Vol. B15 (1984) ACTA PHYSICA POLONICA No 8

RENORMALIZATION AND WARD IDENTITIES IN A NON--LINEAR SPINOR MODEL

By S. I. Kruglov

Institute of Physics of the BSSR Academy of Sciences, Minsk*

(Received May 26, 1983; revised version received November 22, 1983)

Problems of renormalization and dynamic mass formation in a four-fermion model with scalar-scalar, pseudoscalar-pseudoscalar and vector-vector interactions are investigated by the method of functional integration in collective variables. Ward-Takahashi identities and Schwinger-Dyson equations have been obtained for fermion and boson Green's functions. It is shown that all infinities are absorbed by the finite number of renormalization constants. The matrix elements of the processes of interaction between fermions and their bound states are independent of renormalization constants.

PACS numbers: 11.15.-q, 11.15.Bt

1. *Introduction*

Four-fermion models are known to be non-renormalized when expanded by the dimensional coupling constant. Yet, as shown in [1–3], such theories can be renormalized by using mean-field expansion, the requirement that the theory be self-congruent leading to a condition similar to that for the energy gap in the superconductivity theory [4, 5]. Non-trivial solutions of this equation determine the limits of applicability of mean-field expansions and give dynamic fermion masses, provided that the bare masses are zero. Such non-trivial solutions do exist [4, 5] (and correspond to the superconductive state) and are non-analytical by the dimensional coupling constant. Hence, it becomes clear why the expansion by the dimensional coupling constant leads to the existence of unavoidable infinities and non-renormalizability of the model.

In [6], problems of renormalization of the four-fermion theory with $(\bar{\psi}\psi)^2$ interaction in the Lagrangian are studied in detail with the use of mean-field expansion. In the present paper we investigate a model with four-fermion scalar-scalar, pseudoscalar-pseudoscalar and vector-vector interactions, thus actually taking into consideration the pseudovector--pseudovector $((\bar{\psi}\gamma_\mu\gamma_5\psi)^2)$ and tensor-tensor $((\bar{\psi}\gamma_{[\mu}\gamma_{\nu]}\psi)^2)$ interactions as well, since they can be excluded from the Lagrangian with the aid of the Fierz trasformation. This only

* Address: Institute of Physics, Lenin Avenue 70, Minsk, 220602, USSR.

leads to redetermination of the original interaction constants. Thus, we shall consider the general Heisenberg model which contains however no internal spaces, i.e. multiplets of spinor fields.

In Section 2 we formulate the perturbation theory, find the fermion and the collective excitation propagators. In Section 3 we conduct the procedure of renormalization in lowest order of the perturbation theory and find the spectrum of collective excitations. In Section 4 two Ward-Takahashi identities are derived which are associated with symmetry with respect to conventional phase transformations and γ_5-transformations. In Section 5 Schwinger-Dyson equations are obtained in the form containing functional derivatives and in the integral form. Also discussed are problems of renormalization in the general case which is not connected with the perturbation theory. In Section 6 we summarize our conclusions.

2. *Model*

Let us study a Lagrangian-based model

$$\mathscr{L} = -\bar{\psi}\gamma_\mu\partial_\mu\psi + \frac{\kappa}{2}(\bar{\psi}\psi)^2 - \frac{\varrho}{2}(\bar{\psi}\gamma_\mu\psi)^2 - \frac{\lambda}{2}(\bar{\psi}\gamma_5\psi)^2. \tag{1}$$

Using the functional integration method [7] and adding the constant

$$\int D\varphi D\tilde{\varphi} DA_\mu \prod_x \delta(\partial_\mu A_\mu) \exp\left\{-i\int dx\left[\frac{\mu_0^2}{2}\left(\varphi - \frac{g_0\bar{\psi}\psi}{\mu_0^2}\right)^2 + \frac{\mu_0'^2}{2}\left(\tilde{\varphi} - \frac{ig_0'\bar{\psi}\gamma_5\psi}{\mu_0'^2}\right)^2 + \frac{M_0^2}{2}\left(A_\mu - \frac{ie_0\bar{\psi}\gamma_\mu\psi}{M_0^2}\right)^2\right]\right\}$$

the generating functional for Green's functions is written in the form

$$Z[\bar{\eta}, \eta, j] = N_1 \int D\varphi D\tilde{\varphi} DA_\mu D\bar{\psi} D\psi \prod_x \delta(\partial_\mu A_\mu) \exp\left\{i\int dx \left[-\bar{\psi}\gamma_\mu\partial_\mu\psi + \bar{\psi}(g_0\varphi + ig_0'\tilde{\varphi}\gamma_5 + ie_0A_\mu\gamma_\mu)\psi - \frac{\mu_0^2}{2}\varphi^2 - \frac{\mu_0'^2}{2}\tilde{\varphi}^2 - \frac{M_0^2}{2}A_\mu^2 + \bar{\eta}\psi + \bar{\psi}\eta + j_\varphi\varphi + j_{\tilde{\varphi}}\tilde{\varphi} + j_\mu A_\mu\right]\right\}, \tag{2}$$

where collective scalar φ, pseudoscalar $\tilde{\varphi}$ and vector A_μ neutral fields have been introduced, $\kappa = g_0^2/\mu_0^2$, $\lambda = g_0'^2/\mu_0'^2$, $\varrho = e_0^2/M_0^2$; g_0, g_0', e_0 are non-dimensional constants and the constants μ_0, μ_0' and M_0 have the mass dimensionalities m; $\bar{\eta}$, η; j_φ, $j_{\tilde{\varphi}}$, j_μ are external sources, $D\varphi = \prod_x d\varphi$.

Lagrangian (1) is invariant with respect to the global phase transformations $\psi' = \exp(i\varphi)\psi$ leading to the law of conservation of current $I_\mu = i\bar{\psi}\gamma_\mu\psi$, i.e., to the current 4-vector transversality: $\partial_\mu I_\mu = 0$.

Let us decompose the current I_μ into the transverse (I^t_μ) and longitudinal (I^l_μ) parts according to the relations

$$I_\mu = I^t_\mu + I^l_\mu, \quad I^t_\mu = I_\mu - \frac{\partial_\mu(\partial_\nu I_\nu)}{\Box}, \quad I^l_\mu = \frac{\partial_\mu(\partial_\nu I_\nu)}{\Box}. \tag{3}$$

Similar relations can be written for the field A_μ. The validity of equalities

$$A_\mu I_\mu = A^t_\mu I^t_\mu + A^l_\mu I^l_\mu, \quad A_\mu I^t_\mu = A^t_\mu I_\mu, \quad A^t_\mu I^l_\mu = A^l_\mu I^t_\mu = 0 \tag{4}$$

can be easily checked.

It follows from (4) that instead of taking into account the current transversality it is possible to use the field A_μ transversality and vice versa.

The factor $\prod_x \delta(\partial_\mu A_\mu)$ in the functional integral (2) takes into account the field A_μ transversality [3].

Let us now present the δ-function in the form

$$\prod_x \delta(\partial_\mu A_\mu) = \lim_{\alpha\to 0} \exp\left\{-\frac{i}{2\alpha}\int dx(\partial_\mu A_\mu)^2\right\}. \tag{5}$$

Substituting (5) into (2) and integrating over the Fermi-fields $\bar{\psi}$, ψ, we obtain

$$Z[\bar{\eta}, \eta, j] = \lim_{\alpha\to 0} N \int D\phi \exp(iW_0),$$

$$W_0 = \int dxdy \left\{\bar{\eta}(x)S(x, y)\eta(y) - \delta(x-y)\left[\frac{\mu_0^2}{2}\varphi^2 + \frac{\mu_0'^2}{2}\tilde{\varphi}^2\right.\right.$$

$$\left.\left. + \frac{M_0^2}{2}A_\mu^2 + \frac{1}{2\alpha}(\partial_\mu A_\mu)^2 - j_A\phi_A\right]\right\} - i \operatorname{tr}\ln(1+\hat{G}_0 g_A\phi_A\gamma_A),$$

$$\phi_A = (A_\mu, \varphi, \tilde{\varphi}), \quad j_A = (j_\mu, j_\varphi, j_{\tilde{\varphi}}), \quad g_A = (e_0, g_0, g_0'),$$

$$\gamma_A = (i\gamma_\mu, I, i\gamma_5), \quad g_A\phi_A\gamma_A = ie_0 A_\mu\gamma_\mu + g_0\varphi + ig_0'\tilde{\varphi}\gamma_5. \tag{6}$$

If in (6) α is not allowed to approach zero we have arbitrary gauge and the field A_μ has an arbitrary longitudinal part. In this case both quanta with spin 1 and quanta with spin 0 correspond to the field A_μ. The Lorentz gauge corresponds to the case $\alpha \to 0$ and the field A_μ transversality is restored. Then the field A_μ describes the quanta with pure spin 1.

$S(x, y)$ is the Green fermion function in the external collective fields, which satisfies the equation

$$(\gamma_\mu\partial_\mu - g_A\phi_A\gamma_A)S(x, y) = \delta(x-y) \tag{7}$$

and the function $\hat{G}_0$ obeys the equation

$$\gamma_\mu\partial_\mu\hat{G}_0(x, y) = -\delta(x-y). \tag{8}$$

As seen from (2) and (6), the dimensional constants μ_0, μ'_0, M_0 have entered into the mass terms of the Lagrangian, and the non-dimensional constants g_0, g'_0, e_0 have entered as constants of interaction between the collective fields φ, $\tilde{\varphi}$, A_μ and the spinor field. In a single-loop approximation when the collective fields φ, $\tilde{\varphi}$, A_μ are constant ($\phi_A = \text{const}$), the solution of equation (4) in the momentum space will be the expression [8]

$$S(p) = \frac{-i\hat{p} - g_0\varphi + ig'_0\tilde{\varphi}\gamma_5}{p^2+m^2}, \tag{9}$$

where $\hat{p} = p_\mu\gamma_\mu$, $m^2 = g_0^2\varphi^2 + g_0'^2\tilde{\varphi}^2$.

In obtaining (9), the substitution of the variables $p_\mu \to p_\mu - A_\mu$ was made.

The equations of motion for the collective fields $\delta W_0/\delta\phi_A = 0$ at $\bar{\eta} = \eta = j_\varphi = j_{\tilde{\varphi}} = j_\mu = 0$ will have the form

$$\varphi = -\frac{ig_0^2}{4\pi^4\mu_0^2}\int\frac{\varphi dp}{p^2+m^2}, \quad \tilde{\varphi} = -\frac{ig_0'^2}{4\pi^4\mu_0'^2}\int\frac{\tilde{\varphi} dp}{p^2+m^2}. \tag{10}$$

The collective vector field has a trivial solution $\underset{0}{A_\mu} = 0$. The possibility of the existence of the nontrivial solution $\underset{0}{A_\mu} \neq 0$ in the case of current redistribution is discussed in [3].

Non-trivial non-analytic solutions, $\varphi_0 \neq 0$, $\tilde{\varphi}_0 \neq 0$, of equations (10) exist under the conditions $g_0 = g'_0$, $\mu_0 = \mu'_0$, $\mu_0\Lambda^2 > 4\pi^2$, where Λ is the truncation momentum [4, 5]. When the above conditions are met the fermion acquires the mass $m_0^2 = g_0^2(\varphi^2+\tilde{\varphi}^2)$.

Expanding the fields ϕ_A in (6) near the statistical solutions $\varphi(x) \to \varphi_0 + \varphi(x)$, $\tilde{\varphi}(x) \to \tilde{\varphi}_0 + \tilde{\varphi}(x)$, $A_\mu(x) \to A_\mu(x)$ where φ_0, $\tilde{\varphi}_0$ are subject to equations (10) and expanding expression (6) in terms of constants g_A, we obtain a perturbation theory series [8]

$$Z[\bar{\eta}, \eta, j] = \lim_{\alpha\to 0} N\int D\phi \exp i\left\{\int\int dxdy[\bar{\eta}(x)S(\phi_0+\phi)\eta(y)\right.$$

$$\left. + \delta(x-y)j_A\phi_A - \tfrac{1}{2}\phi_A(x)\underset{0}{\Delta}{}^{-1}_{AB}(x, y)\phi(y)] + \sum_{n=3}^{\infty}\frac{i}{n}\operatorname{tr}(S_0 g_A\phi_A\gamma_A)^n\right\}, \tag{11}$$

where $S_0 = S(\phi_0)$ is determined by expression (9) at $\varphi = \varphi_0$, $\tilde{\varphi} = \tilde{\varphi}_0$, $g_0 = g'_0$ and the propagator for the collective fields in the momentum space is given by the expression

$$\underset{0}{\Delta}{}^{-1}_{AB}(p^2) = ig_Ag_B\operatorname{tr}\int\frac{dk}{(2\pi)^4}S_0(p+k)\gamma_AS_0(k)\gamma_B + \frac{1}{\alpha}p_\mu p_\nu\delta_{\mu A}\delta_{\nu B} + \delta_{AB}M_A,$$

$$M_A = (M_0, \mu_0, \mu_0). \quad \text{(n.s.)} \tag{12}$$

(n.s.) denotes in this paper the absence of summation over the repeating index.

Substituting expression (9) into (12) and calculating the trace of matrices, we have

$$\underset{0}{\Delta}{}^{-1}_{\mu\nu}(p^2) = M_0^2\delta_{\mu\nu} + \frac{1}{\alpha}p_\mu p_\nu$$

$$+\frac{ie_0^2}{4\pi^4}\int\frac{dq[\delta_{\mu\nu}(q(q-p)+m_0^2)+p_\mu q_\nu+p_\nu q_\mu-2q_\mu q_\nu}{[(q-p)^2+m_0^2]\,(q^2+m_0^2)},$$

$$\underset{0}{\Delta}{}^{-1}_{55}(p^2)=\mu_0^2-\frac{ig_0^2}{4\pi^4}\int\frac{dq[g_0^2(\varphi_0^2-\tilde\varphi_0^2)+(p-q)q]}{[(q-p)^2+m_0^2]\,(q^2+m_0^2)},$$

$$\underset{0}{\Delta}{}^{-1}_{66}(p^2)=\mu_0^2-\frac{ig_0^2}{4\pi^4}\int\frac{dq[g_0^2(\tilde\varphi_0^2-\varphi_0^2)+(p-q)q]}{[(q-p)^2+m_0^2]\,(q^2+m_0^2)},$$

$$\underset{0}{\Delta}{}^{-1}_{56}(p^2)=\underset{0}{\Delta}{}^{-1}_{65}(p^2)=-\frac{ig_0^2}{4\pi^4}\int\frac{dq2g_0^2\varphi_0\tilde\varphi_0}{[(q-p)^2+m_0^2]\,(q^2+m_0^2)},$$

$$\underset{0}{\Delta}{}^{-1}_{5\mu}(p^2)=\underset{0}{\Delta}{}^{-1}_{\mu5}(p^2)=\underset{0}{\Delta}{}^{-1}_{6\mu}(p^2)=\underset{0}{\Delta}{}^{-1}_{\mu6}(p^2)=0. \quad (13)$$

3. Renormalization in the lowest order of the theory

Calculating integrals in (13) and making use of equations (10), we find

$$\underset{0}{\Delta}{}^{-1}_{55}(p^2)=(p^2+4g_0^2\varphi_0^2)\left(Z_\varphi^{-1}-\frac{g_0^2}{8\pi^2}J_1(p)\right),$$

$$\underset{0}{\Delta}{}^{-1}_{66}(p^2)=(p^2+4g_0^2\tilde\varphi_0^2)\left(Z_{\tilde\varphi}^{-1}-\frac{g_0^2}{8\pi^2}J_1(p)\right),$$

$$\underset{0}{\Delta}{}^{-1}_{56}(p^2)=4g_0^2\varphi_0\tilde\varphi_0\left(Z_\varphi^{-1}-\frac{g_0^2}{8\pi^2}J_1(p)\right),$$

$$\underset{0}{\Delta}{}^{-1}_{\mu\nu}(p^2)=\left(M_0^2-\frac{\Lambda^2-m_0^2}{2}\cdot\frac{e_0^2}{4\pi^2}\right)\delta_{\mu\nu}+\frac{1}{\alpha}p_\mu p_\nu$$

$$+\frac{2e_0^2}{3g_0^2}Z_\varphi^{-1}(\delta_{\mu\nu}p^2-p_\mu p_\nu)-\frac{e_0^2}{4\pi^2}J_{\mu\nu}(p),$$

$$J_1(p)=\int_0^1 dx\ln\left[1+\frac{p^2}{m_0^2}x(1-x)\right],\qquad Z_\varphi^{-1}=\frac{g_0^2}{8\pi^2}\left(\ln\frac{\Lambda^2}{m_0^2}-1\right),$$

$$J_{\mu\nu}(p)=\tfrac{2}{9}p^2\delta_{\mu\nu}+\tfrac{1}{18}p_\mu p_\nu+(\delta_{\mu\nu}p^2-p_\mu p_\nu)\int_0^1 dx2x(1-x)\ln\left[1+\frac{p^2}{m_0^2}x(1-x)\right]. \quad (14)$$

Let us introduce renormalized values

$$\underset{0}{\Delta}{}'_{AB}(p^2)=Z_A^{-1}\underset{0}{\Delta}_{AB}(p^2),\qquad \phi'_A=Z_A^{-1/2}\phi_A,$$

$$g_A'^2=Z_Ag_A^2,\qquad Z_A=(Z_v,Z_\varphi,Z_{\tilde\varphi}),\qquad \text{(n.s.)} \quad (15)$$

where Z_φ, $Z_v = (3g_0^2/2e_0^2)Z_\varphi$ are constants of renormalization of fields φ_0, $\tilde{\varphi}_0$ and A_μ. It follows from (14) that

$$\underset{0}{\Delta}{}_{55}^{-1}(-4g_0^2\varphi_0^2) = 0, \quad \underset{0}{\Delta}{}_{66}^{-1}(-4g_0^2\tilde{\varphi}_0^2) = 0, \tag{16}$$

whence we find the masses of the collective fields φ, $\tilde{\varphi}$

$$m_\varphi^2 = 4g_0^2\varphi_0^2, \quad m_{\tilde{\varphi}}^2 = 4g_0^2\tilde{\varphi}_0^2. \tag{17}$$

We obtain from (17) the formula $m_\varphi^2 + m_{\tilde{\varphi}}^2 = 4m_0^2$ where m_0 is the fermion mass. Along with the renormalization of the fields and coupling constants of (15), we define the renormalized mass of the vector neutral collective field A_μ

$$M^2 = \left(M_0^2 - \frac{e_0^2}{4\pi^2}\frac{\Lambda^2 - m_0^2}{2}\right)Z_v. \tag{18}$$

Using relations (15) and (17) we find from (14), to the accuracy of terms of the order $g_0^2/4\pi^2$, $e_0^2/4\pi^2$, which determine radiation corrections, the Lagrangian which is bilinear over the compound (collective) fields

$$\begin{aligned}\mathscr{L}_\phi = & -\tfrac{1}{2}\left[(\partial_\mu\varphi)^2 + (\partial_\mu\tilde{\varphi})^2 + 4g_0^2(\varphi_0\varphi + \tilde{\varphi}_0\tilde{\varphi})^2\right] \\ & -\tfrac{1}{4}F_{\mu\nu}^2 - \tfrac{1}{2}M^2A_\mu^2 - \frac{1}{2a}(\partial_\mu A_\mu)^2,\end{aligned} \tag{19}$$

where $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$, $a = \alpha Z_v^{-1}$. Note that the most convenient is the gauge in which, e.g. $\tilde{\varphi}_0 = 0$, $\varphi_0 \neq 0$. As follows from (19), the field $\tilde{\varphi}$ corresponds in this case to a massless particle (Goldstone) and the field φ describes a particle with the mass $2m_0$. We used ambiguity in the Lagrangian selection to an accuracy of divergence-type terms. Note that in [9–11] a model was investigated which was a particular case of (1) at $\kappa = \lambda = 0$ and $\varrho \to \infty$. In this model equivalence with quantum electrodynamics was established. At the same time a massless vector field appeared which was identified with the photon field. In our notation this limiting case corresponds to $M^2 \to 0$. We shall return to this question later when discussing the Ward identities.

From (19) (or (14)) we find the expression for the renormalized propagator in the lowest order of the perturbation theory

$$\underset{0}{\Delta}{}'_{\mu\nu}(p^2) = \frac{1}{p^2 + M^2}\left(\delta_{\mu\nu} + p_\mu p_\nu\frac{a-1}{aM^2 + p^2}\right). \tag{20}$$

Hence, in the limit $\alpha \to 0$ ($a \to 0$), which must be observed according to (11), we obtain a transverse propagator which behaves like $O(1/p^2)$.

Thus, we have renormalized single-particle Green's functions in the lowest order of the perturbation theory. Consider now the question of divergence of the highest (multiparticle) Green's functions. Taking into account that $\underset{0}{\Delta}{}_{AB}(p^2) \sim O(1/p^2)$, $S_0(p) \sim O(1/p)$

at $p^2 \to \infty$, it is possible to derive in the conventional way [12] a formula for the degree of the diagram divergence

$$D = 4-\tfrac{3}{2}F-B_{\varphi}-B_{\tilde{\varphi}}-B_A, \tag{21}$$

where B_{φ}, $B_{\tilde{\varphi}}$, B_A are the numbers of external lines of the boson fields φ, $\tilde{\varphi}$, A_μ respectively and F is the number of external fermion lines of the diagram. At $D < 0$ the integrals converge. As the value D is independent of the order of the perturbation theory, the initial model is of the renormalizable type [13]. The number of types of diverging diagrams is finite and independent of the order of the perturbation theory. In (15), we have provided the renormalization of a single-particle Green's function. According to (21), the diagrams in Fig. 1a, 1b will also be divergent

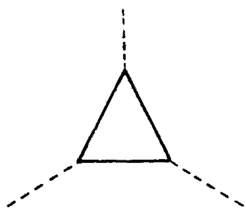

Fig. 1a

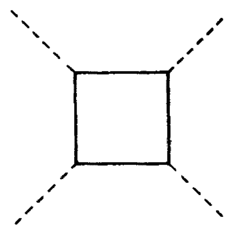

Fig. 1b

where the set of fields ϕ_A corresponds to the line ———. We find from (11) the vertex function in the lowest order of the perturbation theory, which corresponds to Fig. 1a.

$$\Gamma^0_{ABC}(x, y, z) = \frac{\delta^3 W_0}{\delta\phi_A(x)\delta\phi_B(y)\delta\phi_C(z)},$$

$$\Gamma^0_{ABC}(x, y, z) = ig_A g_B g_C \operatorname{tr}\{S_0(z, x)\gamma_A S_0(x, y)\gamma_B S_0(y, z)\gamma_C + S_0(z, y)\gamma_B S_0(y, x)\gamma_A S_0(x, z)\gamma_C\}. \quad \text{(n.s.)} \tag{22}$$

Going into the momentum space

$$\Gamma^0_{ABC}(x, y, z) = \int \frac{dpdq}{(2\pi)^8}\Gamma^0_{ABC}(p, q)e^{ip(y-x)}e^{iq(x-z)} \tag{23}$$

we find from (22)

$$\Gamma^0_{ABC}(p, q) = ig_C g_A g_B \operatorname{tr}\left\{\int \frac{dk}{(2\pi)^4}[S_0(k+p-q)\gamma_A S_0(k)\gamma_B S_0(k+p) + S_0(k-p)\gamma_B S_0(k)\gamma_A S_0(k+q-p)]\gamma_C\right\}. \quad \text{(n.s.)} \tag{24}$$

Taking into account linear divergence of integrals (24), it is possible to regularize the vertex function of (24) in the following way

$$\Gamma^0_{ABC}(p, q) = \lambda_{ABC}Z_A^{-1/2}Z_B^{-1/2}Z_C^{-1/2} + ig_A g_B g_C \operatorname{reg}\Gamma^0_{ABC}(p, q), \text{(n.s.)} \tag{25}$$

where

$$\Gamma^0_{ABC}(0,0) = \lambda_{ABC} Z_A^{-1/2} Z_B^{-1/2} Z_C^{-1/2}\ \text{(n.s.)}, \qquad Z_A = \begin{cases} Z_v & A = \mu, \\ Z_\varphi & A = 5{,}6. \end{cases}$$

The renormalized vertex function is determined by the relations

$$\Gamma^{0\prime}_{ABC} = \Gamma^0_{ABC} Z_A^{1/2} Z_B^{1/2} Z_C^{1/2}. \quad \text{(n.s.)} \tag{26}$$

Using (24), it is easy to calculate the values λ_{ABC} in (25). The four-point Green function described by the diagram given in Fig. 1b is regularized in a similar fashion [6]. Note that for the field A_μ Furry's theorem is fulfilled: the total matrix element corresponding to diagrams with closed fermion loops having an odd number of external vector lines is equal to zero.

4. *Ward-Takahashi's identities*

Let us introduce, in the conventional way [14, 15], the generating functional of Green's connection functions

$$W[\bar\eta, \eta, j] = -i \ln Z[\bar\eta, \eta, j]. \tag{27}$$

To uniquely define the Green function, it is necessary to go into Euclidean space and return into Minkovski space on completing the functional integration [16]. We shall formally work in Minkovski space assuming the necessity of the above procedure. The generating functional (27), (2) (at $g_0 = g_0'$, $\mu_0 = \mu_0'$) is invariant with respect to γ_5-transformations of the sources

$$\eta' = e^{-i\theta\gamma_5}\eta, \qquad \bar\eta' = \bar\eta e^{-i\theta\gamma_5},$$

$$j_5' = j_5 \cos 2\theta + j_6 \sin 2\theta,$$

$$j_6' = -j_5 \sin 2\theta + j_6 \cos 2\theta, \qquad (j_5 \equiv j_\varphi, \quad j_6 \equiv j_{\tilde\varphi}), \tag{28}$$

which can be easily checked by substituting the variables

$$\psi' = e^{i\theta\gamma_5}\psi, \qquad \bar\psi' = \bar\psi e^{i\theta\gamma_5},$$

$$\varphi' = \varphi \cos 2\theta + \tilde\varphi \sin 2\theta,$$

$$\tilde\varphi' = -\varphi \sin 2\theta + \tilde\varphi \cos 2\theta. \tag{29}$$

Taking this into account, we find from the condition of the generating functional independence with regard to the parameter θ of the transformations under consideration, i.e. $dW/d\theta = 0$

$$\int dx \left\{ \frac{\delta W[J]}{\delta \eta'(x)} \frac{\delta \eta'(x)}{\delta\theta} + \frac{\delta W[J]}{\delta \bar\eta'(x)} \frac{\delta \bar\eta'(x)}{\delta\theta} + \frac{\delta W[J]}{\delta j_5'(x)} \frac{\delta j_5'(x)}{\delta\theta} + \frac{\delta W[J]}{\delta j_6'(x)} \frac{\delta j_6'(x)}{\delta\theta} \right\} = 0, \tag{30}$$

where $J = (j_A, \bar\eta, \eta)$.

Using the definitions of [17]

$$\langle\phi_A(x)\rangle = \frac{\delta W[J]}{\delta j_A(x)}, \quad \langle\psi(x)\rangle = \frac{\delta W[J]}{\delta\bar{\eta}(x)},$$

$$\langle\bar{\psi}(x)\rangle = \frac{\delta W[J]}{\delta\eta(x)}, \quad \frac{\delta\Gamma[\phi]}{\delta\langle\phi_A(x)\rangle} = -j_A(x),$$

$$\frac{\delta\Gamma[\phi]}{\delta\langle\psi(x)\rangle} = -\bar{\eta}(x), \quad \frac{\delta\Gamma[\phi]}{\delta\langle\bar{\psi}(x)\rangle} = -\eta(x), \tag{31}$$

where $\Gamma[\phi] = W[J] - \int dx(j_A\langle\phi_A\rangle + \bar{\eta}\langle\psi\rangle + \langle\bar{\psi}\rangle\eta)$ is the effective action, we find from (30), taking into consideration (28) at $\eta = \bar{\eta} = 0$, the Ward-Takahashi identity

$$\int dx\left\{\langle\varphi(x)\rangle\frac{\delta\Gamma[\phi]}{\delta\langle\tilde{\varphi}(x)\rangle} - \langle\tilde{\varphi}(x)\rangle\frac{\delta\Gamma[\phi]}{\delta\langle\varphi(x)\rangle}\right\} = 0. \tag{32}$$

(Similar derivation was used in the σ-model [18]). Differentiating functionally with respect to the mean fields $\langle\varphi\rangle$, $\langle\tilde{\varphi}\rangle$ we obtain from (32) connection between the propagators and the vertex functions

$$\begin{aligned}
\Delta_{66}^{-1}(x, y) - \Delta_{55}^{-1}(x, y) &= \int dz\{\langle\varphi(z)\rangle\Gamma_{566}(x, z, y) - \langle\tilde{\varphi}(z)\rangle\Gamma_{556}(x, z, y)\} \\
&= \int dz\{\langle\varphi(z)\rangle\Gamma_{665}(x, z, y) - \langle\tilde{\varphi}(z)\rangle\Gamma_{655}(x, z, y)\}, \\
\Delta_{56}^{-1}(x, y) + \Delta_{65}^{-1}(x, y) &= \int dz\{\langle\tilde{\varphi}(z)\rangle\Gamma_{656}(x, z, y) - \langle\varphi(z)\rangle\Gamma_{666}(x, z, y)\} \\
&= \int dz\{\langle\varphi(z)\rangle\Gamma_{565}(x, z, y) - \langle\tilde{\varphi}(z)\rangle\Gamma_{555}(x, z, y)\},
\end{aligned} \tag{33}$$

where

$$\Delta_{AB}^{-1}(x, y) = -\frac{\delta^2\Gamma[\phi]}{\delta\langle\phi_A(x)\rangle\delta\langle\phi_B(y)\rangle}, \quad \Gamma_{ABC}(x, y, z) = \frac{\delta^3\Gamma[\phi]}{\delta\langle\phi_A(x)\rangle\delta\langle\phi_B(y)\rangle\delta\langle\phi_C(z)\rangle}.$$

Equalities (33) are exact relations for complete Green's functions. If we now use loop expansion and take into account that it retains the properties of the generating functional symmetry, it can readily be concluded that relations similar to (33) hold for any n-loop approximation. In a single-loop approximation when $\langle\phi_A(x)\rangle = \phi_A$ the constant and vertex functions are given by relations (22), (24). We have from (33) in the momentum space

$$\begin{aligned}
\underset{0}{\Delta}{}_{66}^{-1}(p^2) - \underset{0}{\Delta}{}_{55}^{-1}(p^2) &= \varphi_0\Gamma^0_{566}(0, p) - \tilde{\varphi}_0\Gamma^0_{556}(0, p) \\
&= \varphi_0\Gamma^0_{665}(0, p) - \tilde{\varphi}_0\Gamma^0_{655}(0, p),
\end{aligned} \tag{34a}$$

$$\underset{0}{\Delta}{}_{56}^{-1}(p^2) + \underset{0}{\Delta}{}_{65}^{-1}(p^2) = \tilde{\varphi}_0\Gamma^0_{656}(0, p) - \varphi_0\Gamma^0_{666}(0, p) = \varphi_0\Gamma^0_{565}(0, p) - \tilde{\varphi}_0\Gamma^0_{555}(0, p). \tag{34b}$$

Using (13), (A1), the fulfilment of naive Ward-Takahashi identities (34) can be easily checked for the unrenormalized values.

Passing to the renormalized values, according to (15), (26), we make sure that Ward-Takahashi identities (34a) also hold for renormalized values. Similarly, it is possible to check the fulfilment of (34b) for renormalized values.

Derive now the Ward-Takahashi identities connecting the vertex function

$$\Gamma_\mu(z, x, y) = \frac{\delta\langle S(x, y)\rangle^{-1}}{e_0\delta\langle A_\mu(z)\rangle}, \quad \left(\langle A_\mu(z)\rangle = \frac{\delta W[J]}{\delta j_\mu(z)}\right) \tag{35}$$

with the complete reverse fermion propagator $\langle S(x, y)\rangle^{-1}$ where $\langle S(x, y)\rangle = \delta^2 W[J]/ /\delta\bar{\eta}(x)\delta\eta(y)$. To do this, write (35) in the form

$$\Gamma_\mu(z, x, \xi) = -\int dydt\langle S(x, y)\rangle^{-1}\frac{\delta\langle S(y, t)\rangle}{e_0\delta\langle A_\mu(z)\rangle}\langle S(t, \xi)\rangle^{-1}. \tag{36}$$

Now, taking into account the equation of motion for the field $A_\mu(x)$

$$\frac{\delta W_0}{\delta A_\mu(x)} = 0, \tag{37}$$

we obtain, making use of (4) and the equality $\delta\langle A_\mu(x)\rangle/\delta A_\nu(y) = \delta_{\mu\nu}\delta(x-y)$,

$$\frac{\delta\langle S(y, t)\rangle}{e_0\delta\langle A_\mu(z)\rangle} = \frac{\int S(y, z)i\gamma_\mu S(z, t)e^{iW_0}D\phi}{\int e^{iW_0}D\phi}. \tag{38}$$

Differentiating (38) with respect to z_μ and taking into account (4), we have

$$\frac{\partial}{\partial z_\mu}\frac{\delta\langle S(y, t)\rangle}{e_0\delta\langle A_\mu(z)\rangle} = i(\langle S(y, z)\rangle\delta(z-t)-\langle S(z, t)\rangle\delta(y-z)). \tag{39}$$

Differentiating (36) with respect to z_μ and using (39), we arrive at the Ward-Takahashi identity

$$\frac{\partial\Gamma_\mu(z, x, y)}{\partial z_\mu} = i[\langle S(x, z)\rangle^{-1}\delta(z-y)-\langle S(z, y)\rangle^{-1}\delta(x-z)]. \tag{40}$$

Passing in (40) into the momentum space according to formula (23), we obtain

$$(p'_\mu - p_\mu)\Gamma_\mu(p, p') = \langle S(p)\rangle^{-1}-\langle S(p')\rangle^{-1}. \tag{41}$$

5. *Schwinger-Dyson equations*

For convenience of loop expansion, the generating functional (27) is represented in Minkovski space in the form

$$W[\bar{\eta}, \eta, j] = -i\varepsilon\ln\int D\phi\exp\left(\frac{iW_0}{\varepsilon}\right). \tag{42}$$

Here we have introduced the parameter ε which is assumed to be equal to 1 at the end of computations [16]. Expansion in terms of the parameter ε corresponds to loop expansion. Using definitions of (31) and formulae (42), (6), (7), it is possible to make sure that the following Schwinger-Dyson equation is true

$$\left(\gamma_\mu \partial_\mu - g_A \langle \phi_A(x) \rangle \gamma_A + i\varepsilon g_A \gamma_A \frac{\delta}{\delta j_A}\right) \langle \psi(x) \rangle = \eta(x). \tag{43}$$

Taking the functional derivative $\delta/\delta\eta$ of both sides of equation (43) and assuming $\eta = \bar{\eta} = 0$, we obtain a Schwinger-Dyson equation for the single-particle Green's function

$$\left(\gamma_\mu \partial_\mu - g_A \langle \phi_A(x) \rangle \gamma_A + i\varepsilon g_A \gamma_A \frac{\delta}{\delta j_A}\right) \langle S(x, y) \rangle = \delta(x-y). \tag{44}$$

Similarly, an equation for multiparticle Green's functions can be obtained from (43).

Using the definitions

$$\Delta_{AB}(x, y) = \frac{\delta \langle \phi_A(x) \rangle}{\delta j_B(y)}, \quad \Gamma_A(z, x, y) = \frac{\delta \langle S(x, y) \rangle^{-1}}{g_A \delta \langle \phi_A(z) \rangle} \quad \text{(n.s.)} \tag{45}$$

and (44), we have an integral form of Schwinger-Dyson equations

$$\langle S(x, y) \rangle^{-1} - (\gamma_\mu \partial_\mu - g_A \gamma_A \langle \phi_A \rangle) \delta(x-y) = \Sigma(x, y), \tag{46}$$

where $\Sigma(x, y) = -\varepsilon g_A^2 \gamma_A \int dt dz \langle S(x, t) \rangle \Delta_{AB}(x, z) \Gamma_B(z, t, y)$ is a mass operator. The equation of motion for the mean fields follow from the condition $\delta W_0/\delta\phi_A = 0$

$$M_A^2 \langle \phi_A(x) \rangle = j_A + i g_A \operatorname{tr} \gamma_A \langle S(x, x) \rangle. \quad \text{(n.s.)} \tag{47}$$

Consider that $\delta j_A(x)/\delta\langle\phi_B(y)\rangle = \Delta_{AB}^{-1}(x, y)$. Then, making use of (47), (45) and the equality

$$\frac{\delta \langle S(x, x) \rangle}{\delta \langle \phi_A(y) \rangle} = -g_A \int dz dt \langle S(x, t) \rangle \Gamma_A(y, t, z) \langle S(z, x) \rangle \quad \text{(n.s.)}$$

we find

$$M_A^2 \delta_{AB} \delta(x-y) = \Delta_{AB}^{-1}(x, y) - i g_A^2 \operatorname{tr} \gamma_A \int dz dt \langle S(x, t) \rangle \Gamma_B(y, t, z) \langle S(z, x) \rangle. \quad \text{(n.s.)} \tag{48}$$

We obtain from definitions (45)

$$\Gamma_A(z, x, y) = -\gamma_A \delta(x-y) \delta(x-z) - \frac{\delta \Sigma(x, y)}{g_A \delta \langle \phi_A(z) \rangle}. \quad \text{(n.s.)} \tag{49}$$

In a similar fashion, the vertex functions are determined

$$\Gamma_{ABC}(z, x, y) = -\frac{\delta \Delta_{BC}^{-1}(x, y)}{\delta \langle \phi_A(z) \rangle}, \quad \Gamma_{ABCD}(t, z, x, y) = -\frac{\delta^2 \Delta_{CD}^{-1}(x, y)}{\delta \langle \phi_A(t) \rangle \delta \langle \phi_B(z) \rangle}. \tag{50}$$

Note that from (48) it follows, by virtue of the Ward identities (40), that

$$\frac{\partial \Delta_{\mu\nu}^{-1}(x, y)}{\partial y_\nu} = M_0^2 \frac{\partial \delta(x-y)}{\partial y_\mu}. \tag{51}$$

Thus, the propagator for the collective fields $\Delta_{\mu\nu}^{-1}$ does not satisfy the transversality conditions, which is due to the field A_μ massivity. At $M_0 = 0$, $\Delta_{\mu\nu}^{-1}$ satisfies the transversality condition, which is in agreement with the results obtained by other authors [9–11]. Passing into the momentum space, we obtain from (46), (48), taking into account (23)

$$\langle S(p)\rangle^{-1} = i\hat{p} - g_A\langle\phi_A\rangle\gamma_A - \Sigma(p),$$

$$\Sigma(p) = i\varepsilon\gamma_A \int \frac{dk}{(2\pi)^4} \langle S(p-k)\rangle \Delta_{AB}(k)\Gamma_B(p-k, p), \quad \text{(n.s.)}$$

$$\Delta_{AB}^{-1}(p^2) = M_A^2\delta_{AB} + i \operatorname{tr} \gamma_A \int \frac{dk}{(2\pi)^4} \langle S(p+k)\rangle \Gamma_B(p+k, k) \langle S(k)\rangle. \tag{52}$$

The renormalization procedure based on the Schwinger-Dyson equations (48), (52) and relations (50), which is not connected with the perturbation theory, is carried out in the initial model in a way similar to that of the case of scalar-scalar interactions considered in [6]. At the same time, the relations in the momentum space will take on the form

$$\Delta'_{AB}(p^2) = \Delta_{AB}(p^2)Z_A^{-1}, \quad \Gamma'_A(p, q) = \Gamma_A(p, q)Z_1, \quad \psi' = \psi Z_2^{-1/2},$$

$$\langle S(p)\rangle' = \langle S(p)\rangle Z_2^{-1}, \quad \langle\phi_A\rangle' = \langle\phi_A\rangle Z_A^{-1/2}, \quad \text{(n.s.)}$$

$$g_A'^2 = g_A^2 Z_A \left(\frac{Z_2}{Z_1}\right)^2, \quad \Delta_{AB}^{-1}(0) = M_A^2\delta_{AB}Z_A^{-1}, \quad \langle S(0)\rangle^{-1} = mZ_2^{-1} \tag{53}$$

plus similar relations for three- and four-vertex functions.

The non-renormalized matrix element corresponding to the n-vertex diagram and including m vector-fermion vertices can schematically be written in the form

$$\mathcal{M} \sim e_0^m g_0^{n-m} \int (\Gamma_A)^n \langle S\rangle^{F_i} \Delta^{B_i^A} \Delta^{B_i^\varphi} \psi^{F_e} \langle A\rangle^{B_e^A} \langle\varphi\rangle^{B_e^\varphi} \psi^{F_e}, \tag{54}$$

where F_i, F_e, B_e^A, B_e^φ, B_i^A, B_i^φ are the number of internal fermion lines, external fermion lines, external vector lines, external scalar and pseudoscalar lines, internal vector lines and internal scalar and pseudoscalar lines, respectively. Substituting the renormalized values of (52) into (53) and taking into account that $n = F_i + \frac{1}{2}F_e$, $m = B_e^A + 2B_i^A$, $n-m = B_e^\varphi + 2B_i^\varphi$, we find the regularized matrix element

$$\mathcal{M}_R \sim (e')^m (g')^{n-m} \int (\Gamma'_A)^n (\langle S\rangle')^{F_i} (\Delta')^{B_i^\varphi} (\Delta')^{B_i^A} (\psi')^{F_e} (\langle A\rangle')^{B_e^A} (\langle\varphi\rangle')^{B_e^\varphi} (\psi')^{F_e}. \tag{55}$$

Thus, the matrix element of any process does not contain divergent renormalization constants and depends only on the renormalized physical values. In the model under consideration, renormalization of charges, mass-fermion and collective fields leads to the elimination of divergencies in all orders of the perturbation theory.

6. Conclusion

Thus, the initial model including scalar-scalar, pseudoscalar-pseudoscalar and vector--vector interaction has been reformulated by the method of functional integration in terms of interaction between fermions and collective massive scalar, pseudoscalar and vector fields. At the same time, kinetic terms of collective fields appear from the vacuum polarization diagrams. Perturbation theory has been considered which corresponds to a loop-expansion and leads to renormalizability of interaction. All infinities in the model are absorbed by the finite number of renormalization constants. The matrix elements of the processes of the interaction of fermions with their states (collective field) are independent of renormalization constants.

APPENDIX

After computations, we obtain from (24), making use of (9), the following vertex functions (for A, B, $C = 5, 6$)

$$\Gamma^0_{555}(p,q) = \frac{ig_0^4\varphi_0}{4\pi^4}\int\frac{dk}{F}\left\{\frac{1}{R}\left[g_0^2(3\tilde{\varphi}_0^2-\varphi_0^2)+2pk-pq+3k^2-4kq+q^2\right]\right.$$
$$\left.+\frac{1}{H}\left[g_0^2(3\tilde{\varphi}_0^2-\varphi_0^2)-2pk-2kq+qp+3k^2\right]\right\},$$

$$\Gamma^0_{655}(p,q) = \frac{ig_0^4\tilde{\varphi}_0}{4\pi^4}\int\frac{dk}{F}\left\{\frac{1}{R}\left[g_0^2(\tilde{\varphi}_0^2-3\varphi_0^2)-2qk+k^2-qp+q^2\right]\right.$$
$$\left.+\frac{1}{H}\left[g_0^2(\tilde{\varphi}_0^2-3\varphi_0^2)-pq+k^2\right]\right\},$$

$$\Gamma^0_{666}(p,q) = \frac{ig_0^4\tilde{\varphi}_0}{4\pi^4}\int\frac{dk}{F}\left\{\frac{1}{R}\left[(3\varphi_0^2-\tilde{\varphi}_0^2)g_0^2+2pk-pq+3k^2-4kq+q^2\right]\right.$$
$$\left.+\frac{1}{H}\left[g_0^2(3\varphi_0^2-\tilde{\varphi}_0^2)+3k^2-2pk+pq-2kq\right]\right\},$$

$$\Gamma^0_{566}(p,q) = \frac{ig_0^4\varphi_0}{4\pi^4}\int\frac{dk}{F}\left\{\frac{1}{R}\left[g_0^2(\varphi_0^2-3\tilde{\varphi}_0^2)+k^2-2qk-qp+q^2\right]\right.$$
$$\left.+\frac{1}{H}\left[g_0^2(\varphi_0^2-3\tilde{\varphi}_0^2)+k^2-qp\right]\right\},$$

$$\Gamma^0_{656}(p,q) = \frac{ig_0^4\varphi_0}{4\pi^4}\int\frac{dk}{F}\left\{\frac{1}{R}\left[g_0^2(\varphi_0^2-3\tilde{\varphi}_0^2)+qp-q^2+k^2\right]\right.$$
$$\left.+\frac{1}{H}\left[g_0^2(\varphi_0^2-3\tilde{\varphi}_0^2)+k^2-2qk+qp\right]\right\},$$

$$\Gamma^0_{556}(p,q) = \frac{ig_0^4\tilde{\varphi}_0}{4\pi^4}\int\frac{dk}{F}\left\{\frac{1}{R}\left[g_0^2(\tilde{\varphi}_0^2-3\varphi_0^2)+2pk-pq+k^2+q^2-2kq\right]\right.$$
$$\left.+\frac{1}{H}\left[g_0^2(\tilde{\varphi}_0^2-3\varphi_0^2)-2pk+pq+k^2\right]\right\},$$

$$\Gamma^0_{665}(p,q) = \frac{ig_0^4\varphi_0}{4\pi^4}\int\frac{dk}{F}\left\{\frac{1}{R}\left[g_0^2(\varphi_0^2-3\tilde{\varphi}_0^2)+2pk+(k-q)^2-qp\right]\right.$$
$$\left.+\frac{1}{H}\left[g_0^2(\varphi_0^2-3\tilde{\varphi}_0^2)+k^2-2pk+qp\right]\right\},$$
$$\Gamma^0_{565}(p,q) = \frac{ig_0^4\tilde{\varphi}_0}{4\pi^4}\int\frac{dk}{F}\left\{\frac{1}{R}\left[g_0^2(\tilde{\varphi}_0^2-3\varphi_0^2)+k^2+q(p-q)\right]\right.$$
$$\left.+\frac{1}{H}\left[g_0^2(\tilde{\varphi}_0^2-3\varphi_0^2)+k^2-2qk+qp\right]\right\}, \qquad \text{(A1)}$$

where notations $F = [(k-q)^2+m_0^2]\,(k^2+m_0^2)$, $R = (p+k-q)^2+m_0^2$, $H = (k-p)^2+m_0^2$ are introduced.

Assuming that $p = 0$ in expressions (A1) and integrating, we obtain with the aid of notations (14)

$$\Gamma^0_{566}(0,q) = \Gamma^0_{665}(0,q) = -4g_0^2\varphi_0 Z_\varphi^{-1}+\frac{g_0^4\varphi_0}{2\pi^2}J_1(q)-g_0^6\varphi_0\tilde{\varphi}_0^2J_2(q),$$
$$\Gamma^0_{556}(0,q) = \Gamma^0_{655}(0,q) = -4g_0^2\tilde{\varphi}_0 Z_\varphi^{-1}+\frac{g_0^4\tilde{\varphi}_0}{2\pi^2}J_1(q)-g_0^6\tilde{\varphi}_0\varphi_0^2J_2(q), \qquad \text{(A2)}$$

where

$$J_2(q) = \frac{i}{\pi^4}\int dk\left[\frac{1}{[(k-q)^2+m_0^2]^2\,(k^2+m_0^2)}+\frac{1}{[(k-q)^2+m_0^2]\,(k^2+m_0^2)^2}\right]$$

is a finite integral.

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