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CALCULATION OF NEUTRON REGISTRATION EFFICIENCY FOR EXPERIMENTAL INVESTIGATION OF $tt\mu \to {}^4He+2n+\mu^-$ AND $dt\mu \to {}^4He+n+\mu^-$ MUON-CATALYSED FUSION REACTIONS

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Neutron registration efficiency in experiments on muon-catalysed fusion reaction $tt\mu \to {}^4He+2n+\mu^-$ and $dt\mu \to {}^4He+n+\mu^-$ is calculated. The dependence of light output response of the detectors on energy threshold of the registration apparatus is obtained. For fusion reaction in the $tt\mu$ molecule the values of registration efficiency are determined for several types of final-state interaction between the reaction products.

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1. *Introduction*

The theoretical prediction of the resonance mechanism of $dt\mu$ and $tt\mu$ molecule formation [1–3] and its experimental confirmation [4, 5] have given rise to considerable growth of interest in the investigation of muon-catalysis of nuclear fusion of hydrogen isotopes. So far only one experiment has been performed in which the formation rate of $dt\mu$ molecules has been measured [5]. Large values obtained for these rates will undoubtedly stimulate further studies in this direction, aimed at the investigation of the possibility of using the muon-catalysis of nuclear fusion as a source of nuclear energy. However, to determine the effectiveness of muon-catalysis in a deuterium-tritium mixture in which the dt fusion takes place, it is necessary to have complete information about the competitive channels. One of these channels which leads to the decrease of energy yield due to the principal reaction

$$t\mu + d \to dt\mu \begin{cases} \nearrow {}^4He + n + \mu^- \\ \searrow {}^4He\mu + n \end{cases} + 17.6\ \text{MeV} \qquad (1)$$

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is the creation of the $tt\mu$ molecules and subsequent fusion

$$t\mu + t \rightarrow tt\mu \begin{cases} {}^4He + 2n + \mu^- \\ {}^4He\mu + 2n \end{cases} + 11.33\ \text{MeV}. \quad (2)$$

Therefore, the determination of the characteristics of reactions (1) and (2) constitutes an up-to-date task for experimentalists working in this field.

Since the determination of the parameters characterising the μ-molecular processes (1) and (2) is based on the analysis of total yields and time distributions of the final-state neutrons, the knowledge of neutron registration efficiency becomes necessary[1]. In case of reaction (2) where two neutrons are produced in the final state, registration efficiency is referred to fusion events rather than single neutrons.

Fig. 1 presents the main features of the experimental set-up for the investigation of reactions (1) and (2). A detailed description of the apparatus can be found in Ref. [6]. Muons are stopped in the target which is a stainless-steel container (24 mm in diameter

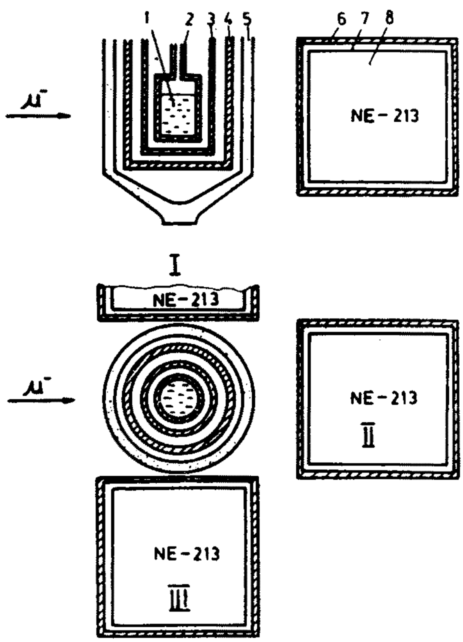

Fig. 1. Side and top views of the target and detectors. 1 — Liquid tritium or liquid D_2–T_2 mixture, 2 — Steel target wall, 3 — Vacuum chamber wall, 4 — Proportional wire chamber, 5 — Plastic scintillator, 6 — Steel detector walls, 7 — Teflon containers, 8 — Liquid NE-213 scintillators

[1] It has been suggested [18] that parameters characterising the μ-molecular processes (1) and (2) can be determined without knowing registration efficiency. However, the proposed method is based on the assumption that fusion rate, λ_f, is much larger than the corresponding μ molecule formation rate. This, according to theoretical estimates, is not necessarily the case for fusion in the $tt\mu$ molecule. Additionally the $tt\mu$ formation rate has to be sufficiently large ($\lambda_{tt\mu} \gg 10^6\ s^{-1}$). Otherwise (even for $\lambda_f \rightarrow \infty$) the required statistics may render the method difficult in practical application.

and 70 mm high) filled with liquid tritium or liquid D_2–T_2 mixture. The outgoing neutrons are registered in three (I–III) liquid NE-213 scintillators. Additionally, a proportional wire chamber and a plastic scintillator are placed around the target to detect beam muons and μ decay electrons.

The aim of the present work is to determine registration efficiency ε of neutrons from fusion acts in ttμ and dtμ molecules for the apparatus shown in Fig. 1. Taking into account the complexity of the measurement geometry and the presence of substantial quantity of matter between the target and detectors, the calculation of registration efficiency has been performed using the Monte-Carlo method [7].

The calculation procedure is described below in Section 2. Further details and calculation results are presented in Section 3. Section 4 contains the discussion of errors.

2. *Calculation procedure*

The algorithm of the calculation included:

1. Random sampling of the coordinates of fusion events. A homogeneous space distribution of reaction vertices was assumed.
2. Random sampling of the momenta of reaction products, which was carried out using a modified version of programme FOWL [8].
3. Simulation of neutron interactions within the target.
4. Simulation of neutron interactions in target and detector walls, and in matter placed between the target and detectors.
5. Simulation of neutron interactions in the NE-213 scintillators.
6. Inclusion of neutrons registered after being previously scattered in scintillator walls or scintillators themselves.
7. Constructions of the resulting pulse-height distributions for neutrons registered in detectors I–III.

The following neutron interaction processes in the NE-213 scintillator were taken into account;

1) elastic scattering of neutrons on protons and carbon nuclei,

2) inelastic scattering of neutrons on carbon: $^{12}C(n, n'\gamma)^{12}C$,

3) nuclear reaction: $^{12}C(n, \alpha)^9Be$,

4) additionally, for fusion in the dtμ molecule, reactions: $^{12}C(n, n')^{12}C^*(3\alpha)$; $^{12}C(n, \alpha)^9Be^*(n)^8Be(2\alpha)$.

Other processes such as interactions of the 4.43 MeV γ rays from inelastic scattering of neutrons on carbon nuclei or — for fusion in the ttμ molecule — reactions mentioned in point 4 above, were neglected as their influence on the value of registration efficiency has been found to be negligible.

Considering small dimensions of the NE-213 scintillators no more than two consecutive collisions of neutrons with hydrogen or carbon nuclei were taken into account. The contribution of the remaining multiple-scattering processes was estimated to be less than one per cent. For each neutron collision in the NE-213 scintillators final particle momenta and energies were determined and the ensuing light output from the recoil proton, α particle or carbon nucleus was found. For double-collision events the resulting pulse height was a sum of the individual amplitudes. Also only single and double-collision events were taken into account for interactions in matter filling the target, while for remaining elements of the apparatus (steel walls, plastic scintillator, teflon walls of the NE-213 scintillators) only single interactions were included.

The relevant total cross sections and angular distributions were taken from Ref. [9–12] and an empirical relationship between particle energy and light output [13–15] was used in the determination of pulse-height spectra.

3. *Results*

A. Reaction $tt\mu \to {}^4He + 2n + \mu^-$

Since fusion in the ttμ molecule involves the production of three strongly interacting particles, the energy and angular distributions of the outgoing neutrons depend on the characteristics of the final-state interaction between reaction products. Taking this interaction into account may change the value of registration efficiency in comparison with the value calculated using pure phase-space. Unfortunately, the existing data [16, 17] do not provide sufficient information about the shape of the final-state interaction in reaction (2), especially in the region of "incident" energy close to zero which is the case for muon-catalysed fusion. Therefore, in this work the calculations of registration efficiency for ttμ fusion were performed for three different hypotheses:

a) absence of the final-state interaction (pure phase space),

b) only the n–n interaction included,

c) only the α–n interaction included.

The differential cross section for production of three particles in the final state with two particles (i, j) interacting strongly can be written as:

$$d\sigma_{123} = \text{const}\, F_{ij}^2(k_{ij})\, dR, \tag{3}$$

where dR represents the phase space element, $\hbar k_{ij}$ is a relative momentum of interacting particles and F_{ij}^2 is the enhancement factor describing the interaction.

In the present calculations the formulae representing F_{ij}^2 were taken from Ref. [17]:

i) for the n–n interaction F_{ij}^2 was given by the Jost-function enhancement factor:

$$F_{nn}^2 = \frac{\left(\frac{r_0}{2}\right)^2 \left\{k_{nn}^2 + \left[\frac{1}{r_0} + \sqrt{\frac{1}{r_0^2} + \frac{2}{r_0 a_{nn}}}\right]^2\right\}^2}{k_{nn}^2 + \left[\frac{1}{2} r_0 k_{nn}^2 - \frac{1}{a_{nn}}\right]^2}, \tag{4}$$

where $r_0 = 2.65$ fm is an effective interaction radius and $a_{nn} = -17.0$ fm is the corresponding scattering length.

ii) for the α–n interaction two enhancement factors giving slightly different results were tried:

$$F_{\alpha n}^2 = \left[k_{\alpha n}^2 + \left(\frac{0.04599}{k_{\alpha n}^2} + 0.05553 + 0.5890 k_{\alpha n}^2\right)^2\right]^{-1} \tag{5}$$

(the Watson-Migdal enhancement factor) and

$$F_{\alpha n}^2 = \frac{(k_{\alpha n}^2 + 0.1342)(k_{\alpha n}^2 + 3.018)}{[(k_{\alpha n} - 0.2856)^2 + 0.04113][(k_{\alpha n} + 0.2856)^2 + 0.04113]} \tag{6}$$

(Jost-function enhancement factor).

The initial energy spectra and angular correlations of neutrons produced in reaction (2) are presented in Figs. 2 and 3 respectively. Different curves correspond to hypotheses a), b) and c), listed above. For the α–n interaction Eq. (6) gives smaller deviations from the corresponding values obtained for pure phase-space, therefore, only results obtained using Eq. (5) are shown.

Fig. 4 presents pulse height distribution for one of the detectors (calculated using pure phase-space). The events of two neutrons registered simultaneously in one detector are taken into account. Inclusion of such events changes the shape of pulse height distribution which in turn influences the value of ε in a degree depending on the energy cut-off.

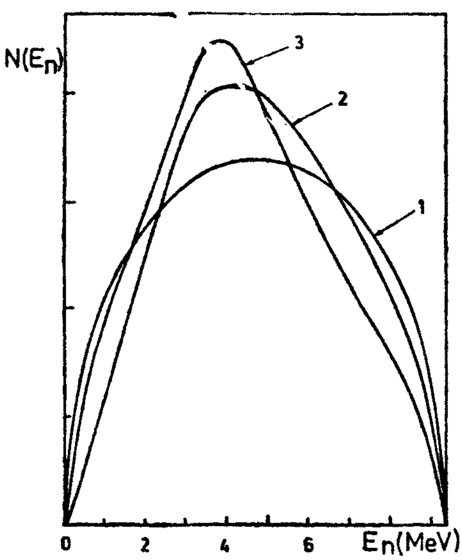

Fig. 2

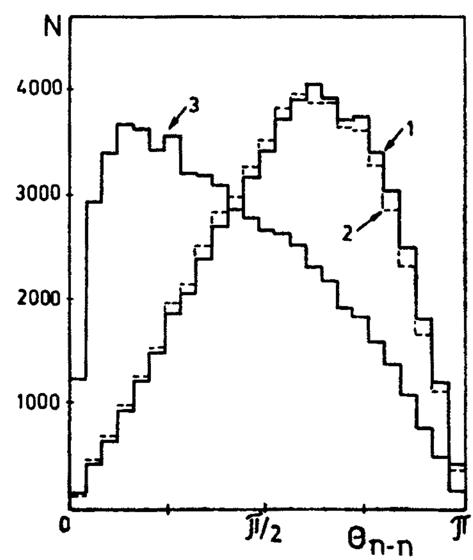

Fig. 3

Fig. 2. Initial energy spectra of neutrons produced in reaction (2). The distributions are normalized to the same area, N is in arbitrary units. Curves 1, 2 and 3 correspond respectively to: pure phase-space, pure (α–n) and pure (n–n)-interaction. The (α–n)-curve corresponds to enhancement factor given by Eq. (5)
Fig. 3. Angular correlations between neutrons produced in reaction (2). The assignment of histograms is as in Fig. 2

The dependence of registration efficiency on energy cut-off obtained for hypotheses a), b) and c), is shown in Fig. 5. It is seen that the influence of the final-state interaction increases with increasing energy threshold. This effect is more clearly seen in Fig. 6 which presents the ratio of ε to its phase space value ε_{ps} for the α–n and n–n interactions. The observed tendency is easily explained by the difference in neutron energy spectra seen in

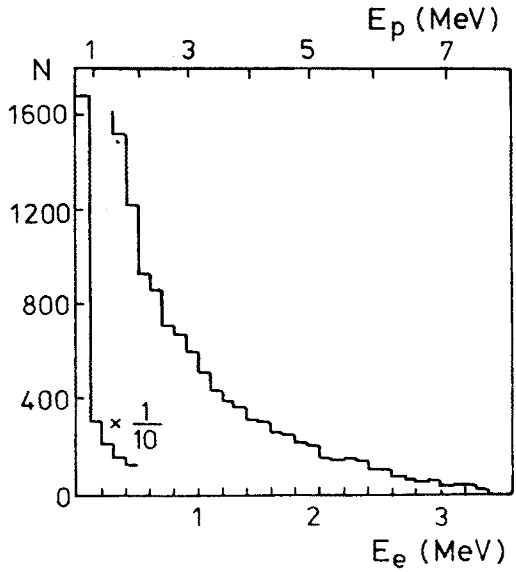

Fig. 4

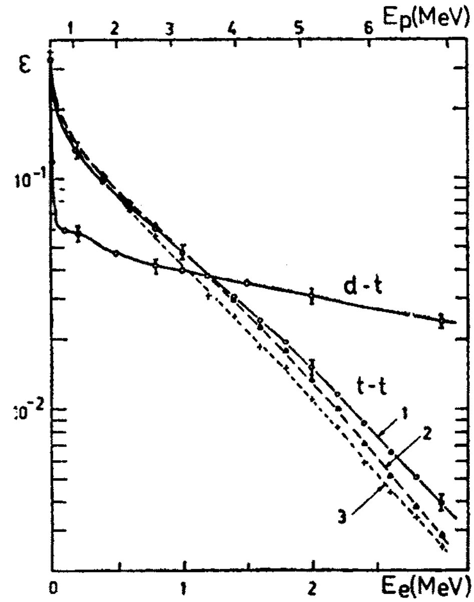

Fig. 5

Fig. 4. Pulse-height distribution for one detector corresponding to neutrons produced in reaction (2). Pulse height is given in units of light-output-equivalent energy of electrons (lower scale) and protons (upper scale). The vertical scale gives the number of events in intervals of $E_e = 0.1$ MeV

Fig. 5. Dependence of neutron registration efficiency on energy threshold of the apparatus, E_{th}, for reactions (1) and (2). Horizontal scales are as in Fig. 4. The curves are assigned as in Fig. 2

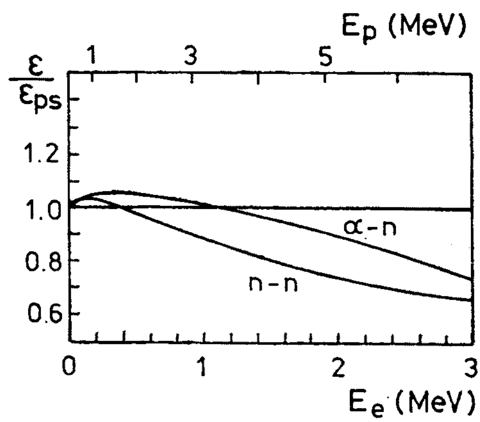

Fig. 6. Ratio of neutron registration efficiency ε to its phase-space value ε_{ps} vs energy threshold for different hypotheses. Horizontal scales are as in Fig. 4

Fig. 2. Angular correlations also contribute to the changes of ε with final interaction characteristics, however, in our geometry their effect is significantly less important. Let us remark here that, as seen in Figs. 5 and 6, the analysis of the threshold dependence of registration efficiency may, in principle, provide some insight into the type of the final-state interaction. Moreover, measuring the neutron angular correlations can provide useful additional information in this respect. For example, in our calculations the ratio of coincidences in detectors I and II to I–III coincidences for hypothesis b) differs significantly from the corresponding ratios for hypotheses a) and c). It seems that such possibility of looking at nuclear fusion processes in this unique energy region deserves further attention.

B. Reaction $dt\mu \to {}^4He + n + \mu^-$

In contrast to fusion in the $tt\mu$ molecule neutrons produced in reaction (1) are monoenergetic ($E_n = 14.1$ MeV) which makes the situation considerably simpler. The registration efficiency was calculated according to the procedure described in Section 2.

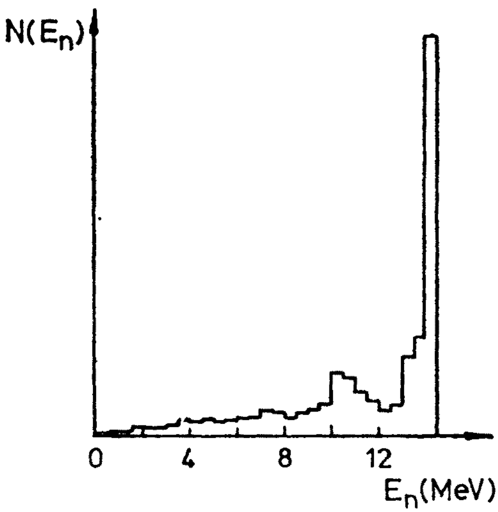

Fig. 7

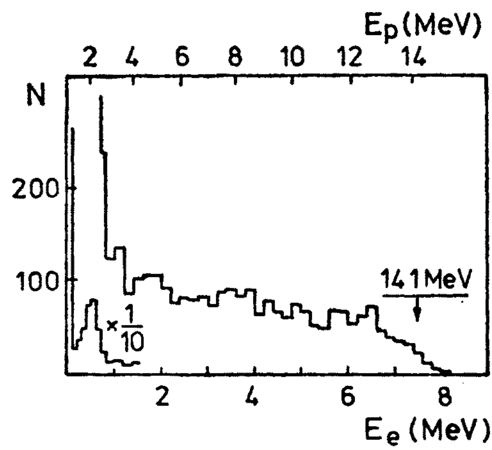

Fig. 8

Fig. 7. Energy spectrum of neutrons produced in reaction (1) incident on NE-213 scintillator after traversing layers of material between the target and detector. $N(E_n)$ is in arbitrary units

Fig. 8. Pulse height distribution for neutrons produced in reaction (1). The scales are as in Fig. 4

Fig. 7 presents the energy spectrum of neutrons entering the NE-213 scintillators after traversing the layers of material placed between the target and detectors. The peak at $E_n \simeq 10$ MeV is due to inelastic scattering of neutrons on iron and carbon nuclei. The light output spectrum is shown in Fig. 8. The peak at $0.4 \div 0.6$ MeV can be attributed to α particles from reactions ${}^{12}C(n, \alpha){}^9Be$, ${}^{12}C(n, n'){}^{12}C^*(3\alpha)$ and ${}^{12}C(n, \alpha){}^9Be^*(n){}^8Be(2\alpha)$. The dependence of the registration efficiency on energy threshold is shown in Fig. 5 together with the results for reaction (2).

4. *Discussion of errors*

The error of neutron registration efficiency for reactions (1) and (2) is determined by the following factors:

a) errors of input neutron cross sections,
b) uncertainties in the relations:
particle energy — light output for protons,
α-particles and carbon nuclei,
c) statistical errors,
d) geometrical uncertainty of relative configuration of the target and detectors,
e) simplifying assumptions used in the calculation algorithm,
f) additionally, for reaction (2) — ambiguity connected with the presence of final-state interaction between reaction products.

To estimate the error of registration efficiency due to points (a) and (b), calculations were repeated with the corresponding input values changed by two standard deviations. The resulting registration efficiency was found to change within the statistical error which for energy threshold equal to zero was about 0.5%. The geometrical uncertainty (d) and simplifying assumptions (e) were estimated to produce errors of about five and four per cent respectively.

In the actual experimental determination of parameters of reaction (2) one has to resort to some value of efficiency averaged over different final-state interaction models. The above-mentioned ambiguity produces an additional error which does not exceed 6% for energy thresholds below 1 MeV. Thus, the total error of ε for reaction (2) is in this case below 9%, and for reaction (1) correspondingly smaller.

Such accuracy allows one to use the values of registration efficiency determined in this work to interpret the data on muon-catalysed fusion processes in the $tt\mu$ and $dt\mu$ molecules.

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