THE QUADRUPOLE DEFORMATION PARAMETERS OF CHARGE DISTRIBUTIONS IN NUCLEI*

By E. Wesołowski**

Nuclear Structure Research Laboratory, University of Rochester, Rochester, New York 14627, USA

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A semi-empirical formula for calculating quadrupole deformation parameters of charge distributions in the nuclei with Z, $N \geqslant 6$ is presented. The formula is applied for calculating the reduced E2 transition probabilities — $B(E2\uparrow)$ between ground- and first 2^+ excited states in even-even nuclei. An attempt is made to explain this formula in a two-liquid-drop and a microscopic model.

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1. Introduction

The picture of a nucleus as a system in which proton and neutron spatial distributions have the same geometrical form, with a constant ratio of densities and the same deformation parameters, is one of the basic assumptions of the classical nuclear collective models [1]. Although there is no direct experimental evidence, some qualitative arguments are usually used to support this assumption. The observation of M1 transitions between collective states in nuclei and the fact that measured g_R factors are smaller than expected Z/A values [2] are significant experimental evidence against the above assumption. The most direct evidence for different proton and neutron distributions comes from a disparity between the measured root mean square (rms) radii of proton and neutron distributions [3]. This difference can not be explained as being due to the different numbers of protons and neutrons in nuclei; different geometrical parameters have to be used. We would like to mention two models dealing with that problem from which can follow that neutron matter and proton matter in nuclei behave independently, having different deformation parameters, only as a whole influencing each other.

In order to explain magnetic nuclear properties Greiner [4] introduced the idea that there is the difference between proton and neutron deformation parameters, due to the

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** Present address: Instytut Fizyki Doświadczalnej, Uniwersytet Warszawski, Hoża 69, 00-681 Warszawa, Poland.

different strength of the pairing forces for protons and neutron. Such a generalized collective model (GCM) has been used several times [5, 6] giving considerable evidence for different proton and neutron deformations. In that model the deformation parameters for protons and neutrons represent independent degrees of freedom; proton and neutron fluids are coupled strongly via symmetry energy which suppresses large differences in the spatial distributions of protons and neutrons.

In the Droplet Model (DM) introduced by Meyers and Swiatecki [7] the proton and neutron densities in the nucleus are treated independently; in the total nuclear energy formula those densities are taken to be parameters. The mass formula derived consists of two parts: a smooth macroscopic part and an oscillating microscopic part. One of the components of the microscopic part is a shell correction calculated by a method developed by Swiatecki [8]. In this method one assumes that protons and neutrons in a nucleus (in its ground state) behave independently and that the shell correction, due to the deviation of the nucleon level distribution from uniformity, is a sum of such corrections calculated separately for the proton and neutron components, each being a function of the number of nucleons of one kind and shell-model magic numbers. The Fermi gas model was applied for calculating the shell corrections.

A simple relation between the density of states near the Fermi surface of the Fermi gas system and a deformation parameter of that system [9] allows us to think about the shell correction in the DM as an additional energy connected with the deformations of proton and neutron liquids (the shell correction is unaffected by closed shells). Accordingly we expect that deformation parameters for protons and neutrons can be written as a function of the number of nucleons of one kind and their shell-model magic numbers, as can the shell-correction term in the DM. These functions have values zero at magic numbers of particles and positive bumps in between. The strong coupling between proton and neutron degrees of freedom and the expected increased deformability of the system with increasing deformation parameter imply that the deformation parameter of the nucleus can be written as a product of unperturbed deformation parameters of proton and neutron distributions.

This qualitative picture we applied in calculating the reduced E2 transition probabilities — $B(\mathrm{E2}\uparrow)$ for even-even nuclei.

2. *The formula for calculating B(E2) values*

The reduced E2 transition probabilities for even-even nuclei were taken from the literature [10]. Values of experimental errors have been taken from the same sources, but we have assumed that those errors are not lower than 5 percent [11].

To make the calculations as simple as possible the $B(\mathrm{E2})$ values were calculated according to the formula for axially symmetric deformed nuclei [12].

$$B(\mathrm{E2}\uparrow) = \frac{5}{9\pi} Z^2 \langle r^2 \rangle^2 (0.945\beta)^2 \ [\mathrm{e^2 fm^4}], \tag{1}$$

where β is a deformation parameter of proton distribution. The rms radii of proton distributions $\langle r^2 \rangle^{1/2}$ were calculated using the empirical formula developed earlier [13].

We assume that the square of the deformation parameters for protons can be written simple as a product of two fully separable functions of protons and neutrons variables:

$$\beta^2 \sim f_p(p, p_i) \cdot f_n(n, n_i). \tag{2}$$

The formula introduced for calculating the deformation parameters of the proton distribution has the form:

$$\beta^2 = \eta Z^{\pi} N^{\nu} \left(1 - \xi_p \frac{\Lambda_L - \Lambda_u}{2} - \delta_p\right)\left(1 - \xi_n \frac{\pi_L - \pi_u}{2} - \delta_n\right) \tag{3}$$

with

$$\Lambda_L = \exp\{-\alpha_p (Z - Z_L)^2 / \Delta Z\}, \tag{3a}$$

$$\Lambda_u = \exp\{-\beta_p (Z - Z_u)^2 / \Delta Z + \gamma_p (Z - Z_u)\}, \tag{3b}$$

$$\pi_L = \exp\{-\alpha_n (N - N_L)^2 / \Delta N + \gamma_n (N - N_L)\}, \tag{3c}$$

$$\pi_u = \exp\{-\beta_n (N - N_u)^2 / \Delta N\}, \tag{3d}$$

where Z, N are proton and neutron numbers in a nucleus, Z_L, Z_u, N_L, N_u are proton and neutron shell-model magic numbers closest to the Z and N numbers ($Z_L < Z \leqslant Z_u$ and $N_L < N \leqslant N_u$) and $\Delta Z, \Delta N$ are shell sizes ($\Delta Z = Z_u - Z_L$ and $\Delta N = N_u - N_L$).

The magic numbers used in calculations are: 2, 8, 20, 28, 40, 50, 82, 114 for protons and 2, 8, 20, 28, 38, 50, 82, 126, 184 for neutrons. All the above magic numbers are the commonly accepted ones except the number 38 for neutrons. This number was assumed on a basis of a minimum-χ^2 criterion.

The free parameters: two sets of six, for protons and neutrons (α_x, β_x, γ_x, δ_x, ξ_x, π/ν) and one common (η) were found by a best-fit procedure. In fact the parameters of the formula are functions of the shell (see Table I). The parameter η found from the fit

TABLE I

The formula parameters

	α	β	γ	ξ	δ	π
$Z =$ 6–20	0.4127	0.6027	−0.3346	0.8917	0.3636	1.219
22–40	0.8953	0.2813	−0.0316	1.425	0.0829	1.219
42–50	0.3643	0.4634	−0.0795	1.546	0.1319	−2.057
52→	0.3453	0.1657	+0.0046	1.651	0.1319	−2.057
						ν
$N =$ 6–20	0.2024	0.0154	−1.151	1.241	(0.3636)	−2.217
22–50	1.457	0.1281	−0.0671	1.375	(0.0829)	−2.217
52–82	0.1553	0.2873	−0.0717	1.562	(0.1319)	0.9031
84→	1.018	0.3175	+0.1034	1.572	(0.1319)	0.9031
	$\eta = 11.696$					

in $Z > 40$, $N > 50$ region was used for all regions of fitting. The parameters δ_n and δ_p are zero for all but closed-shell nuclei (in the fitted A-regions we always assumed $\delta_n = \delta_p$). Because of the discontinuities of the values of the π parameter at $Z = 40$ and ν parameter at $N = 50$ only one zirconium isotope (^{90}Zr) has been included in our calculation. Nevertheless, the A-dependence of the B(E2) values and the values of the moment of inertia for nuclei in this A-region indicate a subshell closure at $N = 56$ (or 58).

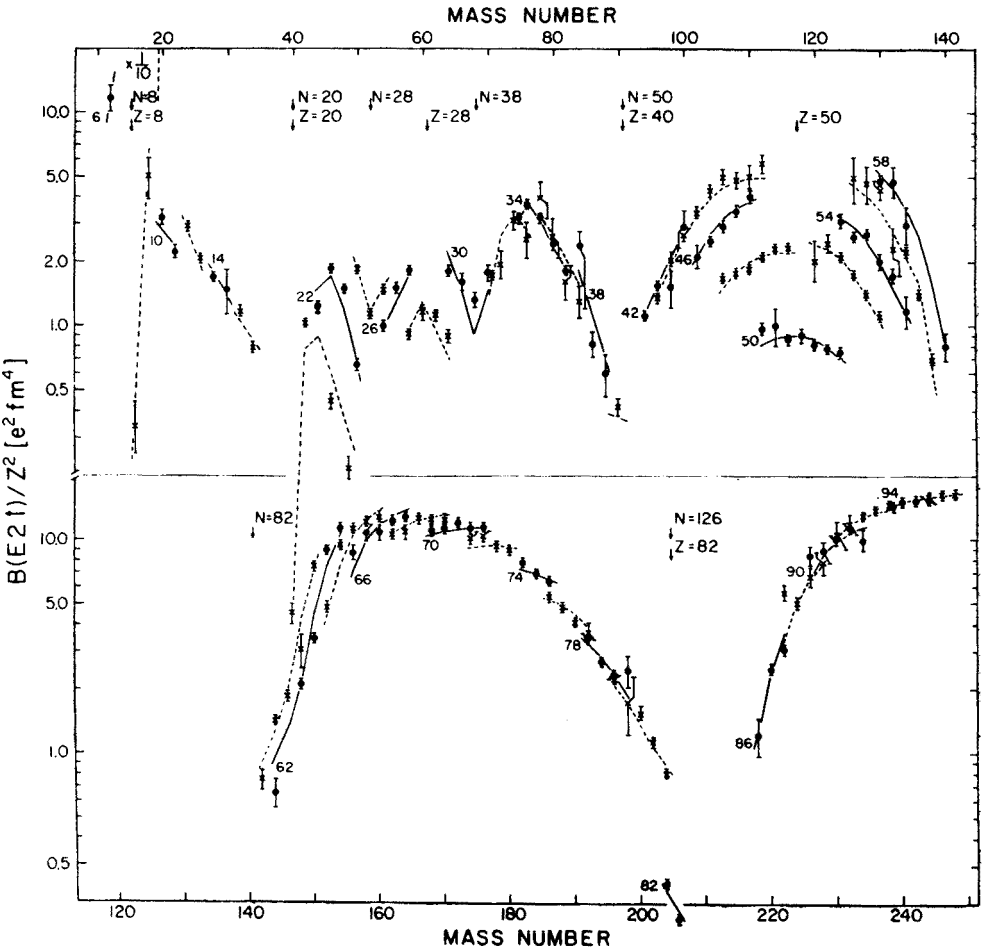

Fig. 1. Fit of the $B(E2\uparrow)/Z^2$ values, calculated according to formulas (1) and (3), to the experimental data

The calculated $B(E2\uparrow)$ values together with the experimental data are shown in figure 1. The quality of the fit can be seen from figures 2 and 3, where the distributions of the differences between experimental an calculated $B(E2\uparrow)$ values are shown. The error of the formula, defined as an additional error which has to be added quadratically to the experimental errors to get the value of χ^2 per number of experimental points equal 1, is 8.5 percent, thus slightly less than the mean experimental error (8.8%).

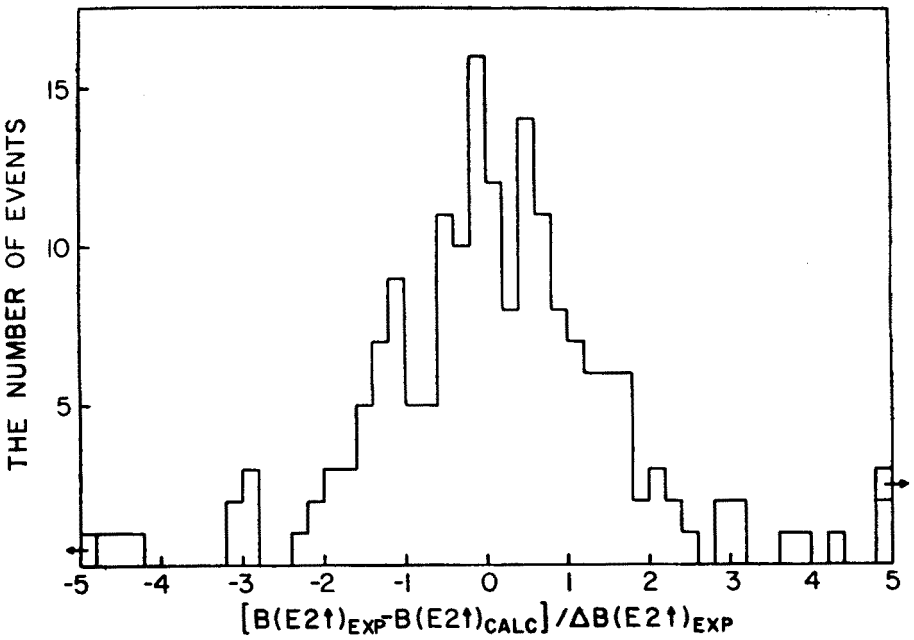

Fig. 2. Distribution of the standard deviations of the calculated values, according to (1) and (3), from the experimental data

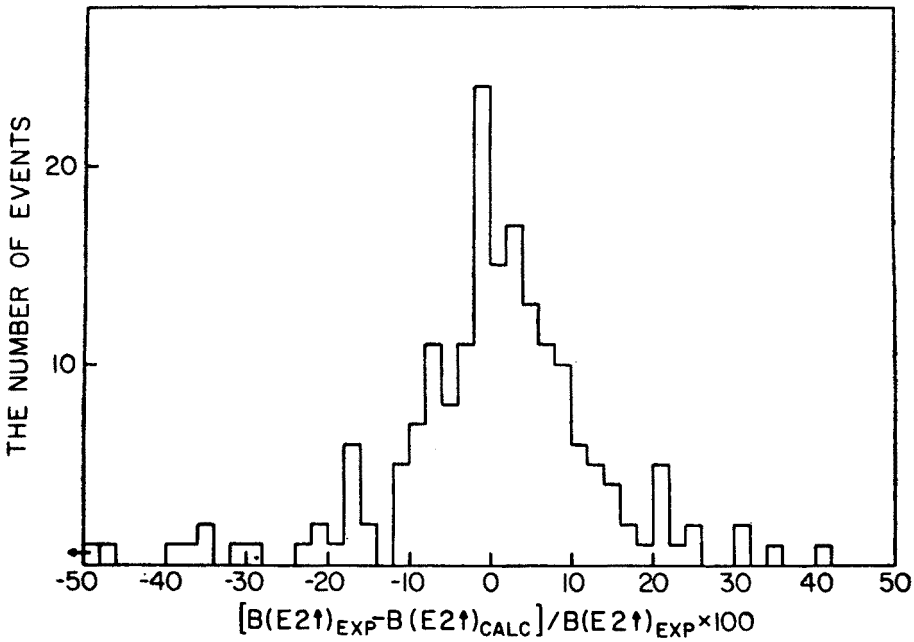

Fig. 3. Distribution of the percentage deviations of the calculated values, according to (1) and (3), from the experimental data

3. Discussion

It has been shown that the A-dependence of the reduced E2-transition probabilities can be explained with reference to the shell model. The electric quadrupole moment can be written as a product of two functions as the separated contributions of proton and neutron nuclear components.

This fact could be explained in the framework of a two-liquid drop nuclear model resembling the Greiner [4] and Bertsch-Janecke [14] models.

3.1. Two-liquid-drop model

Let us ussume that the protons and neutrons in a nucleus form two independent drops. In the absence of any interaction between these two liquids each of them shows its own shell-model N-behavior being of spherical form, when the number of particles (N) equals a magic number and being deformed in between magic numbers. If symmetry forces are switched on, these two liquid drops tend to assume the same shapes — the same deformations. (The finite range of the symmetry forces allows for slightly different deformation parameters). The deformation obtained for this two-liquid system can be calculated from the relation: $\Delta S_1 \cdot \tilde{F}_1 = \Delta S_2 \cdot \tilde{F}_2$ where $\Delta S_{1,2}$ are surface area differences (before and after switching on the symmetry forces) for proton and neutron drops and $\tilde{F}_{1,2}$ are the mean values of a deformation-conserving forces. Assuming that effective radii $R_{1,2}$ remain constant and that the shape-conserving forces are inversely proportional to the square of a deformation parameter ($F = C/\beta^2$) [15] we can write down

$$R_1^2(\beta^2 - \beta_1^2)\frac{c_1}{\beta\beta_1} = R_2^2(\beta_2^2 - \beta^2)\frac{c_2}{\beta\beta_2}, \qquad [4]$$

where $\beta_{1,2}$ are deformation parameters of the non-interacting distributions and β is a deformation parameter of the equilibrium neutron-proton system (nucleus).

The above equation can be rewritten in the form

$$\beta = s \cdot \beta_1 \cdot \beta_2 \qquad (5)$$

with the prefactor s sightly and slowly varying, comparing with the variation of $\beta_1 \cdot \beta_2$ term, along the A-axis.

It follows from the above that the functions f_p and f_n in the formula developed can be treated in first order as deformation parameters of non-interacting proton- and neutron distributions in nuclei. The A-dependence of these parameters along β-stability valley is shown in figure 4.

The neutron-proton deformation difference was calculated by Seiwert et al. [6] for ^{232}Th and ^{234,236}U isotopes from the observed β- to ground-state band γ-transitions. The obtained value for β_n/β_p, 1.67 ± 0.1, is comparable with the value 1.42 which can be obtained from figure 4 (the lower value obtained in our calculations may speak in favor of larger neutron centrifugal stretching pointed out by the authors).

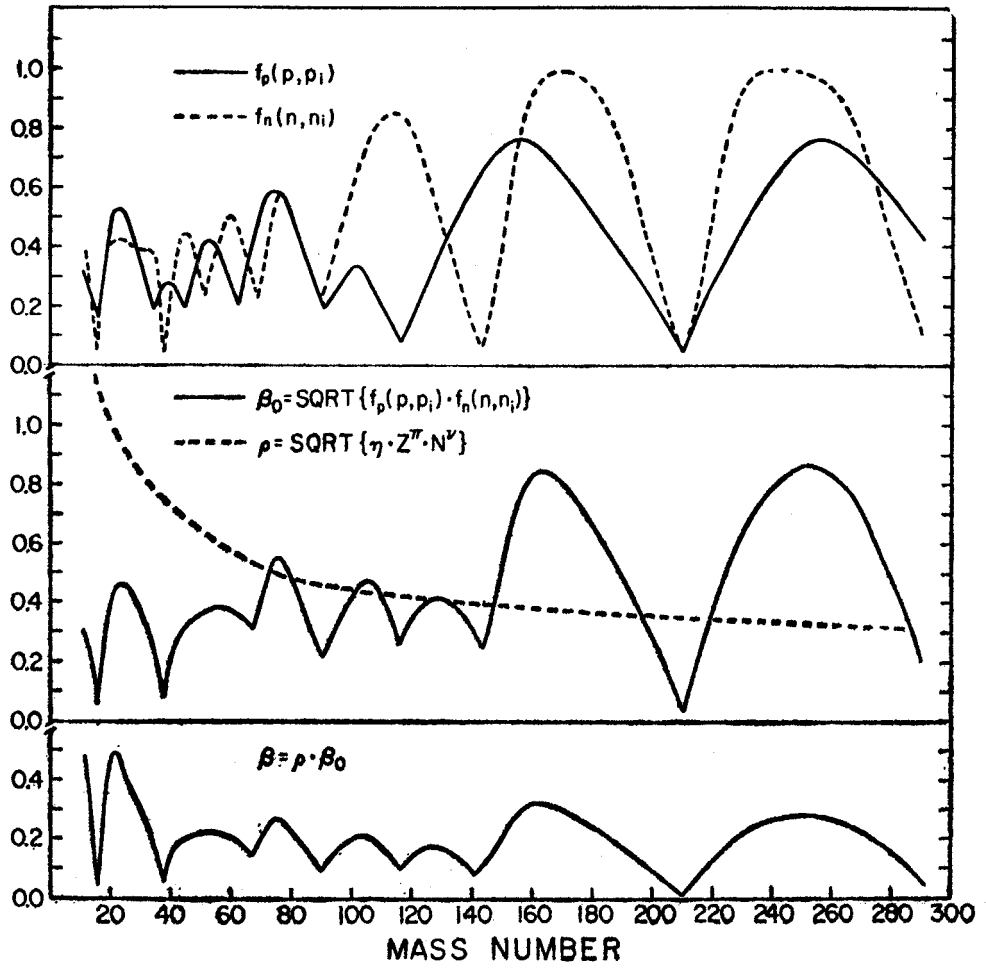

Fig. 4. A-dependence of the different factors of the formula (3) along the β-stability valley

3.2. Microscopic model

The model shown above suffers a serious problem: it can not be accepted by the microscopic theory (a unified shell-model picture of nuclear deformation [17]). It is well established [16–18] that nuclear deformation is produced by the isoscalar n-p interaction between valence particles, while the pairing interaction between $T = 1$ pairs of valence nucleons favors spherical shapes. Having these facts in the mind and making use from interacting bosons model we will show, that our finding can be also explained in the frame of the microscopic nuclear model.

Without the n-p attractive interaction, the neutron and proton matters are fully independent, so their distributions. The spherical distribution of each component is expected, due to the pairing forces. The energy of each component is equal to a sum of an energy of the core and the energy of a pairing interaction between valence nucleons. By switching

on the n-p interaction the distributions of protons and neutrons become dependent: the out of core nucleons become spatially correlated and the nuclear deformation (β) develops. The energy of nuclear deformation can be written as:

$$E_{\mathrm{def}} = E_{\mathrm{np}} + \delta_{\mathrm{pair}}, \tag{6}$$

where E_{np} is an energy of n-p interaction and

$$\delta_{\mathrm{pair}} = (E_{\mathrm{p\,pair}} + E_{\mathrm{n\,pair}})_{\beta} - (E_{\mathrm{p\,pair}} + E_{\mathrm{n\,pair}})_{0} \tag{7}$$

accounts for the dependence of the pairing energy $E_{\mathrm{p(n)\,pair}}$ on the nuclear shape [19]. From the point of view of the macroscopic liquid drop model the deformation of the nucleus changes only the surface and Coulomb energies [20], namely

$$E_{\mathrm{def}} = \left(2SR_0^2 - \frac{3}{20\pi}\frac{Z^2e^2}{R_0}\right)\beta^2 \equiv c\beta^2, \tag{8}$$

where S is the surface tension parameter (3.76 MeV/fm^2) and R_0 is the equilibrium radius (1.20 A$^{1/3}$ fm). Comparing Eqs. (6) and (8) we find

$$\beta^2 = \frac{1}{c}(E_{\mathrm{np}} + \delta_{\mathrm{pair}}). \tag{9}$$

The energy of n-p interaction can be obtained employing the proton-neutron interacting bosons model (IBA — 2) [21]. The neutron-proton interaction, in this model, is represented by a neutron boson–proton boson interaction of the form $Q_{\pi}^{(2)} \cdot Q_{\nu}^{(2)}$. Since the quadrupole operator $Q_{\pi}^{(2)}$ ($Q_{\nu}^{(2)}$) depends on proton (neutron) variables only, and since δ_{pair} is expected to be small, a product of the f_{p} and f_{n} functions in formula (2) can be recognized as the energy of the neutron-proton quadrupole interaction.

The fully phenomenological calculations of β^2 along an A-axis were performed in the frame of the above sketched microscopic model. The quadrupole operator $Q_{\varrho}^{(2)}$ ($\varrho = \pi, \nu$), in the single-j approximation has the form [22]

$$Q_{\varrho}^{(2)} = K_{\varrho 0}(\Omega_{\varrho} - N_{\varrho})^{1/2}(d^{+} \times s + s^{+} \times \tilde{d})^{(2)} + \chi_{\varrho 0}(\Omega_{\varrho} - 2N'_{\varrho})\,(d^{+} \times \tilde{d})^{(2)}. \tag{10}$$

The Ω_{ϱ} is a maximum possible number of bosons in an active shell, N_{ϱ} is a total number of bosons and N'_{ϱ} is a number of particle-like bosons. The operator in the first term counts the total number of bosons, while the operator in the second term counts d-bosons only (the proportionality of a number of d-bosons to the total number of bosons was assumed in further calculations).

The square of the deformation parameters was written in the form:

$$\beta^2 = \frac{1}{c}(G_{\pi}^{i} \cdot G_{\nu}^{j} + P^{i} + P^{j}), \tag{11}$$

where

$$G_{\pi}^{i} = K^{i}(\Omega^{i} - N_{\pi})^{1/2}N_{\pi} + \chi^{i}(\Omega^{i} - 2N'_{\pi})N_{\pi}, \tag{11a}$$

$$G_{\nu}^{j} = K^{j}(\Omega^{j} - N_{\nu})^{1/2}N_{\nu} - \chi^{j}(\Omega^{j} - 2N'_{\nu})N_{\nu}. \tag{11b}$$

In these formulas we assumed that the values of parameters K, χ, and P are shell dependent only. The opposite sign of χ parameters for protons and neutrons (suggested by Eqs. (3a)–(3d)) accounts only for neutron-proton difference in these calculations [23]. The parameters K, χ and P obtained from the fitting the experimental values of B(E2) by formulas (6) and (11) are listed in Table II. The fit of the predicted to the experimental values of β_2 is shown in Fig. 5. Having in mind all the simplifications we have made in these calculations this fit is very satisfactory.

TABLE II

The parameters of microscopic model calculations

Shell	K	χ	P	
22– 28	0.993	0.067	2.321	
30– 38 (40)	0.581	0.020	3.018	
40– 50	0.898	−0.230	−0.901	neutrons
42– 50	0.898	0.230	1.674	protons
52– 82	0.197	0.014	1.058	
84–126	0.236	0.007	−0.589	
128–184	0.066	−0.028	−2.903	

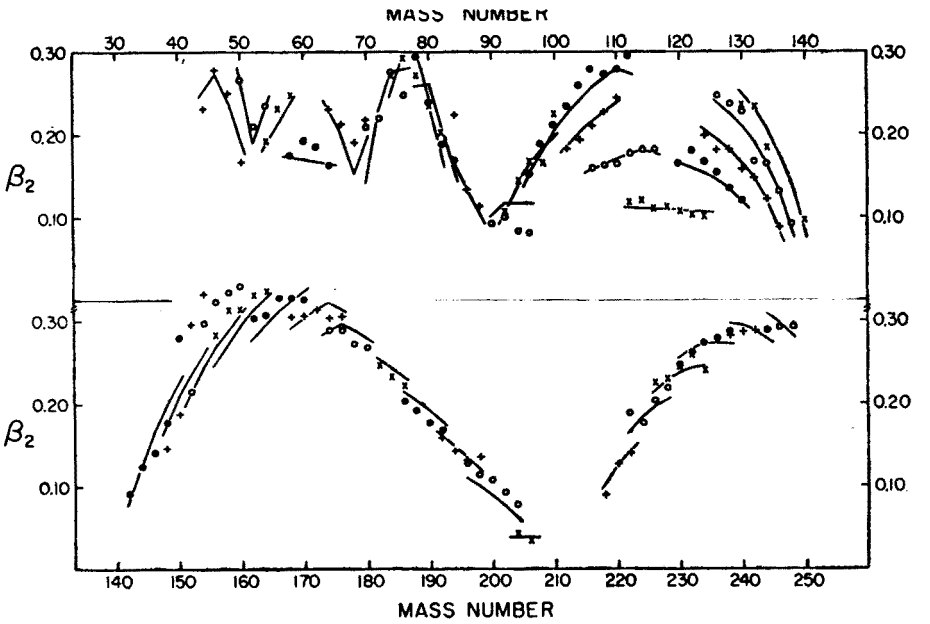

Fig. 5. Fit of the deformation parameters calculated according to formula (11) to the values obtained from the experimental B(E2) values using Eq. (1)

4. Conclusion

It has been shown that for even-even stable nuclei or those close to the β-stability valley, the $B(E2)$ values can be calculated with reference to the shell model. The deformation of the proton matter distribution depends on the degree of filling of the neutron and proton shells.

The macroscopic two-liquid-drop model and microscopic model of nuclear deformation used for explaining this effect are equally acceptable by a quality-of-fit criterion, even though a basically different assumption about nuclear-deformation development is made in these models. Although the microscopic model is superior as being a physically based, the macroscopic model can serve successfully as the equivalent phenomenological picture of nuclear deformation.

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