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ON THE THEORY OF FIELDS IN FINSLER SPACES — II

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Continued from a previous paper (*Acta Phys. Pol.* **B13**, 321 (1982)), some structural observations are further made on the intrinsic behaviour of the vectorial internal vector (y) associated with each point (x), which is shown to be represented by the base connection of y (i.e., δy), different from the ordinary absolute differential of y (i.e., Dy). Then, corresponding to the difference between Dy and δy, a new Finsler metric such as $g_{\lambda\kappa}(x, y) = \gamma_{\lambda\kappa}(x) + h_{\lambda\kappa}(x, y)$ is introduced, which is induced by "unifying" the purely Riemannian metric $\gamma_{\lambda\kappa}(x)$ of the original gravitational field in Einstein's sense and the internal Finsler metric $h_{\lambda\kappa}(x, y)$ of the internal space spanned by vectors $\{y\}$. Some fundamental considerations are also made on the metrical Finsler connections with respect to the metric $g_{\lambda\kappa}(x, y)$.

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1. Introduction

The geometrical structure of the gravitational field in Einstein's sense [2] is, of course, four-dimensional Riemannian (R_4), which is entirely governed by the Riemannian metric (= gravitational potential) $\gamma_{\lambda\kappa}(x)$ ($\kappa, \lambda = 1, 2, 3, 4$). In other words, this R_4-structure may be regarded as "point space-like", "macroscopic" and "local", because only the point x is chosen as the independent variable (cf. [3]).

On the other hand, in the theory of gravitational field in Finsler spaces [1, 4], the line-element (x, y), instead of the point (x), is adopted as the independent variable and the (tangent) vector y ($= y^\lambda$; $\lambda = 1, 2, 3, 4$) is regarded as the internal variable associated with each point x ($= x^\kappa$; $\kappa = 1, 2, 3, 4$). Therefore, it may be said that the Finsler structure (F_4) is more microscopic than the Riemannian structure (R_4) in Einstein's sense and the "deviation" of the F_4-structure from the R_4-structure is caused by the y-dependence. Or else, it may be considered that the Finslerian gravitational field is obtained by attaching the vector y to each point x of the Einstein's Riemannian gravitational field at some microscopic stage. This corresponds to the "nonlocalization" of gravitational field by the vector y, as has been considered in Section 2 of [1] (cf. also [3]). In this sense, the F_4-structure

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of the gravitational field may be regarded as "line-element space-like", "microscopic" and "nonlocal".

The vector y belongs to the so-called internal symmetry space spanned by vectors $\{y\}$, which will be named the (y)-field. The (y)-field itself has, in general, a somewhat "non-" -Riemannian structure ($\bar{R}_4$), including Riemannian (R_4) or Minkowskian (M_4) structure, while the original gravitational field in Einstein's sense, which will be named the (x)-field spanned by points $\{x\}$, has the R_4-structure. Therefore, the resulting Finsler structure (F_4) is induced by "unifying" the $\bar{R}_4$-structure of the (y)-field with the R_4-structure of the (x)-field. One typical example of this "unification" will be given in Section 2, and a new Finsler metric represented by (2.5) will be proposed concretely.

The vector y, as the internal variable, shows its own intrinsic behaviour, different from an arbitrary vector, say X $(= X^\kappa;\ \kappa = 1, 2, 3, 4)$, that is, y is endowed with the internal symmetry such as Poincaré or Lorentz transformation. This intrinsic behaviour is grasped geometrically by its intrinsic "parallelism" or connection (i.e., δy), which is nothing but the base connection (cf. [1, 3, 5]), and is different from the ordinary absolute differential of y (i.e., Dy). In Section 3, the relation between Dy and δy will be clarified and then, on the basis of the difference between Dy and δy, some fundamental observations will be made on the metrical Finsler connection with respect to the metric of (2.5).

In Section 4, some other comments will be made on the Finsler structures with respect to the metric tensors of (2.5) and (2.6), the latter has been introduced in the previous papers [6]. In particular, a new physical interpretation of the special form of torsion given by (4.2) will be proposed. Section 5 will be devoted to conclusions.

2. *On the Finsler structure*

As mentioned above, the Finsler structure (F_4) of the gravitational field is caused by attaching the internal vector y to each point x, so that F_4-structure is derived from the "unification" of the (x)- and (y)-fields. Concerning this "unification", the R_8- or $\bar{R}_8$-structure is first obtained and then, the final F_4-structure is derived from this R_8- or $\bar{R}_8$-structure (cf. [6]). This way of thinking is justified by the contact tensor calculus or the theory of tangent bundles [7], in which it has been proved that the R_{2n}- or $\bar{R}_{2n}$-structure can be arranged to become the F_n-structure ($n = 4$ in our case). But it should be remarked that the "unified" field in our sense is not necessarily compared to the tangent bundle over the Finsler manifold, because the intrinsic behaviour of y, that is, the intrinsic "parallelism" of y can be chosen in various ways according to our physical demands and the metric for the internal (y)-field can be chosen differently from the whole "unified" Finsler metric (see Section 3).

In this Section, in order to emphasize the physical function of y as the internal variable and distinguish the external x and the internal y explicitly, we shall denote $x = (x^\kappa)$ $(\kappa = 1, 2, 3, 4)$ and $y = (y^i)$ $(i = 1, 2, 3, 4)$ respectively, the latter will be shown to be brought in the "unified" field as the vector y^κ (see (2.1)).

Now, in general, the (y)-field has some "non"-Riemannian structure ($\bar{R}_4$), including R_4- or M_4-structure. The flat (y)-field with M_4-structure may be likened to the isotopic space [8]. In order to "unify" the (x)- and (y)-fields, it is necessary to reflect (or map) the

$\bar{R}_4$-structure of the (y)-field in (or on) the R_4-structure of the (x)-field. For that purpose, we shall here introduce the following mapping relation:

$$y^\kappa = e_i^\kappa(x) y^i, \quad y^i = e_\lambda^i(x) y^\lambda, \tag{2.1}$$

by which the vector y^i of the (y)-field is reflected or embedded in the "unified" field as the vector y^κ. The transformation matrix e satisfies the relations $e_i^\kappa e_\lambda^i = \delta_\lambda^\kappa$ and $e_i^\kappa e_\kappa^j = \delta_i^j$, and y^i plays the role of 1-form corresponding to y^κ (cf. [9]).

Let the metric tensor of the internal (y)-field be denoted by $h_{ij}(y)$, which presents a Riemannian aspect, including a Minkowskian one (η_{ij}). By the mapping relation of (2.1), $h_{ij}(y)$ is reflected or embedded in the "unified" field in the form

$$h_{\lambda\kappa}(x, y) = e_\lambda^j(x) e_\kappa^i(x) h_{ji}(y), \tag{2.2}$$

which becomes, of course, Finslerian depending on y as well as x. In fact, this metric resembles the 1-form Finsler metric derived from the 1-form metric function $L(y^i) = L(e_\lambda^i(x) y^\lambda)$, i.e. $h_{\lambda\kappa} = \frac{1}{2} \dfrac{\partial^2 L^2}{\partial y^\lambda \partial y^\kappa}$ (cf. [9]). If the (y)-field is Minkowskian, then (2.1) and (2.2) are reduced to

$$y^\kappa = \varepsilon_i^\kappa(x, y)\eta^i, \quad \eta^i = \varepsilon_\lambda^i(x, y) y^\lambda, \tag{2.3}$$

$$h_{\lambda\kappa}(x, y) = \varepsilon_\lambda^j(x, y)\varepsilon_\kappa^i(x, y)\eta_{ij}, \tag{2.4}$$

where ε denotes the Finslerian vierbein and the relation $\eta_{ij} = (\eta_i\eta_j + \eta_j\eta_i)/2$ holds good.

The whole "unified" Finsler metric $g_{\lambda\kappa}(x, y)$ of the "unified" field is given by "unifying" the external Riemannian $\gamma_{\lambda\kappa}(x)$ and the internal Finslerian $h_{\lambda\kappa}(x, y)$. We can consider several kinds of "unifications", but here we shall choose, as a typical example, the following formula:

$$g_{\lambda\kappa}(x, y) = \gamma_{\lambda\kappa}(x) + h_{\lambda\kappa}(x, y). \tag{2.5}$$

It is found that the internal vector y^i is fused into the "unified" field through the term $h_{\lambda\kappa}(x, y)$. In the previous papers [6], another "unification" such as

$$g_{\lambda\kappa}(x, y) = \gamma_{\lambda\kappa}(x) \cdot \exp 2\sigma(x, y) \tag{2.6}$$

has been proposed. Comparing (2.5) and (2.6), we can obtain one relation

$$\exp 2\sigma(x, y) = (\gamma_{\lambda\kappa} + h_{\lambda\kappa})\gamma^{\kappa\lambda} \tag{2.7}$$

which means that the conformal scalar σ is also prescribed by the internal metric $h_{\lambda\kappa}$, and vice versa. This corresponds to the microscopic character of our conformal scalar σ introduced in [6], different from Brans-Dicke's scalar [10] or Dirac's conformal scalar [11]. Some other structural features related with (2.5) and (2.6) will be mentioned in Section 4.

Returning to (2.5), we can consider, with emphasis on the physical function of the vector y as the internal variable, that the metric $h_{\lambda\kappa}$ is used to measure the length of y alone (i.e., $y^2 = h_{\lambda\kappa}y^\kappa y^\lambda$), while the metric $g_{\lambda\kappa}$ is used to measure the length of an arbitrary vector

$X(\neq y)$ (i.e., $X^2 = g_{\lambda\kappa}X^\kappa X^\lambda$). Therefore, corresponding to the difference between $g_{\lambda\kappa}$ and $h_{\lambda\kappa}$, we must introduce two different kinds of metrical Finsler connections $Dg_{\lambda\kappa} = 0$ (and $Dh_{\lambda\kappa} \neq 0$) and $\delta h_{\lambda\kappa} = 0$ (and $\delta g_{\lambda\kappa} \neq 0$). Their relation will be considered in detail in the next Section.

3. *On the intrinsic behaviour of the internal variable* (y)

As mentioned at the end of Section 2, we must consider two different metrical connections D and δ for $g_{\lambda\kappa}$ and $h_{\lambda\kappa}$ respectively. This copes with the fact that the intrinsic behaviour of y is different from that of an arbitrary vector $X\,(\neq y)$. That is to say, the "parallelism" of y is represented by the connection δy, not by Dy, while that of X is given by DX (see Section 3 of [1]).

Now, let DX be denoted by [12]

$$DX^\kappa = dX^\kappa + \Gamma^\kappa_{\mu\lambda}X^\mu dx^\lambda + C^\kappa_{\mu\lambda}X^\mu dy^\lambda, \tag{3.1}$$

where $\Gamma^\kappa_{\mu\lambda}$ and $C^\kappa_{\mu\lambda}$ are the ordinary Finsler coefficients of connection. On the other hand, let δy ($\neq Dy$) be written as

$$\delta y^\kappa = dy^\kappa + \Xi^\kappa_{\mu\lambda}y^\mu dx^\lambda + \Delta^\kappa_{\mu\lambda}y^\mu dy^\lambda, \tag{3.2}$$

where $\Xi^\kappa_{\mu\lambda}$ and $\Delta^\kappa_{\mu\lambda}$ are other Finsler coefficients of connection different from $\Gamma^\kappa_{\mu\lambda}$ and $C^\kappa_{\mu\lambda}$ respectively. In (3.1) and (3.2), such homogeneity conditions as $C^\kappa_{\mu\lambda}y^\mu = 0$, $\Delta^\kappa_{\mu\lambda}y^\mu = 0$, etc. may be assumed, if necessary. Then, in order to consider the relation between Dy and δy, we shall here reconsider the relation ($Dg_{\lambda\kappa} = 0$ and $Dh_{\lambda\kappa} \neq 0$) and ($\delta h_{\lambda\kappa} = 0$ and $\delta g_{\lambda\kappa} \neq 0$) as follows [1, 13]: The connection D is a metrical connection for $g_{\lambda\kappa}$ (i.e., $Dg_{\lambda\kappa} = 0$) derived from the non-metrical one δ (i.e., $\delta g_{\lambda\kappa} \neq 0$). From this, by virtue of Kawaguchi's theorem [14], the desired relation can be obtained as follows:

$$Dy^\kappa = \delta y^\kappa + \tfrac{1}{2}\, g^{\kappa\nu}(\delta g_{\nu\lambda})y^\lambda, \tag{3.3}$$

by which the following relations can be obtained from (3.1) and (3.2):

$$\Gamma^\kappa_{\mu\lambda} = \Xi^\kappa_{\mu\lambda} + \tfrac{1}{2}\, g^{\kappa\nu}\left(\frac{\partial g_{\nu\mu}}{\partial x^\lambda} - \Xi^\iota_{\nu\lambda}g_{\iota\mu} - \Xi^\iota_{\mu\lambda}g_{\nu\iota}\right),$$

$$C^\kappa_{\mu\lambda} = \Delta^\kappa_{\mu\lambda} + \tfrac{1}{2}\, g^{\kappa\nu}\left(\frac{\partial g_{\nu\mu}}{\partial y^\lambda} - \Delta^\iota_{\nu\lambda}g_{\iota\mu} - \Delta^\iota_{\mu\lambda}g_{\nu\iota}\right). \tag{3.4}$$

Thus, the relation between Dy and δy has been clarified. Almost all the geometricians assume $Dy = \delta y$. Therefore, the condition $Dy \neq \delta y$ is very different from many special connections treated by geometricians (cf. [9, 12, 15]). Therefore, for example, the Gauss equation stipulating the relation between indicatrix and tangent Riemannian space must be used in its most general form, and the conservation laws, etc. obtained previously in the case of Cartan's connection (see (4.11), (4.12) and (4.14) in [1]) do not hold for this case.

As an interesting special example of δy, we can obtain one "parallelism" of y, (i.e., $\delta y = 0$) from (1.1) in the form

$$\delta y^\kappa = dy^\kappa + \Xi^\kappa_{\mu\lambda} y^\mu dx^\lambda = 0;$$

$$\Xi^\kappa_{\mu\lambda} = -\frac{\partial e^\kappa_i}{\partial x^\lambda} e^i_\mu = e^\kappa_i \frac{\partial e^i_\mu}{\partial x^\lambda}, \tag{3.5}$$

where the homogeneity condition $\Delta^\kappa_{\mu\lambda} y^\mu = 0$ has been assumed. Starting from (3.5), we can determine several kinds of metrical Finsler connections for $h_{\lambda\kappa}$ (i.e., $\delta h_{\lambda\kappa} = 0$) such as the 1-form linear connection $(\Xi^\kappa_{\mu\lambda}, \Xi^\kappa_{0\lambda}, \Delta^\kappa_{\mu\lambda})$[1] (cf. [9]). If the (y)-field is Minkowskian, then the quantity e in (3.5) must be replaced by ε of (2.3). Of course, in those cases, $\delta g_{\lambda\kappa} \neq 0$, even if $\delta h_{\lambda\kappa} = 0$, so that the whole metrical Finsler connection for $g_{\lambda\kappa}$ (i.e., $Dg_{\lambda\kappa} = 0$) must be newly constructed by taking account of (3.3) and (3.4) (see below).

Now, the intrinsic behaviour of y (i.e., δy) is reflected in the whole Finsler structure by replacing dy in (3.1) with δy of (3.2) as follows:

$$DX^\kappa = dX^\kappa + F^\kappa_{\mu\lambda} X^\mu dx^\lambda + \Theta^\kappa_{\mu\lambda} X^\mu \delta y^\lambda, \tag{3.6}$$

where

$$\begin{aligned} F^\kappa_{\mu\lambda} &= \Gamma^\kappa_{\mu\lambda} - N^\nu_\lambda C^\kappa_{\mu\nu}, \\ \Theta^\kappa_{\mu\lambda} &= (P^{-1})^\kappa_\nu C^\nu_{\mu\lambda}, \\ N^\kappa_\lambda &= (P^{-1})^\kappa_\nu Q^\nu_\lambda, \\ P^\kappa_\lambda &= \delta^\kappa_\lambda + \Delta^\kappa_{\mu\lambda} y^\mu, \\ Q^\kappa_\lambda &= \Xi^\kappa_{\mu\lambda} y^\mu. \end{aligned} \tag{3.7}$$

The quantity P^κ_λ is, of course, assumed to be non-singular. The quantity N^κ_λ plays the role of nonlinear connection (cf. [12]), and the quantity $F^\kappa_{\mu\lambda}$ embodies the "unified" symmetry or "unified" gauge field from the standpoint of gauge field theory (cf. [1, 3, 4]). The resulting metrical Finsler connection for $g_{\lambda\kappa}$ (i.e., $Dg_{\lambda\kappa} = 0$) is characterized by the three quantities $(F^\kappa_{\mu\lambda}, N^\kappa_\lambda, \Theta^\kappa_{\mu\lambda})$. From (3.6), the two kinds of covariant derivatives with respect to x and y (with $g_{\lambda\kappa|\mu} = 0$ and $g_{\lambda\kappa}|_\mu = 0$) can be introduced, and then, by use of three kinds of Ricci--identities with respect to those covariant derivatives, three kinds of curvature tensors and five kinds of torsion tensors can be defined, in general; these, however, are all omitted here (cf. [12, 15]). Geometricians have been considering many kinds of specializations of metrical Finsler connections along the theory of special Finsler spaces (cf. [9, 12, 15]). For example, what conditions are necessary and sufficient in order that the quantity $F^\kappa_{\mu\lambda}$ may depend on x alone, etc. These specializations, however, cannot always be accepted for physical problems, much to our regret.

It should be noticed here that several kinds of metrical Finsler connections $(F^\kappa_{\mu\lambda}, N^\kappa_\lambda, \Theta^\kappa_{\mu\lambda})$ can be constructed for the metric tensor $g_{\lambda\kappa}$ of (2.5) under some convenient conditions such as $F^\kappa_{\mu\lambda} = F^\kappa_{\lambda\mu}$ and $N^\kappa_\lambda = F^\kappa_{0\lambda}$ and $\Theta^\kappa_{\mu\lambda} = \Theta^\kappa_{\lambda\mu}$, etc., and that the set of all metrical Finsler

[1] The symbol 0 means the contraction by y, e.g., $\Xi^\kappa_{0\lambda} = \Xi^\kappa_{\mu\lambda} y^\mu$.

connections for $g_{\lambda\kappa}$ (such as $Dg_{\lambda\kappa} = 0$) derived from a fixed non-metrical Finsler connection $(\mathring{F}^{\kappa}_{\mu\lambda}, \mathring{N}^{\kappa}_{\lambda}, \mathring{\Theta}^{\kappa}_{\mu\lambda})$ for $g_{\lambda\kappa}$ (such as $\delta g_{\lambda\kappa} \neq 0$) is given by (cf. [15])

$$F^{\kappa}_{\mu\lambda} = \mathring{F}^{\kappa}_{\mu\lambda} + \mathring{\Theta}^{\kappa}_{\mu\nu} X^{\nu}_{\lambda} + \tfrac{1}{2} g^{\kappa\nu}(g_{\nu\mu\mathring{|}\lambda} + g_{\nu\mu}\mathring{|}_{\iota} X^{\iota}_{\lambda}) + \Omega^{\kappa\nu}_{\iota\mu} Y^{\iota}_{\nu\lambda},$$

$$N^{\kappa}_{\lambda} = \mathring{N}^{\kappa}_{\lambda} - X^{\kappa}_{\lambda},$$

$$\Theta^{\kappa}_{\mu\lambda} = \mathring{\Theta}^{\kappa}_{\mu\lambda} + \tfrac{1}{2} g^{\kappa\nu} g_{\nu\mu}\mathring{|}_{\lambda} + \Omega^{\kappa\nu}_{\iota\mu} Z^{\iota}_{\nu\lambda},$$

where X^{κ}_{λ}, $Y^{\kappa}_{\mu\lambda}$ and $Z^{\kappa}_{\mu\lambda}$ are arbitrary Finsler tensor fields, and $g_{\lambda\kappa\mathring{|}\mu}$ and $g_{\lambda\kappa}\mathring{|}_{\mu}$ denote the covariant derivatives with respect to the fixed connection, which vanish if the fixed connection is metrical for $g_{\lambda\kappa}$, and $\Omega^{\kappa\nu}_{\iota\mu} = \frac{1}{2}(\delta^{\kappa}_{\iota}\delta^{\nu}_{\mu} - g_{\iota\mu}g^{\kappa\nu})$.

4. *Other comments*

In the previous papers [6], the author, bearing in mind the conformal structure of Brans-Dicke theory [10] or the Weyl-Dirac theory with torsion [11], has proposed the Finslerian conformal structure in the form of (2.6), where the conformal scalar σ is constructed by y properly and represents the difference between F_4- and R_4-structures. In those papers [6] (and also in Section 5 of [1]), the vector y is regarded as the so-called space--time fluctuation at some more microscopic stage than in Einstein's sense, which is assumed to be summarized as the metrical fluctuation in the form of (2.6). Starting from this metric (2.6), we can also determine several kinds of metrical Finsler connections for $g_{\lambda\kappa}$ under some convenient conditions. In fact, one special connection has been determined concretely by us [16] under the assumption that $\delta y = Dy$, $F^{\kappa}_{\mu\lambda} = F^{\kappa}_{\lambda\mu}$, $\Theta^{\kappa}_{\mu\lambda} = \Theta^{\kappa}_{\lambda\mu}$ and $N^{\kappa}_{\lambda} = F^{\kappa}_{0\lambda}$.

As in the previous papers [6], if y is reduced to a function of x (i.e., $y = y(x)$), then those two metrical Finsler connections derived from $g_{\lambda\kappa}$ of (2.5) and $g_{\lambda\kappa}$ of (2.6) become also Riemannian or non-Riemannian according as the torsion $(T^{\kappa}_{\lambda\mu}(x) = F^{\kappa}_{\lambda\mu} - F^{\kappa}_{\mu\lambda}) = 0$ or $\neq 0$. It should be noticed that if the relation $y^{\kappa}(x) = \sum_{i=1}^{4} e^{\kappa}_{i}(x)$ is chosen as the osculating condition, then $y^{i} = e^{i}_{\lambda} y^{\lambda} = 1$ holds good (see (2.1)), so that $h_{ij}(y)$ and corresponding coefficient $\Delta^{i}_{jk} (= e^{i}_{\kappa} e^{j}_{\lambda} e^{\mu}_{k} \Delta^{\kappa}_{\lambda\mu})$ become constant (cf. [9]). The osculating (non-) Riemannian coefficient of connection can be written formally in the form

$$F^{\kappa}_{\lambda\mu}(x) = \{^{\kappa}_{\lambda\mu}\} + \Phi^{\kappa}_{\lambda\mu}, \tag{4.1}$$

where $\{^{\kappa}_{\lambda\mu}\}$ denotes the Christoffel three-index symbol derived from $\gamma_{\lambda\kappa}(x)$ and $\Phi^{\kappa}_{\lambda\mu}$ is defined as the rest; the latter contains the contortion tensor constructed by the torsion $T^{\kappa}_{\lambda\mu}(x)$. In the case of (2.5), the term $\Phi^{\kappa}_{\lambda\mu}$ contains the contributions by $h_{\lambda\kappa}(x, y(x))$ too, while in the case of (2.6), it contains the contributions by $\sigma(x, y(x))$. Therefore, it is obvious that those effects of $\Phi^{\kappa}_{\lambda\mu}$ still remain after the averaging process $\langle\rangle$ with respect to y considered in the previous papers [6] is performed, so that the resulting macro-field becomes somewhat "non"-Riemannian, in general. Those "non"-Riemannian features may serve as the source term at the stage of field equation (see Section 5 in [1]). It should be remarked that in Section 5 in [1], the osculating process ($y = y(x)$) itself is regarded as the averaging process with respect to y, but in [6], a new averaging process $\langle\rangle$ satisfying the conditions $\langle \exp \sigma \rangle = 1$ and $\langle \sigma_\lambda \rangle = 0$ is used. However, the averaging process $\langle\rangle$ itself cannot yet be "geometrized".

As to the torsion $T^{\kappa}_{\lambda\mu}(x)$ appearing in (4.1), the following interesting special form has been introduced in the previous papers [6]:

$$T^{\kappa}_{\lambda\mu}(x) = \delta^{\kappa}_{\lambda}\sigma_\mu - \delta^{\kappa}_{\mu}\sigma_\lambda, \tag{4.2}$$

where $\sigma_\lambda = \dfrac{\partial \sigma}{\partial x^\lambda}$, which is influenced by the conformal transformation. Concerning this, we shall here propose another physical interpretation: Now, from the standpoint of thermodynamics of irreversible processes (TIP), we shall treat the space-time fluctuation $y(x)$ as the thermodynamical variable connected with entropy production. And we shall also assume that $y(x)$ is summarized as the metrical fluctuation in the same form as (2.6), i.e.,

$$g_{\lambda\kappa}(x, y(x)) = \gamma_{\lambda\kappa}(x) \cdot \exp 2\sigma(x, y(x)), \tag{4.3}$$

which may be regarded, in this case, as a geometrical "unification" between the (x)-field with R_4-structure and the (y)-field with $\bar{R}_4$-structure. That is to say, (4.3) embodies a geometrical "unification" between the framework of general relativity and gravitation (GRG) and that of TIP.

Concerning the $\bar{R}_4$-structure of TIP, the author has already shown [17], on the basis of the relation between the cycle theory and the holonomy group theory, that the concept of torsion is connected with entropy production. Therefore, corresponding to the non--Riemannian structure (with torsion $\neq 0$) of TIP, the "unified" field prescribed by (4.3) becomes conformally non-Riemannian.

In general, by analogy with TIP [18], the entropy change (ΔS) in the "unified" field (= thermodynamical gravitational field) is stipulated as

$$\Delta S = \alpha_I \Delta X^I, \tag{4.4}$$

where ΔX^I means the "unified" discrepancy, $X^I = (x^\kappa, y^\lambda)$ the "unified" coordinate, and α_I may be likened to the ideal vector defined by (4.4) itself. Since ΔS contains some irreversible component (i.e., entropy production) caused by $y(x)$, ΔS is not completely integrable. Therefore, α_I becomes, in general, non-integrable, so that the non-holonomic object O^I_{JK} is introduced as

$$\partial_K \alpha_J - \partial_J \alpha_K = -O^I_{JK}\alpha_I. \tag{4.5}$$

This fact means that the torsion T^I_{JK} of the "unified" field is introduced by

$$T^I_{JK} = \alpha^I(\partial_K \alpha_J - \partial_J \alpha_K) \,(\equiv -O^I_{JK}). \tag{4.6}$$

It should be considered that the coordinate X^I is so chosen that the relation T^I_{JK} $(= F^I_{JK} - F^I_{KJ}) = -O^I_{JK}$ may be satisfied. The "unified" metric tensor g_{IJ}, which is nothing but $g_{\lambda\kappa}(x, y(x))$ of (4.3), is defined by α_I as follows:

$$g_{IJ} = (\alpha_I \alpha_J + \alpha_J \alpha_I)/2, \tag{4.7}$$

which gives $(\Delta S)^2 = g_{IJ}\Delta X^I \Delta X^J$. In this case, the "unified" coefficient of connection corresponding to $F^{\kappa}_{\lambda\mu}(x)$ of (4.1) is given by

$$F^I_{JK}(X) = \{^{\ I}_{JK}\} + \Phi^I_{JK}, \tag{4.8}$$

where $\{^{\ I}_{JK}\}$ represents the Christoffel three-index symbol formed with g_{IJ} and Φ^I_{JK} the contortion tensor formed with T^I_{JK}.

From (4.3) and (4.7), we can extract

$$(\alpha_I \equiv) \qquad \alpha_\lambda = \gamma_\lambda \exp \sigma, \tag{4.9}$$

so that from (4.6) and (4.8), we can put

$$(T^I_{JK} \equiv)\, T^{\kappa}_{\lambda\mu} = \delta^{\kappa}_{\lambda}\sigma_\mu - \delta^{\kappa}_{\mu}\sigma_\lambda, \tag{4.10}$$

$$(F^I_{JK} \equiv) F^{\kappa}_{\lambda\mu} = \{^{\ \kappa}_{\lambda\mu}\} + \delta^{\kappa}_{\lambda}\sigma_\mu. \tag{4.11}$$

Thus, the special form of torsion given by (4.2) has been obtained in (4.10), and its interesting thermodynamical meaning as entropy production has been given. At any rate, one geometrical "unification" between GRG and TIP has been proposed.

5. *Conclusions*

Thus, we have made a structural study of the intrinsic behaviour of y. In Section 1, we have mentioned our standpoint to regard the Finslerian gravitational field as one kind of "nonlocal" field. In Section 2, the internal (y)-field has been embedded in the external (x)-field and the Finsler metric $g_{\lambda\kappa}(x, y) = \gamma_{\lambda\kappa}(x) + h_{\lambda\kappa}(x, y)$ has been introduced. In Section 3, the relation between Dy and δy has been clarified and the whole spatial structure has been considered. In Section 4, some other comments have been made, and a new physical interpretation with respect to the torsion $T^{\kappa}_{\lambda\mu}(x) = \delta^{\kappa}_{\lambda}\sigma_\mu - \delta^{\kappa}_{\mu}\sigma_\lambda$ has been given.

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