FORMATION OF GALAXIES IN *G*-VARIABLE COSMOLOGIES: II. THE HOYLE-NARLIKAR AND THE BRANS-DICKE GRAVITATIONAL THEORIES

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The growth of small linear gravitational instabilities in the Hoyle–Narlikar and the Brans–Dicke theories of gravitation is discussed. It is shown that small linear adiabatic density fluctuations can grow faster (or protogalaxies could form earlier) in the *G*-variable cosmologies than in the general theory of relativity.

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1. Introduction

Canuto and Goldman [1] have shown that the variability of the gravitational parameter G reflects a very serious problem, namely the question whether the Strong Equivalence Principle is valid. Up to now no one can answer this question, since we do not know if the rate of gravitational (two orbiting planets) and atomic (e.g. hydrogen atom) clocks has remained the same for the whole history of the Universe. Therefore, one may and should consider the G-variable theories of gravitation.

The evolution of linear density fluctuations in the Dirac cosmologies has been discussed by Krempeć-Krygier [2, Paper I] and Davidson [3]. It has been shown that the density fluctuations grow faster (or the protogalaxies become gravitationally bound systems earlier) in the Dirac additive creation theory than in the standard general relativity theory. The fate of the density fluctuations in the Dirac multiplicative creation theory depends upon a free parameter $\gamma^\sigma = G_0\varrho_0/H_0^2$. Therefore, for the acceptable present values of the gravitational parameter G_0, of the density ϱ_0 and of the Hubble parameter H_0, the density perturbations seem to grow slower in the multiplicative creation version than in the additive one.

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In the present paper we shall discuss the evolution of the linear adiabatic density fluctuations in the Hoyle–Narlikar and the Brans-Dicke cosmologies. We shall restrict our considerations to the Newtonian approximation and flat cosmological models only. A brief description of the Hoyle–Narlikar theory of gravitation and the discussion of the fate of the density fluctuations in this theory will be given in Section 2. Section 3 deals with the evolution of linear density perturbations in the Brans–Dicke theory of gravitation. The concluding remarks are given in Section 4.

2. The Hoyle–Narlikar theory of gravitation

Generally, the field equations of the general relativity equate the tensors describing the geometrical properties of the space-time to the physical quantities known as the energy-momentum tensor. In 1964, Hoyle and Narlikar [4] formulated the theory of gravitation, in which they have introduced into the energy-momentum tensor the C-field — the scalar function of position. The C-field can be considered either as a pure field or as a field coupled to the particles.

The Hoyle–Narlikar theory of gravitation is based upon the Mach principle which states that the properties of local matter (e.g. its mass, inertia etc.) are determined by the properties of the rest of the Universe. Hence, for example the mass-energy of a particle $m(x)$ is given by the mass-energy of the rest matter of the Universe $M(x)$, i.e. $m(x) = \lambda M(x)$ (where λ is a coupling constant of a given particle and it can be $\gtrless 0$). The field equations calculated from the action principle [5–7] have the general form, as follows:

$$R_{ik}-\tfrac{1}{2}\, g_{ik}R = -8\pi G[(T_{ik}^{(m)}+J_{ik})+T_{ik}^{(rad)}].$$

Here T_{ik} is the usual energy-momentum tensor, J_{ik} stands for the inertial terms in the energy-momentum tensor, R_{ik} is the Ricci tensor, g_{ik} — the metric tensor, R — the scale factor or the curvature scalar, G — the gravitational parameter, (m) and (rad) refer to matter and radiation respectively.

We shall limit our considerations to matter-dominated epochs of the homogeneous and isotropic cosmological models of the Universe (dust-filled models with $p_m = 0$). Hence, the conservation equation of the energy-momentum is:

$$[G(T_{(m)}^{ik}+J^{ik})]_{;} = 0.$$

The semicolon stands for the covariant derivative.

Solutions of the field equations and of the conservation laws given in the atomic units for flat dust-filled cosmological models of the Universe are (see [7]):

$$R(t) = (2H_0 t)^{1/2}, \qquad \varrho_{(m)} \propto (R^3 G^{1/2})^{-1}.$$

Here t is the atomic time, H is the Hubble parameter and subscripts "0" refer to the present values.

Therefore, the line element in the atomic coordinate frame for the flat dust-filled cosmological models of the Universe is as follows:

$$ds^2 = c^2dt^2 - 2H_0 t[dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\varphi^2)].$$

c is the light velocity and r, θ, φ are the spherical coordinates. Since, in the Hoyle–Narlikar theory of gravitation the gravitational parameter changes with the atomic time in the way $G = \gamma/t$, one has:

$$\varrho_{\rm m} = \frac{\sigma}{t}, \quad H = \frac{\dot{R}}{R} = \frac{1}{2t}, \quad \frac{\dot{\varrho}_{(m)}}{\varrho_{(m)}} = -\frac{1}{t} = -2H,$$

$$(1+z) = \frac{R_0}{R} = \left(\frac{t_0}{t}\right)^{1/2}, \quad G \propto (1+z)^2. \tag{1}$$

Here γ and σ are constants.

We shall consider the expanding homogeneous and isotropic medium (Universe) described by the above relations as an unperturbed state. Then, we impose the radial adiabatic density perturbations. The fate of the density fluctuations is described by the hydrodynamic equations, which have the general form, as follows:

— the continuity equation

$$\frac{\partial \varrho_t}{\partial t} + \nabla(\varrho_t \vec{V}_t) = 0; \tag{2}$$

— equations of motion, which in the case of no dissipation are the Euler equations

$$\frac{\partial \vec{V}_t}{\partial t} + (\vec{V}_t \nabla)\vec{V}_t = -\frac{1}{\varrho_t}\nabla p_t + \vec{g}_t; \tag{3}$$

— the equations of the gravitational field

$$\nabla \vec{g}_t = -4\pi G \varrho_t,$$

$$\nabla \times \vec{g}_t = 0. \tag{4}$$

Here ϱ_t, V_t, p_t are the total values of the mass-energy density, of velocity and of pressure, ∇ is the Nabel operator, x stands for the vector multiplication and $\vec{g}_t = -4\pi G \varrho_t \frac{\vec{r}}{3}$ for the Newtonian approximation considered in this paper. Our goal is to investigate the properties of the unperturbed expanding medium and the radial linear and adiabatic perturbations (ϱ_1, V_1, p_1, g_1):

$$\varrho_t(r, t) = \varrho_{\rm m}(t) + \varrho_1(r, t),$$

$$\vec{V}_t(r, t) = \vec{r}H(t) + \vec{V}_1(r, t),$$

$$p_t(r, t) = p_{\rm m}(t) + p_1(r, t),$$

$$\vec{g}_t(r, t) = \vec{g}(t) + \vec{g}_1(r, t). \tag{5}$$

Since we have assumed the radial motion and the spherical symmetry we take the Hubble motion, i.e. $\vec{V} = \vec{r}H$ as unperturbed motion in the above equations. In fact, $p_{\rm m}(t) = 0$

and $p_1(r,t) = V_s^2 \varrho_1(r,t)$ for the considered adiabatic density fluctuations in the dust-filled models (where V_s is the sound velocity).

Substituting (5) into (2–4) and neglecting all the second order terms we have in spherical coordinates:

$$\dot{\varrho}_1 + \dot{\varrho}_m + \vec{r} H \varrho_1' + 3H\varrho_m + \varrho_m V_1' + \frac{2V_1 \varrho_m}{r} + 3H\varrho_1 = 0, \tag{6}$$

$$\vec{r}\dot{H} + \dot{\vec{V}}_1 + \vec{r}H^2 + \vec{r}H\vec{V}_1' + H\vec{V}_1 = \vec{g}_t - V_s^2 \left(\frac{\varrho_1'}{\varrho_m}\right), \tag{7}$$

Here: $\dot{} = \partial/\partial t$ and $' = \partial/\partial r$.

We have assumed that the Hubble parameter satisfies the following condition:

$$3H^2 + 3\dot{H} = -4\pi G \varrho_m. \tag{8}$$

Hence, we have $\gamma\sigma = 3/(16\pi)$. Putting (8) and $\dot{\varrho}_m = -2H\varrho_m$ into (6), we have

$$\varrho_m V_1' + \frac{2\varrho_m V_1}{r} = -\dot{\varrho}_1 - H\varrho_m - \vec{r}H\varrho_1' - 3H\varrho_1. \tag{9}$$

Since $\frac{\partial}{\partial r}(r^2 V_1) = r^2\left(V_1' + \frac{2V_1}{r}\right)$, (9) takes a form:

$$\frac{\partial}{\partial r}(r^2 V_1) = -\frac{r^2}{\varrho_m}(\dot{\varrho}_1 + \vec{r}H\varrho_1' + 3H\varrho_1 + H\varrho_m). \tag{10}$$

Supplying ∇ to (7) and using (8) we have

$$\frac{1}{r^2}\frac{\partial^2}{\partial r \partial t}(r^2 V_1) + \frac{H}{r}\frac{\partial^2}{\partial r^2}(r^2 V_1) = -4\pi G \varrho_1 - \frac{V_s^2}{\varrho_m}\left(\varrho_1'' + \frac{2\varrho_1'}{r}\right). \tag{11}$$

Here $'' = \partial^2/\partial r^2$. Putting (10) into (11) one derives the following equation for the evolution of the density fluctuations:

$$\ddot{\varrho}_1 + 7H\dot{\varrho}_1 + r^2 H^2 \varrho_1'' + \varrho_1'(r\dot{H} + 8rH^2) + \varrho_1(6\dot{H} + 15H^2) + 2rH\dot{\varrho}_1'$$
$$= V_s^2\left(\varrho_1'' + \frac{2\varrho_1'}{r}\right). \tag{12}$$

Here: $\ddot{} = \partial^2/\partial t^2$. Introducing the relative density fluctuations defined as $\delta(r,t) = \varrho_1(r,t)/\varrho_m(t)$ and putting $\dot{\varrho}_m$ and $\ddot{\varrho}_m$ as given by (1) we have calculated for the evolution of the density perturbations the following equation:

$$\ddot{\delta} + 3H\dot{\delta} + \delta(4\dot{H} + 5H^2) + \delta''(r^2H^2 - V_s^2) + \delta'\left(r\dot{H} + 4rH^2 - \frac{2V_s^2}{r}\right) + 2rH\dot{\delta}' = 0. \tag{13}$$

Next, we separate the dependence upon the spatial (radial) coordinate and the time according to the formula $\delta(r, t) = \omega(t)\,[\exp(ikr/R(t))]/r$, which represents the spherical wave. In this way, we derive the following equation for the evolution of the density fluctuations (for the compressional modes) or to potential velocity perturbations):

$$\ddot{\omega}+H\dot{\omega}+\omega\left(3\dot{H}+3H^2+\frac{k^2V_s^2}{R^2}\right) = 0. \tag{14}$$

Here $(k/R(t))$ represents the wave number. Since we assume that the galaxies are formed from the small density perturbations, which have survived up to the recombination from the previous phase of the acoustic damped oscillations, we can neglect the pressure term $(V_s^2k^2/R^2)$, as small in comparison with the gravitational one, namely with $(3\dot{H}+3H^2 = -4\pi G\varrho_m)$ for the epoch after the recombination [8]. It corresponds to the case, when the wave number (k/R) is much smaller than the Jeans number (or the mass of the density fluctuations is much larger than the Jeans mass). Then, after substitution of $H = 1/(2t)$ and $\dot{H} = -1/(2t^2)$ into (14), the evolution of the density fluctuations is described by the equation

$$\ddot{\omega}+\frac{1}{2t}\dot{\omega}-\frac{3}{4t^2}\omega = 0. \tag{15}$$

In this case, the growth of the density fluctuations with the atomic time is given by the power law, i.e. $\omega_\pm \propto t^{n_{1,2}}$ (where $n_1 = 1.15$ and $n_2 = -0.65$).

However, we have also considered the case, when one has to take the pressure term into account. Then, the evolution of the density fluctuation is described by the following equation:

$$\ddot{\omega}+\frac{1}{2t}\dot{\omega}+\omega\left(\frac{k^2V_s^2}{R^2}-\frac{3}{4t^2}\right) = 0. \tag{16}$$

Here $R = (2H_0t)^{1/2}$ as given by (1).

The solution of this equation is expressed by the cylindrical functions Z_ν and it is: $\omega_\pm = t^{0.25}Z_{\pm 1.8028}(kV_s(2t/H_0)^{1/2})$. Generally, the cylindrical functions of a given order $\nu: Z_\nu$ are the linear combinations of the Bessel functions of the first J_ν and of the second kinds Y_ν, namely: $Z_\nu(x) = c_1J_\nu(x)+c_2Y_\nu(x)$ (where c_1 and c_2 are constants). Since we have put the adiabatic perturbations into the unperturbed medium we may take $p = K\varrho^{\gamma_1}$ (where $\gamma_1 = c_p/c_v$) and $V_s^2 = (\partial p/\partial\varrho)_{ad} = K\gamma_1\varrho^{\gamma_1-1} = K_1t^{1-\gamma_1}$ (where K_1 is a constant for a given γ_1). Introducing $A^2 = k^2V_s^2t^{\gamma_1}/R^2$ we have derived for the evolution of the density perturbations the equation

$$\ddot{\omega}+\frac{1}{2t}\dot{\omega}+\omega\left(\frac{A^2}{t^{\gamma_1}}-\frac{3}{4t^2}\right) = 0. \tag{17}$$

The solution of the above equation is given by cylindrical Bessel functions and has the form: $\omega_\pm \propto t^{-0.25}Z_\nu(At^\kappa/\kappa)$ (where $\nu = \pm(13)^{1/2}/(4\kappa)$ and $\kappa = (2-\gamma_1)/2$). For the interesting cases, i.e. $\gamma_1 = 4/3$ and $5/3$, we have $\kappa > 0$ and for $t \ll A^{-1/\kappa}$ the density fluctuations could grow ($\omega_\pm \propto t^{0.25\pm(13)^{1/2}/4}$) while for $t \gg A^{-1/\kappa}$ — there would be the damping oscillations only. The condition for the growth of the density perturbations is equivalent to the physical situation, when the influence of the gravitational forces upon the density perturbations is larger than the pressure effect, namely $A^2/t^{\gamma_1} < 3/4t^2$.

3. *The Brans-Dicke scalar-tensor theory of gravitation*

Brans and Dicke [9] have formulated the scalar-tensor theory of gravitation. In this theory, known as the Brans-Dicke theory of gravitation, besides the tensor gravitational field, there is the scalar field ϕ, whose density ϱ_λ and pressure p_λ are given as follows: $p_\lambda = \varrho_\lambda c^2 = (2\Omega+3)c^2(\dot{\lambda})^2/(32\pi G\lambda^2)$ (where $\phi = \log\lambda$) [10]. Ω is the coupling constant between the scalar and tensor fields. In the scalar-tensor theory of gravitation the gravitational parameter is determined as $G = G_0(\phi_0/\phi)$. The field equations were derived by putting $G \propto \phi^{-1}$ and inserting the term connected with the scalar field — T^ϕ_{ik} into the energy-momentum tensor. For the isotropic and homogeneous cosmological models of the ideal fluid with the energy-momentum tensor: $T_{ik} = -(\varrho+p)u_i u_k + pg_{ik}$ the field equations have a general form as follows:

$$R_{ik} - \tfrac{1}{2} g_{ik}R = -\frac{8\pi G_0}{c^4\phi}(T^m_{ik} + T^\phi_{ik}). \tag{18}$$

Here T^m_{ik} and T^ϕ_{ik} are the energy-momentum tensors of matter and of the scalar field respectively. The scalar field ϕ is determined from the wave equation:

$$\Box\phi = (-g)^{1/2}[(-g)^{1/2}\phi^{,i}]_{,i} = -\frac{8\pi G_0}{(3+2\Omega)c^4}T, \tag{19}$$

T is the trace of the energy-momentum and "," stands for partial derivatives. These two equations and the conservation equation of energy-momentum take the form (for $c = G_0 = 1$):

$$\left(\frac{\dot{R}}{R}\right)^2 + \frac{k}{R^2} = \frac{8\pi\varrho}{\phi} - \frac{\dot{\phi}\dot{R}}{R} + \frac{\Omega(\dot{\phi})^2}{6\phi^2},$$

$$\frac{d}{dt}(\phi R^3) = \frac{8\pi}{3+2\Omega}(\varrho-3p)R^3,$$

$$\dot{\varrho} = -\frac{3\dot{R}}{R}(\varrho+p). \tag{20}$$

Here dot stands for d/dt and k is the curvature constant. When Ω, k, $\dot{R}_0$, R_0, ϕ_0 and ϱ_0 are known, one can derive $R(t)$, $\phi(t)$ and $\varrho(t)$. For $\Omega \to \infty$, the scalar-tensor theory is equivalent to the general relativity. We shall discuss the flat dust-filled cosmological models ($k = 0$,

$p = 0$) only. Hence, the solutions of (20) are [9, 11, 12]:

$$R = R_0 \left(\frac{t}{t_0}\right)^{2(1+\Omega)/(4+3\Omega)}, \quad \varrho = \varrho_0 \left(\frac{t}{t_0}\right)^{-6(1+\Omega)/(4+3\Omega)},$$

$$G = G_0 \left(\frac{t}{t_0}\right)^{-2/(4+3\Omega)}, \quad H = \frac{2(1+\Omega)}{(4+3\Omega)t}, \quad (1+z) = \left(\frac{t_0}{t}\right)^{2(1+\Omega)/(4+3\Omega)}. \tag{21}$$

It is possible that at the earlier epochs of the Universe, the density of the energy-momentum of the scalar field ϱ_λ was larger than the density of mass-energy of matter and could accelerate the rate of cosmic expansion. The evolution of the density fluctuations in the Brans-Dicke theory of gravitation has already been considered by Bandyopadhyay [16] by a different method. In his discussion Bandyopadhyay used the extended Raychaudhuri equation derived by Banerji [17]. He obtained the power law for the evolution of the density fluctuations with time in the flat dust-filled cosmological models of the Universe. However, the values of this Q-parameter depend upon the values of b and A. b and A are as follows [11]: $b = \varrho_0 t_0^{6(1+\Omega)/(3\Omega+4)}$; $A = [8\pi\varrho_0/(2\Omega+3)t_0^{6(1+\Omega)/(3\Omega+4)}$. ($\varrho_0$ is the average unperturbed density at the present time t_0.) One obtains the damping density oscillations only for the mentioned values of b and A using the formula given by Bandyopadhyay.

Hence, it is worth to discuss the evolution of the density fluctuations in the Brans-Dicke theory of gravitation for the unperturbed state given by (21) using the same method as in the previous Section. We have obtained that the evolution of the density perturbations, in the case when the pressure term is small in comparison with the gravitational one, is described by:

$$\ddot{\omega} + \frac{4(1+\Omega)\dot{\omega}}{(4+3\Omega)t} - \frac{6}{t^2}\left\{\frac{(1+\Omega)(2+\Omega)}{(4+3\Omega)^2}\omega\right\} = 0. \tag{22}$$

Deriving the above equation we have assumed that the condition $(3H^2+3\dot{H} = -4\pi G\varrho)$ is satisfied. It gives us, together with (21), that:

$$G_0\varrho_0 t_0^2 = (3/2\pi)(1+\Omega)(2+\Omega)/(4+3\Omega)^2.$$

The solution of equation (22) has the following power form:

$$\omega_\pm \propto t^{n_{1,2}} \propto (1+z)^{-[(4+3\Omega)/[2(1+\Omega)]]n_{1,2}}. \tag{23}$$

Here $n_{1,2} = [-\Omega \pm [\Omega^2+24(1+\Omega)(2+\Omega)]^{1/2}]/[2(4+3\Omega)]$ and z is the redshift. In order to obtain the growing modes of the density fluctuations, i.e. $n_1, 0$, one should restrict the discussion to the values of Ω larger than -1 ($\Omega > -1$). For the example, $n_{1,2}$ take the following values:

(*i*) for $\Omega = -0.9$, $n_1 = 1.06$ and $n_2 = -0.37$;
(*ii*) for $\Omega = 1.0$, $n_1 = 0.79$ and $n_2 = -0.93$;
(*iii*) for $\Omega = 30.0$, $n_1 = 0.68$ and $n_2 = -0.996$.

On the other hand, observations of the solar gravitational deflection of the radio waves [13] as well as modern lunar laser experiments [14, 15] have given the limitation for Ω, i.e. $|\Omega| \geqslant 30$. When one takes into account the pressure term, the evolution of the density perturbations takes the form

$$\ddot{\omega}+\frac{4(1+\Omega)}{(4+3\Omega)t}\dot{\omega}+\omega\left[\frac{k^2V_s^2}{R^2}-\frac{6(1+\Omega)(2+\Omega)}{(4+3\Omega)^2t^2}\right]=0. \tag{24}$$

For the adiabatic density fluctuations with $V_s^2 = K\gamma_1\varrho^{\gamma_1-1}$, the above equation takes a form:

$$\ddot{\omega}+\frac{4(1+\Omega)}{(4+3\Omega)t}\dot{\omega}+\omega\left(\frac{A^2}{t^{2(1-\Omega)(3\gamma_1-1)/(4+3\Omega)}}-\frac{6(1+\Omega)(2+\Omega)}{(4+3\Omega)^2t^2}\right)=0, \tag{25}$$

Here $A^2 = \frac{k^2V_s^2}{R^2}t^{2(1+\Omega)(3\gamma_1-1)/(4+3\Omega)}$.

For the special case, namely for $\gamma_1 = \frac{5+4\Omega}{3(1+\Omega)}$ or $\Omega = \frac{5-3\gamma_1}{3\gamma_1-4}$, the above equation reduces to the equation:

$$\ddot{\omega}+\frac{4(1+\Omega)}{(4+3\Omega)t}\dot{\omega}+\frac{\omega}{t^2}\left[A^2-\frac{6(1+\Omega)(2+\Omega)}{(4+3\Omega)^2}\right]=0. \tag{26}$$

The solution of this equation is described by the power law, as follows: $\omega_\pm \propto t^{n_{1,2}}$ (where $n_{1,2} = -\Omega/[2(4+3\Omega)]\pm 0.5\{[\Omega^2+24(1+\Omega)(2+\Omega)]/(4+3\Omega)^2-4A^2\}^{1/2}$. Therefore, one has:

(*i*) the damping density oscillations only for $A^2 > \frac{\Omega^2+24(1+\Omega)(2+\Omega)}{4(4+3\Omega)^2}$;

(*ii*) the decreasing density fluctuations for

$$\frac{6\Omega^2+18\Omega+12}{(4+3\Omega)^2} < A^2 < \frac{\Omega^2+24(1+\Omega)(2+\Omega)}{4(4+3\Omega)^2},$$

(*iii*) the growing and decreasing modes of the density perturbations for $A^2 < \frac{6\Omega^2+18\Omega+12}{(4+3\Omega)^2}$.

The discussed special case corresponds to the following values of the coupling constant:

(*i*) for $\gamma_1 = 4/3$ (the relativistic fluid), we have $\Omega\to\infty$. It is known that for $\Omega\to\infty$ the Brans-Dicke theory of gravitation reduces to the general relativity. Hence, we have for $\Omega\to\infty$ the damping density oscillations for $A^2 > 25/36$, the decreasing density fluctuations for $2/3 < A^2 < 25/36$ and the growing as well as decreasing modes of the density perturbations for $A^2 < 2/3$ in agreement with the general relativity (see [8]).

(*ii*) for $\gamma_1 = 5/3$ (the non-relativistic fluid), we obtain $\Omega = 0$ and the initial density perturbations can grow for $A^2 < 3/4$. The general solution of equation (25) is: $\omega_\pm = t^{-\Omega/[2(4+3\Omega)]}Z_{\pm\nu}(At^\kappa/-\kappa)$ (where $\nu = [\Omega^2+24(1+\Omega)(2+\Omega)]^{1/2}/[2(4+3\Omega)]$ and $\kappa = [3\gamma_1(1+\Omega)-(5+4\Omega)]/(4+3\Omega)$. For $\kappa>0$, i.e. for $\gamma_1 > (5+4\Omega)/[3(1+\Omega)]$, the density perturbations can grow for $t \gg A^{1/\kappa}$, namely $\omega_\pm \propto t^{-\Omega/[2(4+3\Omega)]\pm(\Omega^2+24(1+\Omega)(2+\Omega))^{1/2}/[2(4+3\Omega)]}$, while for $t \ll A^{1/\kappa}$ — the $Z_{\pm\nu}$ functions oscillate.

The condition for the growth of the density fluctuations is equivalent to the Jeans condition and corresponds to the case, when the influence of the pressure upon the density fluctuations is overcome by the gravitational forces.

4. *Concluding remarks*

It is known that in general relativity the initial random density fluctuations could not grow into the protogalaxies, when one accepts the power law for their growth [18, 19]. In fact, the small density perturbations may grow during the matter era only. Therefore, the initial adiabatic density fluctuations should be of the order of $\sim (10^{-5} \div 10^{-4})$ to produce the protogalaxies at times about $\sim (10^8 \div 10^9)$ yrs [20, 21]. On the other hand, the initial thermal density fluctuations could be: $\omega_0 \sim N^{-1/6} \approx 10^{-11}$ for the evolution through the critical point (where N is the number of particles contained in the density perturbations). Hence, the formation of protogalaxies or protoclusters of galaxies is still an open question. Nevertheless, one tries to avoid this difficulty assuming that the non-linear effects giving the faster growth of the density perturbations appear after the growth of the linear adiabatic density fluctuations to the order of 1 [22, 23].

However, the discussion of the G-variable cosmologies (see Paper I and the present paper) indicates that the linear adiabatic density fluctuations can grow faster in these cosmologies. Hence, smaller initial density fluctuations are required for the formation of protogalaxies at the same time as in general relativity or protogalaxies appear as the gravitationally bound systems earlier in the G-variable cosmologies than in general relativity for the same initial density fluctuations. If one assumes that the protogalaxies were formed at $z = 4$, we have derived the times: 3.6×10^8, 4.8×10^7 (see also [3]), 6.2×10^8 yrs (for $\Omega = 1.0$) and 7.9×10^4 yrs (for $\Omega = -0.9$) for their formation in the Hoyle-Narlikar, the Dirac additive creation and the Brans-Dicke theories of gravitation respectively compared to 1.1×10^9 yrs for the flat Friedman cosmological models.

Generally, the ratio of the growth rate of the density fluctuations in the G-variable cosmologies (GV) to that of general relativity (GR) is described by: $\omega_+(\mathrm{GV})/\omega_+(\mathrm{GR}) \propto (t^{\mathrm{GR}})^n/(t^{\mathrm{GV}})^m \propto (t_0^{\mathrm{GR}})^n(1+z)^{n_1}/[(t_0^{\mathrm{GV}})^m(1+z)^{m_1}]$. The values of the power indices for the discussed flat dust-filled cosmological models are as follows: *i)* $n = 2/3$, $n_1 = -1$ for GR; *ii)* $m = 1.15$, $m_1 = -2.30$ for the Hoyle-Narlikar theory (H–N); *iii)* $m = 1.81$, $m_1 = -5.43$ for the additive creation version of the Dirac cosmology (DAC); *iv)* $m = 1.06$, $m_1 = -6.89$ and $m = 0.79$, $m_1 = -1.38$ for the Brans-Dicke cosmology (B–D) with $\Omega = -0.9$ and 1.0 respectively. In order to eliminate the constant factors in the above formula, we have estimated the relative ratios of the growth rate of the density fluctuations in the G-variable cosmologies, e.g. at $z = 20$ and 4: $\omega_+{}^{\mathrm{GV}}(z = 4)/\omega_+{}^{\mathrm{GV}}(z = 20)$ to compare them with those of GR. We have obtained the values of these ratios for the (H–N) theory and the (B–D) cosmology with $\Omega = 1.0$ comparable with that of GR. On the other hand, the density perturbations at $z = 20$ might be 4.7×10^3 and 5.8×10^2 times smaller in the (B–D) cosmology with $\Omega = -0.9$ and the (DAC) theory respectively than in GR in order to obtain the same density fluctuations at $z = 4$. Hence, it seems that the G-variable cosmologies can help in the problem of the galaxies formation from the linear adiabatic density perturbations.

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