ON THE RELATION BETWEEN COUPLING AND MASS

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We study quark-lepton mass pattern in a phenomenological manner. We consider the self interaction to be responsible for generation of the mass. We find that the ratio of average current quark mass and charged lepton mass bears a constant ratio for each family. We predict top quark mass to be 25 GeV.

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1. *Introduction*

In the last decade, the understanding of various interactions of elementary particles had advanced via the development of grand unified theories (GUTs). This seems to present, for the first time, an unified view of the cosmos. But still the theory has many free parameters, like fermion masses, mixing angles etc. The common belief is that quark and lepton masses and other parameters would be calculable in the final theory, but it still remains to be realized [1]. In Quantum Electrodynamics, lepton mass being infinitely renormalized, remains a free parameter. Similar is the case with Glashow-Weinberg-Salam model where Yukawa coupling is infinitely renormalized [2]. However, in GUTs, fermion masses can be generated by the same Yukawa coupling, so the mass ratios are finitely renormalized [3]. But the obtained mass ratios are not well satisfied. Thus, from theoretical point of view, situation at present is not much encouraging. Therefore, it may be useful to approach the problem in a phenomenological manner, as the available information is not small. Earlier Bjorken has studied quark-lepton mass pattern empirically, considering mass to be a smooth function of charge and generation [4].

The study of empirical relations has led to many interesting ideas which can even connect macro and micro universes [5]. For instance, it has been observed [6] that the ratio of hadron radius and Hubble radius of the universe match well with the ratio of gravitational and strong couplings, i.e.,

$$\frac{r_{\text{had}}}{R_{\text{Hub}}} \approx \frac{\text{grav. coupling}}{\text{strong coupling}} (\sim 10^{-41}) \tag{1.1}$$

suggesting that our cosmos and hadrons may be considered as similar systems. Similarly, ratio of strong and electromagnetic coupling match with ratio of nucleon and electron masses as follows:

$$\frac{G^2_{NN\pi}/\hbar c}{e^2/\hbar c} \approx \frac{m_{\text{nucleon}}}{m_{\text{electron}}} (\sim 1840). \tag{1.2}$$

This relation suggests a link between mass and interacting charge of a particle. The idea is in accordance with the view where self interactions are designated to be dynamical source of the particle mass [7]. We, in the present note, extend such relationship to quark--lepton level.

2. *Mass generation*

We assume:

(1) Mass of each particle is generated mainly through self interaction [7], i.e.,

$$m = m_{st} + m_{em} + \ \dots \ . \tag{2.1}$$

(2) Mass generation is dominated by those interactions which obey exact local symmetries.

(3) Since strong coupling changes with energy, quark masses are energy dependent [8].

$$m_i^{\text{long}}(Q^2) = m_i^{\text{short}}(Q^2) + U_i(Q^2), \tag{2.2}$$

where long and short distance masses represent constituent and current masses respectively. $U_i(Q^2)$ is energy dependent contribution due to gluon cloud surrounding the quark. So at very high energy (near grand unification) one sees current masses effectively.

3. *Quark and charged lepton masses*

Considering the exact local interaction, i.e., strong and electromagnetic ones, we extend the relationship (1.2) to the quark-lepton level in each family as:

$$\frac{\text{average quark mass}}{\text{charged lepton mass}} \approx \frac{\text{strong coupling contribution}}{\text{electromagnetic coupling contribution}} = \frac{g^2 N_c}{e^2}, \tag{3.1}$$

where N is the color degree of freedom. In order to relate all the mass ratios to one coupling, we choose it to be the fine structure constant (α). We choose the energy scale where grand unification occurs [3, 9], (i.e., $\alpha_s = \frac{8}{3}\alpha$). At this scale, we need considering current masses for the quarks and the physical masses of the leptons. We then obtain the following:

$$\frac{(m_u + m_d)/2}{m_e} \approx \frac{(m_c + m_s)/2}{m_\mu} \approx \frac{(m_t + m_b)/2}{m_\tau} \approx 8 \tag{3.2}$$

leading to:

$$m_u + m_d \approx 8.2 \text{ MeV},$$

$$m_c + m_s \approx 1.7 \text{ GeV},$$

$$m_t + m_b \approx 30 \text{ GeV}. \tag{3.3}$$

The obtained values agree well with other estimates [1] for the first and the second generations. For the third generation, we then predict top quark mass $m_t \approx 25$ GeV using $m_b = 5$ GeV.

4. *Neutrino masses*

So far we have considered masses to arise through exact interactions (assumption 1 and 2) which keeps neutrinos massless, i.e.,

$$m_{\nu_e} = m_{\nu_\mu} = m_{\nu_\tau} = 0 \tag{4.1}$$

in accordance with normal expectations. But some grand unified models do predict massive neutrinos [3]. Experimentally, a Russian group [10] claims to find electron-neutrino mass to be around 25 eV. If the neutrino is really massive, such contribution, in our picture, may arise through the weak interaction. This contribution being small ($\sim$ 25 eV) does not affect the quark-lepton relations (3.2) significantly. However, it may be interesting to point out that the ratio of neutrino mass to electron mass ($\sim 5 \times 10^{-5}$) also coincides with the ratio of strengths of weak and electromagnetic interactions i.e.

$$\frac{m_{\nu_e}}{m_e} \approx \frac{\text{weak coupling}}{\text{electromagnetic coupling}} . \tag{4.2}$$

Extending this to other generations:

$$\frac{m_{\nu_e}}{m_e} \approx \frac{m_{\nu_\mu}}{m_\mu} \approx \frac{m_{\nu_\tau}}{m_\tau} (\sim 5 \times 10^{-5}) \tag{4.3}$$

we obtain:

$$m_{\nu_\mu} \sim 5 \text{ keV},$$

$$m_{\nu_\tau} \sim 90 \text{ keV}, \tag{4.4}$$

which are well below the experimental upper limits [11]:

$$m_{\nu_\mu} < 520 \text{ keV} \quad \text{and} \quad m_{\nu_\tau} < 250 \text{ MeV}.$$

5. *A new particle*

All the fermions (matter-particles) can be classified by the kinds of interactions in which they participate (Table I). Extending the series in this manner, we postulate a new type of null charge fermion say, "gravinon" which interacts only through theg ravitation. If it is massive, we expect it to satisfy the following empirical relation:

$$\frac{m_{\text{gravinon}}}{m_{\text{electron}}} \approx \frac{\text{grav. coupling}}{\text{electromagnetic coupling}} \sim 10^{-40}. \tag{5.1}$$

TABLE I

Particles	Interaction
Quarks	Gravitation, weak, Electromagnetic, strong
Charged leptons	Gravitation, weak, Electromagnetic
Neutrinos	Graviiation, weak
?	Gravitation

The effects of this particle, obviously, will not be significant in the energy range where the strength of the gravitation is negligible. At the Planck mass scale, where the gravitational effects become dominant, it may play an important role, e.g., at the very early stages of universe.

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