SIX-NUCLEON SPECTROSCOPIC AMPLITUDES FOR 1p-SHELL NUCLEI. PART 1. FRACTIONAL PARENTAGE COEFFICIENTS

By E. Kwaśniewicz

Institute of Physics, Silesian Technical University, Katowice*

M. Siemaszko

Institute of Physics, Silesian University, Katowice**

A. Szczurek and L. Jarczyk

Institute of Physics, Jagellonian University, Cracow***

(Received February 9, 1984)

A method is given for calculations of the fractional parentage coefficients (FPC) for separation of six 1p-shell nucleons with the space symmetry [42] or [411]. It is shown that the FPC's $\langle p^n|p^{n-6}, p^6\ [42]\rangle$ and $\langle p^n|p^{n-6}, p^6\ [411]\rangle$ may be written as a product of the weight factor, orbital and isospin-spin coefficients. The compact formulas which express the six-nucleon FPC's through two and four-nucleon ones are presented. The orbital coefficients, together with the weight factors, are calculated and tabulated.

PACS numbers: 21.60.-n, 21.60.Cs

1. Introduction

In the last years the experiments in which transfer of six nucleons exists have been reported in several papers (see for example Refs. [1–6]).

In the DWBA description of multinucleon transfer reactions one of the essential components which enter into the formula for the differential cross section are the multinucleon spectroscopic amplitudes [7–9].

The theory of m-nucleon ($m \leqslant 4$) spectroscopic amplitudes for 1p-shell ($A \leqslant 16$) nuclei has been developed in many papers (see for example Refs. [8, 10–13]).

* Address: Instytut Fizyki, Politechnika Śląska, Krasińskiego 4, 40-019 Katowice, Poland.
** Address: Instytut Fizyki, Uniwersytet Śląski, Uniwersytecka 4, 40-007 Katowice, Poland.
*** Address: Instytut Fizyki, Uniwersytet Jagielloński, Reymonta 4, 30-059 Kraków, Poland.

Recently Smirnov and Tchuvilsky [14] (see also Ref. [15]) have developed a theory of m-nucleon ($m > 4$) spectroscopic amplitudes for 1p-shell nuclei. The application of this theory in the calculations of multinucleon spectroscopic amplitudes requires the knowledge of the multinucleon fractional parentage coefficients (FPC).

In the direct multinucleon transfer reactions one assumes that the multinucleon clusters are transferred in the ground and/or low-lying excited states. The ground and low--lying states of the most probable six-nucleon clusters i.e. ^{6}Li and ^{6}He correspond to the shell model configuration $(1s)^4(1p)^2$. They are described completely by the Young diagrams [42] and [411]. Therefore in the present work the six-nucleon FPC's for this symmetry were calculated.

In general m-nucleon ($m \geqslant 2$) FPC's can be expressed in terms of the FPC's for the separation of less than m nucleons. This method has been applied in computations of two, three and four-nucleon FPC's for p^n configuration [16–18]. Recently Chlebowska et al. [19] have calculated five-nucleon FPC's $\langle p^n | p^{n-5}, p^5\ [41]\rangle$ with the aid of one and four--nucleon FPC's.

In Sect. 2 the formulas are derived for calculations of the orbital and isospin-spin FPC'S $\langle p^n |^{n-6}, p^6\ [42]\rangle$ and $\langle p^n | p^{n-6}, p^6\ [411]\rangle$ with the aid of two and four-nucleon orbital and isospin-spin FPC's.

The application of these FPC's in calculations of six-nucleon spectroscopic amplitudes for 1p-shell nuclei will be considered in detail in the next work (Part 2) of the present paper.

2. *Theory*

The totally antisymmetric shell-model state of n particles can be expressed in terms of the totally antisymmetric states of m nucleons vector-coupled to the parent states (totally antisymmetric) of the remaining $n-m$ nucleons. The coefficients of this expansion are called the fractional parentage coefficients (FPC) [20, 21].

The six-nucleon FPC's belonging to the p^n configuration can be calculated with the aid of four and two-nucleon FPC's by using the following formula [22]

$$\begin{aligned} &\langle p^n[f]\alpha LTS | p^n[f_1]\alpha_1 L_1 T_1 S_1,\ p^6[f_2]\alpha_2 L_2 T_2 S_2\rangle \\ &= \sum_{\substack{\alpha' L' T' S' \\ \alpha_0 L_0 T_0 S_0 \\ \alpha_{00} L_{00} T_{00} S_{00} \\ [f'] [f_0] [f_{00}]}} \langle p^n[f]\alpha LTS | p^{n-2}[f']\alpha' L' T' S',\ p^2[f_0]\alpha_0 L_0 T_0 S_0\rangle \\ &\times \langle p^{n-2}[f']\alpha' L' T' S' | p^{n-6}[f_1]\alpha_1 L_1 T_1 S_1,\ p^4[f_{00}]\alpha_{00} L_{00} T_{00} S_{00}\rangle \\ &\times \langle p^6[f_2]\alpha_2 L_2 T_2 S_2 | p^4[f_{00}]\alpha_{00} L_{00} T_{00} S_{00},\ p^2[f_0]\alpha_0 L_0 T_0 S_0\rangle \\ &\times U(L_1 L_{00} L L_0;\ L' L_2) U(S_1 S_{00} S S_0;\ S' S_2) U(T_1 T_{00} T T_0;\ T' T_2), \end{aligned} \tag{1}$$

where $[f]$, $[f_1]$, $[f_2]$, $[f']$, $[f_0]$ and $[f_{00}]$ are the Young diagrams describing the irreducible representations of the permutation groups S^n, S_{n-6}, S_6, S_{n-2}, S_2 and S_4, respectively.

$T, T_1, T_2, T', T_0, T_{00}, S, S_1, S_2, S', S_0, S_{00}$, L, L_1, L_2, L', L_0 and L_{00} are the isospin, spin and orbital angular momenta of $n, n-6, 6, n-2, 2$ and 4 p-shell nucleons, respectively. The symbols $\alpha, \alpha_1, \alpha_2, \alpha', \alpha_0$ and α_{00} stand for the sets of additional quantum numbers required for complete labelling of the nuclear states considered. U represents the angular momentum recoupling coefficient [23].

Because the representation $[f]$ is contained once in the outer product $[f_1]\times[f_2]$, i.e. in the transformation

$$[f_1]\times[f_2] = \sum a_{[f]}[f],$$

coefficients $a_{[f]}$ is equal to unity, theorefore the FPC $\langle p^n[f]\alpha LTS|p^{n-m}[f_1]\alpha_1 L_1 T_1 S_1, p^m[f_2]\alpha_1 L_2 T_2 S_2\rangle$ can be expressed as a product of the weight factor, orbital and isospin-spin FPC's [22] i.e.

$$\langle p^n[f]\alpha LTS|p^{n-m}[f_1]\alpha_1 L_1 T_1 S_1, p^m[f_2]\alpha_2 L_2 T_2 S_2\rangle$$

$$= \sqrt{\frac{n_{[f_1]}n_{[f_2]}}{n_{[f]}}}\langle p^n[f]\alpha L|p^{n-m}[f_1]\alpha_1 L_1, p^m[f_2]\alpha_2 L_2\rangle$$

$$\times\langle (ts)^n[\tilde{f}]\alpha TS|(ts)^{n-m}[\tilde{f}_1]\alpha_1 T_1 S_1, p\ [\tilde{f}_2]\alpha_2 T_2 S_2\rangle, \tag{2}$$

where $n_{[f_1]}, n_{[f_2]}$ and $n_{[f]}$ are the dimensions of the corresponding representations. In the case of p^n configuration this condition is fulfilled when the outer product is equal to $[f_1]\times[42]$ or $[f_1]\times[411]$ [24, 25]. Thus the total FPC $\langle p^n|p^{n-6}, p^6[f_2]\rangle$, in which we assume that $[f_2]$ = [42] or [411], may be factorized according to formula (2). The orbital FPC's $\langle p^n[f]\alpha L|p^{n-6}[f_1]\alpha_1 L_1, p_6[f_2]\alpha_2 L_2\rangle$ and the isospin-spin FPC's $\langle (ts)^n[f]\alpha TS|(ts)^{n-6}[f_1]$ $\alpha_1 T_1 S_1, (ts)^6[f_2]\alpha_2 T_2 S_2\rangle$ can be calculated separately.

For practical computations of these coefficients we have derived the appropriate formulas starting from the following equation [21, 22, 19]

$$\langle (r)|[f_2](\bar{r}), (r')\rangle\langle p^n[f]\alpha L|p^{n-6}[f_1]\alpha_1 L_1, p^6[f_2]\alpha_2 L_2\rangle$$

$$= \langle p^{n-6}[f_1](r')\alpha_1 L_1, p^6[f_2](\bar{r})\alpha_2 L_2; LM|p^n[f](r)\alpha LM\rangle, \tag{3}$$

where $(r) = (r_n r_{n-1} \ldots r_1) = (r_n r_{n-1} \ldots r_{n-6+1}, r'), (r') = (r_{n-6}\, r_{n-6-1} \ldots r_1)$ and $(\bar{r})$ are the standard Yamanouchi symbols for the representations of the groups S_n, S_{n-6} and S_6, respectively. The factor $\langle (r)|[f_2](\bar{r}), (r')\rangle$ comes from the set of the transformation coefficients for the transition from the basis functions of the standard Young-Yamanouchi representation of S_n to those of the representation of $S_{n-6}\times S_6$.

The wave function spreading out the representation $[f]$ of S_n can be expressed in terms of two-nucleon FPC's, i.e.,

$$|p^n[f](r)\alpha LM\rangle = \sum_{\alpha' L'\alpha_0 L_0 [f_0](r_0)}\langle (r)|[f_0](r_0), (\bar{r}')\rangle\langle p^n[f]\alpha L|p^{n-2}[f']\alpha' L', p^2[f_0]\alpha_0 L_0\rangle$$

$$\times|p^{n-2}[f'](\bar{r}')\alpha' L', p^2[f_0](r_0)\alpha_0 L_0;\ LM\rangle, \tag{4}$$

where $[f']$ is determined by $(\bar{r}')$. Similarly the wave function $|p^{n-2}[f'](\bar{r}')\alpha' L' M'\rangle$ can be expressed in terms of four-nucleon FPC's, i.e.,

$$|p^{n-2}[f'](\bar{r}')\alpha' L' M'\rangle = \sum_{\alpha_1 L_1 \alpha_{00} L_{00} [f_{00}](r_{00})} \langle(\bar{r}')|[f_{00}](r_{00}),(r')\rangle \times \langle p^{n-2}[f']\alpha' L'|p^{n-6}[f_1]\alpha_1 L_1, p^4[f_{00}]\alpha_{00}L_{00}\rangle \times |p^{n-6}[f_1](r')\alpha_1 L_1, p^4[f_{00}](r_{00})\alpha_{00}L_{00}; L'M'\rangle. \quad (5)$$

Inserting Eq. (5) into Eq. (4) we obtain

$$|p^n[f](r)\alpha LM\rangle = \sum_{\substack{\alpha' L' \alpha_0 L_0 \alpha_1 L_1 \alpha_{00} L_{00} \\ [f_0](r_0)[f_{00}](r_{00})}} \langle(r)|[f_0](r_0),(\bar{r}')\rangle \langle(\bar{r}')|[f_{00}](r_{00}),(r')\rangle$$

$$\times \langle p^n[f]\alpha L|p^{n-2}[f']\alpha' L', p^2[f_0]\alpha_0 L_0\rangle \langle p^{n-2}[f']\alpha' L'|p^{n-6}[f_1]\alpha_1 L_1, p^4[f_{00}]\alpha_{00}L_{00}\rangle \times |p^{n-6}[f_1](r')\alpha_1 L_1, p^4[f_{00}](r_{00})\alpha_{00}L_{00}\{L'\}: p^2[f_0](r_0)\alpha_0 L_0; LM\rangle. \quad (6)$$

Because in our considerations the six separated nucleons are described by the Young diagram $[f_2] = [42]$ (or $[411]$) it is sufficient to take the Yamanouchi symbol $(\bar{r}) = (221111)$ (or (321111)) as the one from the set of nine (ten) corresponding to this diagram. Then the wave function of the six separated nucleons can be expressed as follows

$$|p^6[f_2](\bar{r})\bar{\alpha}_2\bar{L}_2\bar{M}_2\rangle = \sum_{\bar{\alpha}_0\bar{L}_0\bar{\alpha}_{00}\bar{L}_{00}} \langle p^6[f_2]\bar{\alpha}_2\bar{L}_2|p^4[4]\bar{\alpha}_{00}\bar{L}_{00}, p^2[\bar{f}_0]\bar{\alpha}_0\bar{L}_0\rangle \times |p^4[4]\bar{\alpha}_{00}\bar{L}_{00}, p^2[\bar{f}_0]\bar{\alpha}_0\bar{L}_0; \bar{L}_2\bar{M}_2\rangle, \quad (7)$$

where $[\bar{f}_0]$ is determined by $(\bar{r})$.

Making use of the U-recoupling coefficients of angular momenta and taking into account the equation (7) we have

$$|p^{n-6}[f_1](r')\bar{\alpha}_1\bar{L}_1, p^6[f_2](\bar{r})\bar{\alpha}_2\bar{L}_2: LM)$$

$$= \sum_{\bar{\alpha}_0\bar{L}_0\bar{\alpha}_{00}\bar{L}_{00}} \langle p^6[f_2]\bar{\alpha}_2\bar{L}_2|p^4[4]\bar{\alpha}_{00}\bar{L}_{00}, p^2[\bar{f}_0]\bar{\alpha}_0\bar{L}_0\rangle \times |p^{n-6}[f_1](r')\bar{\alpha}_1\bar{L}_1: p^4[4]\bar{\alpha}_{00}\bar{L}_{00}, p^2[\bar{f}_0]\bar{\alpha}_0\bar{L}_0\{\bar{L}_2\}; LM\rangle$$

$$= \sum_{\bar{L}'\bar{\alpha}_{00}\bar{L}_{00}\bar{\alpha}_0\bar{L}_0} U(\bar{L}_1\bar{L}_{00}L\bar{L}_1; \bar{L}'\bar{L}_2) \langle p^6[f_2]\bar{\alpha}_2\bar{L}_2|p^4[4]\bar{\alpha}_{00}\bar{L}_{00}, p^2[\bar{f}_0]\bar{\alpha}_0\bar{L}_0\rangle \times |p^{n-6}[f_1](r')\bar{\alpha}_1\bar{L}_1, p^4[4]\bar{\alpha}_{00}\bar{L}_{00}\{\bar{L}'\}: p^2[\bar{f}_0]\bar{\alpha}_0\bar{L}_0; LM\rangle. \quad (8)$$

Inserting expressions (6) and (8) into the equation (3) we obtain the following formula for the orbital FPC's

$$\langle p^n[f]\alpha L|p^{n-6}[f_1]\alpha_1 L_1, p^6[f_2]\alpha_2 L_2\rangle$$

$$= \langle(r)|[f_2](\bar{r}),(r')\rangle^{-1}\langle(r)|[f_0](r_0),(\bar{r}')\rangle \langle(\bar{r}')|[4],(r')\rangle \times \sum_{\alpha_0 L_0 \alpha_{00} L_{00} \alpha' L'} U(L_1 L_{00} L L_0; L' L_2) \langle p^n[f]\alpha L|p^{n-2}[f']\alpha' L', p^2[f_0]\alpha_0 L_0\rangle$$

$$\times \langle p^{n-2}[f']\alpha' L'|p^{n-6}[f_1]\alpha_1 L_1, p^4[4]\alpha_{00}L_{00}\rangle \langle p^6[f_2]\alpha_2 L_2|p^4[4]\alpha_{00}L_{00}, p^2[f_0]\alpha_0 L_0\rangle. \quad (9)$$

The isospin-spin coefficients can be derived in a way similar to that for the orbital ones and are expressed by the formula

$$\begin{aligned}
&\langle (ts)^n[\tilde{f}]\alpha TS|\,(ts)^{n-6}[\tilde{f}_1]\alpha_1 T_1 S_1,(ts)^6[\tilde{f}_2]\alpha_2 T_2 S_2\rangle\\
&= \langle(\tilde{r})|[\tilde{f}_2]\,(\tilde{\bar{r}}),(\tilde{r}')\rangle^{-1}\langle(\tilde{r})|[\tilde{f}_0]\,(\tilde{r}_0),(\tilde{\bar{r}}')\rangle\,\langle(\tilde{\bar{r}}')|[\tilde{4}],(\tilde{r}')\rangle\\
&\times \sum_{\alpha_0 T_0 S_0 \alpha_{00} T_{00} S_{00}\alpha' T' S'} U(T_1 T_{00} T T_0;\, T' T_2) U(S_1 S_{00} S S_0;\, S' S_2)\\
&\times \langle (ts)^n[\tilde{f}]\alpha TS|(ts)^{n-2}[\tilde{f}']\alpha' T' S',(ts)^2[\tilde{f}_0]\alpha_0 T_0 S_0\rangle\\
&\times \langle (ts)^{n-2}[\tilde{f}']\alpha' T' S'|(ts)^{n-6}[\tilde{f}_1]\alpha_1 T_1 S_1,(ts)^4[\tilde{4}]\alpha_{00} T_{00} S_{00}\rangle\\
&\times \langle (ts)^6[\tilde{f}_2]\alpha_2 T_2 S_2|(ts)^4[\tilde{4}]\alpha_{00} T_{00} S_{00},(ts)^2[\tilde{f}_0]\alpha_0 T_0 S_0\rangle,
\end{aligned}\tag{10}$$

where $[\tilde{f}]$ are the diagrams adjoint to $[f]$ ones.

The coefficients $\langle(r)[f_2]|(\bar{r}),(r')\rangle^{-1}\langle(r)|[f_0]\,(r_0),(\bar{r}')\rangle\,\langle(\bar{r}')|[4],(r')\rangle$ in Eq. (9) as well as the similar ones in Eq. (10) can be calculated from the orthonormality conditions of the orbital and isospin-spin FPC's. In our case these coefficients are equal to unity for the similar reasons as it was pointed out by Chlebowska et al. [19].

From the tables given by Elliott et al. [16] and Rotter [18] one can find that the following relations hold

$$\langle (ts)^{n-2}[\tilde{f}']\alpha' T' S'|(ts)^{n-6}[\tilde{f}_1]\alpha_1 T_1 S_1,(ts)^4[\tilde{4}]\alpha_{00} T_{00} S_{00}\rangle = (-1)^{n-2}$$

and

$$\langle (ts)^6[\tilde{f}_2]\alpha_2 T_2 S_2|(ts)^4[\tilde{4}]\alpha_{00} T_{00} S_{00},(ts)^2[\tilde{f}_0]\alpha_0 T_0 S_0\rangle = 1.$$

Therefore the formula (10) can be simplified considerably. The six-nucleon isospin-spin FPC's can be expressed directly by means of two nucleon ones, i.e.

$$\begin{aligned}
&\langle (ts)^n[\tilde{f}]\alpha TS|(ts)^{n-6}[\tilde{f}_1]\alpha_1 T_1 S_1,(ts)^6[\tilde{f}_2]\alpha_2 T_2 S_2\rangle\\
&= (-)^{n-2}\langle (ts)^n[\tilde{f}]\alpha TS|(ts)^{n-2}[\tilde{f}']\alpha_1 T_1 S_1,(ts)^2[\tilde{f}_0]\alpha_2 T_2 S_2\rangle,
\end{aligned}\tag{11}$$

where $[\tilde{f}_0] = [\tilde{2}]$ for $[\tilde{f}_2] = [\widetilde{42}]$, $[\tilde{f}_0] = [\widetilde{11}]$ for $[\tilde{f}_2] = [4\widetilde{11}]$ and $[\tilde{f}']$ result from the condition $[\tilde{f}] \subset [\tilde{f}'] \times [\tilde{f}_0]$ and $[\tilde{f}'] \subset [\tilde{f}_1] \times [\tilde{4}]$.

We have calculated all orbital six-nucleon FPC's with the space symmetry [42] and [411] according to the formula (9). The two and four-nucleon orbital FPC's required in these calculations were taken from the tables published by Elliott et al. [16] and Rotter [18]. The isospin-spin six-nucleon FPC's can be obtained directly from tables of Elliott et al. [16] via Eq. (11).

The calculated orbital coefficients and weight factors are listed in Table I[1].

The autors would like to thank dr. B. Kamys for helpful discussions.

[1] The calculated six-nucleon orbital coefficients $\langle p^7|p^1,p^6\rangle$, $\langle p^8|p^2,p^6\rangle$, $\langle p^9|p^3,p^6\rangle$, $\langle p^{10}|p^4\,[4],p^6\rangle$ and $\langle p^{11}|p^5\,[41],p^6\rangle$ differ from corresponding 1–5 nucleon ones [20, 16, 26, 18, 19] only by the phase factor. But for completeness they are also included in Table I.

TABLE I

The weight factors $\langle [f] \| [f_1], [f_2] \rangle$ ($[f_2] = [42]$ or $[411]$) and orbital fractional parentage coefficients $\langle p^n [f] L | p^{n-6} [f_1] L_1, p^6 [f_2] L_2 \rangle$. First column specifies the value of the angular momentum L corresponding to the diagram $[f]$. First row specifies the values of angular momenta L_1 and L_2 corresponding to the diagrams $[f_1]$ and $[f_2]$, respectively

$\langle [43] \| [1], [42] \rangle = \sqrt{\frac{9}{14}}$

[43] \ [1]	PS	PD$_I$	PD$_{II}$	PF	PG
P	$-\sqrt{\frac{16}{54}}$	$-\sqrt{\frac{3}{54}}$	$-\sqrt{\frac{35}{54}}$		
D		$\sqrt{\frac{35}{54}}$	$\sqrt{\frac{3}{54}}$	$\sqrt{\frac{16}{54}}$	
F		$-\sqrt{\frac{56}{378}}$	$\sqrt{\frac{120}{378}}$	$\sqrt{\frac{175}{378}}$	$-\sqrt{\frac{27}{378}}$
G				$-\sqrt{\frac{1}{6}}$	$\sqrt{\frac{5}{6}}$

$\langle [421] \| [1], [42] = \sqrt{\frac{9}{35}}$

[421] \ [1]	PS	PD$_I$	PD$_{II}$	PF	PG
P	$\sqrt{\frac{5}{27}}$	$\sqrt{\frac{15}{27}}$	$-\sqrt{\frac{7}{27}}$		
D		$-\sqrt{\frac{5}{54}}$	$-\sqrt{\frac{21}{54}}$	$\sqrt{\frac{28}{54}}$	
F		$-\sqrt{\frac{35}{378}}$	$-\sqrt{\frac{3}{378}}$	$-\sqrt{\frac{70}{378}}$	$-\sqrt{\frac{270}{378}}$

$\langle [44] \| [2], [42] \rangle = \sqrt{\frac{9}{14}}$

[44] \ [2]	SS	SD$_I$	SD$_{II}$	SG	DS	DD$_I$	DD$_{II}$	DF	DG
S	$-\sqrt{\frac{16}{54}}$					$-\sqrt{\frac{3}{54}}$	$-\sqrt{\frac{35}{54}}$		
D		$-\sqrt{\frac{735}{3780}}$	$\sqrt{\frac{175}{3780}}$		$\sqrt{\frac{196}{3780}}$	$\sqrt{\frac{945}{3780}}$	$\sqrt{\frac{625}{3780}}$	$\sqrt{\frac{1050}{3780}}$	$-\sqrt{\frac{54}{3780}}$
G				$-\sqrt{\frac{14}{126}}$		$\sqrt{\frac{7}{126}}$	$-\sqrt{\frac{15}{126}}$	$-\sqrt{\frac{35}{126}}$	$\sqrt{\frac{55}{126}}$

TABLE I (continued)

$\langle[422]|[2],\ [42]\rangle = \sqrt{\frac{9}{56}}$

[422] \ [2]	SS	SD_I	SD_{II}	DS	DD_I	DD_{II}	DF	DG
S	$\sqrt{\frac{5}{27}}$				$\sqrt{\frac{15}{27}}$	$-\sqrt{\frac{7}{27}}$		
D		$-\sqrt{\frac{15}{135}}$	$-\sqrt{\frac{7}{135}}$	$-\sqrt{\frac{1}{135}}$		$\sqrt{\frac{16}{135}}$	$-\sqrt{\frac{42}{135}}$	$-\sqrt{\frac{54}{135}}$

$\langle[431]|[2],\ [42]\rangle = \sqrt{\frac{9}{70}}$

[431] \ [2]	SD_I	SD_{II}	SF	DS	DD_I	DD_{II}	DF	DG
P					$-\sqrt{\frac{5}{54}}$	$\sqrt{\frac{21}{54}}$	$-\sqrt{\frac{28}{54}}$	
D	$\sqrt{\frac{5}{108}}$	$-\sqrt{\frac{21}{108}}$		$\sqrt{\frac{12}{108}}$	$\sqrt{\frac{35}{108}}$	$\sqrt{\frac{3}{108}}$	$-\sqrt{\frac{14}{108}}$	$\sqrt{\frac{18}{108}}$
F			$\sqrt{\frac{70}{378}}$		$-\sqrt{\frac{35}{378}}$	$-\sqrt{\frac{27}{378}}$	$-\sqrt{\frac{21}{378}}$	$-\sqrt{\frac{225}{378}}$

$\langle[431]|[11],\ [42]\rangle = \sqrt{\frac{9}{70}}$

[431] \ [11]	PS	PD_I	PD_{II}	PF	PG
P	$\sqrt{\frac{5}{27}}$	$-\sqrt{\frac{15}{27}}$	$-\sqrt{\frac{7}{27}}$		
D		$\sqrt{\frac{5}{54}}$	$-\sqrt{\frac{21}{54}}$	$-\sqrt{\frac{28}{54}}$	
F		$\sqrt{\frac{35}{378}}$	$-\sqrt{\frac{3}{378}}$	$\sqrt{\frac{70}{378}}$	$-\sqrt{\frac{270}{378}}$

TABLE I (continued)

$\langle[441]|[3], [42]\rangle = \sqrt{\frac{3}{22}}$

[441] \ [3]	PS	PD_I	PD_{II}	PF	PG	FS	FD_I	FD_{II}	FF	FG
P	$\sqrt{\frac{16}{810}}$	$\sqrt{\frac{108}{810}}$	$-\sqrt{\frac{140}{810}}$				$-\sqrt{\frac{7}{810}}$	$\sqrt{\frac{135}{810}}$	$-\sqrt{\frac{350}{810}}$	$\sqrt{\frac{54}{810}}$
F		$-\sqrt{\frac{7}{1890}}$	$-\sqrt{\frac{135}{1890}}$	$\sqrt{\frac{350}{1890}}$	$\sqrt{\frac{54}{1890}}$	$\sqrt{\frac{84}{1890}}$	$-\sqrt{\frac{378}{1890}}$	$-\sqrt{\frac{90}{1890}}$		$-\sqrt{\frac{792}{1890}}$

$\langle[432]|[3], [42]\rangle = \sqrt{\frac{3}{56}}$

[432] \ [3]	PS	PD_I	PD_{II}	PF	FD_I	FD_{II}	FF	FG
P	$-\sqrt{\frac{80}{810}}$	$-\sqrt{\frac{135}{810}}$	$\sqrt{\frac{7}{810}}$		$-\sqrt{\frac{140}{810}}$	$\sqrt{\frac{108}{810}}$	$-\sqrt{\frac{70}{810}}$	$-\sqrt{\frac{270}{810}}$
D		$\sqrt{\frac{125}{1350}}$	$-\sqrt{\frac{189}{1350}}$	$\sqrt{\frac{112}{1350}}$		$\sqrt{\frac{96}{1350}}$	$-\sqrt{\frac{378}{1350}}$	$-\sqrt{\frac{450}{1350}}$

$\langle[441]|[21], [42]\rangle = \sqrt{\frac{3}{14}}$

[441] \ [21]	PS	PD_I	PD_{II}	PF	PG	DD_I	DD_{II}	DF	DG
P	$-\sqrt{\frac{80}{648}}$	$-\sqrt{\frac{135}{648}}$	$\sqrt{\frac{7}{648}}$			$-\sqrt{\frac{125}{648}}$	$-\sqrt{\frac{189}{648}}$	$-\sqrt{\frac{112}{648}}$	
F		$-\sqrt{\frac{140}{1512}}$	$\sqrt{\frac{108}{1512}}$	$\sqrt{\frac{70}{1512}}$	$-\sqrt{\frac{270}{1512}}$		$\sqrt{\frac{96}{1512}}$	$\sqrt{\frac{378}{1512}}$	$-\sqrt{\frac{450}{1512}}$

TABLE I (continued)

$$\langle[432]|[21],\ [42]\rangle = \sqrt{\tfrac{3}{28}}$$

[432] \ [21]	PS	PD_I	PD_{II}	PF	DS	DD_I	DD_{II}	DF	DG
P	$-\sqrt{\frac{5}{81}}$		$-\sqrt{\frac{28}{81}}$			$-\sqrt{\frac{20}{81}}$		$\sqrt{\frac{28}{81}}$	
D		$\sqrt{\frac{20}{135}}$		$-\sqrt{\frac{28}{135}}$	$\sqrt{\frac{3}{135}}$		$-\sqrt{\frac{12}{135}}$		$-\sqrt{\frac{72}{135}}$

$$\langle[442]|[4],\ [42]\rangle = \sqrt{\tfrac{1}{28}}$$

[442] \ [4]	SS	SD_I	SD_{II}	DS	DD_I	DD_{II}	DF	DG	GD_I	GD_{II}	GF	GG
S	$-\sqrt{\frac{32}{270}}$				$-\sqrt{\frac{105}{270}}$	$\sqrt{\frac{25}{270}}$						$-\sqrt{\frac{108}{270}}$
D		$-\sqrt{\frac{42}{9450}}$	$-\sqrt{\frac{490}{9450}}$	$\sqrt{\frac{196}{9450}}$	$\sqrt{\frac{945}{9450}}$	$\sqrt{\frac{625}{9450}}$	$-\sqrt{\frac{1050}{9450}}$	$-\sqrt{\frac{54}{9450}}$	$\sqrt{\frac{378}{9450}}$	$-\sqrt{\frac{810}{9450}}$	$\sqrt{\frac{1890}{9450}}$	$\sqrt{\frac{2970}{9450}}$

$$\langle[442]|[31],\ [42]\rangle = \sqrt{\tfrac{3}{28}}$$

[442] \ [31]	PD_I	PD_{II}	PF	DS	DD_I	DD_{II}	DF	DG	FD_I	FD_{II}	FF	FG
S					$\sqrt{\frac{5}{54}}$	$-\sqrt{\frac{21}{54}}$					$\sqrt{\frac{28}{54}}$	
D	$\sqrt{\frac{30}{1350}}$	$-\sqrt{\frac{126}{1350}}$	$-\sqrt{\frac{168}{1350}}$	$\sqrt{\frac{60}{1350}}$	$\sqrt{\frac{175}{1350}}$	$\sqrt{\frac{15}{1350}}$	$\sqrt{\frac{70}{1350}}$	$\sqrt{\frac{90}{1350}}$	$\sqrt{\frac{70}{1350}}$	$\sqrt{\frac{54}{1350}}$	$-\sqrt{\frac{42}{1350}}$	$\sqrt{\frac{450}{1350}}$

TABLE I (continued)

$\langle[433]|[31], [42]\rangle = \sqrt{\frac{9}{70}}$

[433] \ [31]	PS	PD_I	PD_{II}	DD_I	DD_{II}	DF	FD_I	FD_{II}	FF	FG
P	$\sqrt{\frac{30}{810}}$	$-\sqrt{\frac{90}{810}}$	$-\sqrt{\frac{42}{810}}$	$-\sqrt{\frac{25}{810}}$	$\sqrt{\frac{105}{810}}$	$-\sqrt{\frac{140}{810}}$	$\sqrt{\frac{35}{810}}$	$-\sqrt{\frac{3}{810}}$	$-\sqrt{\frac{70}{810}}$	$-\sqrt{\frac{270}{810}}$

$\langle[442]|[22], [42]\rangle = \sqrt{\frac{1}{14}}$

[442] \ [22]	SS	SD_I	SD_{II}	DS	DD_I	DD_{II}	DF	DG
S	$\sqrt{\frac{5}{27}}$				$-\sqrt{\frac{15}{27}}$	$-\sqrt{\frac{7}{27}}$		
D		$\sqrt{\frac{15}{135}}$	$-\sqrt{\frac{7}{135}}$	$-\sqrt{\frac{1}{135}}$		$\sqrt{\frac{16}{135}}$	$\sqrt{\frac{42}{135}}$	$-\sqrt{\frac{54}{135}}$

$\langle[443]|[41], [42]\rangle = \sqrt{\frac{6}{77}}$

[443] \ [41]	PS	PD_I	PD_{II}	DD_I	DD_{II}	DF	FD_I	FD_{II}	FF	FG	GF	GG
P	$-\sqrt{\frac{48}{1296}}$	$-\sqrt{\frac{9}{1296}}$	$-\sqrt{\frac{105}{1296}}$	$-\sqrt{\frac{175}{1296}}$	$-\sqrt{\frac{15}{1296}}$	$\sqrt{\frac{80}{1296}}$	$-\sqrt{\frac{56}{1296}}$	$\sqrt{\frac{120}{1296}}$	$-\sqrt{\frac{175}{1296}}$	$-\sqrt{\frac{27}{1296}}$	$-\sqrt{\frac{81}{1296}}$	$-\sqrt{\frac{405}{1296}}$

$\langle[443]|[32], [42]\rangle = \sqrt{\frac{15}{154}}$

[443] \ [32]	PS	PD_I	PD_{II}	DD_I	DD_{II}	DF	FD_I	FD_{II}	FF	FG
P	$\sqrt{\frac{30}{810}}$	$\sqrt{\frac{90}{810}}$	$-\sqrt{\frac{42}{810}}$	$\sqrt{\frac{25}{810}}$	$\sqrt{\frac{105}{810}}$	$\sqrt{\frac{140}{810}}$	$-\sqrt{\frac{35}{810}}$	$-\sqrt{\frac{3}{810}}$	$\sqrt{\frac{70}{810}}$	$-\sqrt{\frac{270}{810}}$

TABLE I (continued)

$\langle[444]|[42],[42]\rangle = \sqrt{\frac{27}{154}}$

[444] \ [42]	SS	D_ID_I	$D_{II}D_{II}$	FF	GG
S	$\sqrt{\frac{1}{27}}$	$-\sqrt{\frac{5}{27}}$	$\sqrt{\frac{5}{27}}$	$-\sqrt{\frac{7}{27}}$	$\sqrt{\frac{9}{27}}$

$\langle[421]|[1],[411]\rangle = \sqrt{\frac{2}{7}}$

[421] \ [1]	PP	PF
P	-1	
D	$\sqrt{\frac{3}{10}}$	$-\sqrt{\frac{7}{10}}$
F		-1

$\langle[431]|[2],[411]\rangle = \sqrt{\frac{1}{7}}$

[431] \ [2]	SP	SF	DP	DF
P	$-\sqrt{\frac{20}{45}}$		$\sqrt{\frac{4}{45}}$	$\sqrt{\frac{21}{45}}$
D			$-\sqrt{\frac{8}{15}}$	$\sqrt{\frac{7}{15}}$
F		$\sqrt{\frac{5}{30}}$	$\sqrt{\frac{1}{30}}$	$-\sqrt{\frac{24}{30}}$

$\langle[422]|[11],[411]\rangle = \sqrt{\frac{5}{28}}$

[422] \ [11]	PP	PF
S	1	
D	$\sqrt{\frac{4}{25}}$	$\sqrt{\frac{21}{25}}$

$\langle[441]|[3],[411]\rangle = \sqrt{\frac{5}{42}}$

[441] \ [3]	PP	PF	FP	FF
P	$-\sqrt{\frac{8}{15}}$			$\sqrt{\frac{7}{15}}$
F		$\sqrt{\frac{3}{15}}$	$\sqrt{\frac{3}{15}}$	$-\sqrt{\frac{9}{15}}$

$\langle[432]|[21],[411]\rangle = \sqrt{\frac{5}{42}}$

[432] \ [21]	PP	PF	DP	DF
P	$\sqrt{\frac{5}{15}}$		$\sqrt{\frac{3}{15}}$	$-\sqrt{\frac{7}{15}}$
D	$\sqrt{\frac{3}{25}}$	$-\sqrt{\frac{7}{25}}$	$-\sqrt{\frac{1}{25}}$	$-\sqrt{\frac{14}{25}}$

TABLE I (continued)

$\langle[442]|[31], [411]\rangle = \sqrt{\frac{10}{84}}$

[442] \ [31]	PP	PF	DP	DF	FP	FF
S	$\sqrt{\frac{8}{15}}$					$-\sqrt{\frac{7}{15}}$
D	$-\sqrt{\frac{8}{375}}$	$-\sqrt{\frac{42}{375}}$	$-\sqrt{\frac{80}{375}}$	$\sqrt{\frac{70}{375}}$	$-\sqrt{\frac{7}{375}}$	$\sqrt{\frac{168}{375}}$

$\langle[433]|[22], [411]\rangle = \sqrt{\frac{2}{21}}$

[433] \ [22]	SP	DP	DF
P	$-\sqrt{\frac{5}{30}}$	$-\sqrt{\frac{4}{30}}$	$-\sqrt{\frac{21}{30}}$

$\langle[443]|[32], [411]\rangle = \sqrt{\frac{25}{231}}$

[443] \ [32]	PP	DP	DF	FF
P	$-\sqrt{\frac{6}{30}}$	$-\sqrt{\frac{3}{30}}$	$\sqrt{\frac{7}{30}}$	$-\sqrt{\frac{14}{30}}$

$\langle[444]|[33], [411]\rangle = \sqrt{\frac{25}{231}}$

[444] \ [33]	PP	FF
S	$\sqrt{\frac{3}{10}}$	$\sqrt{\frac{7}{10}}$

REFERENCES

[1] J. D. Garret, F. Ajzenberg-Selove, H. G. Bingham, *Phys. Rev.* **C10**, 1730 (1974).
[2] H. T. Fortune, H. G. Bingham, *Phys. Rev.* **C10**, 2174 (1974).
[3] H. T. Fortune, L. R. Medsker, *Phys. Rev.* **C12**, 1364 (1975).
[4] F. Ajzenberg-Selove, R. R. Betts, *Phys. Rev.* **C14**, 357 (1976).
[5] K. Kato, S. Kubono, T. Yamaya, H. Endo, K. Takimoto, K. Fujii, J. Schinzu, N. Takahashi, Y. Iwasaki, J. Kasagi, *Phys. Lett.* **62B**, 153 (1976).
[6] S. N. Abramovich, A. I. Baz, B. Y. Guzhovsky, *Yad. Fiz.* **32**, 402 (1980).
[7] N. Austern, *Direct Nuclear Reaction Theories*, John Wiley and Sons, New York 1970.
[8] M. Ichimura, A. Arima, E. C. Halbert, T. Teresawa, *Nucl. Phys.* **A204**, 225 (1973).
[9] D. Kurath, I. S. Towner, *Nucl. Phys.* **A222**, 1 (1974).
[10] I. Rotter, *Nucl. Phys.* **A122**, 567 (1968).
[11] I. Rotter, *Nucl. Phys.* **A135**, 378 (1969).
[12] D. Kurath, *Phys. Rev.* **C7**, 1390 (1973).
[13] D. Kurath, D. J. Millener, *Nucl. Phys.* **A238**, 269 (1975).
[14] Y. F. Smirnov, Y. M. Tchuvilsky, *Phys. Rev.* **C15**, 84 (1977).
[15] Y. Kudo, T. Honda, H. Horie, *Prog. Theor. Phys.* **59**, 101 (1978).
[16] J. P. Elliott, J. Hope, H. A. Jahn, *Phil. Trans. Roy Soc.* **A246**, 241 (1953).
[17] D. Chlebowska, *Acta Phys. Pol.* **25**, 513 (1964)
[18] I. Rotter, *Ann. Phys. (Germany)* **16**, 242 (1965).
[19] D. Chlebowska, E. Kwaśniewicz, M. Siemaszko, *J. Phys. G: Nucl. Phys.* **7**, 623 (1981).
[20] H. A. Jahn, H. van Wieringen, *Proc. Roy. Soc.* **A209**, 502 (1951).
[21] H. Horie, *J. Phys. Soc. Jpn.* **1910**, 1783 (1964).
[22] V. G. Neudatchin, Y. F. Smirnov, *Nucleon Clusters in the Light Nuclei*, Nauka, Moscow 1969 (in Russian).
[23] H. A. Jahn, *Proc. Roy. Soc.* **A205**, 192 (1951).
[24] D. E. Littlewood, *The Theory of Group Characters and Matrix Representations of Group*, Clarendon Press, Oxford 1940.
[25] V. Vanagas, *Algebraic Methods in Nuclear Theory*, Mintis, Vilnius 1971 (in Russian).
[26] D. Chlebowska, Report 449/7, Institute of Nuclear Research, Świerk, Poland 1963.