SU(3) DECOUPLETS OF MESONS

BY M. MAJEWSKI AND T. TYBOR

Institute of Physics, Łódź University*

(*Received February 3, 1984*)

Applying Exotic Commutator Method we obtain the mass formula for the meson decouplet composed of an octet and two singlets SU(3). The mesons 0^+, 2^+, 1^{++} and probably 4^+, 1^{+-}, 3^- can be classified as decouplets. Almost pure $s\tilde{s}$ structure for S^* (975) can be easily obtained. A possible candidate for the second mixing singlet is gluonium.

PACS numbers: 12.70.+q

1. *Introduction*

In the previous paper [1] basing on the exotic commutator method (ECM) we investigated the mass formulae for the SU(3) meson nonet. The conditions to obtain the ideal mixing have been formulated. It has been mentioned that admixture of larger number of SU(3) singlets could destroy the ideal mixing of a nonet. This effect is of special interest for the problem of gluonium states, widely discussed recently (e.g. [2, 3]). Such states, if exist, would mix with the $q\tilde{q}$ octet and singlet states thus destroying the ideal mixing of a nonet. In the present paper the ECM is applied to discuss mixing of an SU(3) meson octet and two SU(3) singlets with isospin $T = 0$ and hypercharge $Y = 0$ irrelevant of their nature (quarkonium, gluonium, hybrid e.t.c.).

2. *Mass formula for* $8+n$*-plets of mesons*

Let $|\eta_8\rangle$ be the isosinglet state from octet. If there exist n SU(3) singlets that mix with $|\eta_8\rangle$, then

$$|\eta_8\rangle = \sum_{i=1}^{n+1} \lambda_i|\eta_i\rangle, \tag{1}$$

where

$$\sum_{i=1}^{n+1} \lambda_i^2 = 1$$

* Address: Instytut Fizyki, Uniwersytet Łódzki, Nowotki 149/153, 90-236 Łódź, Poland.

and $|\eta_i\rangle$ ($i = 1, 2, ..., n+1$) are physical states. Let us introduce the momenta

$$[\eta_8]_r = \langle\eta_8|\,(m^2)^r|\eta_8\rangle \quad (r = 0, 1, 2, ...), \tag{2}$$

where m^2 is the mass operator equared. The momenta have been calculated in the ECM [1]

$$[\eta_8]_r = \tfrac{1}{3}\,a^r + \tfrac{2}{3}\,b^r$$

with $a = \pi$, $b = 2K-\pi$, where $\pi(K)$ is the mass of pion (kaon) squared.

From Eqs. (1) and (2) we obtain

$$\sum_{i=1}^{n+1} \lambda_i^2\eta_i^r = [\eta_8]_r \quad (r = 0, 1, 2, ...), \tag{3}$$

where

$$\eta_i = \langle\eta_i|m^2|\eta_i\rangle \quad (i = 1, 2, ..., n+1).$$

If we consider the system of $n+2$ equations (3) ($r = 0, 1, ..., n+1$) then excluding $n+1$ parameters λ_i^2 we obtain the mass formula

$$[\eta_8]_{n+1} - A_1^{(n+1)}[\eta_8]_n + A_2^{(n+1)}[\eta_8]_{n-1} - \dots + (-1)^{n+1}A_{n+1}^{(n+1)} = 0, \tag{4}$$

where the quantities $A_k^{(n+1)}$ ($k = 1, 2, ..., n+1$) are invariants of the mass matrix (under orthogonal transformation of mixing states) and they can be expressed in terms of physical masses η_i. Indeed, assuming $A_1^{(1)} = \eta_1$, we have the recurrence ($n \geqslant 1$)

$$A_1^{(n+1)} = A_1^{(n)} + \eta_{n+1}. \quad A_2^{(n+1)} = A_2^{(n)} + A_1^{(n)}\eta_{n+1} \cdots,$$

$$A_n^{(n+1)} = A_n^{(n)} + A_{n-1}^{(n)}\eta_{n+1}, \quad A_{n+1}^{(n+1)} = A_n^{(n)}\eta_{n+1},$$

If one more equation (3) (for $r = n+2$) is taken into account then $\lambda_1^2 = \frac{1}{3}$, $\lambda_2^2 = \frac{2}{3}$, $\lambda_i^2 = 0$ ($i = 3, \dots n+1$), $\eta_1 = \pi$, $\eta_2 = 2K-\pi$ and η_i ($i = 3, ..., n+1$) become undetermined. It is obvious that additional equations (3) (for $r > n+2$) are obeyed as identities. So taking into account the sufficient number of momenta $[\eta_8]_r$ we restore the ideal mixing of a nonet, $n-1$ singlets become disconnected and there are no restrictions on their masses.

For the cases $n = 0$ and $n = 1$ Eq. (4) is well known. If $n = 0$, we get the GMO mass formula

$$\eta_1 = \tfrac{1}{3}(4K-\pi). \tag{5}$$

If $n = 1$, we get Schwinger mass formula for a nonet

$$[\eta_8]_2 - (\eta_1+\eta_2)\,[\eta_8]_1 + \eta_1\eta_2 = 0. \tag{6}$$

3. *Mass formula and mixing for meson decouplet*

If $n = 2$, then the mass of the tenth meson can be calculated

$$\eta_3 = \frac{S_3(1, 2)}{S_2(1, 2)}, \tag{7}$$

where

$$S_3(1,2) = [\eta_8]_3 - (\eta_1+\eta_2)[\eta_8]_2 + \eta_1\eta_2[\eta_8]_1, \tag{8}$$

$$S_2(1,2) = [\eta_8]_2 - (\eta_1+\eta_2)[\eta_8]_1 + \eta_1\eta_2. \tag{9}$$

and the mixing parameters can be put in the form

$$\lambda_2^2 = \frac{S_2(3,1)}{(\eta_1-\eta_2)(\eta_1-\eta_3)},$$

$$\lambda_1^2 = \frac{S_2(2,3)}{(\eta_2-\eta_3)(\eta_2-\eta_1)},$$

$$\lambda_3^2 = \frac{S_2(1,2)}{(\eta_3-\eta_1)(\eta_3-\eta_2)}. \tag{10}$$

The parameter λ_i specifies the content of the octet in the physical state $|\eta_i\rangle$.

Let us discuss some properties of Eqs. (7) and (10). If η_1 and η_2 obey the Schwinger mass formula then $\eta_3 \to \infty$ and $|\eta_3\rangle$ decouples, if $\eta_1 = a$ then $\eta_3 = b$ and the second

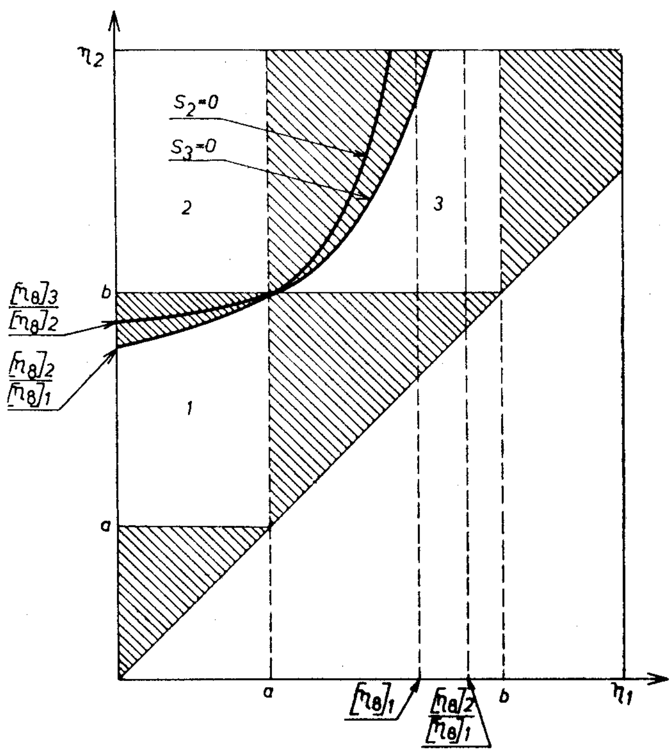

Fig. 1. $\eta_2 > \eta_1$ sector of (η_1, η_2) plane. The curves $S_3 = 0$, $S_2 = 0$ (see formulae (6) and (7)) as well as horizontal and vertical straight lines $\eta_1, \eta_2 = a, b$ bound the regions 1, 2, 3 of values η_1, η_2 at which the third isosinglet can admix

isosinglet decouples ($\lambda_2^2 = 0$ at any value of η_2). The tenth particle can be admixed if $\eta_3 \geqslant 0$ and all λ^2's are positive. It appears that these conditions can be obeyed only under some restrictions on the masses of states $|\eta_1\rangle$ and $|\eta_2\rangle$. To formulate these restrictions we arrange the physical particles in such a way that $\eta_1 < \eta_2$. From analysis of Eqs. (7) and (10) it becomes evident that the masses η_1 and η_2 must belong to one of three unlined regions 1, 2 or 3 on Fig. 1. Consequently, three mixing isosinglets (after proper rearrangement of them) comply with the inequalities

$$\eta_1 < a < \eta_2 < b < \eta_3. \tag{11}$$

To end the general consideration we introduce the orthogonal matrix Θ connecting physical states with unphysical ones

$$\begin{pmatrix} |\eta_1\rangle \\ |\eta_2\rangle \\ |\eta_3\rangle \end{pmatrix} = \Theta \begin{pmatrix} |\eta_8\rangle \\ |\eta_{01}\rangle \\ |\eta_{02}\rangle \end{pmatrix},$$

where $|\eta_{0i}\rangle$ $(i = 1, 2)$ are SU(3) singlets. It can be parametrised by three angles ϑ_j $(j = 1, 2, 3)$

$$\Theta = \begin{pmatrix} c_1 & -s_1c_2 & s_1s_2 \\ s_1c_3 & c_1c_2c_3 - s_2s_3 & -c_1s_2c_3 - c_2s_3 \\ s_1s_3 & c_1c_2s_3 + s_2c_3 & -c_1s_2s_3 + c_2c_3 \end{pmatrix}, \tag{12}$$

where $c_j = \cos\vartheta_j$ and $s_j = \sin\vartheta_j$. The elements of the first column are equal λ_1, λ_2, and λ_3 respectively. Thus two angles ϑ_1 and ϑ_3 can be determined but ϑ_2 remains free.

The state of the physical particle, say $|\eta_1\rangle$, being superposed of three mixing isosinglet states may have essentially different stiucture than the one superposed of two mixing isosinglet states (nonet). This can be easily seen in the case of 0^+ mesons.

4. *Decouplets of* 0^+, 2^+, 1^{++} *and* 4^+ *mesons*

Let us apply the $n = 2$ mass formula to known meson multiplets.

a. 0^+ mesons

Putting $\delta = (0.983\text{ GeV})^2$, $\kappa = (1.350\text{ GeV})^2$, $S_1 = S^* = (0.975\text{ GeV})^2$, $S_2 = \varepsilon = (1.300\text{ GeV})^2$ [4] we get $S_3 = (1.64\text{ GeV})^2$, $\lambda_1^2 = 0.3204$, $\lambda_2^2 = 0.0211$, $\lambda_3^2 = 0.6585$. S_3 weakly depends on the mass of ε (1300) which is not very well determined but it is strongly correlated with the other not too well measured mass of κ (1350). It is so, because the masses of the particles δ (980), κ (1350), S^* (975) and S_3 (1640) satisfy well enough the Schwinger mass formula for a nonet. A good candidate for S_3 is G (1590) with $I^G = 0^+$, $J^{PC} = 0^{++}$, $M = (1592 \pm 25)$ MeV and $\Gamma = (210 \pm 40)$ MeV [5]. The structures of the physical states (assuming that $|S_{01}\rangle$ is the usual SU(3) $q\tilde{q}$ singlet and leaving undefined the structure of $|S_{02}\rangle$) are

$$\begin{pmatrix} |S^*\rangle \\ |\varepsilon\rangle \\ |S_3\rangle \end{pmatrix} = V \begin{pmatrix} |S_u\rangle \\ |S_s\rangle \\ |S_{02}\rangle \end{pmatrix},$$

where $|S_u\rangle = \left|\frac{1}{\sqrt{2}}(u\tilde{u}+d\tilde{d})\right\rangle$, $|S_s\rangle = |s\tilde{s}\rangle$ and

$$V = \begin{pmatrix} 0.327-0.673c_2; & -0.462-0.476c_2; & 0.824s_2 \\ 0.084+0.081c_2+0.804s_2; & -0.119+0.058c_2+0.568s_2; & 0.984c_2-0.100s_2 \\ 0.469+0.455c_2-0.144s_2; & -0.663+0.322c_2-0.102s_2; & -0.176c_2-0.557s_2 \end{pmatrix}.$$

For $\vartheta_2 = 0$ $|S^*\rangle$, $|\varepsilon\rangle$ and $|S_3\rangle$ are almost pure $|s\tilde{s}\rangle$, $|S_{02}\rangle$ and $|S_u\rangle$ states respectively. We see that in spite of the fact that the masses of S^* and S_3 are nearly ideal their structures can be far from ideality.

b. 2^+ mesons

Putting $A_2 = (1.318\ \text{GeV})^2$, $K^* = (1.434\ \text{GeV})^2$, $f_1 = f = (1.273\ \text{GeV})^2$, $f_2 = f' = (1.520\ \text{GeV})^2$ [4] we get $f_3 = (1.93\ \text{GeV})^2$ and $\lambda_1^2 = 0.2213$, $\lambda_2^2 = 0.7752$, $\lambda_3^2 = 0.0035$. A good candidate for f_3 is X (1850) [4]. However, we notice that the mass of f_3 strongly depends on the masses of particles from nonet and in the limits of experimental errors one can get also the value corresponding to the meson θ (1640) $f_3 = (1.7\ \text{GeV})^2$ [3].

c. 1^{++} mesons

If D (1285), E (1420) and recently reported D′ (1526) [6] together with A_1 (1270) and Q_A form the meson decouplet, then $A_1 > D$ (Ineq. (11)). Putting $A_1 = (1{,}290\ \text{GeV})^2$, $Q_A = (1.359\ \text{GeV})^2$, $D_1 = D = (1.283\ \text{GeV})^2$, $D_2 = E = (1.418\ \text{GeV})^2$ [4] we get $D_3 = (1.518\ \text{GeV})^2$, $\lambda_1^2 = 0.2937$, $\lambda_2^2 = 0.6921$, $\lambda_3^2 = 0.0142$.

d. 4^+ mesons

Suppose that the tenth particle is mixing. We accept $\delta = (2.034\ \text{GeV})^2$, $K^* = (2.037\ \text{GeV})^2$ and $h_3 = \varepsilon = (2.300\ \text{GeV})^2$ [4]. With these masses the value of h_2 is strongly restricted (formula (11))

$$(2.034\ \text{GeV})^2 < h_2 < (2.040\ \text{GeV})^2.$$

On the other hand the experimental masses assigned to the particle h (2040) are distributed so widely that both the h_1 and h_2 obeying Eq. (7) can be taken from the region covered with them. Thus the evidences for h (2040) may correspond to two different isosinglets. λ_1^2 and λ_3^2 are very small since $h_2 \cong [h_8]_1$, i.e. it obeys well the GMO mass formula for octet (5).

In any of the multiplets discussed the masses of one pair of isosinglets obey well the mass formula for a nonet (6). One isosinglet state of the pair is the $|\eta_1\rangle$, nearly degenerated in mass with the isotriplet. Therefore one can try to make a prediction for two more multiplets.

e. 3^- mesons

We have $g = (1.691\ \text{GeV})^2$, $K^* = (1.775\ \text{GeV})^2$, $\omega_1 = \omega = (1.6679\ \text{GeV})^2$ [4]. So only one isosinglet is known, but $\omega_1 < g$ is typical for meson decouplet. From Eq. (6) we find that one of the two unknown isosinglets should have the mass about 1.85 GeV.

f. 1^{+-} mesons

Putting $B = (1.233\ \text{GeV})^2$, $Q_B = (1.324\ \text{GeV})^2$ (the value correlated with Q_A, since they mix) and $H = (1.190\ \text{GeV})^2$ [4] we find for one of the two unknown isosinglets $H' \approx (1.40\ \text{GeV})^2$.

The question is what is the nature of an extra-singlet state mixing with the $q\tilde{q}$ nonet. One possibility, widely discussed recently (see e.g. [2, 3]), is that this state is gluonium. If it is so, then the physical isosinglet states would be the quarkonia-gluonium mixtures with the rates described by the mixing matrix Θ (12). It would be no pure physical gluonia and the existing estimations of the lowest lying pure gluonium mass for various J^{PC} (see e.g. [3]) should be regarded not too literally.

Another possibility is that the singlet is built up of the quarks (radially excited, 4-quark state or hybrid) and there exist also the related octet states, but the singlet state lies apparently lower and therefore mixes mainly with the ground nonet states. However, the existence of the additional flavored multiplets should disturb in general the decuplet classification independently of the nature of the singlet mixed to nonet.

5. *What about the* 0^- *and* 1^- *mesons?*

In these cases we have a quite different situation. Let us discuss them shortly.

(*i*) 0^- mesons

The tenth meson does not mix with the nonet π, K, η, η' (958) since Eq. (7) gives negative value for the mass squared of the third isosinglet. However we observe

1. The mass of η meson much better complies with the octet GMO formula ($\eta = (0.567\ \text{GeV})^2$ than with the Schwinger one for nonet ($\eta = (0.495\ \text{GeV})^2$).

2. The mass of η' is nearer the mass scale of the higher lying (radially excited?) flavored states than the ground state one.

Therefore we may guess that the mixing with the higher lying states determines the mass of η' to a greater extent than the mixing with the ground state octet.

Unfortunately, the masses of these higher lying states are poorly established. It we want to have the masses of the isosinglets η' (958), ς (1275), ι (1440) [4] compatible with the formula (7) we must choose proper values for the masses of π' and K'. Choosing $\pi' = (1.250\ \text{GeV})^2$, $K' = (1.350\ \text{GeV})^2$, $\eta_1 = \eta' = 0.917\ \text{GeV}^2 = (0.9576\ \text{GeV})^2$, $\eta_2 = (1.275\ \text{GeV})^2$ we obtain $\eta_3 = (1.450\ \text{GeV})^2$ and $\lambda_1^2 = 0.0060$, $\lambda_2^2 = 0.3913$, $\lambda_3^2 = 0.6027$.

(*ii*) 1^- mesons

The tenth meson cannot be admixed to the ground nonet since $\lambda_3^2 < 0$. Besides of this the situation with the masses of the higher lying isosinglets is far to be clear.

In both the cases apart of the ground state octet ($\underline{8}$) there is known a higher lying one ($\underline{8}'$). A singlet state ($\underline{1}$) should mix with both octets and the octets themselves should mix with one another. If the $\underline{8}+\underline{1}$ mixing is comparable with the $\underline{8}'+\underline{1}$ mixing, than it would be impossible to separate any nonet. The same argument can be repeated for the polimultiplet $\underline{9}+\underline{1}+\underline{9}$. Of course, it is also possible that there is no extra singlet at the mass region in question and we have to do with some kind of mixing of $\underline{8}+\underline{8}+\underline{1}+\underline{1}$.

6. *Conclusion*

Many meson multiplets: 0^+, 2^+, 1^{++} and probably 4^+, 1^{+-}, 3^- can be understood as decouplets rising from mixing of the $q\tilde{q}$ nonet with an extra SU(3) singlet of unknown nature.

Two of the three physical isosinglets belonging to a decouplet have the masses obeying well enough the nonet mass formula and almost equal to the ones expected for ideal mixing. In this respect the mixing of an extra singlet weakly disturbes the ideality. But only in this respect, because these two isosinglets can have quite different structures than the two isosinglets from the nonet ideally mixed. The structures of the physical isosinglets are defined not completely. They depend on the nature of an extra singlet and on one parameter — mixing angle ϑ_2.

By proper choice of ϑ_2 in the mixing matrix of the 0^+ decouplet we can obtain almost pures $s\tilde{s}$ structure for isosinglet S* (975) thus removing the main difficulty in interpretation of this multiplet. The difficulty with the mass spectrum does not arise since the mass formula for decouplet is satisfied well enough.

A good candidate for an extra singlet is gluonium. If it is the case then the physical isosinglets belonging to the decouplet would contain the gluonium component in their structures but none of them would be pure gluonium state.

The decouplet classification would be disturbed or not appear at all if there exists a higher lying octet competiting with the ground octet in the mixing with singlets. Such a situation can occur in the cases of 0^- and 1^- mesons, since the existence of higher lying octets seems to be sure for them. Therefore it is necessary to consider the octet + octet + n singlets mixings. This problem will be discussed elsewhere.

The authors thank Profs. S. B. Gerasimov and A. B. Govorkov for information and comments on new experimental data.

Note added. After completion of this work we noticed the contribution of T. Armstrong et al. [7], where the mass of $3^-\phi$-like meson is given as 1.87 GeV. Then the mass of the tenth meson would be 1.79 GeV.

REFERENCES

[1] M. Majewski, T. Tybor, *Acta Phys. Pol.* **B15**, 267 (1984).
[2] E. D. Bloom, Proc. 21 st Int. Conf. High Energy Phys., Paris 1982; *J. Phys. C* **43**, 3 (1982).
[3] K. Gottfried, Rapporteur's talk at the Int. Europhysics Conf. on High Energy Physics, Brighton, July 1983.
[4] Particle Data Group, *Phys. Lett.* **111B**, (1982).
[5] F. Binon et al., IHEP preprint 83-84, Serpukhov 1983.
[6] Ph. Gavillet et al., *Z. Phys.* **C16**, 119 (1982).
[7] T. Armstrong et al., *Phys. Lett.* **110B**, 77 (1982).