USE OF ISOSPIN IN $\bar{p}p$-ANNIHILATION

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The consequences of isospin invariance for $\bar{p}p \to \bar{N}\Delta$, $\bar{\Delta}N$, $\bar{\Sigma}\Sigma$ and $\bar{\Delta}\Delta$ are derived. We obtain the isospin bounds on $\sigma_{tot}(\bar{\Sigma}^0\Sigma^0)$ from $\sigma_{tot}(\bar{\Sigma}^+\Sigma^+)$ and $\sigma_{tot}(\bar{\Sigma}^-\Sigma^-)$ and compare them to experiment. The present experimental upper bound on $\sigma_{tot}(\bar{\Sigma}^0\Sigma^0)$ are roughly as strong as the upper isospin bounds we obtain. The lower isospin bounds imply $\sigma_{tot}(\bar{\Sigma}^0\Sigma^0) \geqslant 2\ \mu b$ at a $\bar{p}$-momentum around 3 GeV/c. No violation of isospin symmetry is observed. We also obtain the isospin bounds on differential, polarized and unpolarized cross sections. The consequences of saturation of these bounds are derived.

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Strong interactions are assumed to be almost isospin invariant. In this paper, the experimental consequences of isospin invariance are derived for $\bar{p}p \to \bar{B}B'$ with p the proton and B, B' baryons. Some experimental information on the reactions we consider already exists [1]. Our main motivation derives from the fact that detailed experimental results on these reactions are expected from LEAR (the low-energy antiproton machine at CERN); P. D. Barnes et al. in Ref. [1]. Near threshold, the experimental cross sections must be Coulomb-corrected before the isospin bounds can be applied. Direct application will be possible to data to be obtained well above threshold using the $\bar{p}p$-collider version of LEAR (which is planned to be built at CERN).

Our results are straightforward extensions of the isospin bounds valid for πN-scattering [2, 3]. We believe our method to be more general than any to be found in the literature. In particular, it holds with modifications concerning Clebsch-Gordan coefficients only for any hadron-hadron reaction with two outgoing hadrons. It yields bounds not only on cross sections but also on the expectation values of positive functions of the kinematical reaction variables. This will be seen below.

In πN physics, elaborate tests of isospin symmetry have been performed [2, 3] with the result that no indications of a violation of this symmetry remain once the different

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masses of p and n and of Δ^{++}, Δ^{+}, Δ^{0} and Δ^{-} are properly taken into account (compare Ref. [3] for a more detailed discussion). To perform similar tests for reactions other than $\pi N \rightarrow \pi N$, in particular for strange particles, is of obvious interest. In view of the experimental possibilities at LEAR, the reaction $\bar{p}p \rightarrow \bar{\Sigma}\Sigma$ we consider is a good candidate for such a project. Presumably, as is the case in πN physics, it will turn out that isospin is a good symmetry to be used mainly as a constraint on cross section data.

We consider $\bar{p}p \rightarrow \bar{N}\Delta$, $\bar{\Sigma}\Sigma$ and $\bar{\Delta}\Delta$. The results for $\bar{\Sigma}\Sigma$ also apply to $\bar{\Sigma}^{*}\Sigma$ and $\bar{\Sigma}^{*}\Sigma^{*}$. Furthermore, the amplitudes for the charge-conjugate final states (i.e. $\bar{\Delta}N$ and $\bar{\Sigma}\Sigma^{*}$) obey the same relations.

We denote the amplitude of a reaction $\bar{p}p \rightarrow \bar{B}B'$ by $T(\bar{B}B'; j_\alpha)$ with j_α a set of variables specifying the reaction completely. A possible choice of variables is $j_1 = \sqrt{s}$, $j_2 = t$ and $j_3, \ldots, j_6$ the helicities of $\bar{p}$, p, $\bar{B}$ and B'. We use the customary definition of $\sqrt{s}$ and t as total energy in the c.m.s. and momentum-transfer squared, respectively. The scattering angle will be denoted by Ω; a well-known relation connects these three quantities.

Another choice of the j_α would be $j_1 = \sqrt{s}$, $j_2 = t$ and $j_3, \ldots, j_6$ the components of the spins of the $\bar{p}$, p, $\bar{B}$ and B' normal to the scattering plane. Evidently, there is an infinity of possible choices for the variables j_α. Since all possible measurements can be performed by measuring cross sections, all possible experimental information can be derived from measuring the cross sections for all possible choices of j_α. For example, even though the completely polarized differential cross sections $\sigma_{CPD}(\sqrt{s}, t; \bar{p}(j_3)p(j_4) \rightarrow \bar{B}(j_5)B'(j_6))$ for *any particular* definition of the $j_3, \ldots, j_6$ do not imply the complete experimental information, measuring these cross section for *different* definitions of the j_α exhausts all possible measurements at fixed values of $\sqrt{s}$ and t (i.e. no "wave packets" in $\sqrt{s}$ and Ω).

Choosing $j_1 = \sqrt{s}$, $j_2 = t$ and $j_3, \ldots, j_6$ polarizations in an arbitrary, however fixed basis, the completely polarized differential cross section (i.e. for polarized $\bar{p}$, p, $\bar{B}$ and B') is given by

$$\sigma_{CPD}(\bar{B}B'; j_\alpha) \equiv \frac{d\sigma}{d\Omega}(\bar{B}B'; j_\alpha) = \omega_{j_1} |T(\bar{B}B'; j_\alpha)|^2 \tag{1}$$

with ω_{j_1} (the phase space) a *positive* function of $j_1 = \sqrt{s}$. We shall work in the limit that all members of any baryon isospin multiplet have the same mass. Thus ω_{j_1} only depends on the isospin multiplets of $\bar{B}$ and B' (and not on the members considered).

The results we obtain will be valid for the expectation value $\sigma_f(\bar{B}B')$ of any non-negative function f_{j_α} of the kinematical variables j_α,

$$\sigma_f(\bar{B}B') = \sum\!\!\!\!\!\!\int_{j_\alpha} f_{j_\alpha} \sigma_{CPD}(\bar{B}B'; j_\alpha)$$

with $\sum\!\!\!\!\!\int_{j_\alpha}$ a sum (integral) over the discrete (continuous) variables j_α. This may or may not include an integration over s. For the special case that σ_f denotes a cross section, there is no s-integration. In that case, f_{j_α} is constant for a subset of (values of) the j_α and zero for all other. To be able to treat $\sum\!\!\!\!\!\int_{j_\alpha}$ as a sum, we assume that the continuous variables (at most $\sqrt{s}$ and t; with the choice j_2 = total angular momentum only summation would be involved in computing cross sections anyhow) are collected in bins and summed over. Choosing $f_{j_\alpha} = \delta_{j_\alpha' j_\alpha}$ with $\delta_{j_\alpha' j_\alpha}$ the unit kernel of $\sum_{j_\alpha} f$, the σ_f in Eq. (2) would be $\sigma_{CPD}(\bar{B}B'; j_\alpha')$.

We shall explicitly treat below four more types of cross sections: the polarized differential cross sections $\sigma_{PD}(\bar{B}B')$ obtained from σ_{CPD} by summing and/or averaging over some of the polarization variables $j_3, ..., j_6$; the differential cross section $\sigma_D(\bar{B}B'; j_1 = \sqrt{s}, j_2 = t)$ obtained from σ_{PD} by summing over the remaining polarization variables; the polarized (angle-integrated) cross sections σ_P, obtained from σ_{PD} by integrating over an Ω-interval; and finally the total cross section $\sigma_T(\bar{B}B'; j_1 = \sqrt{s})$ resulting from σ_D by integrating over the polar angle Ω (or by summing σ_P over the polarizations variables it contains).

The baryons $\bar{B}$ and B′ are members of isospin multiplets $\{\bar{B}\}$ and $\{B'\}$. Allowing any pair of members of these multiplets as our $\bar{B}$ and B′, the amplitudes $T(\bar{B}B'; j_\alpha)$ can be expressed as linear combinations with Clebsch-Gordan-coefficients as coefficients of at most two amplitudes $T^0(j_\alpha)$ and $T^1(j_\alpha)$, correspondig to total s-channel isospin 0 and 1, respectively. (The T^0 and T^1 of course depend on the multiplets $\{\bar{B}\}$ and $\{B'\}$ considered.) This follows since the isospin of p is $\frac{1}{2}$. Since Δ has isospin $\frac{3}{2}$ in case of $\bar{p}p \to \bar{N}\Delta$ only T^1 can be nonvanishing and one finds, suppressing the variables j_α;

$$|T(\bar{n}\Delta^0)| = |T(\bar{p}\Delta^+)|, \tag{2a}$$

such that

$$\sigma_x(\bar{n}\Delta^0) = \sigma_x(\bar{p}\Delta^+) \tag{2b}$$

for $x = f$, CPD, PD, D, P and T, i.e. the expectation value of any non-negative function and thus any one of the cross section introduced in the above.

In case of the $\bar{\Sigma}\Sigma$ or $\bar{\Delta}\Delta$ final states, both T^0 and T^1 can contribute. Since three (i.e. $\overline{\Sigma^+}\Sigma^+$, $\overline{\Sigma^0}\Sigma^0$ and $\overline{\Sigma^-}\Sigma^-$) and four (i.e. $\overline{\Delta^{++}}\Delta^{++}$, $\overline{\Delta^+}\Delta^+$, $\overline{\Delta^0}\Delta^0$, $\overline{\Delta^-}\Delta^-$) different final states are possible in case of $\bar{\Sigma}\Sigma$ and $\bar{\Delta}\Delta$, respectively, there is one linear relation between the three $\bar{\Sigma}\Sigma$-amplitudes and there are two such relations between the four $\bar{\Delta}\Delta$-amplitudes. Using standard Clebsch-Gordan-coefficients these relations are (suppressing the variables j_α; Eqs. (3) and (4) hold for any choice of the j_α and all values of these variables)

$$T(\overline{\Sigma^+}\Sigma^+) + T(\overline{\Sigma^-}\Sigma^-) + 2T(\overline{\Sigma^0}\Sigma^0) = 0 \tag{3}$$

for $\bar{\Sigma}\Sigma$ and

$$T(\overline{\Delta^{++}}\Delta^{++}) + 2T(\overline{\Delta^+}\Delta^+) + T(\overline{\Delta^0}\Delta^0) = 0 \tag{4a}$$

and

$$T(\overline{\Delta^{++}}\Delta^{++}) - 3T(\overline{\Delta^0}\Delta^0) - 2T(\overline{\Delta^-}\Delta^-) = 0 \tag{4b}$$

for $\bar{\Delta}\Delta$. There are many equivalent forms to Eqs. (4a) and (4b). We can, for example, eliminate any one of the four $\bar{\Delta}\Delta$-amplitudes from these relations and obtain

$$2T(\overline{\Delta^{++}}\Delta^{++}) + 3T(\overline{\Delta^+}\Delta^+) - T(\overline{\Delta^-}\Delta^-) = 0 \tag{4c}$$

and

$$T(\overline{\Delta^+}\Delta^+) + 2T(\overline{\Delta^0}\Delta^0) + T(\overline{\Delta^-}\Delta^-) = 0 \tag{4d}$$

in addition to Eqs. (4a) and (4b). It will be most convenient to treat any member of the set of Eqs. (4) independently. We remark that these relations for *any choice of variables* j_α imply the same relations for *any other* choice. Thus all observable implications of isospin symmetry are implied by sufficient conditions for Eqs. (3) and (4).

In particular, therefore, the inequalities in Eq. (8) and (9) below for cross sections with a *certain fixed definition* of the polarization variables $j_3, \ldots, j_6$ are nevertheless complete expressions of the observable consequences of isospin symmetry, including those consequences that can only be measured by cross section resulting from a different definition of $j_3, \ldots, j_6$. These statements are easily derived by noting that changing the definition of $j_3, \ldots, j_6$ yields amplitudes that are linear combinations of the previous amplitudes with coefficient that do not depend on the particular channel (i.e. $\overline{\Sigma^+}\Sigma^+$, $\overline{\Sigma^0}\Sigma^0$ or $\overline{\Sigma^-}\Sigma^-$; etc.). Any such set of three complex numbers thus forms a triangle for all values of the arguments if one of them does.

Any one of Eqs. (3) and (4) states that a set of three complex numbers, if represented by vectors, forms a triangle. This triangle can also be degenerate, meaning that the three vectors are parallel to each other. We remark here that, if one of Eqs. (4) describes a degenerate triangle, so do the other three.

The restriction on the amplitudes in any one of Eqs. (3) or (4) can be written as

$$A+B+C = 0 \tag{5}$$

with $A = T(\overline{\Sigma^+}\Sigma^+)$, $B = T(\overline{\Sigma^-}\Sigma^-)$ and $C = 2T(\overline{\Sigma^0}\Sigma^0)$ in case of Eqs. (3); etc. The completely polarized differential cross sections σ_{CPD} in Eq. (1) are in this case

$$\sigma_{\text{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha) = \omega_{j_1} a_{j_\alpha}^2,$$

$$\sigma_{\text{CPD}}(\overline{\Sigma^-}\Sigma^-; j_\alpha) = \omega_{j_1} b_{j_\alpha}^2,$$

and

$$\sigma_{\text{CPD}}(\overline{\Sigma^0}\Sigma^0; j_\alpha) = \frac{\omega_{j_1}}{4} c_{j_\alpha}^2, \tag{6}$$

where we have mentioned again the dependences on j_α and introduced the abbreviation $a_{j_\alpha} = |A(j_\alpha)|$, and similarly for b and c.

As has been already stated, Eq. (5) says that the vectors described by the complex numbers A, B and C form a triangle. This is the case if and only if the triangle inequality

$$|a_{j_\alpha} - b_{j_\alpha}| \leqslant c_{j_\alpha} \leqslant a_{j_\alpha} + b_{j_\alpha}, \tag{7a}$$

or, equivalently,

$$-2a_{j_\alpha} b_{j_\alpha} \leqslant a_{j_\alpha}^2 + b_{j_\alpha}^2 - c_{j_\alpha}^2 \leqslant 2a_{j_\alpha} b_{j_\alpha} \tag{7b}$$

holds for the moduli of A, B and C. Due to Eqs. (6), the relations have direct observable consequences for the completely polarized differential cross sections. It will be seen that the same relations hold for σ_f, σ_{PD}, σ_{D}, σ_{P} and σ_{T} introduced above and thus we write them already here in terms of σ_x with x still restricted to CPD

$$\left(\sqrt{\sigma_x(\overline{\Sigma^+}\Sigma^+)} - \sqrt{\sigma_x(\overline{\Sigma^-}\Sigma^-)}\right)^2 \leqslant 4\sigma_x(\overline{\Sigma^0}\Sigma^0) \leqslant \left(\sqrt{\sigma_x(\overline{\Sigma^+}\Sigma^+)} + \sqrt{\sigma_x(\overline{\Sigma^-}\Sigma^-)}\right)^2. \tag{8a}$$

There are many equivalent forms of this. One of them is Eq. (7b), i.e.

$$-2\sqrt{b_x(\overline{\Sigma^+}\Sigma^+)\sigma_x(\overline{\Sigma^-}\Sigma^-)} \leqslant \sigma_x(\overline{\Sigma^+}\Sigma^+)+\sigma_x(\overline{\Sigma^-}\Sigma^-)-4\sigma_x(\overline{\Sigma^0}\Sigma^0) \leqslant 2\sqrt{\sigma_x(\overline{\Sigma^+}\Sigma^+)\sigma_x(\overline{\Sigma^-}\Sigma^-)} \tag{8b}$$

and others obtained by exchanging $4\sigma_x(\overline{\Sigma^0}\Sigma^0)$ with $\sigma_x(\overline{\Sigma^+}\Sigma^+)$ or $\sigma_x(\overline{\Sigma^-}\Sigma^-)$ in Eqs. (8a) or (8b).

In complete analogy to the above we obtain for $\overline{\Delta}\Delta$

$$\left(\sqrt{\sigma_x(\overline{\Delta^{++}}\Delta^{++})} - \sqrt{4\sigma_x(\overline{\Delta^+}\Delta^+)}\right)^2 \leqslant \sigma_x(\overline{\Delta^0}\Delta^0) \leqslant \left(\sqrt{\sigma_x(\overline{\Delta^{++}}\Delta^{++})} + \sqrt{4\sigma_x(\overline{\Delta^+}\Delta^+)}\right)^2, \tag{9a}$$

$$\left(\sqrt{\sigma_x(\overline{\Delta^{++}}\Delta^{++})} - \sqrt{4\sigma_x(\overline{\Delta^-}\Delta^-)}\right)^2 \leqslant 9\sigma_x(\overline{\Delta^0}\Delta^0) \leqslant \left(\sqrt{\sigma_x(\overline{\Delta^{++}}\Delta^{++})} + \sqrt{4\sigma_x(\overline{\Delta^-}\Delta^-)}\right)^2, \tag{9c}$$

$$\left(\sqrt{4\sigma_x(\overline{\Delta^{++}}\Delta^{++})} - \sqrt{9\sigma_x(\overline{\Delta^+}\Delta^+)}\right)^2 \leqslant \sigma_x(\Delta^-\Delta^-) \leqslant \left(\sqrt{4\sigma_x(\overline{\Delta^{++}}\Delta^{++})} + \sqrt{9\sigma_x(\overline{\Delta^+}\Delta^+)}\right)^2, \tag{9c}$$

and

$$\left(\sqrt{\sigma_x(\overline{\Delta^+}\Delta^+)} - \sqrt{\sigma_x(\overline{\Delta^-}\Delta^-)}\right)^2 \leqslant 4\sigma_x(\overline{\Delta^0}\Delta^0) \leqslant \left(\sqrt{\sigma_x(\overline{\Delta^+}\Delta^+)} + \sqrt{\sigma_x(\overline{\Delta^-}\Delta^-)}\right)^2. \tag{9d}$$

Since any two of Eqs. (4) imply the other two, the same holds for Eqs. (9).

To derive Eqs. (8) and (9) for $x = f$, PD, D, P and T, we use Eq. (7b) as an example. For $x = f$, we multiply this relation by f_{j_α} and sum over all j_α; for x = PD, D, P and T Eq. (7b) is summed over the appropriate values of j_α. For generality, we write out the resulting formula containing f_{j_α}

$$-2\sum_{j_\alpha}\!\!\!\!\!\!\int f_{j_\alpha}\omega_{j_1}a_{j_\alpha}b_{j_\alpha} \leqslant \sigma_f(\overline{\Sigma^+}\Sigma^+)+\sigma_f(\overline{\Sigma^-}\Sigma^-)-4\sigma_f(\Sigma^0\Sigma^0) \leqslant 2\sum_{j_\alpha}\!\!\!\!\!\!\int f_\alpha\omega_{j_1}a_{j_\alpha}b_{j_\alpha}. \tag{10}$$

To preceed, we note that the quantities

$$k_{j_\alpha} \equiv \sqrt{f_{j_\alpha}\omega_{j_1}}\, a_{j_\alpha} \quad \text{and} \quad l_{j_\alpha} \equiv \sqrt{f_{j_\alpha}\omega_{j_1}}\, b_{j_\alpha} \tag{11}$$

are real vectors $\boldsymbol{k}$ and $\boldsymbol{l}$ with non-negative components in a space with the scalar product

$$\boldsymbol{k}\cdot\boldsymbol{l} = \sum_{j_\alpha}\!\!\!\!\!\!\int k_{j_\alpha}l_{j_\alpha}. \tag{12}$$

Thus the Schwarz inequality is valid with the result that

$$\boldsymbol{k}\cdot\boldsymbol{l} \leqslant \sqrt{\boldsymbol{k}^2}\cdot\sqrt{\boldsymbol{l}^2} \tag{13}$$

and the equality sign holding if and only if either

$$\boldsymbol{k} = \lambda_1\boldsymbol{l} \quad \text{or} \quad \boldsymbol{l} = \lambda_2\boldsymbol{k} \tag{14}$$

with λ_1 and λ_2 non-negative numbers. (Of course, if e.g. $\lambda_2 \neq 0$, then both relations are equivalent with $\lambda_1 = 1/\lambda_2$.)

Since

$$k \cdot k = \sum_{j_\alpha} f_{j_\alpha} \omega_{j_1} a_{j_\alpha}^2 \equiv \sigma_f(\overline{\Sigma^+}\Sigma^+) \tag{15}$$

etc., Eq. (10) implies

$$-2\sqrt{\sigma_f(\overline{\Sigma^+}\Sigma^+)\sigma_f(\overline{\Sigma^-}\Sigma^-)} \leqslant \sigma_f(\overline{\Sigma^+}\Sigma^+)+\sigma_f(\overline{\Sigma^-}\Sigma^-)-4\sigma_f(\overline{\Sigma^0}\Sigma^0) \leqslant 2\sqrt{\sigma_f(\overline{\Sigma^+}\Sigma^+)\sigma_f(\overline{\Sigma^-}\Sigma^-)}. \tag{16}$$

Thus we have derived Eqs. (8) and (9) for $x = f$ and an obvious modification of the argument implies these relations for x = PD, D, P and T, too. An equality sign can be valid in any one of the relations for σ_f only if

$$\sqrt{f_{j_\alpha}\omega_{j_1}}\, a_{j_\alpha} = \lambda_1 \sqrt{f_{j_\alpha}\omega_{j_1}}\, b_{j_\alpha}, \tag{17a}$$

or

$$\sqrt{f_{j_\alpha}\omega_{j_1}}\, b_{j_\alpha} = \lambda_2 \sqrt{f_{j_\alpha}\omega_{j_1}}\, a_{j_\alpha}. \tag{17b}$$

Thus, if any one of the two inequalities in Eq. (16) is saturated, we may conclude that either

$$f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha) = \lambda_1^2 f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^-}\Sigma^-; j_\alpha), \tag{18a}$$

or

$$f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^-}\Sigma^-; j_\alpha) = \lambda_2^2 f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha). \tag{18b}$$

These relations are equivalent if neither $f_{j_\alpha}\,\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha)$ nor $f_{j_\alpha}\,\sigma_{\mathrm{CPD}}(\overline{\Sigma^-}\Sigma^-; j_\alpha)$ vanishes. It is seen from Eq. (8b) that vanishing of $f_{j_\alpha}\,\sigma_{\mathrm{CPD}}(\overline{\Sigma^-}\Sigma^-; j_\alpha)$ implies

$$f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha) = 4 f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^0}\Sigma^0; j_\alpha), \tag{19a}$$

whereas

$$f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^-}\Sigma^-; j_\alpha) = 4 f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^0}\Sigma^0; j_\alpha) \tag{19b}$$

from vanishing of $f_{j_\alpha}\,\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha)$. Summing over j_α, the result

$$\sigma_f(\overline{\Sigma^+}\Sigma^+) = 4\sigma_f(\overline{\Sigma^0}\Sigma^0), \tag{20a}$$

or

$$\sigma_f(\overline{\Sigma^-}\Sigma^-) = 4\sigma_f(\overline{\Sigma^0}\Sigma^0) \tag{20b}$$

follows. If both Eqs. (18a) and (18b) hold, introducing Eq. (18b) into Eq. (8b) yields

$$-2\lambda_2 f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha) \leqslant (1+\lambda_2^2) f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha) - 4 f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^0}\Sigma^0) j_\alpha)$$
$$\leqslant 2\lambda_2 f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha). \tag{21}$$

Assume for definiteness that the lower inequality in Eq. (16) is saturated. It is then easy to see that the lower inequality in Eq. (21) must be saturated, too. Namely, if this were not the case, Eq. (21) summed over j_α would yield

$$-2\lambda_2 \sum_{j_\alpha} f_{j_\alpha}\omega_{j_1} a_{j_\alpha}^2 = -2\sqrt{\left(\sum_{j_\alpha} f_{j_\alpha}\omega_{j_1} a_{j_\alpha}^2\right)\left(\sum_{j_\alpha} f_{j_\alpha}\omega_{j_1} b_{j_\alpha}^2\right)}$$

$$\equiv -2\sqrt{\sigma_f(\overline{\Sigma^+}\Sigma^+)\sigma_f(\overline{\Sigma^-}\Sigma^-)} < (1+\lambda_2^2)\sigma_f(\overline{\Sigma^+}\Sigma^+) - 4\sigma_f(\overline{\Sigma^0}\Sigma^0)$$

$$= \sigma_f(\overline{\Sigma^+}\Sigma^+) + \sigma_f(\overline{\Sigma^-}\Sigma^-) - 4\sigma_f(\overline{\Sigma^0}\Sigma^0), \tag{22}$$

i.e. the inequality in Eq. (16) that was assumed to be saturated would turn out to be not saturated. Thus

$$f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^0}\Sigma^0; j_\alpha) = \frac{(1+\lambda_2)^2}{4} f_{j_\alpha}\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha). \tag{23}$$

If — rather than the lower one — the upper inequality in Eq. (16) is saturated, λ_2 is replaced by $-\lambda_2$ in Eq. (23). If $f_{j_\alpha} \neq 0$ for a particular set of values j_α, it is possible to conclude from Eqs. (18), (19) and (23) that the corresponding results hold for the cross sections, i.e.

$$\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha) = \lambda_1^2\sigma_{\mathrm{CPD}}(\overline{\Sigma^-}\Sigma^-; j_\alpha), \tag{24a}$$

or

$$\sigma_{\mathrm{CPD}}(\overline{\Sigma^-}\Sigma^-; j_\alpha) = \lambda_2^2\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha), \tag{24b}$$

and

$$\sigma_{\mathrm{CPD}}(\overline{\Sigma^0}\Sigma^0; j_\alpha) = \frac{(1+\lambda_2)^2}{4}\sigma_{\mathrm{CPD}}(\overline{\Sigma^+}\Sigma^+; j_\alpha) \tag{25}$$

if $\lambda_2 \neq 0$, etc.

If sufficiently many values of j_α actually occur in the sum representing σ_f, i.e. f_{j_α}, it is possible to derive Eqs. (24) and (25) for cross sections such as σ_{PD}, σ_{P}, σ_{D} and σ_{T} by summing over the corresponding values of j_α. Since our main interest is in cross sections, we will spell out the consequences this has for σ_x with x = CPD, PD, D, P and T if σ_f itself is one of these cross sections.

It is obvious that vanishing of a cross section $\sigma_x(\overline{\mathrm{B}}\mathrm{B})$ for a particular set of its arguments implies vanishing of all cross sections summed over to obtain $\sigma_x(\overline{\mathrm{B}}\mathrm{B})$. Thus, vanishing of $\sigma_{\mathrm{T}}(\overline{\mathrm{B}}\mathrm{B}')$ for a certain s implies vanishing of all $\sigma_x(\overline{\mathrm{B}}\mathrm{B}')$ for that value of s. If $\sigma_{\mathrm{D}}(\overline{\mathrm{B}}\mathrm{B}')$ vanishes for certain values of s and t, $\sigma_{\mathrm{CPD}}(\overline{\mathrm{B}}\mathrm{B}')$ and $\sigma_{\mathrm{PD}}(\overline{\mathrm{B}}\mathrm{B}')$ both must vanish for that s and t.

Vanishing of $\sigma_{\mathrm{P}}(\overline{\mathrm{B}}\mathrm{B}')$ for a certain s and certain values of the polarizations occurring as free variables in $\sigma_{\mathrm{P}}(\overline{\mathrm{B}}\mathrm{B}')$ implies vanishing of $\sigma_{\mathrm{CPD}}(\overline{\mathrm{B}}\mathrm{B}')$ and $\sigma_{\mathrm{PD}}(\overline{\mathrm{B}}\mathrm{B}')$ for these values of s and polarizations and all the values of variables (such as t and polarizations) summed over in obtaining $\sigma_{\mathrm{P}}(\overline{\mathrm{B}}\mathrm{B}')$ from $\sigma_{\mathrm{CPD}}(\overline{\mathrm{B}}\mathrm{B}')$ and $\sigma_{\mathrm{PD}}(\overline{\mathrm{B}}\mathrm{B}')$, respectively. Likewise, if

$\sigma_{PD}(\bar{B}B')$ vanishes, so does $\sigma_{CPD}(\bar{B}B')$. We denote by $\sigma_{y(x)}$ the cross sections that have to vanish if σ_x vanishes.

For definiteness, we consider in what follows $\bar{\Sigma}\Sigma$-cross sections. Vanishing of all three $\sigma_x(\overline{\Sigma^+}\Sigma^+)$, $\sigma_x(\overline{\Sigma^0}\Sigma^0)$ and $\sigma_x(\overline{\Sigma^-}\Sigma^-)$ for a certain x has obvious consequences for $\sigma_{y(x)}(\bar{\Sigma}\Sigma)$. If — as an experimental fact — any two of the three $\sigma_x(\bar{\Sigma}\Sigma)$ cross sections vanish, due to the inequalities in Eq. (8a) the same must be true for the third and as a consequence for all $\sigma_{y(x)}(\bar{\Sigma}\Sigma)$. Thus isospin would be violated if, for example, $\sigma_x(\overline{\Sigma^0}\Sigma^0) = 0$ and $\sigma_x(\overline{\Sigma^-}\Sigma^-) = 0$ and not $\sigma_x(\overline{\Sigma^+}\Sigma^+) = 0$ for certain values of the variables the cross section σ_x depends on.

The notation $y(x)$ introduced above is useful if at most one cross section of the three $\sigma_x(\bar{\Sigma}\Sigma)$ vanishes. Namely, if $\sigma_x(\overline{\Sigma^-}\Sigma^-) = 0$ then $\sigma_{y(x)}(\overline{\Sigma^-}\Sigma^-) = 0$ and thus, from Eq. (8a) for $\sigma_{y(x)}(\bar{\Sigma}\Sigma)$,

$$\sigma_{y(x)}(\overline{\Sigma^+}\Sigma^+) = 4\sigma_{y(x)}(\overline{\Sigma^0}\Sigma^0). \tag{26a}$$

Likewise, $\sigma_x(\overline{\Sigma^0}\Sigma^0) = 0$ implies

$$\sigma_{y(x)}(\overline{\Sigma^-}\Sigma^-) = \sigma_{y(x)}(\overline{\Sigma^+}\Sigma^+) \tag{26b}$$

and from $\sigma_x(\overline{\Sigma^+}\Sigma^+) = 0$ follows

$$\sigma_{y(x)}(\overline{\Sigma^-}\Sigma^-) = 4\sigma_{y(x)}(\overline{\Sigma^0}\Sigma^0). \tag{26c}$$

We finally consider the possibility that one of the inequalities in Eq. (8a) (or, equivalently, (8b)) is saturated with all three cross sections $\sigma_x(\overline{\Sigma^+}\Sigma^+)$, $\sigma_x(\overline{\Sigma^0}\Sigma^0)$ and $\sigma_x(\overline{\Sigma^-}\Sigma^-)$ nonvanishing for certain values of the kinematical variables of σ_x. Then

$$\sigma_{y(x)}(\overline{\Sigma^-}\Sigma^-) = \lambda^2\sigma_{y(x)}(\overline{\Sigma^+}\Sigma^+), \tag{27a}$$

and

$$\sigma_{y(x)}(\overline{\Sigma^0}\Sigma^0) = \tfrac{1}{4}(1 \pm |\lambda|)^2\sigma_{y(x)}(\overline{\Sigma^+}\Sigma^+) \tag{27b}$$

for those values of the kinematical variables of σ_x for which the inequality for σ_x is saturated. The possibility that $\sigma(\overline{\Sigma^-}\Sigma^-) = 0$ is included in Eqs. (27) and corresponds to $\lambda = 0$. If the lower (upper) inequality in Eq. (8a) is saturated, the minus-(plus-) sign in Eq. (27b) applies.

We remind the reader that a triangle inequality is saturated if and only if the corresponding triangle is degenerate, i.e. forms a line. The strongest consequences of saturation follow from saturation of the weekest inequality, the one for σ_T. If Eq. (8a) or (8b) is saturated for σ_T for a certain s_0 the consequences obtained above hold for $\frac{d\sigma}{d\Omega}$ (s_0, any t, any polarization of $\bar{p}$, p, $\bar{\Sigma}$, Σ) and thus for any cross section at energy $s = s_0$. This obviously includes *all* possible cross sections, i.e. also those involving a different definition of $j_3, \ldots, j_6$. Extension of the above consideration to $\bar{\Delta}\Delta$ has to take into account that (1) there are four (rather than three) final channels and (2) the Clebsch-Gordan-coefficients are different. The second difference has already been built into Eqs. (9) and it is easy to read off the changes implied by it for the final formulas to be derived from saturation of Eqs. (9). In particular, Eqs. (27) are replaced by

$$\sigma_{y(x)}(\overline{\Delta^+}\Delta^+) = \lambda_1^2 \sigma_{y(x)}(\overline{\Delta^{++}}\Delta^{++}), \tag{28a}$$

and

$$\sigma_{y(x)}(\overline{\Delta^0}\Delta^0) = (1 \pm 2|\lambda_1|)^2 \sigma_{y(x)}(\overline{\Delta^{++}}\Delta^{++}) \tag{28b}$$

if (9a) is saturated; they are replaced by

$$\sigma_{y(x)}(\overline{\Delta^-}\Delta^-) = \lambda_2^2 \sigma_{y(x)}(\overline{\Delta^{++}}\Delta^{++}), \tag{29a}$$

and

$$\sigma_{y(x)}(\overline{\Delta^0}\Delta^0) = \tfrac{1}{9}(1 \pm 2|\lambda_2|)^2 \sigma_{y(x)}(\overline{\Delta^{++}}\Delta^{++}) \tag{29b}$$

if Eq. (9b) is saturated; they are replaced by

$$\sigma_{y(x)}(\overline{\Delta^+}\Delta^+) = \lambda_3^2 \sigma_{y(x)}(\overline{\Delta^{++}}\Delta^{++}), \tag{30a}$$

and

$$\sigma_{y(x)}(\overline{\Delta^-}\Delta^-) = (2 \mp 3|\lambda_3|)^2 \sigma_{y(x)}(\overline{\Delta^{++}}\Delta^{++}) \tag{30b}$$

if Eq. (9c) is saturated and by

$$\sigma_{y(x)}(\overline{\Delta^-}\Delta^-) = \lambda_4^2 \sigma_{y(x)}(\overline{\Delta^+}\Delta^+), \tag{31a}$$

and

$$\sigma_{y(x)}(\overline{\Delta^0}\Delta^0) = \tfrac{1}{4}(1 - |\lambda_4|)^2 \sigma_{y(x)}(\overline{\Delta^+}\Delta^+) \tag{31b}$$

if Eq. (9d) is saturated.

The first difference between $\bar{\Sigma}\Sigma$ and $\bar{\Delta}\Delta$ implies that two and only two of the inequalities in Eqs. (9) are independent. We have seen that (compare remark after Eq. (4)) if two (or more) of the inequalities in Eq. (9) are saturated, this implies that *all four* isospin triangles are degenerate and thus *all* four inequalities in Eqs. (9) are saturated. This in turn implies validity of all Eqs. (28)–(31) for nonvanishing cross sections. How to generalize the result valid for vanishing cross sections from $\bar{\Sigma}\Sigma$ to $\bar{\Delta}\Delta$ is obvious.

We conclude by deriving the consequences of isospin for $\sigma_T(\overline{\Sigma^0}\Sigma^0)$ that can be obtained from present data [1] on $\sigma_T(\overline{\Sigma^+}\Sigma^+)$ and $\sigma_T(\overline{\Sigma^-}\Sigma^-)$. They assume of course that the electromagnetic contributions to $\sigma_T(\bar{\Sigma}\Sigma)$ can be neglected at the energies considered. At a $\bar{p}$-momentum between 2.5 and 3.5 GeV/c we read off Fig. 5 of Barnes et al. in Ref. [1] that $\sigma_T(\overline{\Sigma^+}\Sigma^+)$ is approximately constant at 35 μb and $\sigma_T(\overline{\Sigma^-}\Sigma^-)$ is also approximately constant at 10 μb; both with large errors. One data point $\sigma_T(\overline{\Sigma^+}\Sigma^+)$ at 3.5 GeV/c is considerably higher at 70 μb. With $\sigma_T(\overline{\Sigma^+}\Sigma^+) = 35$ μb and $\sigma_T(\overline{\Sigma^-}\Sigma^-) = 10$ μb Eq. (8a) implies 2 μb $\leqslant \sigma_T(\overline{\Sigma^0}\Sigma^0) \leqslant$ 21 μb. The upper limit approximately coincides with the experimental upper limit on $\sigma_T(\overline{\Sigma^0}\Sigma^0)$ in this momentum range. The data point mentioned above yields 7 μb $\leqslant \sigma_T(\overline{\Sigma^0}\Sigma^0) \leqslant$ 33 μb.

At a $\bar{p}$-momentum value of 2.25 GeV/c, $\sigma_T(\overline{\Sigma^+}\Sigma^+) \approx 20\ \mu b$ and $\sigma_T(\overline{\Sigma^-}\Sigma^-) \approx 3\ \mu b$ is known implying $2\ \mu b \leqslant \sigma_T(\overline{\Sigma^0}\Sigma^0) \leqslant 10\ \mu b$, to be compared with the observed $\sigma_T(\overline{\Sigma^0}\Sigma^0) \approx 10\ \mu b$ at ≈ 2.4 GeV/c. Since these momenta are near threshold and the cross sections are rising, no conclusion concerning saturation can be drawn from this. Finally, around 5.8 GeV there is a point for each cross section, namely, $\sigma_T(\overline{\Sigma^+}\Sigma^+) \approx 30\ \mu b$, $\sigma_T(\overline{\Sigma^-}\Sigma^-) \approx 2\ \mu b$ and $\sigma_T(\Sigma^0\Sigma^0) = (8 \pm 2)\ \mu b$. The limits implied by $\sigma_T(\overline{\Sigma^+}\Sigma^+)$ and $\sigma_T(\overline{\Sigma^-}\Sigma^-)$ are $4\ \mu b \leqslant \sigma_T(\overline{\Sigma^0}\Sigma^0) \leqslant 12\ \mu b$, in agreement with the experimental value.

It should also be remarked that models such as the one-boson-exchange models or the model of Ref. [4] imply that $\sigma_T(\overline{\Sigma^-}\Sigma^-) = 0$ and $\sigma_T(\overline{\Delta^-}\Delta^-) = 0$. The consequences of this for the remaining $\bar{\Sigma}\Sigma$ and $\bar{\Delta}\Delta$ cross sections easily follow from Eqs. (8) and (9). In particular, this implies for $\bar{\Sigma}\Sigma$ Eqs. (27) with $\lambda = 0$ for $y(x) \equiv y =$ CPD, PD, D, P and T, i.e. $\sigma_y(\overline{\Sigma^+}\Sigma^+) = 4\sigma_y(\overline{\Sigma^0}\Sigma^0)$, and furthermore for $\bar{\Delta}\Delta$ the cross section relations $\sigma_y(\overline{\Delta^{++}}\Delta^{++}) = 9\sigma_y(\overline{\Delta^0}\Delta^0) = \frac{9}{4}\sigma_y(\overline{\Delta^+}\Delta^+)$ with the same y.

For application, our main results are *Eq. (2b) and the inequalities in Eqs. (8) and (9).* These are valid for σ_x the total as well as any type of partial cross section. The discussion of consequences of saturation of the isospin bounds has furthermore been carried over from πN scattering (Ref. [2]) to $\bar{p}p \to B\bar{B}'$ by developing a method that can be applied to any hadron-hadron interaction with two outgoing hadrons.

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