RADIATIVE CORRECTIONS TO THE CABIBBO ANGLE

By J. Pawełczyk

Institute of Theoretical Physics, Warsaw University*

(Received January 17, 1984)

It is shown that in L-R symmetric models with additional horizontal symmetry the one-loop corrections can significantly change the zeroth order predictions of these models.

PACS numbers: 12.10.Ck

1. Introduction

In gauge theories of the electroweak interactions the quark mass matrix is completely arbitrary. Its diagonalization gives the physical quark masses and the so-called weak mixing angles grouped in the Kobayashi-Maskawa matrix. All these parameters are free. On the other hand speculations on a connection between the quark masses and the Cabibbo angle are as old as the Cabibbo theory. They are supported by the well known phenomenological relation for the Cabibbo angle

$$\theta_C \simeq \sqrt{\frac{m_d}{m_s}}$$

where m_d, m_s are current masses of the d and s quark respectively.

Practical realization of this idea was suggested by Weinberg in 1977 [1]. He proposed to impose an additional symmetry upon the Yukawa coupling lagrangian. It allows one to express the weak mixing angles in terms of the quark masses. This additional symmetry must commute with the gauge symmetry of the theory. It is called horizontal symmetry and it will be denoted by G_H in the following. Horizontal symmetry relates each quark family to the others as well as each Higgs multiplet to the others (if, of course, the used representations of G_H are irreducible).

We restrict our considerations to the standard model [2] and the left-right (L-R) symmetric model [3] with a global horizontal symmetry [4]. The most general Yukawa

* Address: Instytut Fizyki Teoretycznej, Uniwersytet Warszawski, Hoża 68, 00-681 Warszawa, Poland.

coupling lagrangian has the form:

$$\mathscr{L}_Y = \sum_{i,j,k} (\bar{\psi}^i_L \Gamma^{(n)}_{kij} \psi^j_R \phi_k + \bar{\psi}^i_L \Gamma^{(p)}_{kij} \psi'^j_R \tilde{\phi}_k) + \text{h.c.} \tag{1}$$

where ψ^i_L are doublets of SU(2), ψ^i_R, $\psi^{i'}_R$ are singlets for the standard model and $\psi^i_R = \psi'^i_R$ are doublets of SU(2) for the L–R symmetric model, $\phi_k = \begin{pmatrix} \phi^+_k \\ \phi^0_k \end{pmatrix}$; $\tilde{\phi}_k = i\tau^2 \phi^*_k$ are Higgses for the standard model, $\phi_k = \begin{pmatrix} \phi^0_{1k} & \phi^+_{1k} \\ \phi^-_{2k} & \phi^0_{2k} \end{pmatrix}$; $\tilde{\phi} = \tau^2 \phi^*_k \tau^2$ are Higgses for the L–R symmetric model.

The G_H-invariance of the L_Y can be expressed as follows:

$$\begin{aligned} K^+_L \Gamma^{(n)}_k K_R &= D^*_{kl} \Gamma^{(n)}_l, \\ K^+_L \Gamma^{(p)}_k K_R &= D'_{kl} \Gamma^{(p)}_l, \end{aligned} \tag{2}$$

where K_L, K_R, D are representations of G_H for the left (L) and the right-handed (R) quarks, and for the Higgses, respectively. Above equations usually determine the matrices Γ_k up to a few free parameters.

There is exhaustive literature on horizontal symmetries. Numerous models have been proposed. The most general discussion of models with a horizontal symmetry leading to phenomenologically acceptable mixing angles has been given by Ecker, Konetschny and Grimus [5].

The aim of this paper is to evaluate the radiative corrections to the Cabibbo angle obtained in the way described above. We do not want to choose any particular horizontal symmetric model because its explicit construction is not necessary for our purpose (except that it should predict phenomenologically acceptable values of the weak mixing angles).

2. *Radiative corrections*

After introducing a horizontal symmetry the weak mixing angles are not free parameters. Spontaneous breakdown of the gauge symmetry breaks down the horizontal symmetry, too. As it has been shown [6], this mechanism induces finite radiative corrections to the relations expressing the weak mixing angles in terms of the quark masses, obtained in the tree approximation. The corrections are finite because we have no counterterms to the weak mixing angles and on the other hand the theory is renormalizable. It is interesting to study how much the radiative processes can change the zeroth order predictions. We only consider the coriections to the Cabibbo angle which is known with a reasonable accuracy:

$$\sin \theta_C = 0.2288 \pm 0.01.$$

For the purpose of our calculations it is covenient to write the interaction of quarks and gauge bosons in the following form:

$$\mathscr{L}_{\text{int}} = \bar{\psi}^0 t_a \gamma_\mu \psi^0 A^{a\mu}, \tag{3}$$

where we assume that the gauge boson fields A^a_μ are mass eigenstates with mass M_a. The quark multiplet ψ^0 has the following structure:

$$\bar{\psi}^0 = (\bar{p}_L, \bar{n}_L, \bar{p}_R, \bar{n}_R) \tag{4}$$

where p, n stand for all up (p) and down (n) quarks. The mass matrix of the multiplet ψ^0 is

$$m = \left(\begin{array}{cc|cc} & & m_p & \\ & & & m_n \\ \hline m_p^+ & & & \\ & m_n^+ & & \end{array}\right), \tag{5}$$

where m_p and m_n are off-diagonal mass matrices for the unmixed quarks p, n, respectively. The interaction of quarks and scalars is given by the lagrangian (1).

The relevant for our calculations Feynman diagrams are presented in Fig. 1. They contribute to the quark mass matrices and therefore to the weak mixing angles, too. We are interested in corrections which do not have symmetry of the group G_H (in the

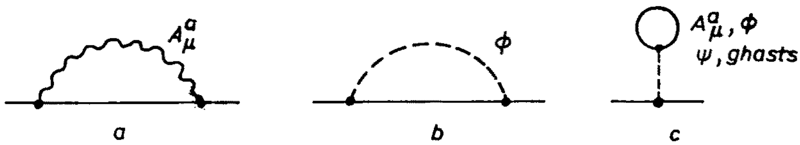

Fig. 1. Diagrams contributing to quark self-energy

sense of Eqs. (2)) — all the others (including infinities) can be absorbed in the redefined constants of the lagrangian. This fact causes that the tadpole diagram 1c does not contribute at all. We expect the largest correction comes from the diagram 1a. The contribution of the diagram 1b should be $\left(\frac{m_q}{M_W}\right)^2$ times smaller than the contribution of the diagram 1a $\left(m_q \text{ is a mass of a quark, } M_W \text{ is a mass of a charged boson; we assume that } \left(\frac{m_q}{M_W}\right)^2 \ll 1\right)$. We will consider the diagram 1b later on.

Let us calculate the diagram 1a in the unphysical basis ψ^0. According to the expression (3) the relevant part of the quark self-energy has the form

$$-i\Sigma^A\Big|_{\not{p}=m} = \frac{-i}{16\pi^2}\sum_a \int_0^1 dx(2mt_a(1-x)-4t_a m)\ln\left[M_a^2(1-x)+m^2x^2\right]t_a, \tag{6}$$

where M_a is the mass of the boson "a". The correction to the quark mass matrix m is then:

$$\Delta m = \Sigma^A\Big|_{\not{p}=m}.$$

The transformation from the unphysical basis to the physical one is done by the unitary matrices $U_L'^p$, $U_R'^n$ ... which diagonalize the relevant quark mass matrices $m_p+\Delta m_p$, ...

The matrices U' should not be very different from the matrices U which diagonalize the bare quark mass matrices m_p, m_n. Then we can write $U' = U \cdot W$ where W is close to the identity matrix (we assume 3 generations of quarks).

$$W_L^p = \begin{pmatrix} 1+i\alpha_4 & \alpha_1 & \alpha_2 \\ -\alpha_1^* & 1+i\alpha_5 & \alpha_3 \\ -\alpha_2^* & -\alpha_3^* & 1+i\alpha_6 \end{pmatrix}; \; |\alpha_i| \ll 1. \tag{7}$$

Parameters $\alpha_1, \alpha_2, \alpha_3$ are complex; $\alpha_4, \alpha_5, \alpha_6$ are real. Corrected Kobayashi-Maskawa matrix has the form:

$$U'_{KM} = U_L'^{p+} U_L'^{n} = W_L^{p+} U_{KM} W_L^n. \tag{8}$$

Matrix U_{KM} contains zeroth order predictions for the weak mixing angles (we use its standard parametrization). Equation (8) enables one to calculate corrections to the weak mixing angles if the matrices W_L^p and W_L^n are known. The latter can be found from the condition that the U's should diagonalize the one loop corrected quark mass matrices. The relevant equation has the form

$$m'_u = W_L^{p+} m_u W_R^p + \Delta m_u, \tag{9}$$

where the m'_u, m_u are diagonal up quark mass matrices: the first one stands for the pysical quark masses, the second — for the bare quark masses. Matrix Δm_u is the correction to the m_u calculated in the physical basis of the quarks i.e.

$$\Delta m_u = \tfrac{1}{2}\, U_L^{p+} \Delta m_p U_R^p + \tfrac{1}{2}\, U_R^{p+} \Delta m_p^h U_L^p,$$

where m_p, m_p^h are corrections to the m_p, m_p^+, respectively, in the notation given by Eqs. (4) and (5). We must remember that the contribution to m_u is given by all the diagrams from Fig. 1a (they are presented in Fig. 2) regardless of the helicities of the external physical

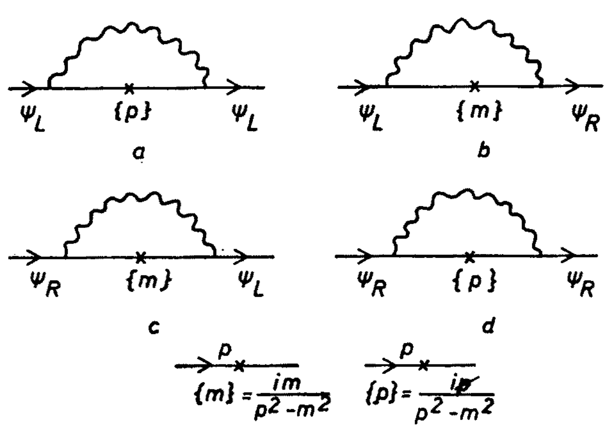

Fig. 2. Contributions to the diagram from Fig. 1a due to different "helicities" of the external quarks

particles. The last conclusion is valid due to the equation:

$$\bar{v}(p)\gamma_5 v(p) = 0,$$

where $v(p)$ is a bispinor describing a free quark.

The final equation for the parameters of the matrix W_L^p is obtained by multiplication $m'_u \cdot m'^+_u$ i.e.

$$m_u^2 = W_L^p m'^2_u W_L^{p+} - \Delta m_u \cdot m'_u - m'_u \cdot \Delta m_u^+ \qquad (10)$$

(we remember that $m'_u = m'^+_u$). Only the off-diagonal terms of the LHS of Eq. (10) play important role — their sum have to give zero. The diagonal terms are absorbed by the renormalization. It is easy to see that the diagonal parameters of W_L^p (i.e. $\alpha_4, \alpha_5, \alpha_6$) have no influence on the off-diagonal part of Eq. (10). We can put $\alpha_4 = \alpha_5 = \alpha_6 = 0$.

Let us introduce a new notation:

$$\Delta m_u \cdot m'_u + m'_u \cdot \Delta m_u^+ = \begin{pmatrix} a_4 & a_1 & a_2 \\ a_1^* & a_5 & a_3 \\ a_2^* & a_3^* & a_6 \end{pmatrix}. \qquad (11)$$

From Eq. (10) we get

$$\alpha_1 = \frac{a_1 - \alpha_2 \alpha_3^* m_t^2}{m_c^2 - m_u^2}; \qquad \alpha_2 = \frac{a_2}{m_t^2 - m_u^2}; \qquad \alpha_3 = \frac{a_3}{m_t^2 - m_c^2}, \qquad (12)$$

where we use standard notation for the masses of the first three up quarks. Analogical expression can be found for the parameters of the matrix W_L^n, $(\alpha_i \to \beta_i)$. Now we can calculate the correction to the Cabibbo angle:

— three quark families

$$\cos\theta'_C = \cos(\theta_C + \Delta\theta_C) \simeq |\cos\theta_C - \sin\theta_C$$

$$\times(\alpha_1 \cos\theta_2 + \alpha_2 \sin\theta_2 + \beta_1^* \cos\theta_3 + \beta_2^* \sin\theta_C)|, \qquad (13)$$

— two quark families

$$\cos\theta'_C = |\cos\theta_C - (\alpha_1 + \beta_1^*)\sin\theta_C|, \qquad (14)$$

θ_2, θ_3 are the angles of the standard parametrization of the Kobayashi-Maskawa matrix.

One can see that radiative processes with flavour conserving neutral currents contribute to the diagonal elements of the matrix Δm_u (Δm_d) because in such processes the external physical quarks must be the same. This means that we can neglect diagrams with an exchange of a neutral vector boson and neutral Higgs which preserves flavour.

3. *Corrections to the Cabibbo angle in the standard model*

In the standard model only the diagram 2a contributes. It is due to the fact that charged vector bosons couple only to left-handed currents. The relevant analytical expression can be written down using Eq. (6) as follows:

$$-i\,\Sigma^A\Big|_{\not p=m} = \frac{-i}{16\pi^2}\sum_{\substack{\text{charged}\\ \text{bosons}}}\int_0^1 dx\; 2mt_a(1-x)\ln\left[M_a^2(1-x)+m^2x^2\right]t_a. \tag{15}$$

We can expand the integral in powers of the small matrix $\frac{m^2}{M_a^2}$ $\left(\text{it is small because } \left(\frac{m_q}{M_a}\right)^2 \ll 1,\ M_a \text{ — mass of the charged boson “a”}\right)$.

$$\Sigma^A\Big|_{\not p=m} \sim \sum_{\substack{\text{charged}\\ \text{bosons}}}\left(mt_at_a+\text{const}\cdot mt_a\left(\frac{m}{M_a}\right)^2 t_a+O\left(\left(\frac{m}{M_a}\right)^4\right)\right), \tag{16}$$

Term mt_at_a does not contribute to the α's and β's because the matrix t_at_a is diagonal and the matrix $m' = m+mt_at_a$ is diagonalized by the same unitary rotation as m. Finally the leading correction is of the order $\frac{g^2}{16\pi^2}\left(\frac{m_q}{M_a}\right)^2$ (g is SU(2) coupling constant included in t_a), so it has the same magnitude as for the diagram 1b. All such contributions will be considerd later on and we expect that they should be small if quark masses are not too big.

Similar results have been obtained by Horetzky [7].

4. *Corrections to the Cabibbo angle in the L–R symmetric model*

In this model the interaction of the charged bosons with quark current has the form:

$$\mathscr{L}_{\text{int}} = gj_{L\mu}W_L^\mu+gj_{R\mu}W_R^\mu+\text{h.c.}\,. \tag{17}$$

The general form of the charged boson mass eigenstates is the following:

$$\begin{aligned} W_1^{\mu+} &= W_L^{\mu+}\cos\xi+W_R^{\mu+}\sin\xi,\\ W_2^{\mu+} &= -W_L^{\mu+}\sin\xi+W_R^{\mu+}\cos\xi. \end{aligned} \tag{18}$$

Present experimental data require [8]

$$|\xi| \leqslant 0.042;\qquad \frac{M_{W_2}}{M_{W_1}} \geqslant 20.6.$$

Theoretical bound for the angle ξ is the following [3] $\left(\text{for } \frac{M_{W_2}}{M_{W_1}} \gg 1\right)$

$$|\xi| < \frac{M_{W_1}}{M_{W_2}}.$$

It is seen that charged vector boson mass eigenstate can be a mixture of the state which couples to the left-handed current and the state which couples to the right-handed current. This can significantly change the result on $\Delta\theta_C$ compared to the standard model. Now all the four diagrams of Fig. 2 contribute to the correction $\Delta\theta_C$. Proceeding similarly as before it can be shown that the leading contribution comes from the diagrams 2b, 2c and it is of the order $\frac{g^2}{16\pi^2} m_q$. We write down the explicit expressions for the α_1 and β_1 for the case of two quark families.

$$\alpha_1 = \frac{g^2}{16\pi^2} 4 \sin 2\xi \frac{m_s - m_d}{m_c + m_u} \ln \frac{M_{W_2}}{M_{W_1}} \sin\theta_C \cos\theta_C \simeq 4.6 \cdot 10^{-5},$$

$$\beta_1 = -\frac{g^2}{16\pi^2} 4 \sin 2\xi \frac{m_c - m_u}{m_d + m_s} \ln \frac{M_{W_2}}{M_{W_1}} \sin\theta_C \cos\theta_C \simeq 4.6 \cdot 10^{-3}$$

(for $\xi = 0.035$, $m_u = 4$ MeV, $m_d = 7$ MeV, $m_s = 150$ MeV, $m_C = 1.5$ GeV).

Then from Eq. (14) we find

$$\Delta\theta_C \simeq \beta_1 = -4.6 \cdot 10^{-3} \text{ rad}.$$

For the three family case the analytical expressions are quite lengthy and we do not present them here. The final results are shown in Fig. 3. Our poor knowledge of the parameters of the Kobayashi-Maskawa matrix (except the Cabibbo angle) forced us to calculate the

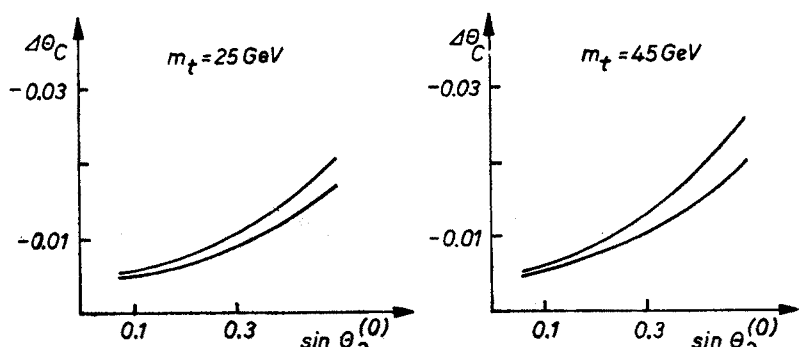

Fig. 3. Correction to the Cabibbo angle as the function of the zeroth order values of $\sin\theta_2^{(0)}$ and $\sin\theta_3^{(0)}$ (upper line — $\sin\theta_3^{(0)} = 0.1$; lower line — $\sin\theta_3^{(0)} = 0.002$) and the mass (still unknown) of the top quark m_t. For the other parameters we assume the following values: $m_u = 4$ MeV, $m_d = 7$ MeV, $m_s = 150$ MeV, $m_c = 1.5$ GeV, $m_b = 4.5$ GeV, $\sin\theta_C^{(0)} = 0.229$, CP violating phase factor $\delta = 0.002$

correction $\Delta\theta_C$ for very wide phenomenologically acceptable range of their values. We should emphasize that the correction shown in Fig. 3 does not depend on the construction of the specific model but it only depends on the zeroth order predictions of the model. The magnitude of the correction may be of the order of present experimental accuracy.

5. Corrections from the Higgs sector

We do not want to choose any particular model with horizontal symmetry because it is not clear which one is the best. In this section we describe general features of the corrections coming from the exchange of the virtual Higgs bosons. As it has been mentioned before such radiative corrections have the magnitude of the order $\frac{g^2}{64\pi^2}\frac{m^3}{M_a^2}$ (M_a^2 is the mass of the lightest charged boson). Then from Eqs. (12), (13) the largest correction to the θ_C has the following form:

$$\Delta\theta_C^H \simeq \beta_1 \sim \frac{g^2}{64\pi^2}\frac{m_t^3}{m_s M_a^2} f(\theta_C, \theta_2, \theta_3)$$

where function $f(\theta_C, \theta_2, \theta_3)$ vanishes when θ_C approaches zero or θ_2 and θ_3 approach zero (i.e. radiative processes cannot produce mixing between families if there is no mixing in the tree approximation), so

$$f(\theta_C, \theta_2, \theta_3) \sim A \sin\theta_C(\sin\theta_2 + B\sin\theta_3).$$

We assume that $A = B = 1$, then for the angles $\theta_2 = \theta_3 = 0.1$ and $m_t = 25$ GeV, $\Delta\theta_C^H$ is about $7.5 \cdot 10^{-4}$ rad. This correction is still small but for the bigger values of θ_2, θ_3 and m_t it can be much larger.

6. Conclusion

We conclude that contrary to the standard model in the L–R symmetric model with a horizontal symmetry the radiative corrections to the Cabibbo angle may be important for the experimental verification of such a theory. The correction $\Delta\theta_C$ may be comparable with the present experimental error for θ_C.

I wish to thank Professor S. Pokorski for stimulating discussions during my work on this problem.

REFERENCES

[1] S. Weinberg, *Festschrift for I. I. Rabi*, Transactions of the N. Y. Academy of Sciences, Series II, Vol. 38 (1977).

[2] S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); A. Salam, in *Elementary Particle Theory*, ed. N. Svarthon, Almqvist and Wiskell, Stockholm 1968.

[3] J. S. Pati, A. Salam, *Phys. Rev.* **D10**, 275 (1974); G. Senjanovic, *Nucl. Phys.* **B153**, 334 (1979).
[4] An exhaustive list of references the reader can find in G. Ecker, W. Konetschny, *Z. Phys.* **C3**, 155 (1979).
[5] G. Ecker, W. Konetschny, *Phys. Lett.* **91B**, 225 (1980); G. Ecker, W. Konetschny, W. Grimus, *Nucl. Phys.* **B177**, 489 (1981).
[6] S. Weinberg, *Phys. Rev.* **D7**, 2887 (1973).
[7] P. Horetzky, *Phys. Lett.* **101B**, 332 (1981).
[8] G. Beall, M. Bander, A. Soni, *Phys. Rev. Lett.* **48**, 848 (1982); G. Beall, A. Soni, Los Angeles preprint UCLA/83/TEP/8, May 1983.