MEAN MULTIPLICITY OF g-PARTICLES IN COLLISIONS OF VARIOUS HADRONS WITH NUCLEI

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Analytic expressions without free parameters that describe the available experimental data on the g-particle mean multiplicities in the interactions of high-energy protons, antiprotons, pions and kaons with nuclei, are obtained. The comparison with available experimental data was made.

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For many years the multiplicity of the so-called g-particles [1] was studied in experiments with emulsions. These particles, with the velocity in the range $0.3 \lesssim \beta \lesssim 0.7$, are in the main protons ejecting from nuclei.

Lately new experimental data on the mean multiplicities of protons with the energy $T_p = 30 \div 300$ MeV (which approximately corresponds to the g-particle energy range) obtained in electronic experiments on various nuclei and under the influence of hadrons of different nature [2, 3] have appeared.

The study of the g-particle mean multiplicity $\bar{n}_g$ is of certain interest since it gives information on the average number of hadron collisions in the nucleus $\bar{\nu}$. The simplest relation between these values is obtained in the leading hadron eikonal model, with the secondary particle cascading being neglected.

Indeed if g-particles are assumed to be recoil protons produced due to successive collisions of the incident hadron h in the nucleus with the atomic weight A, then

$$\bar{n}_g = \bar{\nu}\omega_1, \tag{1}$$

where ω_1 is the probability of g-particle production in the elementary hadron-nucleon collision (averaged over protons and neutrons). The value $\bar{\nu}$ is model-dependent and varies in different models:

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a) In the leading hadron cascade model [4]

$$\bar{\nu} = \frac{A\sigma}{\sigma_A}, \tag{2}$$

where σ, σ_A are the cross sections of the incident hadron interaction with nucleon and nucleus, respectively. In investigating the processes of pion multiple production one may not consider the elastic rescatterings of incident hadron in the nucleus, therefore, the hadron-nucleon interaction inelastic cross section σ^{in} is taken as $\sigma = \sigma^{\mathrm{in}}$, and the cross section of the production (of at least one pion) on the nucleus, σ_A^{prod}, is commonly used as σ_A (in accord with the experiment conditions).

In investigating the g-particle production process, one should take into account the contribution of the incident hadron elastic collisions with nuclear protons with the momentum transfer, exceeding the knock-on threshold of g-particles from the nucleus. For example, for the experiment conditions [2, 3] this threshold corresponds to $|t_{\min}| \approx 0.075$ $(\mathrm{GeV}/c)^2$ (allowing for the proton binding energy in the nucleus). Thus, in the expression (2) one should take $\sigma = \sigma^{\mathrm{tot}} - \Delta\sigma_{\mathrm{el}}$ as the cross section σ, where $\Delta\sigma_{\mathrm{el}}$ is the cross section of the elastic interaction with the momentum transfers $|t| < |t_{\min}|$.

Then, for the mean number of the incident hadron collisions we have:

$$\bar{\nu} = \sum_{n=1}^{A} nP_n = \sum_{n=1}^{A} n \cdot \frac{1}{\sigma_A n!} \int [\sigma T(b)]^n e^{-\sigma T(b)} d^2 b = \frac{A\sigma}{\sigma_A}, \tag{3}$$

where $T(b) = \int \varrho(b, z) dz$ is the projection of one-particle nuclear density $\varrho(r)$ on the impact parameter plane; P_n is the probability of the n-fold collision. The hadron-nucleus cross section σ_A is determined by the expression

$$\sigma_A = \int (1 - e^{-\sigma T(b)}) d^2 b, \tag{4}$$

which practically corresponds to the normalization cross section in the experiment [2, 3], being approximately equal to the total inelastic cross section of hadron-nucleus interaction, σ_A^{in}.

Further we shall consider the values $\bar{\nu}, \sigma, \sigma_A$ in the above sense.

b) In the leading hadron cascade model with the change of the leader type [5] the mean number of collisions is given by the following expression: (see Appendix II)

$$\bar{\nu}' = \frac{A\sigma_2}{\sigma_A} + \frac{\sigma_1 - \sigma_2}{\sigma_A} N(0; \sigma_0) \tag{5}$$

σ_1, σ_2 are the interaction cross sections of the projectile and hadron to which the leading property is transferred respectively, with a nucleon;

$$N(0; \sigma) = \int d^2 b \, \frac{1 - e^{-\sigma T(b)}}{\sigma} \tag{6}$$

is the effective nucleon number; $\sigma_0 = \sigma_1(1-\eta)$ where η is the probability of preserving the leading property in the elementary act of incident hadron-nucleon interaction [5].

c) Using in the additive quark model the probabilities for the quark collisions in the nucleus [6], we obtain the following expression for the mean number of collisions of a hadron composed of constituent quarks:

$$\bar{\nu}_q = n_q \frac{N(0;\sigma_q)}{N(0;\sigma)} = \frac{n_q \sigma_q A}{\sigma_A} \tag{7}$$

σ_q is the cross section of the constituent quark-nucleon interaction. It follows from (1), that the g-particle mean multiplicity in all the three models should possess the scaling property, i.e. depend on the type of the incident particle and the target-nucleus atomic number only through the corresponding $\bar{\nu}$.

TABLE I

Mean number of proton collisions in nucleus in various models

	C	Cu	Pb
Leading hadron cascade model	1.77	2.72	4.00
Leading hadron cascade model with the leader type change	1.64	2.38	3.31
Additive quark model	1.30	1.66	1.98

The probability ω_1 of g-particle production in the hadron-nucleon interaction at sufficiently high energies is practically independent of the incident hadron type [7]. This value was extracted from the available data [7] on the proton inclusive spectra in the reactions $pp \to px$, $p(\pi^+)n \to px$. For the comparison with the experimental data on nuclei [2, 3] where protons with energies $T_{min} = 30$ MeV to $T_{max} = 300$ MeV were registered, the portion of protons was determined in reactions on a free nucleon in the transferred momentum range $|t|_{min} = 2m_p(T_{min}+\varepsilon)$ to $|t|_{max} = 2m_p(T_{max}+\varepsilon)$ where $\varepsilon \approx 10$ MeV is the nucleon binding energy in the nucleus. This portion proved to be equal to $\omega_1 = 0.15 \pm 0.02$ (allowing for the elastic scattering) and practically independent of E_h in the incident hadron energy region E_h about 100 GeV and higher. Further on we neglect also the possible dependence of ω_1 on E_h in the hadron energy region from tens to 100 GeV. Table I presents the values for $\bar{\nu}$ depending on the target atomic number, e.g. for the case of pA--interaction. In calculations the Fermi nuclear density with the parameters from Ref. [8] was used. The comparison of experimental data on $\bar{n}_g$ [1] with the predictions of the expressions (1), (4), (5), (7) shows that experimental data are much higher than theoretical predictions. It is a direct indication of the significant contribution of the secondary particle cascading in the nucleus to the g-particle production process.

Below an attempt is made to calculate analytically the contribution of cascade processes, namely, of secondary interactions of pions produced in the nucleus, to the g-particle mean multiplicity. We shall then assume that the pions generated in the target fragmentation region make the main contribution in the cascading. In the l.s. they have energies not exceeding 3 GeV (at the primary energy of the order 100 GeV); the production lengths $l \sim P_\pi/\mu^2$ ($\mu^2 \sim 0.3$–$0.7\ \mathrm{GeV}^2$, the characteristic mass [9]) of these pions are of the order of internuclear distances in the nucleus, therefore, pions can undergo secondary interactions. The contribution of more energetic secondary pions cascading is suppressed: in this paper we neglect it. The role of other possible processes that can affect the g-particle production, will be discussed below.

The g-particle mean multiplicity in the leading hadron cascade model with regard to secondary interactions of pions from the target fragmentation region is given by the following expression (see Appendix I):

$$\bar{n}_g = \bar{\nu}\omega_1 + \frac{A\sigma_1^{\mathrm{in}}}{\sigma_A} m\omega_2 \left[1 - \frac{N(0;\sigma_3)}{A}\right], \tag{8}$$

where m is the mean multiplicity of pions in the target fragmentation region, that is generated, on the average, in each collision of the leading hadron; σ_3 is the averaged cross section of these pions interaction with nuclear nucleons, and ω_2 is the averaged probability of g-particles generation in these interactions (the values σ_3 and ω_2 are averaged over the pion energy spectra).

As is seen from this expression, the account of secondary interactions leads to the violation of $\bar{\nu}$-scaling, i.e. there occurs an additional A-dependence for $\bar{n}_g$.

In the above approximation one may also obtain the expression for $\bar{n}_g$ with regard to the secondary pion cascading in the model with the change of the leader type (see Appendix II):

$$\bar{n}_g = \omega_1 \left[\frac{A\sigma_2}{\sigma_A} - \frac{\sigma_1 - \sigma_2}{\sigma_A} N(0;\sigma_0)\right] + \omega_2 m \left[\frac{A\sigma_2^{\mathrm{in}}}{\sigma_A} + \frac{\sigma_3(\sigma_1^{\mathrm{in}} - \sigma_2^{\mathrm{in}})}{\sigma_3 - \sigma_0} \cdot \frac{N(0;\sigma_0)}{\sigma_A} - \frac{\sigma_1^{\mathrm{in}}\sigma_3 - \sigma_0\sigma_2^{\mathrm{in}}}{\sigma_3 - \sigma_0} \cdot \frac{N(0;\sigma_3)}{\sigma_A}\right], \tag{9}$$

where σ_0 has the above sense: $\sigma_0 = \sigma_1(1-\eta) = \sigma_1^{\mathrm{in}}(1-\eta^{\mathrm{in}})$; η^{in} is the probability of preserving the leading property in the elementary act of the incident hadron-nucleon inelastic interaction. One may make sure that at $\sigma_0 \to 0$ the expression (9) goes over to (8). The probability of the leading property transfer from nucleon (antinucleon) to pion is estimated in Ref. [5]: $(1-\eta^{\mathrm{in}})_{N\to\pi} \approx 0.35$, for kaons from the data of [10] one may in much the same way estimate $(1-\eta^{\mathrm{in}})_{K^+\to\pi} \approx 0.37$ and $(1-\eta^{\mathrm{in}})_{K^-\to\pi} \approx 0.42$. In the case of incident pion the probability of the leading property transfer to a particle of another type is negligible [5, 10].

Other parameters in expressions (8) and (9) were also estimated from experimental data on nucleons. It should be emphasized that the analysis of the possible dependence of theoretical expressions for $\bar{n}_g$ on the incident hadron energy was not the task of the present paper (note that the available experimental data on emulsions point to the practical absence of such dependence in the wide range of initial energies 20–400 GeV). Therefore, apart from experimental errors, we attribute an additional error, connected with the averaging of these data, to the parameters extracted from the data on hadron-nucleon interactions. To estimate m, the parametrization [5] of the pion inclusive spectra in the target fragmentation region was used; to estimate σ_3 and ω_2 the partial and differential cross sections of the exclusive channel of the low energy pion-nucleon interactions [11] were used. As a result, we obtain the following estimates: $m = 3.0 \pm 0.5$, $\sigma_3 = 27 \pm 4$ mb, $\omega_2 = 0.5 \pm 0.05$. The estimate of ω_1 was given above: $\omega_1 = 0.15 \pm 0.02$. For the elementary act cross sections the following values were used: $\sigma_{pN} = 33.5$, $\sigma_{pN}^{in} = 31.5$, $\sigma_{\bar{p}N} = 36$, $\sigma_{\bar{p}N}^{in} = 34$, $\sigma_{\pi N} = 21$, $\sigma_{\pi N}^{in} = 20$, $\sigma_{K^+N} = 17.5$, $\sigma_{K^+N}^{in} = 16.5$, $\sigma_{K^-N} = 19$, $\sigma_{K^-N}^{in} = 18$ mb.

The results of the g-particle average multiplicity calculation by formulae (8) (for primary pions) and (9) (for primary (anti)protons and kaons) are shown in Figs. 1–3. The

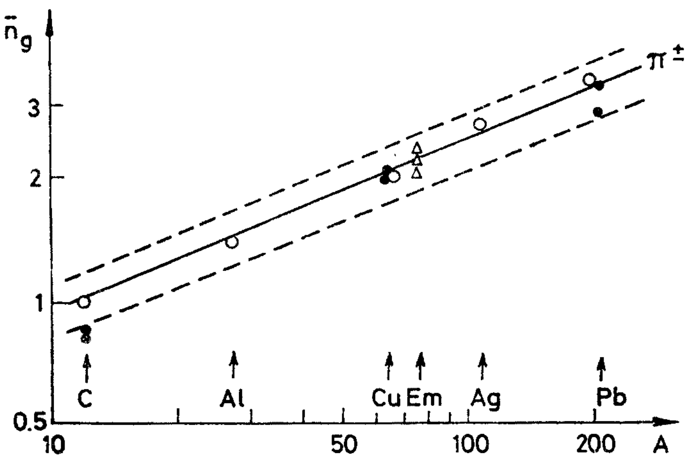

Fig. 1. Mean multiplicity of g-particles in $\pi^{\pm}A$ interactions: ● π^+A (50–150 GeV) [2]; ○ π^-A (37.5 GeV) [3]; Δ π^-Em (50–340 GeV) [1]; lines present theoretical calculations according to the expression (8)

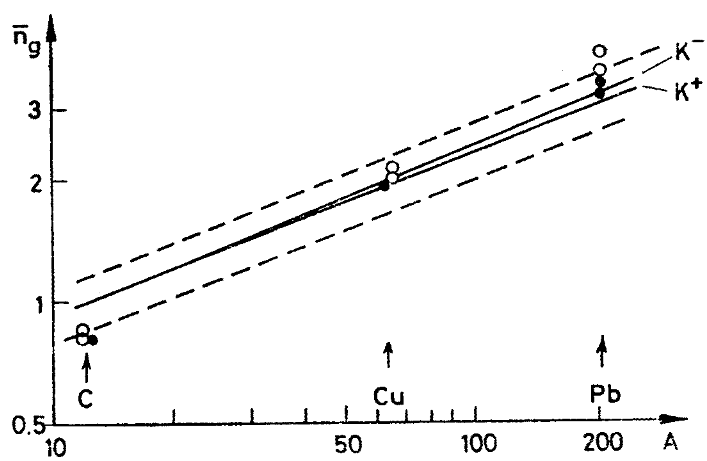

Fig. 2. Mean multiplicity of g-particles in $K^{\pm}A$ interactions: ● K^+A (50–150 GeV) [2]; ○ K^-A (50–150 GeV) [2]; lines present theoretical calculations according to the expression (9)

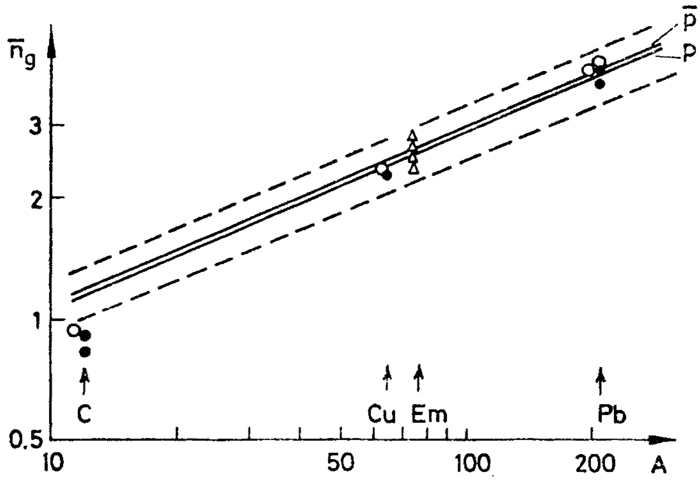

Fig. 3. Mean multiplicity of g-particles in p($\bar{p}$)A interactions: ● pA (50–150 GeV) [2]; ○ $\bar{p}A$ (50–150 GeV) [2]; ▲ pEm (50–400 GeV) [1]; lines present theoretical calculations according to the expression (9)

area corresponding to uncertainties in the parameters from expressions (8), (9) are shown by a dotted line. The experimental data for various nuclear targets at the primary energies 37.5 – 150 GeV [2, 3] are presented ibidem. These data are obtained under the conditions somewhat different from those of photoemulsion experiments. Particularly, in Refs. [2, 3] the proton energy upper limit was 300 MeV, while in the emulsions this limit for g-particles is 400 MeV. Therefore, to compare the electronic and emulsion experiment data, the portion of g-particles with the energies 300–400 MeV, some 10% [12] of their mean multiplicity, was subtracted from the emulsion data [1]. The photoemulsion data in the energy range 50–400 GeV, corrected in this way, are also given in Figs. 1–3.

As is seen from Figs. 1–3, the proposed theoretical scheme represents satisfactorily both absolute values and the A-dependence of g-particles mean multiplicities. Note that A-dependences are different for various primary hadrons; in parametrization of $\bar{n}_g \sim A^{\alpha}$ in the range of medium and heavy nuclei $\alpha = 0.42$ for primary protons (antiprotons), $\alpha = 0.41$ for pions and $\alpha = 0.4$ for kaons.

It should be emphasized, that according to expressions (8) and (9), the cascading of secondary pions from the target fragmentation region (the second term in formulae (8) and (9)) is the main contribution (78–87%) to the g-particle mean multiplicity.

The g-particle multiple production was investigated in a series of theoretical works [13, 14]. In these calculations the free parameters (defined from the hadron-nucleus interaction data) and, as a rule, the Monte Carlo methods are applied. In [14] an analytical expression is obtained by solving equations describing the stochastic process of g-particle "multiplication and death". The secondary low energy pion cascading that makes the main contribution to g-particle multiplicity, according to estimates of this paper, was not considered.

In describing the recoil nucleon secondary interactions in Ref. [14] the length of their free path in the nuclear matter λ_p was taken to be about 2 f, while the experimental estimate for λ_p for protons with energies from tens to hundreds MeV is noticeably higher: $\lambda_p \sim (5\div 10)$ f [15], i.e. one can, in fact, practically neglect their secondary interactions. As to recoil nucleons of higher energies, their secondary interactions can make some contribution to the g-particle production. However, their number is noticeably less (4–5 times), than that of pions from the fragmentation region, and their contribution to $\bar{n}_g$ does not practically exceed the uncertainty in the calculated values of $\bar{n}_g$, presented in Figs. 1–3.

Thus, in the approach stated in this paper one succeeds to obtain a satisfactory description of available experimental data on $\bar{n}_g$ at high energies by simple analytical expressions, without using free parameters. For the quantitative estimation of the degree of the model adequacy more precise experimental data both on nuclear targets and nucleons are required.

APPENDIX I

Calculation of secondary pion contribution to g-particle production

Consider the case when the incident (leading) hadron, at an impact parameter b, undergoes in the nucleus K inelastic collisions with the cross section σ_1^{in} in the points with the longitudinal coordinates $z_1, \ldots, z_K$. In each interaction point on the average m low energy pions are produced (in the target fragmentation region), whose formation time does not exceed mean internucleon distances in nucleus. Each pion interacts with nuclear nucleons with the mean cross section σ_3. Since pion energies are low, we shall assume that each pion can produce a g-particle in one interaction only, with the g-particle production probability being ω_2. The pion produced in the point with the coordinate z_i undergoes a secondary interaction with a probability (in the eikonal approximation)

$$1-\exp\left[-\sigma_3 \int_{z_i}^{\infty} \varrho(z)dz\right].$$

The expression for the mean multiplicity of g-particles, n_K, considering the contribution of pions from all points $z_1, \ldots, z_K$ in the framework of multiple scattering theory (MST) is the following:

$$P_K^{in}(b)\cdot n_K(b) = m\omega_2 e^{-\sigma_1^{in}T(b)}\cdot(\sigma_1^{in})^K \int_{-\infty}^{\infty} dz_1 \varrho(b, z_1)$$

$$\times \int_{z_1}^{\infty} \varrho(b, z_2)dz_2 \ldots \int_{z_{K-1}}^{\infty} \varrho(b, z_K)dz_K \cdot \{\sum_{i=1}^{K} (1-e^{-\sigma_3 \int_{z_i}^{\infty} \varrho(b,z')dz'})\}, \tag{I.1}$$

where $\varrho(b, z)$ is the one-particle nuclear density; $T(b) = \int_{-\infty}^{\infty} \varrho(b, z)dz$ (for the sake of briefness we shall later on omit the argument in $T(b)$); $P_K^{in}(b) = [\sigma_1^{in}T]^K/K!$ defines the probability of K-fold inelastic collision of the incident hadron in nucleus.

Carrying out the integration and summation in the right-hand side of (I.1), we obtain

$$P_K^{in}(b)n_K(b) = m\omega_2 \frac{(\sigma_1^{in}T)^K}{K!} e^{-\sigma_1^{in}T} K \frac{e^{-\sigma_3 T}-1+\sigma_3 T}{\sigma_3 T}. \tag{I.2}$$

Summing (I.2) in K,

$$\mu(b) = \sum_{K=1}^{A} P_K^{\text{in}}(b)\cdot n_K(b) = m\omega_2\sigma_1^{\text{in}}\left(T - \frac{1-e^{-\sigma_3 T}}{\sigma_3}\right). \tag{I.3}$$

Integrating (I.3) over the impact parameter b and normalizing to the total inelastic cross section of hadron-nucleus interaction, σ_A, we obtain the mean number of g-particles produced in interactions of secondary pions in nucleus:

$$\bar{n}_g^{\text{sec}} = m\omega_2 \frac{A\sigma_1^{\text{in}}}{\sigma_A}\left[1 - \frac{N(0;\sigma_3)}{A}\right], \tag{I.4}$$

where $N(0;\sigma) = \frac{1}{\sigma}\int (1-e^{-\sigma T})d^2b$ is the effective nucleon number.

The mean number of g-particles produced immediately in collisions of the incident hadron in nucleus is

$$\bar{n}_g^{\text{inc}} = \bar{\nu}\omega_1, \tag{I.5}$$

where

$$\bar{\nu} = \frac{A\sigma_1}{\sigma_A} \tag{I.6}$$

is the mean number of collisions of the incident hadron in nucleus, ω_1 is the g-particle production probability in the elementary hadron-nucleon interaction.

Thus the mean multiplicity of g-particles in the hadron-nucleus interaction is

$$\bar{n}_g = \bar{n}_g^{\text{inc}} + n_g^{\text{sec}} = \bar{\nu}\omega_1 + \frac{A\sigma_1^{\text{in}}}{\sigma_A} m\omega_2\left[1 - \frac{N(0;\sigma_3)}{A}\right]. \tag{I.7}$$

APPENDIX II

Consideration of the change of the leading particle type

Consider now the case, when the transfer of the leading property from an incident particle to a particle of another type is possible. Let us denote the probability of preserving the leading property in the elementary act by η; the probability of the leading property transfer will be $(1-\eta)$. The expression for the mean number of collisions of the leading particle in nucleus for this case, obtained in the framework of multiple scattering theory, is given in Ref. [5].

$$\nu' = \frac{A\sigma_2}{\sigma_A} + \frac{\sigma_1 - \sigma_2}{\sigma_A} N(0;\sigma_0) \tag{II.1}$$

where $\sigma_0 = (1-\eta)\sigma_1$.

Here is the derivation of (II.1).

The probability that at the given impact parameter b the incident particle, before reaching the point z_j, has interacted (with the cross section σ_1) l times preserving its leading property, and in the point z_j, due to the interaction with a nucleon, has transferred the leading property to a particle of another type, which has then interacted (with the cross section σ_2) $r = (K-l-1)$ times, is

$$P_{lr}(b) = \int_{-\infty}^{\infty} e^{-\sigma_1 \int_{-\infty}^{\infty} \varrho(b,z')dz'} \cdot \frac{1}{l!}\left[\eta\sigma_1 \int_{-\infty}^{z_j} \varrho(b, z')dz'\right]^l \cdot \sigma_0 \varrho(b, z_j)dz_j$$

$$\times \frac{1}{r!}\left[\sigma_2 \int_{-\infty}^{z_j} \varrho(b, z')dz'\right]^r \cdot e^{-\sigma_2 \int_{z_j}^{\infty} \varrho(b,z')dz'} \tag{II.2}$$

From (II.2) we obtain the probability of the K-fold collision of the leading particle ($K = l + r + 1$):

$$P_K(b) = (\sigma_0 T)e^{-\sigma_2 T} \int_0^1 dy e^{-(\sigma_2-\sigma_1)Ty} \cdot \frac{1}{(K-1)!}\left[\eta\sigma_1 Ty + b_2 T(1-y)\right]^{K-1}. \tag{II.3}$$

From (II.3) follows that the probability of the change of the particle leading property is

$$\sum_{K=1}^{A} P_K(b) = 1 - e^{-\sigma_0 T}, \tag{II.4}$$

and the mean number of collisions $\bar{K}(b)$ is defined by the expression

$$(1-e^{-\sigma_0 T}) \cdot \bar{K}(b) = (\sigma_2 T)\left(1 - \frac{\eta\sigma_1}{\sigma_2} e^{-\sigma_0 T}\right) + \frac{\eta\sigma_1 - \sigma_2}{\sigma_0} \cdot (1-e^{-\sigma_0 T}) + (1-e^{-\sigma_0 T}). \tag{II.5}$$

The probability that the leading particle colliding at the given impact parameter b K times, preserves its leading property is

$$P_K^{\mathrm{L}}(b) = \frac{(\eta\sigma_1 T)^K}{K!} e^{-\sigma_1 T}. \tag{II.6}$$

From (II.6) follows that the probability of preserving the leading property is

$$\sum_{K=1}^{A} P_K^{\mathrm{L}}(b) = e^{-\sigma_0 T} - e^{-\sigma_1 T}, \tag{II.7}$$

and the mean number of collisions is then defined by the expression

$$(e^{-\sigma_0 T} - e^{-\sigma_1 T}) \cdot \bar{K}(b) = \eta\sigma_1 T e^{-\sigma_0 T}. \tag{II.8}$$

Integrating the sum of the expressions (II.5) and (II.8) over b and normalizing to the total inelastic cross section of the interaction of the incident particle with nucleus, σ_A, we obtain the final expression (II.1).

Making use of the experimental fact that the g-particle production probability ω_1 in the elementary hadron-nucleon interaction is practically independent of the incident hadron type, we find that the contribution from the leading particle interactions in nucleus (without consideration of secondary pion interactions) to the g-particle multiplicity is

$$\bar{n}_g^L = \omega_1 \bar{\nu}' = \omega_1 \left[\frac{A\sigma_2}{\sigma_A} + \frac{\sigma_1 - \sigma_2}{\sigma_A} N(0; \sigma_0) \right]. \tag{II.9}$$

Note that at $\eta \to 1 (\sigma_0 = (1-\eta)\sigma_1 \to 0)$ or at $\sigma_2 \to \sigma_1$ formula (II.9) transforms into (I.5), where the possibility of the change of the leading particle type is not envisaged.

Let us pass to the calculation of the contribution of secondary pions in recoil proton production. Consider first the case when the incident particle is the leading particle in all inelastic collisions. In that case calculations are similar to those carried out in Appendix I with the difference that the factor $(\sigma_1)^K$ in (I.1) is replaced by $(\eta^{\text{in}}\sigma_1^{\text{in}})^K$. Formula (I.3) then takes the form

$$\mu_{\mathrm{h}\to\mathrm{h}}(b) = m\omega_2 \left[(\eta^{\text{in}}\sigma_1^{\text{in}} T) e^{\sigma_0 T} - \eta^{\text{in}}\sigma_1^{\text{in}} \frac{e^{-\sigma_0 T} - e^{-(\sigma_0+\sigma_3)T}}{\sigma_3} \right], \tag{II.10}$$

where η^{in} is the probability of preserving the leading property in the elementary act of inelastic interaction on a nucleon. Note that $\sigma^{\text{in}}(1-\eta^{\text{in}}) = \sigma(1-\eta) = \sigma_0$. It is obvious that at $\sigma_0 \to 0$ the formula (II.10) turns into (I.3).

Consider now the case when the incident hadron in the j-th collision (in the point z_j) transfers the leading property to a particle of another type, which then undergoes in the nucleus $(k-j)$ collisions. One should here consider separately three cases: a) $j > i$; b) $j < i$; c) $j = i$. For example, for the case a) the i-th term of the expression, similar to the expression (I.1), has the form

$$\mu_{i<j}^{(i,j,K)} = m\omega_2 e^{-\sigma_1^{\text{in}} T(b)} \cdot (\eta^{\text{in}}\sigma_1^{\text{in}})^{j-1} \sigma_0 (\sigma_2^{\text{in}})^{K-j} \int_{-\infty}^{\infty} dz_1 \varrho(z_1) \ldots \int_{z_{i-2}}^{\infty} dz_{i-1} \varrho(z_{i-1})$$
$$\times \int_{z_{i-1}}^{\infty} dz_i \varrho(z_i) \left(1 - e^{-\sigma_2^{\text{in}} \int_{z_i}^{\infty} \varrho(b,z')dz'}\right) \cdot \int_{z_i}^{\infty} dz_{i+1} \varrho(z_{i+1}) \ldots \int_{z_{j-2}}^{\infty} dz_{j-1} \varrho(z_{j-1})$$
$$\times \int_{z_{j-1}}^{\infty} dz_j \varrho(z_j) e^{-(\sigma_3 - \sigma_1^{\text{in}}) \int_{z_j}^{\infty} \varrho(b,z')dz'} \int_{z_j}^{\infty} dz_{j+1} \varrho(z_{j+1}) \ldots \int_{z_{K-1}}^{\infty} dz_K \varrho(z_K). \tag{II.11}$$

After some simple but somewhat awkward calculations, one succeeds in integrating the expression (II. 11) over longitudinal coordinates and summing in $i = 1 \div (j-1)$, $j = 2 \div K$, $K = 2 \div A$; after that the right-hand side of the expression reads

$$\mu_{i<j}(b) = m\omega_2 \eta^{\text{in}} \sigma_1^{\text{in}} \left[-\frac{e^{-\sigma_3 T} - e^{-\sigma_0 T}}{\sigma_0 - \sigma_3} + \frac{e^{-\sigma_0 T} - e^{-(\sigma_0+\sigma_3)T}}{\sigma_3} + \frac{1 - e^{-\sigma_0 T}}{\sigma_0} - T e^{-\sigma_0 T} \right]. \tag{II.12}$$

Similarly for the cases of $j < i$ and $j = i$ we obtain

$$\underset{i>j}{\mu(b)} = m\omega_2\sigma_2^{\text{in}}\left[\frac{\sigma_0(e^{-\sigma_3 T}-e^{-\sigma_0 T})}{\sigma_3(\sigma_0-\sigma_2)} - \left(\frac{\sigma_0}{\sigma_3}+1\right)\cdot\left(\frac{1-e^{-\sigma_0 T}}{\sigma_0}\right)+T\right], \quad \text{(II.13)}$$

$$\underset{i=j}{\mu(b)} = m\omega_2\sigma_0\left[\frac{1-e^{-\sigma_0 T}}{\sigma_0} - \frac{e^{-\sigma_3 T}-e^{-\sigma_0 T}}{\sigma_0-\sigma_3}\right]. \quad \text{(II.14)}$$

Summing the expressions (II.10), (II.12), (II.13), (II.14), integrating them over the impact parameter b and normalizing to the total inelastic cross section σ_A, we obtain the mean multiplicity of g-particles, produced due to interactions of secondary pions in nucleus:

$$\bar{n}_g^{\text{sec}} = m\omega_2\left[\frac{A\sigma_2^{\text{in}}}{\sigma_A} + \frac{\sigma_3(\sigma_1^{\text{in}}-\sigma_2^{\text{in}})}{\sigma_3-\sigma_0}\,\frac{N(0;\sigma_0)}{\sigma_A} - \frac{\sigma_1^{\text{in}}\sigma_3-\sigma_0\sigma_2^{\text{in}}}{\sigma_3-\sigma_0}\,\frac{N(0;\sigma_3)}{\sigma_A}\right]. \quad \text{(II.15)}$$

The total mean multiplicity of g-particles is

$$\bar{n}_g = \bar{\nu}'\omega_1+\bar{n}_g^{\text{sec}}. \quad \text{(II.16)}$$

It is easy to check that the expression (II.16) at $\sigma_0 \to 0$, i.e. when the probability of the leading property transfer approximates zero, goes into the expression (I.7).

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