# FOUR-PARTICLE CORRELATIONS IN NUCLEI ON THE $sd\text{-}f_{7/2}$ LEVELS*

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The four-body $J = 0$ and $T = 0$ or $T = 2$ interacting term was explicitly introduced into the shell-model pairing Hamiltonian for nuclei whose protons and neutrons fill up the same harmonic oscillator shell. Due to the quasi-spin symmetry SO(5) the Hamiltonian was analytically diagonalised for one $j$-shell and it was also applied to the schematic two-level model $sd\text{-}f_{7/2}$. It was found that even the small admixture of the four-body interaction provides a possibility for theoretical description of the excited $0^+$ levels for nuclei with $A \geqslant 40$. State vectors obtained by diagonalisation were then used for transfer probability calculations leading to comparison of the probabilities of the cluster and step by step transfer reactions. The predicted new "four-body superfluidity" could be interpreted as a background for a possible two-paired boson "abnormal state" within Interacting Boson Model.

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## 1. Introduction

The existence of dynamical structures consisting of several nucleons on the nuclear surface seems to be very firmly established. Among those structures the most common are two-particle and four-particle alpha-like clusters. The alpha-decay of heavy nuclei could be interpreted as an evidence for such clusters in nuclei. The large variety of alpha-transfer experiments like $(^6\mathrm{Li}, \alpha)$, $(^7\mathrm{Li}, \mathrm{t})$ or $(^{16}\mathrm{O}, ^{12}\mathrm{C})$ have also pointed out to such clusters. Moreover, the reactions of the type $(\mathrm{p}, \alpha)$ or $(^{12}\mathrm{C}, ^{16}\mathrm{O})$ also need the ready to use (reaction time is about $10^{-22}$ s) alpha-like structures on the nuclear surface. These and other pieces of experimental evidence are well known, see for example [1–4]. Recently [5] the so called energy of the cluster correlations has been separated from the total binding

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energy of nuclei and the conclusion has been drawn that two-nucleon and four-nucleon clusters exist almost in any region of nuclei.

The motivation of the present theoretical work is similar to those explained in the papers [1–3]. Hence, we repeat only the main lines stressing the generalization of the present work as compared to the previous ones.

From the commonly accepted assumption that the two-nucleon interaction is responsible for the behaviour of nuclei, the observed strong four-particle correlations should follow as well. Such programme faces, however, at least two main difficulties. First, so far we do not know the exact two-body nuclear interaction and so we are forced to choose more or less fortunate phenomenological shape for nuclear forces. Moreover, such assumptions have to be taken differently for description of different properties of nucleus, or different kind of nuclei. Secondly, even with the best choice of two-body interaction, the technique to solve the nuclear problems, say within the shell model, needs the configuration space to be cut off. But taking the phenomenological two-body interaction, we are not sure whether or not we lose that part of nuclear properties we are interested in. Let us illustrate this point by a simple example. Assume the nucleons on a single $j$-level and assume the generalised pairing interaction as well among protons, neutrons and protons-neutrons with the same intensity $G$. The pairing energy [6] under such assumptions can be analytically written in the form

$$E_j = -\frac{G}{4}\left[(n-v)\left(2j+4-\frac{n}{2}-\frac{v}{2}\right)-2T(T+1)+2t(t+1)\right], \tag{1}$$

where $n$ is the number of nucleons, $v$ – seniority number, $T$ – total isospin, $t$ – so called reduced isospin.

In Fig. 1 we plot the binding energy of the last two paired particles against the number of particles in a cluster. It can be seen from Fig. 1 that every second pair of particles which closes the alpha-like structure of the cluster has larger binding energy than its neighbouring pairs. However, it becomes clear that the four-body correlations following pair-coupling are too weak to explain the observed four-particle correlations in nuclei. On the other hand, the cut off of the one-particle configuration space can be improved by taking effective interaction which consists not only of two-body, but also, for example, four-body part of interaction.

Having the above arguments in mind, we have chosen the phenomenological Hamiltonian in the shell model approximation consisting of a residual pairing part generalized on both protons and neutrons and a four-body effective part. Then, the shape of a four-body part has been governed, by assumption, by the SO(5) dynamical symmetry the same as for a pairing part in $j-j$ coupling. Under these assumptions the chosen Hamiltonian has as building blocks the generators of SO(5) transformations and hence the matrix elements of the Hamiltonian can be analytically calculated as well for pairing as for the four–body part with the help of the known algebraic techniques.

Hence, the four-body structures considered here are of the form

$$\{(a_j^+ a_j^+)^{J'=0;T'=1}(a_j^+ a_j^+)^{J'=0;T'=1}\}^{J=0;T}_{M=0;T_0}. \tag{2}$$

According to the Pauli principle, the allowed $T$ values in (2) are $T = 0$ and $T = 2$. The isoscalar four–body structures with $T = 0$ have already been considered [1, 2] and some special cases of isotensor $T = 2$ correlation with the seniority zero have also been discussed [7].

The generalization of the present approach is due to the possibility of considering on the same footing both the isoscalar and isotensor parts of the four–body effective interaction for arbitrary seniority. The SO(5) dynamical symmetry is a background enabling such generalization.

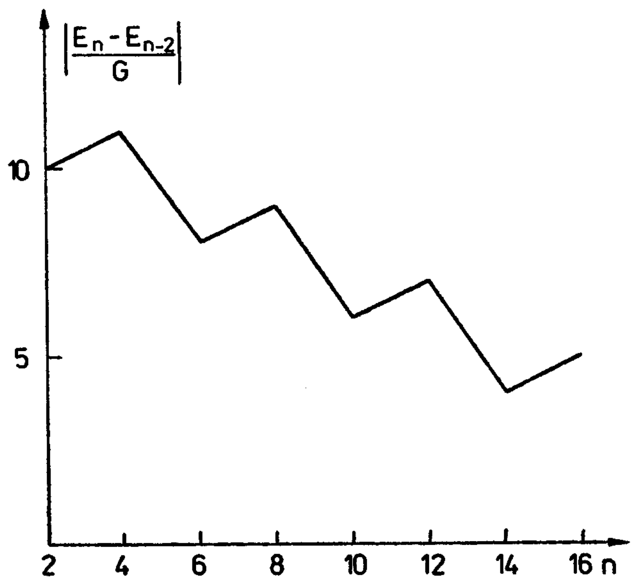


Fig. 1. The binding energy for the last two paired particles ($E_n$–$E_{n-2}$) against the number of particles in a cluster

Although we have applied the calculated matrix elements of such correlated four-particle structures to a schematic two-level model of nuclei, we have also tried to draw some quantitative physical conclusions. This has been done by a comparison of experimental and calculated $0^+$ energy levels. We have also discussed in the framework of the model the enhancement of many neutron transfer against the several step transfer of the same number of neutrons. At the end we will briefly mention a possible connection of the presented model with Interacting Boson Approximation in the domain of a predicted four-body phase transition.

The approach in the chosen Hamiltonian is general enough to be applied to a large region of nuclei and for the description of different nuclear properties. In Part 2 of the paper we shall develop the formalism of the quasi-spin SO(5) symmetry for four-body interactions and Part 3 will contain results and conclusions.

## 2. *Two- and four-body interaction in the quasi-spin formalism*

Let us define the creation and annihilation two-particle, $J = 0$, operators

$$P_k^+(j) = \sum_{m>0,\tau_1,\tau_2} (-1)^{j-m}(\tfrac{1}{2}\tau_1; \tfrac{1}{2}\tau_2|1k)a^+_{jm\tau_1}a^+_{j-m\tau_2}$$
$$= \tfrac{1}{2}(2j+1)^{1/2}(a_j^+ a_j^+)^{J=0;\,T=1}_{M=0;\,k}, \tag{3}$$
$$P_k(j) = [P_k^+(j)]^+,$$

where $\tau_{1(2)} = \pm\frac{1}{2}$ and $k = -1, 0, 1$ stand for the third components of the isospin for one- and two-particle coupling respectively.

The pairing Hamiltonian for both protons and neutrons is now

$$H_{\text{pair}} = -\sum_{j_1,j_2,k} G_k P_k(j_1)P_k(j_2). \tag{4}$$

If we consider only one $j$-level and suppose $G_k = G = \text{const}$ then the Hamiltonian (4) can be diagonalised analytically and the energy formula (1) follows from the diagonalisation. It is due to the observation that the ten coupled creation and destruction operators

$$(a_j^+ a_j^+)^{J=0;\,T=1}_{k}, \quad (\tilde{a}_j\tilde{a}_j)^{J=0;\,T=1}_{k},$$
$$(a_j^+ \tilde{a}_j)^{J=0;\,T=0}, \quad (a_j^+ \tilde{a}_j)^{J=0;\,T=1}_{k}, \tag{5}$$

where $\tilde{a}_{jm\tau} = (-1)^{j+m+\frac{1}{2}+\tau}\, a_{j-m-\tau}$ and $k = 0, \pm 1$ form the closed set under commutation and they form the generators of infinitesimal transformations of the orthogonal quasi-spin group SO(5) [6].

Irreducible representations of the group SO(5) are factorized by two numbers $\lambda_1$ and $\lambda_2$ which have a group theory meaning as numbers of two fundamental representations used in the construction of a given irreducible representation $(\lambda_1, \lambda_2)$. These two group theory numbers are connected, in the quasi-spin formalism, to the seniority number $v$ and the reduced isotopic spin $t$ [8]

$$\lambda_1 = 2t, \quad \lambda_2 = j+\tfrac{1}{2}-\frac{v}{2}-t. \tag{6}$$

The most important physical state vectors form the bases of the irreducible representations $(0, \lambda_2)$ and $(1, \lambda_2)$. For those representations one needs three further quantum numbers for complete specifications of the state-vectors. Those quantum numbers can be chosen with physical meaning as quantum numbers of the total isospin $T$, its third component $T_0$, and the particle number $n$. Then the state vectors are uniquely denoted by $|\lambda_1\lambda_2;\, nTT_0\rangle$.

The four-body part of the Hamiltonian is constructed with the help of the four body creation and destruction operators defined in the following formulas

$$Q^+_{\lambda\mu}(j) = \sum_k (1k;\, 1\mu-k|\lambda\mu)P_k^+(j)P^+_{\mu-k}(j)$$
$$= \tfrac{1}{4}(2j+1)\,\{(a_j^+ a_j^+)^{J=0;\,T'=1}(a_j^+ a_j^+)^{J=0;\,T'=1}\}^{J=0;\,\lambda}_{\mu} \tag{7}$$

and

$$Q_{\lambda\mu}(j) = [Q_{\lambda\mu}(j)]^+.$$

Then the compact shape of the Hamiltonian reads

$$H = \sum_{jm\tau} \varepsilon_j a^+_{jm\tau} a_{jm\tau} - G \sum_{j_1, j_2, k} P^+_k(j_1) P_k(j_2) - \tfrac{1}{4} \sum_{j_1, j_2, \lambda, \mu} \chi_\lambda Q^+_{\lambda\mu}(j_1) Q_{\lambda\mu}(j_2). \tag{8}$$

The operators $P$ and $Q$ have following tensor characters, needed for algebra calculation, in the isospin space:

$$P^+_k(j) = \mathscr{T}^{(1)}_k, \quad (-1)^{1-k} P_k(j) = \mathscr{T}'^{(1)}_{-k},$$
$$Q^+_{\mu\lambda}(j) = \mathscr{T}^{(\lambda)}_\mu, \quad (-1)^\mu Q^{(j)}_{\lambda\mu} = \mathscr{T}'^{(\lambda)}_{-\mu}. \tag{9}$$

For only one $j$-level, after straightforward calculation, the Hamiltonian (8) is shown to be diagonal in the SO(5) basis $|vt;\ nTT_0\rangle$ and the eigenvalue problem has been analytically solved

$$E = E_s + E_{\text{pair}} + E_{T=0} + E_{T=2}, \tag{10}$$

where $E_s = \varepsilon n$ and

$$-\frac{1}{G} E_{\text{pair}} = \tfrac{1}{4} \left\{ (n-v)\left( \Omega + 3 - \frac{n}{2} - \frac{v}{2} \right) - 2T(T+1) + 2t(t+1) \right\},$$

$$-\frac{1}{\chi_0} E_{T=0} = \tfrac{1}{192} (n-v-2T)(n-v+2T+2)(2\Omega+4-n-v-2T) \times (2\Omega+6-n-v+2T),$$

$$-\frac{1}{\chi_2} E_{T=2} = -\frac{2E_{T=0}}{\chi_0} + \tfrac{1}{8}(n-\Omega-4)[(n-v)(2\Omega+6-n-v) + 2T(T+1)(n-\Omega-4)], \tag{11}$$

where $\Omega = 2j+1$ is the pair degeneracy.

For several $j$-levels, a direct product of several irreducible representations of the group SO(5) must be considered. In the many $j$-levels too, the vectors $|vt;\ nTT_0\rangle$ provide a convenient set for the construction of a complete basis, either as "weak" or "strong coupled" in the space SO(5). Previously calculated matrix elements of the operators $P^+_k(j)$ and $P_k(j)$ [8] were then used in calculation of matrix elements of the four particle clusters $Q^+_{\lambda\mu}(j)$ and $Q_{\lambda\mu}(j)$.

The calculations were done in the frame of the very schematic two level model sd-f, Fig. 2, for nuclei above $A = 40$ i.e. for Ca isotopes with $A = 40, 42, 44, 46$ and for Ti isotopes with $A = 42, 44, 46, 48$.

The strong coupled basis

$$|(0\lambda_2)(0\lambda'_2); (\lambda''_1\lambda''_2) nTT_0\rangle_x \tag{12}$$

has some disadvantages: the coupled vectors need an additional quantum number $x$ to be fully factorized and such quantum number with physical meaning has not been found so far. For vector construction one needs the Wigner coefficients for the group SO(5) which are only partially known and, moreover, the one particle part of the Hamiltonian (8) is not diagonal in this basis. We therefore decided to use a weak coupled basis defined by the expression

$$|(0\lambda_2)n_1T_1;(0\lambda_2')n_2T_2;\,nTT_0\rangle = \sum_{T_0'T_0''}(T_1T_0'T_2T_0''|TT_0)$$
$$\times|(0\lambda_2)n_1T_1T_0'\rangle\,|(0\lambda_2')n_2T_2T_0''\rangle, \tag{13}$$

where subcripts 1 and 2 denote the sd and $f_{7/2}$ level respectively.

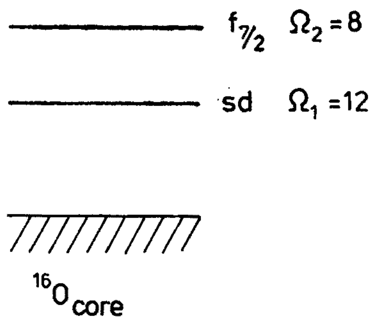


Fig. 2. The schematic two-level shell-model sd–$f_{7/2}$ using the spherical core $^{16}O$

The diagonal and non-diagonal matrix elements of the Hamiltonian (8) in the basis (13) were obtained by straightforward calculation:

$$\langle n_1T_1;\; n_2T_2;\; nTT_0|H|n_1T_1;\; n_2T_2;\; nTT_0\rangle = \tfrac{1}{2}\,\varepsilon(n_2-n_1)+E_1+E_2 \tag{14}$$

where $E_1$ and $E_2$ are the energies (10) taken for the sd and $f_{7/2}$ levels respectively:

$$\langle n_1'T_1';\,n_2'T_2';\,nTT_0|H|n_1T_1;\,n_2T_2;\,nTT_0\rangle = -G(-1)^{T_1+T+T_2'+1}\begin{Bmatrix}T_1T_2T\\T_2'T_1'1\end{Bmatrix}$$
$$\times[\langle n_1'T_1'\|P\|n_1T_1\rangle\,\langle n_2'T_2'\|P^+\|n_2T_2\rangle+\langle n_1'T_1'\|P^+\|n_1T_1\rangle\,\langle n_2'T_2'\|P\|n_2T_2\rangle]$$
$$-\tfrac{1}{4}\sum_{\lambda=0,2}\chi_\lambda(-1)^{T_1+T+T_2}\begin{Bmatrix}T_1T_2T\\T_2'T_1'\lambda\end{Bmatrix}[\langle n_1'T_1'\|Q_\lambda\|n_1T_1\rangle\,\langle n_2'T_2'\|Q_\lambda^+\|n_2T_2\rangle$$
$$+\langle n_1'T_1'\|Q_\lambda^+\|n_1T_1\rangle\,\langle n_2'T_2'\|Q_\lambda\|n_2T_2\rangle]. \tag{15}$$

The reduced matrix elements of the $Q_{\lambda\mu}^+$ and $Q_{\lambda\mu}$ are given in the Appendix. The diagonalisation of the Hamiltonian $H$ is followed by the energy levels $J = 0$ for which the state-vectors were constructed with a given admixtures of the basis vectors (13). The constructed eigenvectors were then used to calculate the particle transfer and to evaluate the enhancement for many neutron transfer.

Exact results and their comparison with experimental data will be given in the next section. However, we want to note now that the very schematic two-level model is not expected to provide exact description of experiments. It is rather worth to stress that in spite of the simplified model, the four-body interaction works rather well.

## 3. *Results*

### 3.1. Energy calculations

The energy calculations were performed for the Ca isotopes ($A$ = 40, 42, 44, 46) and for Ti isotopes (A = 42, 44, 46, 48). For the schematic two-body and four-body interactions (8) it was possible to evaluate the energies of the ground and excited $0^+$ states. There were evaluated, for each nucleus, the energies of the first three excited $0^+$ states taken relatively to the ground state ($E_g = 0$). The calculations were done with pairing ($P$) and four-particle isoscalar ($Q_0$) and isotensor ($Q_2$) interactions. The model Hamiltonian (8) involves four free parameters: the energy difference $\varepsilon$ of two one-particle levels

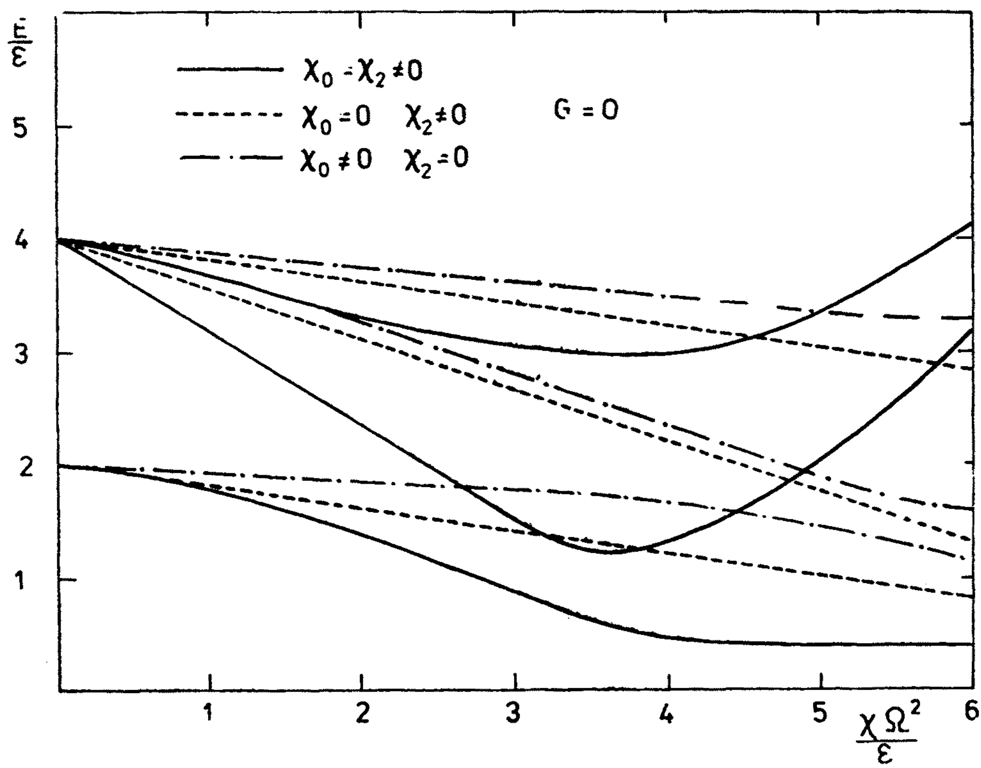


Fig. 3. The energies of the first three excited $0^+$ levels ($E_{\text{ground}} = 0$) against the strength parameter $\chi\Omega^2/\varepsilon$ of the four-body interaction with different admixtures for $^{40}$Ca nucleus. A good fit is provided for $\chi_0 = \chi_2 = 0.043$ MeV with $\Omega^2 = 96$ and $\varepsilon = 2.3$ MeV ($\chi\Omega^2/\varepsilon \approx 1.80$)

and the intensities $G$, $\chi_0$, $\chi_2$ of the considered interactions. The typical dependence of the energy levels versus intensity of the interaction is given in Fig. 3 and Fig. 4.

Analysing the vast numerical data and comparing them with experimental results allowed us to draw the conclusion that the best fit is provided by the $Q_0+Q_2$ interaction alone. The conclusion, on the first sight rather strange, can be physically explained by the remark that the four-body interaction contains already the pairing contribution. We further diminished the number of free parameters taking $\chi_0 = \chi_2$ for two parts of four body interactions. The effective energy difference $\varepsilon$ between the two one-particle levels

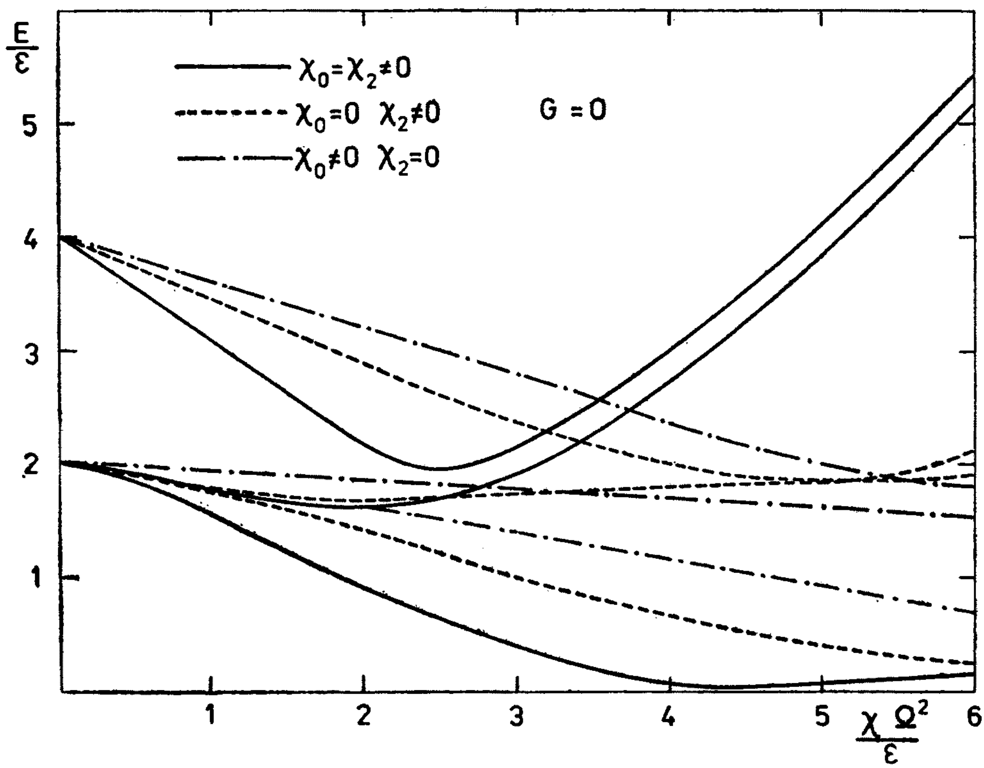


Fig. 4. The same as in Fig. 3 but for the $^{42}$Ca (or $^{42}$Ti) nucleus

$\Omega_1 = 12$ and $\Omega_2 = 8$ ($\Omega_{1,2}$ – is a pair degeneracy) was fixed as $\varepsilon = 2.30$ MeV. Comparison of the calculated energies with experimental data [9–10] is given in Fig. 5 (Ca isotopes) and in Fig. 6 (Ti isotopes). The two parameter model Hamiltonian described the energy levels for the Ca isotopes well and for the Ti isotopes fairly well.

It follows from the comparison of experimental and theoretical results that the four-body scalar plus tensor interaction comprises enough pairing correlations, on the one hand and provides a second order correction due to the configuration space cut-off, as a four-body effective interaction. Certainly it is not a unique model describing the $0^+$ states of the considered nuclei.

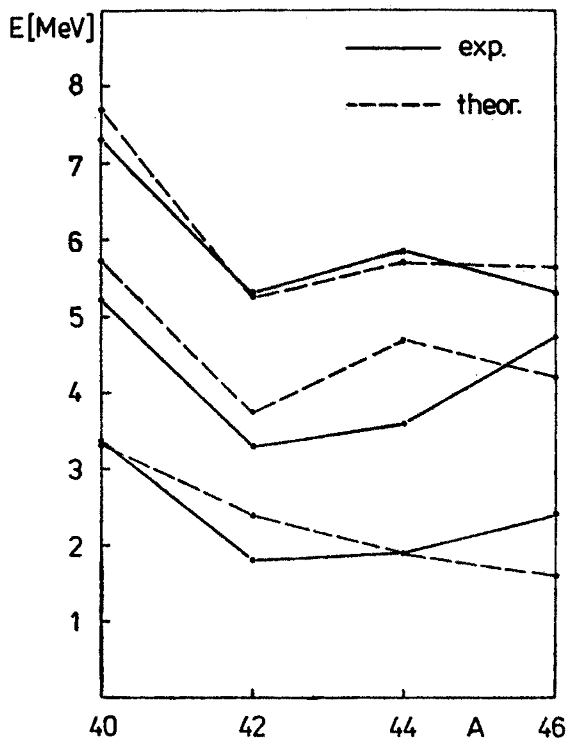


Fig. 5

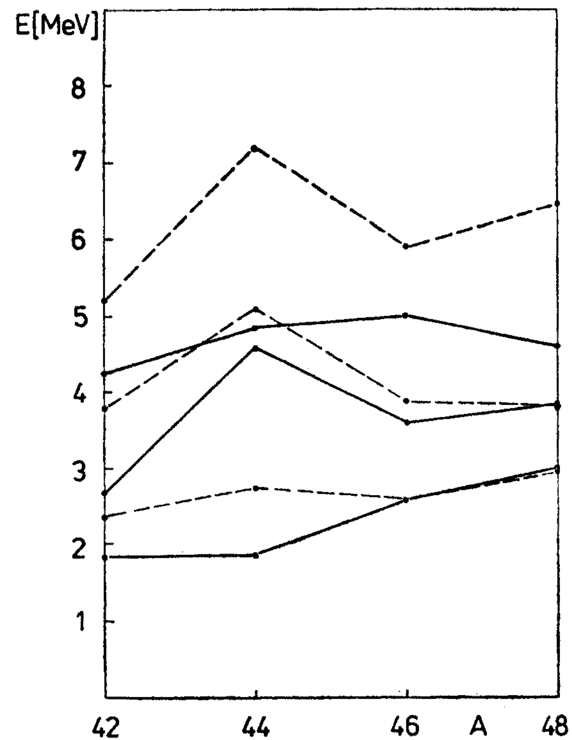


Fig. 6

Fig. 5. Comparison of the calculated $0^+$ energies with experimental data [9–11] for Ca nuclei with the Hamiltonian strength parameters $G = 0$ and $\chi_0 = \chi_2 = 0.043$ MeV

Fig. 6. The same as in Fig. 5 but for the Ti nuclei

### 3.2. Enhancement of many neutron cluster transfer

In a semi-classical description the cross section for a transfer of $n$-nucleons can be factorized into two parts [12] for the heavy ion scattering below the Coulomb barrier

$$\frac{d\sigma}{d\Omega} = P_n \left(\frac{d\sigma}{d\Omega}\right)_{\text{elastic}} . \tag{16}$$

It follows that the transfer probability $P_n$ is a measurable quantity. It has been also noticed [13] that the nucleons are preferentially transfered in separated steps rather than in one step as a cluster. If we assume that the intermediate ground states for step by step transfers are the states which take on the most of the transfer probabilities and that the mean time-independent interaction responsible for the transfer is a good approximation, we may then write

$$P_1' P_1'' \ldots P_1^{(n)} = C_n P_n, \qquad C_n > 1, \tag{17}$$

where $P_1, P_2$ are non-step single (or pair) transfer probabilities and $P_n$ is a transfer probability of a cluster. However, the experimental evidence, as for example in the reaction $^{A1}$Sn($^{A2}$Ni, $^{A3}$Ni)$^{A4}$Sn shows reversed effects, namely $C_n < 1$ or even $C_n \ll 1$ [14].

In the model case considered here, we are in position to search for the theoretical comparison of the probability of $n$-cluster transfer with the probability of several single or pair transfers.

In what follows, we keep the line of shell model transfer consideration by Kurath and Towner [15] with adaptation to our model Hamiltonian as done in the work of Ref. [2]. Hence we can rewrite, under proper simplifications, the formula (7.1) of the paper [2]

$$\frac{d\sigma}{d\Omega} \sim \left| \begin{bmatrix} n_0+n \\ n_0 \end{bmatrix}^{\frac{Q}{2}} \langle n_0+n, L \| \chi_n^+ \| n_0, T \rangle G_n \right|^2 , \tag{18}$$

where $Q$ is the number of oscillator quanta for relative motion of the nucleus $n_0$ and the transfered cluster $n$, $\chi_n^+$ is a transfer operator. The $G_n$ are the proper $n$-cluster product of the Moshinsky-Talmi brackets for the transformation to c.m. coordinates. The reduced matrix elements in (18) can be analytically evaluated using the eigenvectors obtained from the Hamiltonian (8), because the transfer operator $\chi_n^+$ is the product of generators for the SO(5) transformations.

The preliminary numerical results are obtained for the probability of the step-by-step transfers $^{40}$Ca $\to$ $^{42}$Ca $\to$ $^{44}$Ca $\to$ $^{46}$Ca as compared to the cluster transfer $^{40}$Ca $\to$ $^{46}$Ca. The product of the first two factors in (18) has been obtained with almost the same value for both types of transfers. However, the $G_n$ factor in (18) is much lower for the cluster transfer than for the step-by-step reaction and hence we have obtained the relative probability factor $c_n$ equal $c_n \approx 10^3$, which is in accord with the semi-classical prediction [13], but contradicts the interpretation involved in the experimental work of the reference [14]. The approximate evaluation of the $c_n$ factor with the zeroth order shell model functions (no mixing assumed) gives also the enhancement of the step-by-step transfer over the cluster transfer in the region $^{58}Ni \rightarrow {}^{64}Ni$ considered in [14].

These are preliminary results and an enlarged programme for enhancement calculations is in the present time in preparation.

### 3.3. Four-body interaction and Interacting Boson Model

If we literally take the boson of Arima and Iachello [16–17] as a properly correlated pair of nucleons, then the two-boson interaction may be viewed as a four-body part of the fermion Hamiltonian. Hence from this point of view we may consider the four-body isoscalar and isotensor interactions as a background analogy of the interacting boson Hamiltonian with the s-boson only, but having three kinds of boson: neutron bosons, proton bosons and mixed neutron-proton bosons. The boson formalism in this case ought to be slighty more complicated than the $F$-spin formalism introduced by Arima et al. [18] for boson case. Whether or not this analogy is exact, is still a question demanding further, more developed considerations.

In this respect we may make the remark that looking on the four-body correlation more from the physical viewpoint we can assign to it a possibility for nuclei to be in a "normal" or "superfluid" state against the four-body correlation. In the boson description that four-body superfluidity can be viewed as a two-body pairing abnormal state in analogy to

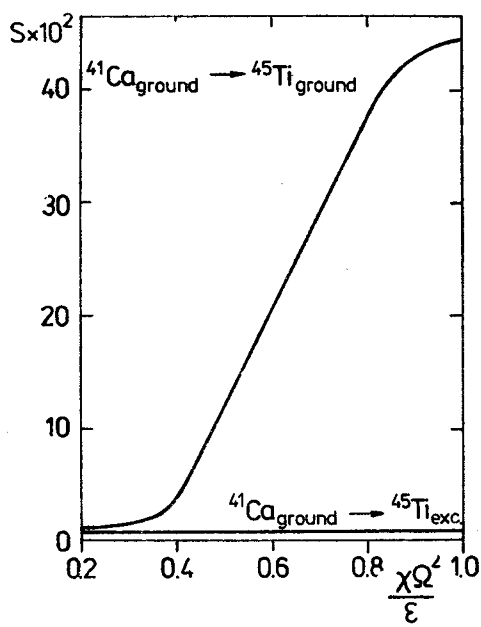


Fig. 7. Spectroscopic factors $^{(4)}S$ for the four body transfer with the constant pairing ($G\Omega/\varepsilon = 0.2$) versus the strength of the four-body interaction ($\chi_0\Omega^2/\varepsilon$). The rapid increase of the factor $^{(4)}S$ for the transfer from the ground to ground state with almost the zero value for the transfer from the ground to excited state of Ti shows a possible "superfluid" alpha-particle state

the pairing fermion case. Then, the four-body spectroscopic transfer factor can be used as a measure of a "normal" or "superfluid" nucleus state. Namely, if it is in the "normal" state, then the levels above the Fermi level are empty and the levels below are fully occupied. The transfer of a four-particle structure will have equal probabilities to any of the empty levels either in the ground or excited states. Otherwise, for the "superfluid" nucleus, still in respect of four-body correlations, the levels are neither empty nor occupied and one can assign to each four-particle level an occupation probability only. The four-particle transfer must then necessarily change the structure of a nucleus and the new structure will be predominantly of the ground state structure.

To summarise, the formalism presented here is able to describe both: the "normal" and "superfluid" states of nuclei. We may mention the results of our previous calculations [2] for the transfer of alpha-like structure from $^{45}$Ti to $^{41}$Ca. In the Fig. 7 the four-body spectroscopic factor is given versus the strength of the isoscalar four-body interaction with inclusion of pairing forces. The rapid change of the spectroscopic factor for the value $\chi_0\Omega^2/\varepsilon \approx 0.7$ shows that the $^{41}$Ca isotope would change its state from "normal" to a "superfluid" condition. This value of the strength parameter seems to play the same role as $G_{crit}$ for the pairing force. A more detailed study of this problem is, however, needed to pursue an experimental search for the "new superfluidity".

## 4. *Conclusions*

To give the proper account for the known experimentally four-body correlations in nuclei, we have proposed the phenomenological Hamiltonian consisting of a pairing part generalized to protons and neutrons and a four-body effective interaction. Both parts of the Hamiltonian were constructed from the generators of orthogonal transformations of the group in 5-dimensional abstract space. Due to the construction it was possible to diagonalize the Hamiltonian for even nucleons on a given $j$-shell level and to obtain analytical formulas for matrix elements of the Hamiltonian in the case of several $j$-levels. The generalization of the present theoretical approach is mainly due to the general treatment on equal footing of the isoscalar ($T = 0$) and isotensor ($T = 2$) parts of the four-body interaction.

The Hamiltonian was then applied to the even nuclei of $40 \leqslant A < 50$ in the two-level space sd-$f_{7/2}$ model. We have concluded that even generalized pairing interaction does not comprise enough four-body correlations to describe the $0^+$ excited levels for the isotopes chosen for comparison with the experimental ones. Then the effective four-body part of the Hamiltonian plays the very essential role.

The same formalism was applied to the comparison of a cluster with a step-by-step transfer reaction of nucleons. The conclusion was drawn that is in qualitative agreement with the semi-classical transfer theory but in opposition to the recent experimental report [14].

At last, we have made comments on the possible existence of the four-body superfluidity in nuclei. It was also suggested that such kind of superfluidity may have the analogy in the boson pairing condensation within the Interacting Boson Model.

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## APPENDIX

### *Matrix elements of the operators $Q^+_{\lambda\mu}$ and $Q_{\lambda\mu}$*

The $Q^+$ ($Q$) (7) operators are quadratic in the operators $P^+$ ($P$) (3) whose matrix elements have been obtained earlier. Hence, using the standard algebraic methods one may obtain in a straightforward way the matrix elements of the operators $Q^+$ ($Q$) between the states which form the basis for the irreducible representations (0, $\lambda_2$) and (1, $\lambda_2$) of the quasi-spin group SO(5). We tabulated below the reduced (in the isospin space) matrix elements of the operators $Q$ from which one can get the matrix elements of both: creation $Q^+$ and destruction $Q$ four-body operators, using the following relations:

$$\langle n'T'\|Q_\lambda\|nT\rangle = (-1)^{T'+\lambda-T}\langle nT\|Q^+_\lambda\|n'T'\rangle, \tag{A1}$$

$$\langle n'T'T_0+\mu|Q^+_{\lambda\mu}|nTT_0\rangle = (2T'+1)^{-1/2}(TT_0\lambda\mu|T'T_0+\mu)\,\langle n'T'\|Q^+_\lambda\|nT\rangle, \tag{A2}$$

$$\langle n'T'T_0-\mu\|Q_{\lambda\mu}\|nTT_0\rangle = (-)^{\lambda-\mu}(2T'+1)^{-1/2}(TT_0\lambda-\mu|T'T_0-\mu)\,\langle n'T'\|Q_\lambda\|nT\rangle. \tag{A3}$$

1. Reduced matrix elements of the operators $Q$ between the basis vectors of the irreducible representation (0, $\lambda_2$)

$$\langle n-4, T\|Q_0\|nT\rangle = 3^{\frac{1}{2}}/24\,[(2T+1)(n-v-2T)(n-v+2T+2)$$

$$\times(2\Omega-n-v+2T+6)(2\Omega-n-v-2T+4)]^{\frac{1}{2}},$$

$$\langle n-4, T+2\|Q_2\|nT\rangle = \tfrac{1}{8}[(T+1)(T+2)/(2T+3)]^{\frac{1}{2}}$$

$$\times[(n-v-2T)(n-v-2T-4)(2\Omega-n-v+2T+6)(2\Omega-n-v+2T+10)]^{\frac{1}{2}},$$

$$\langle n-4, T\|Q_2\|nT\rangle = -\tfrac{1}{8}[2T(T+1)(2T+1)/3(2T-1)(2T+3)]^{\frac{1}{2}}$$

$$\times[(n-v-2T)(n-v+2T+2)(2\Omega-n-v+2T+6)(2\Omega-n-v-2T+4)]^{\frac{1}{2}},$$

$$\langle n-4, T-2\|Q_2\|nT\rangle = \tfrac{1}{8}[T(T-1)/(2T-1)]^{\frac{1}{2}}$$

$$\times[(n-v+2T+2)(n-v+2T-2)(2\Omega-n-v-2T+4)(2\Omega-n-v-2T+8)]^{\frac{1}{2}}.$$

2. Reduced matrix elements of the operators $Q$ between the basis vectors of the irreducible representation (1, $\lambda_2$).

2.1. $T+n/2$ even

$$\langle n-4, T\|Q_0\|nT\rangle = 3^{\frac{1}{2}}/24[(2T+1)(n-v-2T-1)$$
$$\times(n-v+2T+3)(2\Omega-n-v+2T+5)(2\Omega-n-v-2T+5)]^{\frac{1}{2}},$$
$$\langle n-4, T+2\|Q_2\|nT\rangle = \tfrac{1}{32}[(2T+1)(2T+3)(2T+5)/(T+1)(T+2)]^{\frac{1}{2}}$$
$$\times[(n-v-2T-1)(n-v-2T-5)(2\Omega-n-v+2T+5)(2\Omega-n-v+2T+9)]^{\frac{1}{2}},$$
$$\langle n-4, T+1\|Q_2\|nT\rangle = -\tfrac{1}{32}[(2T+1)(2T+3)/2T(T+1)^3(T+2)]^{\frac{1}{2}}$$
$$\times[(n-v+2T+3)(2\Omega-n-v+2T+5)]^{\frac{1}{2}}\{T[(n-v-2T-1)$$
$$\times(2\Omega-n-v+2T+9)]^{\frac{1}{2}}+(T+2)[(n-v-2T-3)(2\Omega-n-v+2T+7)]^{\frac{1}{2}}\},$$
$$\langle n-4, T\|Q_2\|nT\rangle = -\tfrac{1}{32}[(2T-1)(2T+1)(2T+3)/6T^3(T+1)^3]^{\frac{1}{2}}$$
$$\times[(n-v+2T+3)(2\Omega-n-v+2T+5)]^{\frac{1}{2}}\{(2T^2+2T+1)$$
$$\times[(n-v-2T-1)(2\Omega-n-v-2T+5)]^{\frac{1}{2}}-[(n-v+2T+1)(2\Omega-n-v+2T+7)]^{\frac{1}{2}}\},$$
$$\langle n-4, T-1\|Q_2\|nT\rangle = \tfrac{1}{32}[(2T-1)(2T+1)/2T^3(T+1)(T-1)]^{\frac{1}{2}}$$
$$\times[(n-v+2T+3)(2\Omega-n-v+2T+5)]^{\frac{1}{2}}\{(T-1)[(n-v+2T+1)$$
$$\times(2\Omega-n-v-2T+7)]^{\frac{1}{2}}+(T+1)[(n-v+2T-1)(2\Omega-n-v-2T+5)]^{\frac{1}{2}}\},$$
$$\langle n-4, T-2\|Q_2\|nT\rangle = \tfrac{1}{32}[(2T+1)(2T-1)(2T-3)/T(T-1)]^{\frac{1}{2}}$$
$$\times[(n-v+2T+3)(n-v+2T-1)(2\Omega-n-v-2T+5)(2\Omega-n-v-2T+9)]^{\frac{1}{2}}.$$

2.2. $T+n/2$ odd

$$\langle n-4, T\|Q_0\|nT\rangle = 3^{\frac{1}{2}}/24[(2T+1)(n-v-2T+1)(n-v+2T+1)$$
$$\times(2\Omega-n-v+2T+7)(2\Omega-n-v-2T+3)]^{\frac{1}{2}},$$
$$\langle n-4, T+2\|Q_2\|nT\rangle = \tfrac{1}{32}[(2T+1)(2T+3)(2T+5)/(T+1)(T+2)]^{\frac{1}{2}}$$
$$\times[(n-v-2T+1)(n-v-2T-3)(2\Omega-n-v+2T+7)(2\Omega-n-v+2T+11)]^{\frac{1}{2}},$$
$$\langle n-4, T+1\|Q_2\|nT\rangle = -\tfrac{1}{32}[(2T+1)(2T+3)/2T(T+1)^3(T+2)]^{\frac{1}{2}}$$
$$\times[(n-v-2T+1)(2\Omega-n-v-2T+3)]^{\frac{1}{2}}\{T[(n-v-2T-3)$$
$$\times(2\Omega-n-v+2T+7)]^{\frac{1}{2}}+(T+2)[(n-v-2T-1)(2\Omega-n-v+2T+9)]^{\frac{1}{2}}\},$$
$$\langle n-4, T\|Q_2\|nT\rangle = -\tfrac{1}{32}[(2T-1)(2T+1)(2T+3)/6T^3(T+1)^3]^{\frac{1}{2}}$$
$$\times[(n-v-2T+1)(2\Omega-n-v-2T+3)]^{\frac{1}{2}}\{(2T^2+2T+1)$$
$$\times[(n-v+2T+1)(2\Omega-n-v+2T+7)]^{\frac{1}{2}}-[(n-v-2T-1)(2\Omega-n-v-2T+5)]^{\frac{1}{2}}\},$$

$$\langle n-4, T-1\|Q_2\|nT\rangle = \tfrac{1}{32}[(2T-1)(2T+1)/2T^3(T+1)(T-1)]^{\frac{1}{2}}$$
$$\times[(n-v-2T+1)(2\Omega-n-v-2T+3)]^{\frac{1}{2}}\{(T-1)[(n-v+2T-1)$$
$$\times(2\Omega-n-v-2T+5)]^{\frac{1}{2}}+(T+1)[(n-v+2T+1)(2\Omega-n-v-2T+7)]^{\frac{1}{2}}\},$$
$$\langle n-4; T-2\|Q_2\|nT\rangle = \tfrac{1}{32}[(2T+1)(2T-1)(2T-3)/T(T-1)]^{\frac{1}{2}}$$
$$\times[(n-v+2T+1)(n-v+2T-3)(2\Omega-n-v-2T+3)(2\Omega-n-v-2T+7)]^{\frac{1}{2}}.$$

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