by the order parameter $\langle\bar{\psi}\psi\rangle$.

$$\langle\bar{\psi}\psi\rangle \begin{cases} \neq 0 & T < T_{\chi SB}, \\ = 0 & T > T_{\chi SB}. \end{cases} \tag{4.1}$$

The other transition, which is usually discussed only for the pure gauge theory is the confinement-deconfinement transition. For the quarkless theory this transition is characterized by the Wilson line order parameter

$$W(\vec{x}) \equiv \mathrm{tr}\, \mathrm{P} \exp\left[\int_0^{1/T} dt A_0(t, \vec{x})\right]. \tag{4.2}$$

The expectation $\langle W\rangle$ determines the free energy $F_q$ of a single static quark.

$$\langle W\rangle = e^{-F_q/T} = \begin{cases} 0 & T < T_{\mathrm{dec.}} \text{ (confinement)} \\ \text{finite} & T > T_{\mathrm{dec.}} \text{ (no confinement).} \end{cases} \tag{4.3}$$

Initial investigations of $T_{\mathrm{dec}}$ using lattice methods came from strong coupling analysis [13]. Subsequent numerical work also found evidence for a phase transition in the scaling region [14]. We have calculated both $\langle\bar{\psi}\psi\rangle$ and $\langle W\rangle$ for SU(3) gauge theory with fundamental quarks in the quenched approximation. Results are shown in Fig. 6 for $N_t = 2$ and $N_t = 4$. The first fact one notices is that the two transitions, deconfinement and chiral symmetry restoration, occur at the same time. This behaviour is different from what we found for higher representations of SU(2). Presumably the quadratic Casimir for fundamental quarks is too small, so that when confining forces disappear the remaining nonconfining forces alone (characteiized by $g^2(T_{\mathrm{dec}})C_2$) are not strong enough to support $\chi$SB. The next point to notice is that the change in the order parameters is very sharp. We take this as evidence for the first order nature of the transition. From the data of Fig. 6 one can deduce

$$T_{\mathrm{dec}} \approx T_{\chi SB} \approx (0.46 \pm 0.10)\sqrt{\sigma}, \tag{4.4}$$

where $\sigma$ is the string tension. If $\sqrt{\sigma} \approx 450$ MeV as usually assumed, then one obtains the value $T_c \approx 200$ MeV quoted in the Introduction.

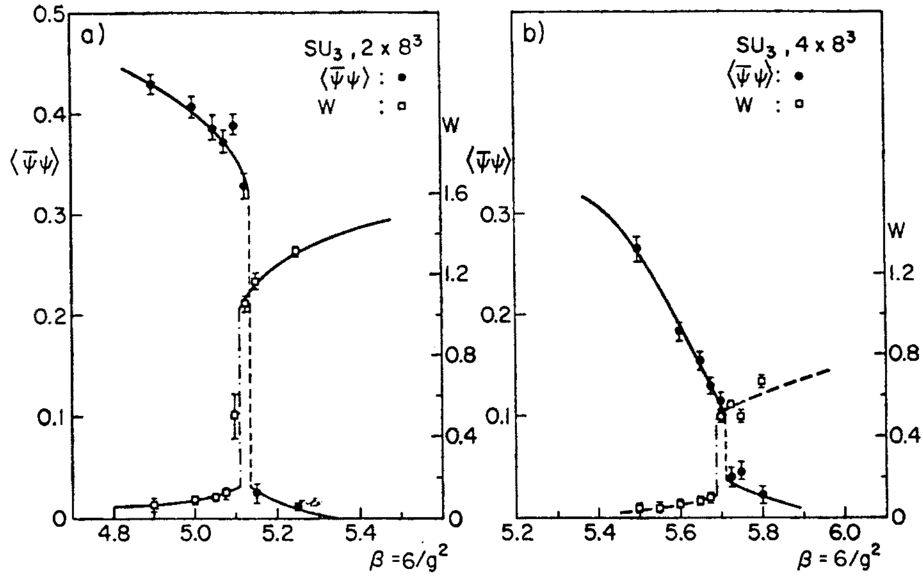


Fig. 6. The order parameters $\langle\bar{\psi}\psi\rangle$ and $\langle W\rangle$ in SU(3) lattice gauge theory with fundamental quarks

Most of the lattice work to date on QCD at finite temperature (including the work presented here) has been carried out in the quenched approximation, in which dynamical quark loops are ignored. So important physics could be missing in the numerical simulations. For instance once dynamical quarks are included the Wilson line is no longer a good order parameter since external static charges can now be screened by dynamical quarks. There is no longer an absolute need for a phase transition. Whether the deconfinement transition will survive as a bona fide phase transition depends crucially on factors such as the order of the transition in the quarkless theory, on the mass of the quarks and on the number of flavors $N_f$. Since quarkless SU(3) theory exhibits a first order transition it was believed that dynamical quarks will not wash away the deconfinement transition as long as the mass is not made too small and $N_f$ is not too large. However a recent calculation by Hasenfratz, Karsch and Stamatescu [15] indicates that the inclusion of relatively massive quarks already destroys the transition. They have incorporated quark loops in their calculations using a lowest order hopping parameter expansion and argue that the deconfinement transition disappears for $m$ smaller than $3.5T_c \sim 4.3T_c$ ($\sim$ order of a GeV). Undoubtedly much effort will be spent in the coming months (years) to obtain a more precise picture of the deconfinement "transition" in a realistic simulation of QCD with quarks.

Dynamical quarks are not expected to affect the chiral symmetry restoring transition quite so dramatically. Here $\langle\bar{\psi}\psi\rangle$ still remains a good order parameter in the presence of quark loops, so the phase transition should not disappear. However both the order of the transition and also the value of $T_{\chi SB}$ could differ from in the quarkless case. My collaborators and I are working hard towards setting up practical schemes for including dynamical quarks so that better estimates of the critical temperatures can be made.

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