AN ESTIMATION OF ENERGY DENSITIES FOR CENTRAL COLLISIONS OF ULTRA-RELATIVISTIC HEAVY NUCLEI

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The energy density distribution in the central region (mean value and dispersion) is calculated for heavy ion collisions in the constituent quark model. The mean energy density increases slowly with the atomic number A for nucleus-nucleus collisions and even for uranium is around 1 GeV/fm^3, below estimations of the critical energy of the transition to quark-gluon plasma. However, the fluctuations are large, so plasma may be formed in tubes of hadronic rather than nuclear size.

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Rencently it has been realized that collisions of ultra-relativistic heavy ions may offer a practical method for studying properties of plasma of quarks and gluons [1]. Numerous attempts have been made to study conditions for plasma formation and possible experimental signatures of this phenomenon in both fragmentation and central regions have been proposed [2].

In Ref. [3] it is argued that for fragmentation regions the net quark density and mean energy are large, and, consequently, the formation of quark matter is very likely. However, the recent study by Kajantie et al. [4] does not confirm this result.

The central region is even less understood. The lack of a reliable model for particle production in nucleus-nucleus collisions is the main source of uncertainties. The number of particles produced in the central region during such a collision is a crucial parameter. A common expectation is that for central collisions of identical nuclei this number is A times greater than for nucleon-nucleon inelastic interactions [5], where A is the atomic number of each nucleus. Once this is assumed, one gets the mean energy density which, for ISR energies, is a little bit larger than estimations of the critical energy density of the transition from hadron to quark gluon matter [4]. So, one can argue that for a slightly more pessimistic choice of parameters, the plasma will not be formed in the central region.

In a recent paper, Ehtamo et al. [6] give arguments that fluctuations of energy densities in the central region can be large if the Low-Nussinov model of hadron-hadron collisions

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[7] is applied to nucleus-nucleus interactions. This is a very encouraging result because, if true, it means that plasma formation can be probable even if the mean energy density is well below the critical value. However, the results of Ref. [6] have been derived for SU_2 rather than SU_3 colour. It is believed that, due to the non-Abelian nature of both groups, the results should be similar. Unfortunately, this is not true. In one respect SU_2 and SU_3 are very different. SU_2 admits only real representations, whereas quarks of SU_3 form complex triplets. Consequently, the results for mean energy densities in nucleus-nucleus and nucleus-antinucleus collisions can be different, whereas quarks and antiquarks of SU_2 are in unitary equivalent representations, and the energy densities in both processes should be the same.

To illustrate this point, I give a new estimation of the energy density in the central region using the coloured [8] constituent quark model [9].

I try to make the following arguments as simple as possible, so I ignore many "trivial" geometrical complications. In particular, I take identical heavy nuclei which collide at zero impact parameter. The incident energies must be so large that the central rapidity plateau is well defined. Following Bjorken [10], I assume boost invariance in the central region, so I consider a slice $(-\Delta y/2, \Delta y/2)$ around the centre-of-mass rapidity $y = 0$.

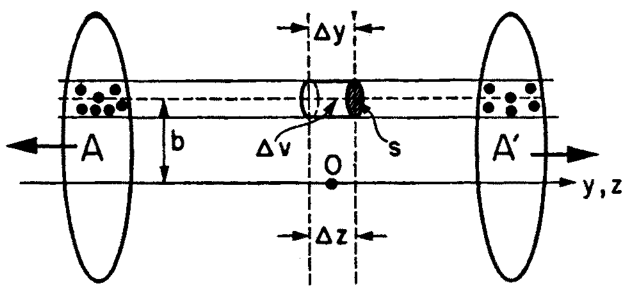

Fig. 1. Definitions of geometrical variables for a central collision of heavy nuclei A and A'

Let us consider a cylinder C parallel to the longitudinal direction 0z, see Fig. 1. I shall call it a tube. S is some typical hadronic area $\sim 1\ \text{fm}^2$, and b is the distance between the axis 0z and the symmetry axis of C. We will calculate the total energy E of particles in a volume $\Delta V = S \cdot \Delta z$. Let G_L denote the group of constituent quarks in the outgoing fireball A which are contained in C. They are moving to the left. G_R denotes the right-moving constituent quarks in the intersection of C and the nuclear fireball A'.

In the constituent quark model [8, 9] each nucleon consists of three constituent quarks which are colour triplets before, as well as after the collision. At high energies, constituent quarks move along trajectories parallel to the longitudinal direction 0z. So, the constituents contained in C interact mainly with themselves. Therefore, it seems reasonable to ignore interactions with the constituents outside C. Then our problem simplifies to the problem of a collison of G_L and G_R. At the moment of collision, when the wave packets of constituents overlap, they interact by exchanging gluons. Consequently, the quarks G_L forget their initial colour state and form some colour state in a representation U. Then the requirement of colour conservation for all the constituents in C implies that the quarks G_R are in a representation U^*.

Let $|\phi\rangle$ denote a colour state of the constituent quarks in C. It is a colour singlet state in a space $V_{L\otimes R}$ which is a tensor product of colour spaces for all G_L and G_R quarks. Let $\hat{X}_L^a$ and $\hat{X}_R^a$ $(a = 1, 2, ..., 8)$ be generators acting on left- and right-moving constituents, respectively. So, for example, $\hat{X}_L^a$ acts as a generator in the subspace V_L of the G_L quarks and as the unit operator in the subspace V_R of the G_R quarks.

The following reasoning is based on two further assumptions:

1) the energy E originating from C in the rapidity slice $(-\Delta y/2, \Delta y/2)$ is determined once the colour state of the constituents in C is given and has the form:

$$E = -E_0\langle\phi| \sum_a (\hat{X}_L^a \hat{X}_R^a)\, |\phi\rangle, \tag{1}$$

where E_0 does not depend on colours of G_L and G_R quarks;

2) all colour states of C which can be obtained from the initial one by permutations of the colour indices of wounded quarks are equally probable.

The first of these assumptions is not new and some arguments for it can be found in the literature. In particular, I recall a successful calculation of the slopes of Regge trajectories in the MIT bag model [11]. In the present context it means that the expression for E can be factorized into a "vertex" part which depends on the colour charges at the ends of C and another factor which does not. Once this is assumed, one can obtain E_0 from the data on nucleon-nucleon collisions and then calculate E, providing the colour state of C is specified, and this is what the second assumption does. One can consider assumption 2) as an analogue of the microcanonical ensemble for configuration in colour space. Explicit calculations for two and three quarks scattered on heavy nuclei suggest that the approach to this distribution in colour space is fast [8].

It should be stressed that the interval of energy in which the model may work is rather limited. The energy should be large enough, for the central region must be well defined. On the other hand, the constituent quark picture presumably cannot account for large E_t events which are frequent at the SPS collider energies. According to recent ideas [12], such events can be a main source of hadrons at very high energies, and, as they come from hard gluon-gluon scattering, cannot be described in terms of constituent quarks. So, the model may work for TEVATRON and ISR energies where three constituent quarks seem to be a good approximation for a nucleon.

Let us make one more assumption, which — though unnecessary — simplifies much the following considerations. The assumption is that for a central collision of heavy nuclei and some central tube C, all the quarks in C participate in exchanges of gluons, i.e., become wounded. Let l_w and r_w denote the numbers of the left- and right-moving (wounded) quarks in C. Then the mean value of the energy E in the volume ΔV reads:

$$\bar{E} = -\frac{E_0}{(r_w + l_w)!} \sum_p \langle\phi_p| \sum_a (\hat{X}_L^a \hat{X}_R^a)\, |\phi_p\rangle, \tag{2}$$

where the summation extends over all $(r_w+l_w)!$ permutations of the colour indices of wounded quarks, and $|\phi_p\rangle$ is a state obtained from the initial one by a permutation P. As the initial state, I take l_w left-moving quarks forming $l_N = \frac{1}{3} l_w$ singlets, and r_w right-moving quarks forming $r_N = \frac{1}{3} r_w$ singlets. Now, after some algebra, one derives:

$$\bar{E} = E_0 C_F \frac{l_w r_w}{l_w+r_w-1}, \tag{3}$$

and

$$\bar{E}^2 = (E_0 C_F)^2 \left\{ \frac{l_w r_w}{l_w+r_w-1} + \tfrac{45}{4} \left[(l_N)_2 + 2 l_N r_N + (r_N)_2\right] \cdot \frac{(l_w)_2 (r_w)_2}{(l_w+r_w)_4} \right\}, \tag{4}$$

where

$$(n)_m = \prod_{k=0}^{m-1} (n-k) \tag{5}$$

and C_F is the quadratic Casimir invariant for the fundamental representation.

It is noteworthy that the leading term in the expression for

$$D^2 = \bar{E}^2 - (\bar{E})^2 = \tfrac{1}{4} (\bar{E})^2 + \dots \tag{6}$$

is proportional to the square of mean energy, i.e. fluctuations of energy densities are large, in agreement with our expectations.

The final expression for the mean energy is obtained after averaging over the numbers of left- and right-moving nucleons in C. For the Poisson distributions for l_N and r_N one obtains:

$$\langle E \rangle = E_0 C_F \left\langle \frac{l_w r_w}{l_w+r_w-1} \right\rangle \approx 3 E_0 C_F \sum_{l,r=1}^{\infty} \frac{l \cdot r}{l+r-1} \cdot \frac{\bar{N}^{l+r}}{l!\, r!} e^{-2\bar{N}} = \tilde{E}_0 \frac{\bar{N}}{2}, \tag{7}$$

where

$$\tilde{E}_0 = 3 C_F E_0, \tag{8}$$

and $\bar{N}$ is the average number of left-moving nucleons. Consequently,

$$\varepsilon = \frac{\langle E \rangle}{\Delta V} = \tfrac{1}{2} \lambda_0 T_A(b), \tag{9}$$

where

$$\lambda_0 = \frac{1}{\Delta z} \tilde{E}_0 \tag{10}$$

and $T_A(b)$ denotes the nuclear density for a nucleus A integrated along the longitudinal direction.

The parameter λ_0, which is a linear energy density for a string-like configuration of colour charges, can be obtained from the energy distributions in nucleon-nucleon collisions.

According to the Low-Nussinov model [7], the dominant contribution to inelastic final states for high energy nucleon-nucleon interactions comes from a string-like configuration of octet colour charges. So, from Eq. (1) for $|\phi\rangle = |\underset{\sim}{8} \times \underset{\sim}{8}\rangle$ and the definition (10) of λ_0 one obtains:

$$\lambda_0 = \frac{3C_F}{C_A} \lambda_{NN} = \tfrac{4}{3} \lambda_{NN}, \tag{11}$$

where λ_{NN} denotes the linear energy density in the central region for nucleon-nucleon inelastic final states. C_A is the quadratic Casimir invariant for the adjoint representation.

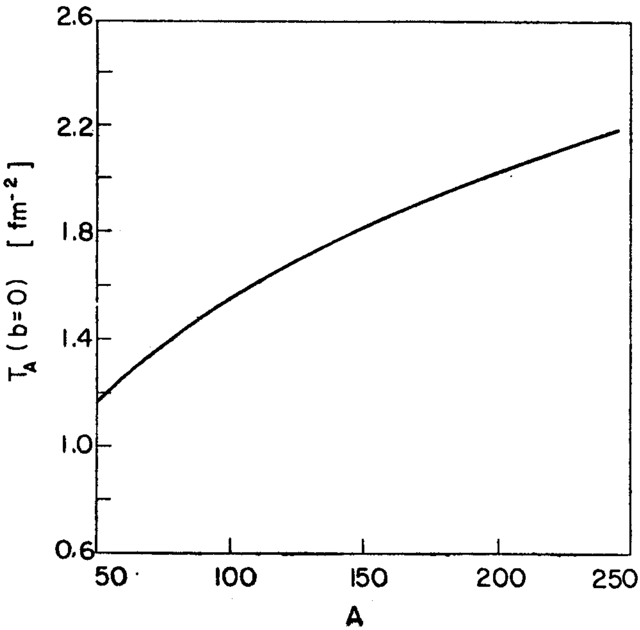

Fig. 2. $T_A(b = 0)$ as a function of atomic weight A. Nuclear parameters are taken from Ref. [13]

Taking 0.3 GeV for pion transverse mass and 2.5 for the height of central plateau, one obtains:

$$\lambda_{NN} = \frac{0.3 \text{ GeV}}{0.4 \text{ fm}} = \tfrac{3}{4} \text{ GeVfm}^{-1}, \tag{12}$$

i.e.,

$$\lambda_0 = 1 \text{ GeVfm}^{-1}. \tag{13}$$

The result (9) is disappointing because even for $b = 0$ and uranium (see Fig. 2)

$$T_U(b = 0) \approx 2.2 \text{ fm}^{-2} \tag{14}$$

and

$$\varepsilon \approx 1.1 \text{ GeVfm}^{-3}, \tag{15}$$

well below estimations of the critical energy density. The dispersion of the distribution is large

$$D \approx \tfrac{1}{2}\varepsilon, \tag{16}$$

so, it is still possible that — due to fluctuations — the quark-gluon plasma will be formed in some tubes. However, these hot regions will have the transverse dimensions of the order of the nucleon rather than nuclear sizes.

Let us consider now nucleus-antinucleus collisions, perhaps an academic exercise. A nucleus moves to the left and an antinucleus to the right. Let us assume that all constituents in a tube C participate in exchanges of colours and define:

$$n = \min(l_w, r_w), \tag{17}$$

where l_w denotes the number of wounded quarks and r_w the number of wounded antiquarks. The assumption (2) must be modified. A possible modification is:

2′) all colour states of quarks corresponding to Young tableaux built from n boxes are equally probable.

The mean energy and energy squared can be easily calculated in this case and read:

$$\bar{E}_n = E_0 C_F n, \tag{18}$$

$$\overline{(E_n)^2} = (E_0 C_F)^2 \left[n + \tfrac{5}{4} n(n-1)\right]. \tag{19}$$

One observes that the mean energy is nearly twice larger than in the nucleus-nucleus case.

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