RISING CROSS-SECTIONS AND RENORMALIZATION OF SECONDARY REGGEONS

By N. A. Kobyliński and E. S. Martynov

Institute for Theoretical Physics, Kiev*

(Received September 15, 1983; final version received January 27, 1984)

In the framework of the reggeon theory with dipole Pomeron the secondary reggeon propagator is studied and the anomalous dimension is evaluated for different structures of an input reggeon and also for different types of its interaction with Pomeron. The results obtained are compared with the experimental data on the ϱ-reggeon exchange.

PACS numbers: 12.40.-y

1. Introduction

In spite of considerable progress in QCD there are many problems in hadron physics which remain outside the range of applicability of the QCD methods. Among them it is necessary to name first of all, the hadron interaction at high energies but small momentum transfers: elastic scattering, charge-exchange processes and so on. In these processes, however, the methods using the reggeon notions proved to be up to the mark.

Over a long period of time both Pomeron and secondary reggeons were considered as simple poles in an angular momentum plane. It was reasonable since total cross-sections seemed to be constant in asymptotics. In this case corrections to a simple reggeon pole induced by its interaction with a simple Pomeron pole are small at high energies. The situation changed when it became clear that Pomeron is harder singularity than a simple j-pole. In the light of new understanding of the Pomeron it became necessary to reconsider all which is known about secondary reggeons. It is obvious that the properties of a renormalized secondary reggeon depend, to a considerable extent, on specific assumptions concerning the structure and properties of Pomeron singularity.

There is a number of approaches to the Pomeron [1-6] differing essentially in predictions for asymptotic behaviour of hadron elastic and inelastic interactions. In the reggeon field theory (RFT) the critical Pomeron is very popular [1]. It results in the total cross-sections rise $\sim(\ln s/s_0)^{\gamma_P}$ where $\gamma_P < 1$ but in the decrease in the ratio σ_{el}/σ_{tot}. Such a behaviour contradicts experiment (this concerns, first of all, the σ_{el}/σ_{tot}). Secondary

* Address: Institute for Theoretical Physics, Metrologicheskaya st. 14b, 252130, Kiev-130, USSR.

reggeons in this theory are transformed from a simple pole to a harder singularity [7, 8] contributing to the amplitude $\sim S^{\alpha}(\ln s/s_0)^{\gamma_R}$. In the first order of ε-expansion one has $\gamma_R = \varepsilon/24$. But the analysis of the data on the ϱ-reggeon exchange gives larger value of γ_R (see Section 3). Furthermore, the ε-expansion converges badly (physical value of ε equals 2) and, therefore, the obtained parameters of the critical Pomeron are unrealistic.

In this paper we study the secondary reggeon in the framework of the dipole Pomeron theory [6] which originates naturally if we accept the following arguments.

Almost in all variants of reggeon theory [1, 3, 4] the renormalized reggeon differs essentially from the input one[1]. Therefore, it makes sense to single out such Pomeron singularities for which the input Pomeron is renormalized to a small degree and then to choose between them the ones being in good agreement with experiment. Solution of this problem leads to the alternative [6, 9]: either Pomeron is a two-fold pole (dipole) or it is a pair of colliding hard branch points with hardness 3/2 (froissaron).

2. *Renormalization-group properties of reggeons in dipole Pomeron theory*

We shall assume the Pomeron to be a two-fold j-pole. Let us consider firstly the case when the secondary reggeon is a simple pole and the vertex for reggeon-reggeon-Pomeron (RRP) interaction is constant and equals λ_0. The Langrangian for such a theory can be represented as

$$\mathscr{L} = \mathscr{L}_P + \mathscr{L}_R + \mathscr{L}_{RP}, \tag{1}$$

where

$$\mathscr{L}_P(\vec{x}, t) = -\tfrac{1}{4}\, \overset{+}{\psi}(\vec{x}, t)\left(\frac{\overleftrightarrow{\partial}}{\partial t}\right)^2 \psi(\vec{x}, t) - i\alpha_0' \nabla \overset{+}{\psi}(\vec{x}, t) \frac{\overleftrightarrow{\partial}}{\partial t} \nabla \psi(\vec{x}, t)$$

$$+(\alpha_0')^2 \nabla^2 \overset{+}{\psi}(\vec{x}, t) \nabla^2 \psi(\vec{x}, t) - i\frac{r_0}{2}\left[\overset{+}{\psi}{}^2(\vec{x}, t) \frac{\partial^2}{\partial t^2} \psi(\vec{x}, t) + \left(\frac{\partial^2}{\partial t^2} \overset{+}{\psi}(\vec{x}, t)\right) \psi^2(\vec{x}, t)\right] \tag{2}$$

is the Lagrangian corresponding to free Pomeron and triple-Pomeron interaction (its form is considered in more detail in Ref. [6]). In $(E, \vec{k})$-representation ($E = 1-j$) the Lagrangian (2) leads to the triple-Pomeron vertex

$$\Gamma^{(1,2;0)}(E_i, \vec{k}_i) = \frac{r_0}{(2\pi)^{3/2}} E^2 \tag{3}$$

(for simplicity we omit in expressions (2), (3) the dependence on $\vec{k}$; as will be seen below this dependence does not change reggeon propagator at zero momentum).

The next term in (1) describes the free secondary reggeon

$$\mathscr{L}_R = \frac{i}{2}\, \overset{+}{\varphi}(\vec{x}, t) \frac{\overleftrightarrow{\partial}}{\partial t} \varphi(\vec{x}, t) - \alpha_{R_0}' \nabla \overset{+}{\varphi}(\vec{x}, t) \nabla \varphi(\vec{x}, t) - \Delta_{R_0} \overset{+}{\varphi}(\vec{x}, t) \varphi(\vec{x}, t), \tag{4}$$

where $\varphi(\vec{x}, t)$ is a reggeon field, α_{R_0}' is a slope and $1-\Delta_{R_0}$ is an intercept for this reggeon.

[1] It is not so for the case of weak coupling [2] but this variant of the theory fails to agree with experiment.

The last term in (1)

$$\mathscr{L}_{\mathrm{RP}} = -i\lambda_0 \overset{+}{\varphi}(\vec{x}, t)\varphi(\vec{x}, t)\,[\overset{+}{\psi}(\vec{x}, t)+\psi(\vec{x}, t)] \tag{5}$$

describes an interaction of reggeons with Pomeron.

Since the number of secondary reggeons is conserved one can made the phase transformation [7]

$$\varphi(\vec{x}, t) \to e^{i\Delta_{\mathrm{R}_0}t}\varphi(\vec{x}, t)$$

which does not change the interaction (5). This allows us to use for secondary reggeons instead of E the variable

$$\mathscr{E} \equiv E-\Delta_{\mathrm{R}_0} = \alpha_{\mathrm{R}_0}-j.$$

Thus, we have the reggeon theory with the input Pomeron propagator

$$G_0^{(1,1;0)}(E, \vec{k}^2) = -\frac{i}{(E-\alpha_0'\vec{k}^2)^2}, \tag{6}$$

the 3P-vertex of the form (3), the input reggeon propagator

$$G_0^{(0,0;1)}(\mathscr{E}, \vec{p}^2) = \frac{i}{\mathscr{E}-\alpha_{\mathrm{R}_0}'\vec{p}^2} \tag{7}$$

and the RRP-vertex

$$\Gamma_0^{(1,0;1)}(E, \vec{k}, \mathscr{E}_i, \vec{p}_i) = \frac{\lambda_0}{(2\pi)^{3/2}}. \tag{8}$$

We follow here the notations of Ref. [7] where the secondary reggeon is considered in the theory with critical Pomeron. We shall not dwell on the rules for diagram calculus which can also be found in Ref. [7].

The aim of the paper is to study the infrared properties of the vertex function $\Gamma^{(n,m;k)}(E_i, \vec{k}_i, \mathscr{E}_i, p_i)$ (the notations are seen from Fig. 1). Unrenormalized vertex functions $\Gamma_{\mathrm{U}}^{(n,m;k)}$ depends, in addition, on the parameters α_0', α_{R_0}', r_0, λ_0.

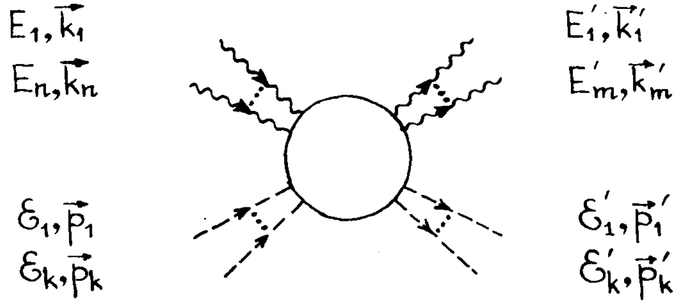

Fig. 1. The vertex function $\Gamma^{(n,m;k)}(E_i, \vec{k}_i, \mathscr{E}_i, \vec{p}_i)$. The wave lines stand for Pomerons, and the dashed lines are secondary reggeons

The renormalization procedure comes to

$$\psi(\vec{x}, t) \to Z^{-1/2}\psi(\vec{x}, t), \quad \varphi(\vec{x}, t) \to Z_R^{-1/2}\varphi(\vec{x}, t),$$

and when the renormalization point is at $E = -E_N < 0$ we have

$$\Gamma^{(n,m;k)}(E_i, \vec{k}_i, \mathscr{E}_i, \vec{p}_i; r, \lambda, \alpha', \alpha'_R, E_N)$$

$$= Z^{(n+m)/2}Z_R^k\Gamma_U^{(n,m;k)}(E_i, \vec{k}_i, \mathscr{E}_i, \vec{p}_i; r_0, \lambda_0, \alpha'_0, \alpha'_{R_0}). \tag{9}$$

The renormalized quantities are defined from the conditions

$$i\Gamma^{(1,1;0)}(E, \vec{k}^2)|_{E=-E_N, \vec{k}^2=0} = -E_N^2, \tag{10}$$

$$\frac{\partial}{\partial \vec{k}^2} i\Gamma^{(1,1;0)}(E, \vec{k}^2)|_{E=-E_N, \vec{k}^2=0} = -2\alpha'(E_N)E_N, \tag{11}$$

$$i\Gamma^{(0,0;1)}(\mathscr{E}, \vec{p}^2)|_{\mathscr{E}=-E_N, \vec{p}^2=0} = -E_N, \tag{12}$$

$$\frac{\partial}{\partial \vec{p}^2} i\Gamma^{(0,0;1)}(\mathscr{E}, \vec{p}^2)|_{\mathscr{E}=-E_N, \vec{p}^2=0} = -\alpha'_R(E_N), \tag{13}$$

$$\Gamma^{(1,2;0)}(E_1, \vec{k}_1, E'_1, \vec{k}'_1, E'_2, \vec{k}'_2)\Big|_{\substack{E_1=2E_1'=2E_2'=-E_N \\ \vec{k}_1'=\vec{k}_2'=\vec{k}_1=0}} = \frac{r(E_N)}{(2\pi)^{3/2}} E_N^2, \tag{14}$$

$$\Gamma^{(0,1;1)}(\mathscr{E}_1, \vec{p}_1, \mathscr{E}'_1, \vec{p}'_1, E'_1, \vec{k}'_1)\Big|_{\substack{\mathscr{E}_1=2\mathscr{E}_1'=2E_1'=-E_N \\ \vec{p}_1'=\vec{k}_1'=\vec{p}_1=0}} = \frac{\lambda(E_N)}{(2\pi)^{3/2}}. \tag{15}$$

Going to dimensionless parameters

$$g(E_N) = \frac{r(E_N)}{[\alpha'(E_N)]^{1/2}}, \tag{16}$$

$$g_R(E_N) = \frac{\lambda(E_N)E_N^{-1}}{\{\frac{1}{2}[\alpha'(E_N)+\alpha'_R(E_N)]\}^{1/2}}, \tag{17}$$

$$v(E_N) = \frac{\alpha'_R(E_N)}{\alpha'(E_N)} \tag{18}$$

it is not difficult to ascertain that

$$\Gamma^{(n,m;k)}(\xi E_i, \vec{k}_i, \xi\mathscr{E}_i, \vec{p}_i; g, g_R, \alpha', v, E_N)$$

$$= \xi^{1+(n+m)/2}\Gamma^{(n,m;k)}\left(E_i, \vec{k}_i, \mathscr{E}_i, \vec{p}_i; g, g_R, \frac{\alpha'}{\xi}, v, \frac{E_N}{\xi}\right)$$

and the renormalized vertex function $\Gamma^{(n,m;k)}$ satisfies the renormalization-group equation

$$\left[\xi \frac{\partial}{\partial \xi} - \beta(g)\frac{\partial}{\partial g} - \beta_R(g)\frac{\partial}{\partial g_R} + \alpha'(1-\zeta/\alpha')\frac{\partial}{\partial \alpha'} + \tfrac{1}{2}(n+m)\gamma \right.$$

$$\left. + k\gamma_R - 1 - \frac{n+m}{2}\right] \Gamma^{(n,m;k)}(\xi E_i, \vec{k}_i, \xi \mathscr{E}_i, \vec{p}_i; g, g_R, \alpha', v, E_N) = 0, \tag{19}$$

where

$$\beta = E_N \frac{\partial g}{\partial E_N}, \quad \gamma = E_N \frac{\partial \ln Z}{\partial E_N}, \tag{20}$$

$$\beta_R = E_N \frac{\partial g_R}{\partial E_N}, \quad \gamma_R = E_N \frac{\partial \ln Z_R}{\partial E_N}, \tag{21}$$

$$\zeta = E_N \frac{\partial \alpha'}{\partial E_N}, \tag{22}$$

$$\sigma = E_N \frac{\partial v}{\partial E_N}. \tag{23}$$

The solution of Eq. (19) reads as

$$\Gamma^{(n,m;k)}(\xi E_i, \vec{k}_i, \xi \mathscr{E}_i, \vec{p}_i; g, g_R, \alpha', v, E_N)$$

$$= \Gamma^{(n,m;k)}(E_i, \vec{k}_i, \mathscr{E}_i, \vec{p}_i; \tilde{g}(-\tau), \tilde{g}_R(-\tau), \tilde{\alpha}'(-\tau), \tilde{v}(-\tau), E_N)$$

$$\times \exp\left\{-\int_0^{-\tau} d\tau' \left[1 + \frac{n+m}{2} - \frac{n+m}{2}\gamma(\tilde{g}(\tau')) - k\gamma_R(\tilde{g}(\tau'), \tilde{g}_R(\tau'), \tilde{v}(\tau'))\right]\right\}, \tag{24}$$

where $\tau = \ln \xi$ and the function depending on τ can be found from the equations

$$\frac{d\tilde{g}(\tau)}{d\tau} = -\beta(g(\tau)), \tag{25}$$

$$\frac{1}{\tilde{\alpha}'(\tau)} \frac{d\tilde{\alpha}'(\tau)}{d\tau} = 1 - \zeta(\tilde{\alpha}'(\tau), \tilde{g}(\tau))/\tilde{\alpha}'(\tau), \tag{26}$$

$$\frac{d\tilde{g}_R(\tau)}{d\tau} = -\beta_R(\tilde{g}(\tau), \tilde{g}_R(\tau), \tilde{v}(\tau)), \tag{27}$$

$$\frac{d\tilde{v}(\tau)}{d\tau} = -\sigma(\tilde{g}(\tau), \tilde{g}_R(\tau), \tilde{v}(\tau)) \tag{28}$$

at the initial conditions $\tilde{g}(0) = g$, $\tilde{g}_R(0) = g_R$, $\tilde{\alpha}'(0) = \alpha'$, $\tilde{v}(0) = v$.

The formulas given above need some remarks. First of all, we note that a Pomeron part of theory can be studied irrespectively of the reggeon one and this has been done to some extent in Ref. [6]. Here it is only important that β-function for Pomeron has zero at $g = 0$. Therefore, in what follows we shall focus on Eqs. (27) and (28).

To find the functions β_R, σ, and also the renormalization constant Z_R we calculate one-loop corrections to the bare propagator (7). These corrections are defined by the diagrams shown in Figs. 2, 3. At $g = 0$ the contribution to RRP-vertex comes only from the first two diagrams in Fig. 3.

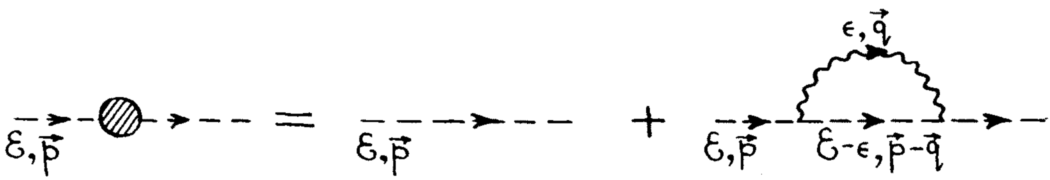

Fig. 2. Diagram representation for the $\Gamma^{(0,0;1)}(\mathscr{E}, \vec{p})$ in the one-loop approximation

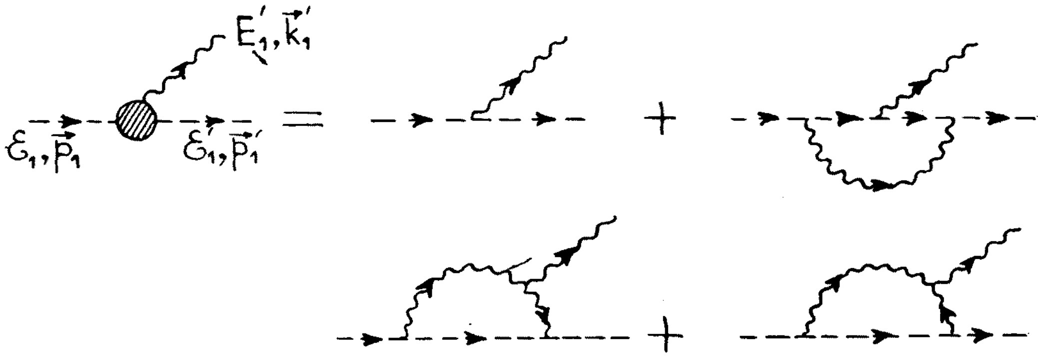

Fig. 3. The same as in Fig. 2 for $\Gamma^{(0,1;1)}(\mathscr{E}_1, \vec{p}_1, \mathscr{E}_1', \vec{p}_1', E_1', \vec{k}_1')$

As a result we obtain for the unrenormalized vertex functions

$$i\Gamma_U^{(0,0;1)}(\mathscr{E}, \vec{p}) = \mathscr{E} - \alpha_{R_0}' \vec{p}^{\,2} + \frac{\lambda_0^2}{4\pi(\alpha_0' + \alpha_{R_0}')} \left(\mathscr{E} - \frac{\alpha_0' \alpha_{R_0}'}{\alpha_0' + \alpha_{R_0}'} \vec{p}^{\,2} \right)^{-1} \tag{29}$$

and

$$i\Gamma_U^{(0,1;1)}\left(-E_N, 0, -\frac{E_N}{2}, 0, -\frac{E_N}{2}, 0 \right) = \frac{\lambda_0}{(2\pi)^{3/2}} - \frac{4\pi^2 \lambda_0^3}{(2\pi)^{9/2}} \frac{E_N^{-2}}{\alpha_0' + \alpha_{R_0}'} . \tag{30}$$

From formulas (9), (12), (13) and (15) we find

$$Z_R^{-1} = 1 + \frac{\lambda_0^2 E_N^{-2}}{8\pi[\frac{1}{2}(\alpha_0' + \alpha_{R_0}')]}$$

or

$$Z_R = 1 - g_R^2/8\pi$$

and taking into account the definitions (17) and (18) we get

$$g_R(E_N) = \frac{\lambda_0 E_N^{-1}}{[\frac{1}{2}(\alpha'_0+\alpha'_{R_0})]^{1/2}} - \frac{3}{8\pi}\frac{\lambda_0^3 E_N^{-3}}{[\frac{1}{2}(\alpha'_0+\alpha'_{R_0})]^{3/2}}, \tag{31}$$

$$v(E_N) = \frac{\alpha'_{R_0}}{\alpha'_0}\left(1 - \frac{\lambda_0^2 E_N^{-2}}{8\pi[\frac{1}{2}(\alpha'_0+\alpha'_{R_0})]}\,\frac{2+\alpha'_{R_0}/\alpha'_0}{1+\alpha'_{R_0}/\alpha'_0}\right). \tag{32}$$

These equations and also Eqs. (21), (23) give

$$\beta_R = -g_R + \frac{3}{4\pi} g_R^3, \tag{33}$$

$$\sigma = g_R^2 \frac{v(2+v)}{4\pi(1+v)}. \tag{34}$$

Putting $\beta_R = 0$ and $\sigma = 0$ we find the only infrared stable point

$$(g_{R_1}^2, v_1) = \left(\frac{4\pi}{3}, 0\right).$$

The fact that this point defines really the infrared properties of the theory is easily seen from the solutions of Eqs. (27), (28)

$$\tilde{g}_R(\tau) = g_{R_1} + (g_R - g_{R_1})e^{-2\tau} \to g_{R_1}, \quad \tau \to \infty, \tag{35}$$

$$v(\tau) = v\exp(-\tfrac{2}{3}\tau) \to 0, \quad \tau \to \infty. \tag{36}$$

The anomalous dimension γ_R can be determined from (21),

$$\gamma_R = \frac{g_R^2}{4\pi}. \tag{37}$$

Evaluating the integral in (24) with the functions (35), (36) we get

$$\Gamma^{(n,m;k)} \sim \xi^{1-k\gamma_{R_1}+\frac{n+m}{2}}, \quad \xi \to 0,$$

where

$$\gamma_{R_1} = \frac{g_{R_1}^2}{4\pi} = \frac{1}{3}.$$

Thus

$$\Gamma^{(n,m;k)}(\xi E_i, \vec{k}_i, \xi\mathscr{E}_i, \vec{p}_i; g, g_R, \alpha', v, E_N) \underset{\xi\to 0}{\sim} \xi^{1+\frac{n+m}{2}-k\gamma_{R_1}}$$
$$\times\Gamma^{(n,m;k)}(E_i, \vec{k}_i, \mathscr{E}_i, \vec{p}_i; 0, g_{R_1}, \alpha', v\xi^{2/3}, E_N).$$

This means that

$$\Gamma^{(n,m;k)}(E_i, \vec{k}_i, \mathscr{E}_i, \vec{p}_i;\ g, g_R, \alpha', \nu, E_N) \sim E_N\left(-\frac{E_N}{E}\right)^{1+\frac{n+m}{2}-k\gamma_{R_1}}$$

$$\times \left(\frac{E_N}{\alpha'}\right)^{\frac{1}{2}(2-n-m-2k)} \Phi_{n,m;k}\left(\frac{E_i}{-E}, \frac{\alpha'\vec{k}_i\cdot\vec{k}_j}{-E};\ \nu\left(-\frac{E}{E_N}\right)^{2/3}, g_{R_1}\right),$$

where

$$E = \sum E_i + \sum \mathscr{E}_i.$$

It follows from the last formula, that the reggeon propagator

$$G^{(0,0;1)}(\mathscr{E}, 0) = [\Gamma^{(0,0;1)}(\mathscr{E}, 0)]^{-1} \sim \mathscr{E}^{-1+\gamma_{R_1}}$$

and, consequently, the reggeon contribution to the forward scattering amplitude is of the form

$$A_R(s, 0) \underset{s\to\infty}{\sim} s^{\alpha_R(0)}(\ln s)^{-\gamma_{R_1}} = s^{\alpha_R(0)}(\ln s)^{-1/3}.$$

In the following section we shall show that this conclusion is not confirmed by the experimental data on the ϱ-reggeon exchange. The data require the degree of $\ln s$ to be positive and, moreover, it should not be too small.

Leaving the assumption on dipole structure of Pomeron in force (which does not contradict the experiment) we have to conclude that one of two suppositions is not valid, namely, either (a) the RRP vertex is not constant, or (b) the input reggeon is not a simple pole.

Remaining in the framework of renormalizable theory with a local Lagrangian we may choose the input RRP vertex in the form

$$\Gamma_0^{(0,1;1)}(\mathscr{E}_1, \vec{p}_1, \mathscr{E}'_1, \vec{p}'_1, E'_1, \vec{k}'_1) \sim \mathscr{E}_1 + a\vec{p}_1^{\,2}, \qquad \mathscr{E}_1, \vec{p}_1^{\,2} \sim 0.$$

After the analysis similar to that for the vertex (8) we find the renormalized reggeon to be a simple j-pole. This conflicts also with the data on ϱ-reggeon exchange (see the following section).

Thus, we must reject the assumption on simple j-pole structure for the input secondary reggeon.

In the case when the secondary reggeon is a two-fold j-pole the interaction is also described by a local Lagrangian. This variant of reggeon theory will be renormalizable if

$$\Gamma_0^{(0,1;1)}(\mathscr{E}_1, \vec{p}_1, \mathscr{E}'_1, \vec{p}'_1, E'_1, \vec{k}'_1) \sim (\mathscr{E}_1 + a\vec{p}_1^{\,2})^{\mu}$$

with $\mu = 0,1$ or 2.

The anomalous dimension at different values of μ is given in Table I. The Table shows also what additional degree of $\ln s$ appears in $A(s, 0)$ for different input reggeons and different RRP-vertices.

TABLE I

Values of anomalous dimension γ_{R_1} and of parameter γ in expression (42) at different input reggeons and different vertices RRP

Reggeon model	Value of μ in RRP vertex	Anomalous dimension γ_{R_1}	Value of γ in Eq. (42)
Simple pole	0	1/3	−1/3
	1	0	0
Two-fold pole	0	2/5	3/5
	1	1/4	3/4
	2	0	1

3. *Analysis of experiment*

We now turn to the data on ϱ-reggeon exchange in order to verify to what extent they confirm the RFT conclusion on the renormalized ϱ-reggeon to have an anomalous dimension and to what type of an input reggeon they correspond to.

As is known the high-energy behaviour of the total $\pi^- p$ and $\pi^+ p$ cross-section difference and also the $\pi^- p \to \pi^0 n$ differential cross-section are completely determined by the exchange of a state with the ϱ-reggeon quantum number. The measurements of the processes during almost 20 years have shown that one singularity dominates which is approximately as hard as a pole and its trajectory is near a straight line going through the ϱ and g mesons. But the accurate analysis of the data reveals some (not large) discrepancy from one-Regge-pole model. This is seen from the following example. $\Delta\sigma_t$ behaviour [10] results in

$$\alpha_\varrho(0) = 0.54 \pm 0.02. \tag{38}$$

The behaviour of $d\sigma/dt(\pi^- p \to \pi^0 n)$ gives [11]

$$\alpha_\varrho(0) = 0.48 \pm 0.01. \tag{39}$$

This points out, in particular, that the real and imaginary parts of πN charge-exchange forward scattering amplitude have, in contrast to predictions of one-Regge-pole model, somewhat different dependence on energy. Fig. 4 shows the ratio Re $A(s, 0)$/Im $A(s, 0)$. In one-Regge-pole model

$$\mathrm{Re}\, A(s, 0)/\mathrm{Im}\, A(s, 0) = \mathrm{tg}\,(\pi\alpha_\varrho(0)/2) \tag{40}$$

and for $\alpha_\varrho = 0.51$ we have Re A/Im A = 1.03. The experiment reveals that this ratio is not constant but decreases with energy. In particular, Refs. [12, 13] turned attention to this fact. To describe the change of the ratio Re A/Im A in the region $p_{lab} \lesssim 20$ GeV/c an additional phenomenological ϱ'-pole with

$$\alpha_{\varrho'}(0) = -0.07 \pm 0.04, \quad \alpha'_{\varrho'} = 0.4 \pm 0.2\ \mathrm{GeV}^{-2}, \tag{41}$$

is usually introduced. The nature of this term is not clear.

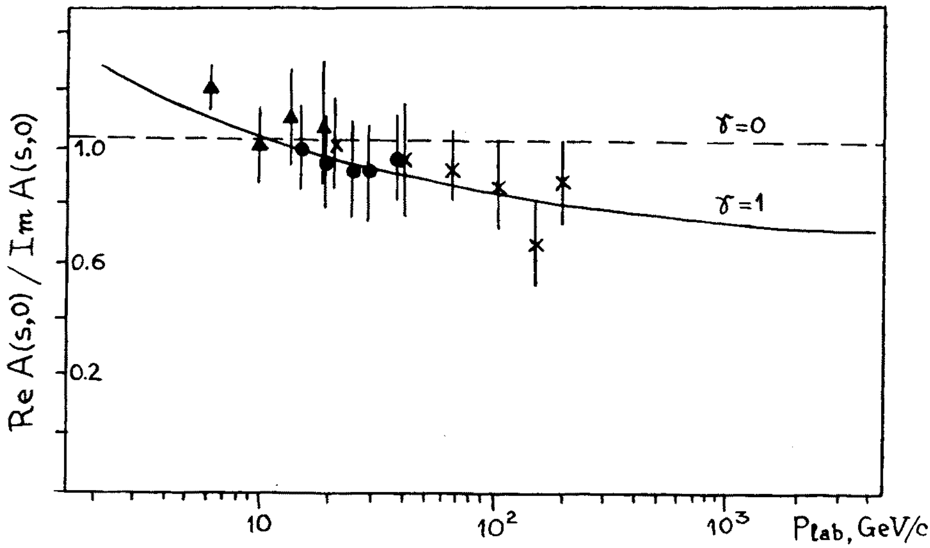

Fig. 4. Ratio of real part of πN charge-exchange forward scattering amplitude to imaginary one. Curves correspond to the model (42) for $\gamma = 0$ (reggeon is a simple pole) and for $\gamma = 1$ (two-fold pole). The data are from Refs. [10–12]

It may be connected with ϱϱ, ϱf, ... branch-points whose intercept and slope are near to (41).

To explain the change of Re A/Im A at $p_{lab} > 20$ GeV/c the assumption has been put forward in Refs. [12, 13] that in asymptotics inelastic processes, similar to elastic ones, tend also to a limit arising from analyticity and unitarity. In Ref. [13], a singularity with unit intercept has been introduced to πN charge exchange amplitude and it has been obtained that its contribution is small at $p_{lab} \lesssim 5$ GeV/c ($\lesssim 5\%$ of whole value of Re A $(s, 0)$) but for $p_{lab} \sim 200$ GeV/c it is 20%. In this model

$$\mathrm{Re}\, A(s, 0)/\mathrm{Im}\, A(s, 0) \sim -\sqrt{s} \to -\infty, \quad s \to \infty.$$

The measurement of polarization in $\pi^- p \to \pi^0 n$ at momenta 100–200 GeV/c is crucial for such a model.

Let us now turn to the expression for the renormalized ϱ-reggeon. Its contribution to πN charge-exchange forward scattering has the form

$$A(s, 0) = -r(-is/s_0)^{\alpha(0)-1}\ln^{\gamma}(-is/s_0). \tag{42}$$

The amplitude is normalized in such a manner that

$$\frac{d\sigma}{dt} = \frac{1}{16\pi}|A|^2,$$

$$\Delta\sigma_t = \sqrt{2}\,\mathrm{Im}\, A(s, 0).$$

Fig. 5 shows how the model (42) agrees with the data on $d\sigma/dt$ ($t = 0$) [11] and $\Delta\sigma_t$ [10] at different values of the parameter γ. For $\gamma \lesssim 0$ we have the χ^2/point $\gtrsim 2.3$ (we use 89 experimental points) that is due to the factors noticed above. The value of χ^2 falls off rapidly when γ increases and the model agrees well with the data for $\gamma \gtrsim 0.2$. In the range $0.6 \lesssim \gamma \lesssim 2.5$ the χ^2 value does not depend appreciably on γ.

In Figs. 4, 6 the model is compared with experiment at $\gamma = 0$ and $\gamma = 1$. Other parameters of the model take in these cases the values in Table II.

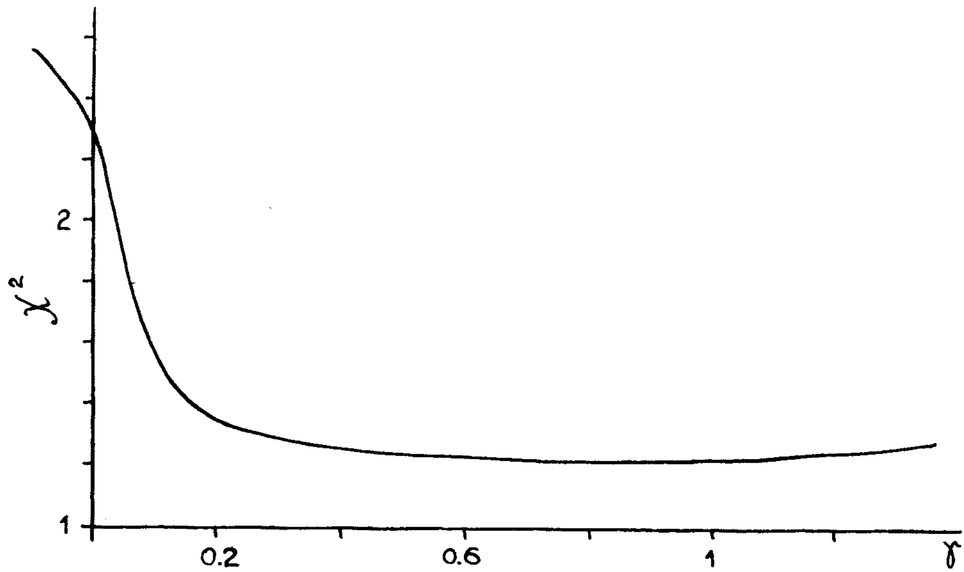

Fig. 5. Dependence of χ^2/point on the value of γ. The curve is obtained from fit to the data [10, 11]

As it is seen from Table I the case $\gamma = 1$ corresponds to the input and renormalized ϱ reggeons to be two-fold Regge poles. This scheme describes well the behaviour of $d\sigma/dt$ as well as the decrease in ratio Re A/Im A with energy.

As a whole, the available data confirms the RFT conclusion that the renormalized secondary reggeons are (at $t \leqslant 0$) harder singularities than a simple pole. But to define the hardness of the reggeon pole one needs the measurements of charge-exchange processes in a wider energy interval.

TABLE II

Values of parameters in expression (42) from fit to the data on $d\sigma/dt$ ($t = 0$) and $\Delta\sigma_t$

γ	r (mb)	s_0 (GeV^2)	α (0)
0	4.30	3.93	0.509
1	11.2	0.178	0.304

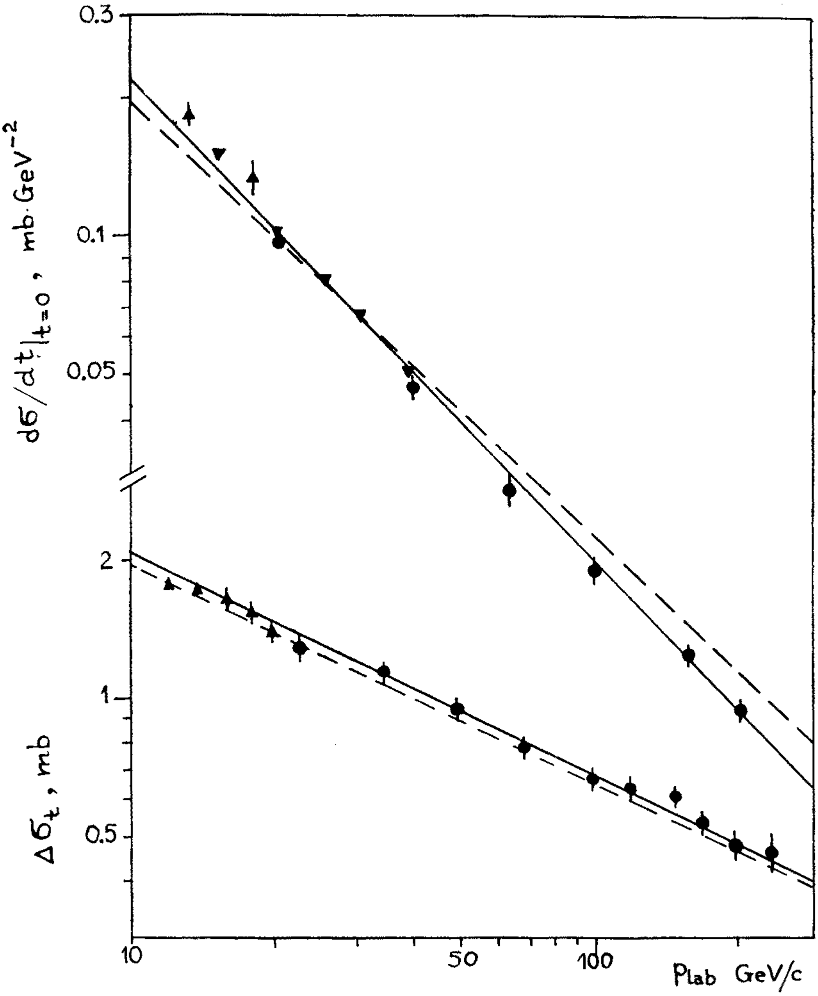

Fig. 6. Differential cross-section of $\pi^- p \to \pi^0 n$ at $t = 0$ and difference of $\pi^- p$ and $\pi^+ p$ total cross-sections. Curves are the model (42) at $\gamma = 0$ (dashed line) and $\gamma = 1$ (solid line). The data are from Refs. [10, 11]

REFERENCES

[1] A. A. Migdal, A. M. Polyakov, K. A. Ter-Martirosyan, *Phys. Lett.* **48B**, 239 (1974); H. D. I. Abarbanel, J B. Bronzan, *Phys. Lett.* **48B**, 345 (1975); M. Moshe, *Phys. Rep.* **37C**, 275 (1978); J. B. Bronzan, J. W. Dash, *Phys. Rev.* **D12**, 4208 (1974).

[2] V. N. Gribov, A. A. Migdal, *Yad. Fiz.* **14**, 1002 (1968) (in Russian).

[3] V. N. Gribov, A. A. Migdal, *Exp. Teor. Fiz.* **55**, 1498 (1968) (in Russian).

[4] H. Cheng, T. T. Wu, *Phys. Lett.* **25B**, 1456 (1970); M. Ciafaloni, G. Marchesini, *Nucl. Phys.* **B109**, 261 (1976); M. S. Dubovikov, B Z. Kopeliovich, L. I. Lapidus, K. A. Ter-Martirosyan, *Nucl. Phys.* **B123**, 147 (1977); V. E. Savrin, N. E. Tyurin, O. L. Chrustalyov, *Elem. Chast. Atom. Yadro* **7**, 21 (1976) (in Russian); N. I. Glushko, N. A. Kobylinsky, Preprint ITP-80-97E, Kiev 1980; V. D. Mur, V. S. Popov, *Yad. Fiz.* **21**, 868 (1975) (in Russian).

[5] R. J. N. Phillips, Preprint RL-73-004, Chilton 1973; A. I. Bugrij, L. L. Jenkovszky, N. A. Kobylinsky, V. P. Shelest, *Lett. Nuovo Cimento* **6**, 577 (1973).
[6] E. S. Martynov, *Yad. Fiz.* **33**, 1431 (1981) (in Russian).
[7] H. D. Abarbanel, R. L. Sugar, *Phys. Rev.* **D10**, 721 (1974).
[8] A. Garcia, C. A. Garcia Canal, N. Parga, *Phys. Rev.* **D18**, 2684 (1978).
[9] H. D. I. Abarbanel, *Phys. Lett.* 295 (1974).
[10] V. Flaminio et al., Preprint CERN-HERA 79-01, 1976.
[11] A. V. Barnes et al., *Phys. Lett.* **37B**, 76 (1976); B. D. Apel et al., *Yad. Fiz.* **30**, 373 (1979).
[12] D. Joynson, E. Leader, B. Nicolescu, C. Lopez, *Nuovo Cimento* **30A**, 345 (1975).
[13] N. A. Kobylinsky, V. V. Timokhin, *Ukr. Fiz. Zh.* **24**, 1142 (1979) (in Russian).
[14] H. Nevelet, P. R. Stevens, *Nucl. Phys.* **B118**, 475 (1977).