STUDY OF THE ^{6}Li NUCLEUS AS A THREE-BODY PROBLEM

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The ^{6}Li nucleus is taken in the present work as composed of an alpha particle, a proton and a neutron. With this structure, the ^{6}Li nucleus, it can be studied as a three-body problem. Three-body integral equations for the three body system are obtained by taking separable potentials for the nuclear interactions between the alpha, the proton and the neutron particles in pairs. The ^{6}Li nucleus is studied in the ground and the first excited states. In the ground state of ^{6}Li nucleus ($J^\pi = 1^+$), the nucleon-nucleon interaction is taken as the triplet $^3S_1$ potential, while for the excited state of the ^{6}Li* nucleus ($J^\pi = 0^+$), these nucleon-nucleon interactions are taken to act in the singlet $^1S_0$ state. The Pauli principle restricts the central part of the nucleon-alpha interaction in the ^{6}Li nucleus to being taken only as a $^1P_1$-interaction. In the present work we have used Gaussian potential forms for the two-body interactions. The various parameters of the two-body interactions are obtained by fitting the corresponding two-body phase shifts. Numerical calculations of the resulting three-body integral equations give the different form factors for both the ground and excited states of the ^{6}Li nucleus. The effective ranges for both states are also calculated. Solving the obtained coupled integral equations numerically, we calculate the biuding energies for the ground and excited states of the ^{6}Li nucleus. Good agreements are found between the theoretically calculated values of the binding energies and the experimental valnes.

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1. Introduction

The nuclear structure for light nuclei with mass number $A = 6$ has been constructed as composed of two nucleons and an elementary alpha particle. This structure is based on the assumption that the alpha particle is treated as an elementary particle, in accordance with two facts [1]: the first reason is that the first excited state and the first breakup channel for the alpha particle lie at about 20 MeV above its ground state [2]; the second reason is that the ^{3}He–^{3}H breakup channel of ^{6}Li lies at about 15.8 MeV above its ground state, while the α-deuteron and np breakup channels lie only at about 1.472 and 3.697 MeV, respectively, above its ground state [3]. Thus for excitation energies of ^{6}Li which are less than 15 MeV, the ^{3}He–^{3}H coupled channel in construction of ^{6}Li is weak and can be neglected. In addition to these assumptions, the internal structure of the alpha particle may be approximately represented by a two level system [4], helping the alpha particle

in the nucleus to maintain its identity although it may be either in its ground state or in its excited state. According to this the structures of $A = 6$ light nuclei, each of which is composed of three particles, help us in studying each of these light nuclei as a three-body problem. Numerous methods of obtaining three-body models of the $A = 6$ system have been introduced following different formalisms based on calculation of deuteron-alpha scattering [5], variational calculation [6], shell models [7], dispersion theory [8], resonating group method [9] and separable potentials [4, 10].

The separable potentials for the two-body interactions in the three-body problem have been found to be very useful in studying the three-body models of the $A = 6$ systems because they greatly simplify extracting an exact solution of the Schrödinger equations, or equivalently the Faddeev equations. This is done by following the separable expansion formalism. The starting point of the separable expansion approach is by using separable potentials for the two-body interactions in the three-body system. The different parameters of the separable two-body interactions are obtained by fitting the experimental low-energy properties and especially the phase shifts of the corresponding two-body systems. With these separable two-body interactions and their defined parameters as input, the binding energies of the ^{6}He and ^{6}Li light nuclei can be calculated by solving the homogeneous Faddeev–Lovelace equations [11]. Starting with the three body Schrödinger's equation, Mitra and his co-workers [10, 12] succeed in reducing the three-body problem to a set of coupled one dimensional integral equations by using separable potentials for the different two-body interactions. These separable potentials have been used [13] in calculating such static properties of the triton as the binding energy, the radius, the Coulomb energy, the form factor and also in calculating the neutron-deuteron scattering cross section.

The alpha particle model has been found to be successful especially in describing the structure of light nuclei. With this alpha particle model, the ^{6}Li nucleus is well described as consisting of an alpha particle, a proton and a neutron. This structure of the ^{6}Li nucleus is very useful since it helps in studying the ^{6}Li nucleus as a three-body problem. A detailed study of the ^{6}Li nucleus has been made by Wackman and Austern [6]. In their study, they solved the three-body Schrödinger's equations by using the Rayleigh–Schrödinger variational technique. Shanley [10] considered the problem by studying the deuteron-alpha particle scattering. In his study, Shanley applied Amado's formalism [14] in solving the three-body problem using separable two-body potentials. In the framework of the Faddeev formalism, Osman [15] solved the ^{6}Li nucleus as a three-body problem. The Faddeev equations are obtained by using the separable expansion. In this approach, the nucleon-nucleon and nucleon-alpha interactions were taken as non-local separable potentials. Using the obtained Faddeev equations, Osman calculated the binding energies for the ground and excited states of the ^{6}Li nucleus by solving the resulting coupled integral equations numerically.

In the present work, the ^{6}Li nucleus is taken to be composed of a structureless alpha particle, a proton and a neutron. With this structure, the ^{6}Li nucleus will be solved as a three-body problem. We use separable type potentials for the nucleon-nucleon and nucleon-alpha interactions. The ^{6}Li nucleus is considered in two different states: the ground state ($J^\pi = 1^+$), and the first excited state ($J^\pi = 0^+$). The wavefunction of the $J = 1$ level

of the ground state of the ^{6}Li nucleus is taken as a combination of the $^{13}S_1$, $^{11}P_1$ and $^{13}D_1$ wavefunctions, while the wavefunction of the $J = 0$ level of the first excited state of the ^{6}Li* nucleus is a combination of the $^{31}S_0$ and $^{33}P_0$ wavefunctions. However, Wackman and Austern [6] found that the admixtures of the $^{33}P_0$ and $^{13}D_1$ states are of the order of 0.05% — so small as to be negligible. This result means that all the tensor force matrix elements could be neglected and so the two-body forces are effectively spin-independent [16]. Then, the form factors and the binding energies of the ^{6}Li nucleus are calculated by solving the resulting three-body integral equations.

In Section 2, the three-body equations for the ^{6}Li nucleus are introduced. Also, the nucleon-nucleon and nucleon-alpha interactions are given. Numerical calculations and results are introduced in Section 3. Discussion and conclusions are given in Section 4.

2. *Two-body interactions and the three-body integral equations*

The dominant part of the neutron-proton interaction for the ground state of the ^{6}Li nucleus ($J^\pi = 1^+$, $T = 0$), is the triplet $^3S_1$ potential. This potential is taken to be of a separable form and could be defined in the momentum space as

$$V_{np}(\boldsymbol{q}, \boldsymbol{q}') = -(\lambda_t/2\pi^2) g_t(q) g_t(q'), \tag{1}$$

where $g_t(q)$ and $g_t(q')$ are well-behaved functions of $2q = |\boldsymbol{P}_1 - \boldsymbol{P}_2|$ and $2q' = |\boldsymbol{P}'_1 - \boldsymbol{P}'_2|$. $\lambda_t = \hbar^2/m$, where m is the nucleon mass, and the subscript t refers to the triplet state. This potential can take different forms with parameters which, in each case fit the corresponding neutron-proton phase shifts. One of these forms is the Gaussian potential form. In the present work, the function $g_t(q)$ is taken to have the Gaussian potential form. The Gaussian potential in space representation is represented as

$$V(r) = -V_0 e^{-a^2 r^2}, \tag{2}$$

where V_0 is the depth of the potential. If the interaction range is r_0, then $a^2 = 1/r_0^2$. This Gaussian potential form as expressed by equation (2) can be transformed to be expressed in momentum representation as

$$V(q) = -A e^{-q^2/4a^2\hbar^2}, \tag{3}$$

where

$$A = \pi^{3/2} V_0 / a^3. \tag{4}$$

The nucleon-alpha interaction is also taken as a separable potential. This potential consists of a central term and a spin-orbit term. Imposing the Pauli principle on the nucleon-alpha interaction, the central term is taken only as a P-wave. Thus, the P-wave function of the nucleon-alpha interaction is represented by a form given by Mitra [10] as

$$V_{N\alpha}(q, q') = -\frac{2\pi}{m_R} g_1(q) g_1(q') \sum_M Y_1^{*M}(\hat{q}) \left[1 + \frac{\boldsymbol{L} \cdot \boldsymbol{\sigma}}{t'}\right] Y_1^M(\hat{q}') \tag{5}$$

m_R is the nucleon-alpha reduced mass given by

$$m_R = mm_\alpha/(m+m_\alpha), \tag{6}$$

where m_α is the mass of the alpha particle. In equation (5), $\boldsymbol{\sigma}$ is the Pauli spin vector for the neutron and $\boldsymbol{L}$ is the relative orbital angular momentum. The function $Y_L^M(\hat{q})$ is the normalized eigenstate of the orbital angular momentum $\boldsymbol{L}$. In the case of $^{11}P_1$ wavefunction, the momenta of the case considered are given by $J = 1$, $T = 0$, $L = 1$ and $S = 0$, which means that the second term of equation (5) is canceled. Therefore, the nucleon-alpha interaction is given by

$$V_{N\alpha}(\boldsymbol{q}, \boldsymbol{q}') = \frac{-2\pi}{m_R} \lambda_\alpha g_1(q) g_1(q') \sum_M Y_1^{*M}(\hat{q}) Y_1^M(\hat{q}'). \tag{7}$$

Equally, the separable nucleon-alpha potential introduced by equation (7) can be given in a suitable form as

$$V_{N\alpha}(\boldsymbol{q}', \boldsymbol{q}') = -\frac{3}{2m_R} \lambda_\alpha g_1(q) g_1(q') \hat{q} \cdot \hat{q}', \tag{8}$$

where λ_α stands for the interaction strength and the function $g_1(q)$ can be expressed to have the Gaussian potential form given by equation (3) but with parameters which fit the corresponding nucleon-alpha phase shifts.

Let us start by deriving the ^{6}Li three-body bound-state equations for $^3S_1$ and $^1P_1$ state from the Schrödinger's equation. In this case the Schrödinger's equation for the present case can be written in momentum representation as

$$(H_0+E)\Psi(\boldsymbol{P}_1, \boldsymbol{P}_2) = -(V_{np}+V_{n\alpha}+V_{p\alpha})\Psi(\boldsymbol{P}_1, \boldsymbol{P}_2), \tag{9}$$

where E is the ^{6}Li nuclear binding energy. The Hamiltonian H_0 is given by

$$H_0 = \frac{1}{2m}(P_1^2+P_2^2) + \frac{1}{2m_\alpha} P_3^2, \tag{10}$$

where $\boldsymbol{P}_1$, $\boldsymbol{P}_2$ and $\boldsymbol{P}_3$ are the momenta of the neutron, the proton and of the alpha particle, respectively. This Hamiltonian H_0 can be expressed in the center-of-mass system as [17]

$$H_0 = P_i^2/2M_i + q_i^2/2\mu_i, \tag{11}$$

where q_i is the relative momentum of the (j, k) subsystem and is given by [18]

$$\boldsymbol{q}_i = \mu_i(\boldsymbol{P}_j/m_j - \boldsymbol{P}_k/m_k) \tag{12}$$

with

$$\mu_i^{-1} = m_j^{-1} + m_k^{-1} \tag{13}$$

i, j and k denote the three particles and are given in cyclic permutations. The momentum P_i stands for the momentum of the particle i relative to the (j, k) subsystem and is given by

$$\boldsymbol{P}_i = M_i\left[\frac{\boldsymbol{P}_i}{m_i} - \frac{\boldsymbol{P}_j+\boldsymbol{P}_k}{m_j+m_k}\right] \tag{14}$$

with

$$M_i^{-1} = m_i^{-1} + (m_j + m_k)^{-1} \tag{15}$$

and

$$\sum_{i=1}^{3} \boldsymbol{P}_i = 0. \tag{16}$$

Therefore, equation (9) can be rewritten in the center-of-mass system as

$$B(\boldsymbol{P}, \boldsymbol{q})\Psi(\boldsymbol{P}, \boldsymbol{q}) = -(V_{np} + V_{n\alpha} + V_{p\alpha})\Psi(\boldsymbol{P}, \boldsymbol{q}), \tag{17}$$

where

$$B(\boldsymbol{P}, \boldsymbol{q}) = \frac{P_i^2}{2M_i} + \frac{q_i^2}{2\mu_i} + E \tag{18}$$

$\Psi(\boldsymbol{P}, \boldsymbol{q})$ is the three-body bound state wavefunction, which forms a coupled set of integral equations in two variables $(\boldsymbol{p}, \boldsymbol{q})$. Introducing an integral notation [19] for the right hand side of equation (17), we can rewrite it as

$$B(\boldsymbol{P}, \boldsymbol{q})\Psi(\boldsymbol{P}, \boldsymbol{q}) = -\iint d\boldsymbol{P}'\boldsymbol{q}d'\langle \boldsymbol{P}\boldsymbol{q}|V_{np} + V_{n\alpha} + V_{p\alpha}|\boldsymbol{P}'\boldsymbol{q}'\rangle \Psi(\boldsymbol{P}', \boldsymbol{q}'). \tag{19}$$

Using these separable potentials [20], the two-body interactions can be simplified in the form

$$(\boldsymbol{P}_i\boldsymbol{q}_i|V|\boldsymbol{P}_i'\boldsymbol{q}_i') = \delta(\boldsymbol{P}_i - \boldsymbol{P}_i')(\boldsymbol{q}_i|V|\boldsymbol{q}_i'). \tag{20}$$

The separable $(\boldsymbol{q}_i|V|\boldsymbol{q}_i')$ potentials stand for the different nucleon-nucleon and nucleon--alpha interactions which are represented by equations (1) and (8). Then, we have for equation (19) an expression

$$\begin{aligned} B(\boldsymbol{P}, \boldsymbol{q})\Psi(\boldsymbol{P}, \boldsymbol{q}) = {} & \frac{h_t}{2\pi^2} \int d\boldsymbol{q}_3' g_t(q_3) g_t(q_3') \Psi(\boldsymbol{P}_3, \boldsymbol{q}_3') \\ & + \frac{3\lambda_\alpha}{2m_R} \int d\boldsymbol{q}_2' g_1(q_2) g_1(q_2') \hat{q}_2 \cdot \hat{q}_2' \Psi(P_2, \boldsymbol{q}_2') \\ & + \frac{3\lambda_\alpha}{2m_R} \int d\boldsymbol{q}_1' g_1(q_1) g_1(q_1') \hat{q}_1 \cdot \hat{q}_1' \Psi(\boldsymbol{P}_1 \cdot \boldsymbol{q}_1'). \end{aligned} \tag{21}$$

The structure of $\Psi(\boldsymbol{P}, \boldsymbol{q})$ can be expressed in the following form

$$\Psi(\boldsymbol{P}, \boldsymbol{q}) = B^{-1}(\boldsymbol{P}, \boldsymbol{q})\,[g_t(q_3)\phi(P) + g_1(q_2)\hat{q}_2 \cdot \boldsymbol{P}_2\phi(\boldsymbol{P}_2) + g_1(q_1)\hat{q}_1 \cdot \boldsymbol{P}_1\phi(\boldsymbol{P}_1)], \tag{22}$$

where $\boldsymbol{P}$ is the sum of the momenta of two of the particles given by

$$\boldsymbol{P} = \boldsymbol{P}_1 + \boldsymbol{P}_2 = -\boldsymbol{P}_3. \tag{23}$$

Introducing the wave function structure $\Psi(\boldsymbol{P}, \boldsymbol{q})$ as given by the expression (22) into equation (21), we get the following integral equations

$$\phi(P) = \frac{h_t}{2\pi^2} \int d\boldsymbol{q}'_3 g_t(q'_3) \Psi(\boldsymbol{P}_3, \boldsymbol{q}'_3), \tag{24}$$

$$\hat{q}_2 \cdot \boldsymbol{P}_2 \phi(\boldsymbol{P}_2) = \frac{3\lambda_\alpha}{2m_R} \int d\boldsymbol{q}'_2 g_1(q'_2) \hat{q}_2 \cdot \hat{q}'_2 \Psi(\boldsymbol{P}_2, \boldsymbol{q}'_2), \tag{25}$$

$$\hat{q}_1 \cdot \boldsymbol{P}_1 \phi(\boldsymbol{P}_1) = \frac{3\lambda_\alpha}{2m_R} \int d\boldsymbol{q}'_1 g_1(q'_1) \hat{q}_1 \cdot \hat{q}'_1 \Psi(\boldsymbol{P}_1, \boldsymbol{q}'_1). \tag{26}$$

Then, we are left with the wavefunctions $\phi(\boldsymbol{P}), \phi(\boldsymbol{P}_1)$ and $\phi(\boldsymbol{P}_2)$ instead of the wavefunction $\Psi(\boldsymbol{P}, \boldsymbol{q})$. These wavefunctions can be solved since equations (24)–(26) are given in one of the following general forms

$$[\lambda_t^{-1} - F_t(P)]\phi(P) = \frac{m}{\pi^2} \int d\xi K(\xi, \boldsymbol{P}) \phi(\xi), \tag{27}$$

$$[\lambda_\alpha^{-1} - G_\alpha(P)]\boldsymbol{P}_2 \phi(P_2) = \tfrac{3}{2} \int d\xi W(\xi, \boldsymbol{P}) \phi(\xi). \tag{28}$$

We have a third integral equation in the function $\phi(P_1)$ which is similar to equation (28), but with the variable P_2 replaced by variable P_1. In equations (27) and (28), we introduced the following expressions

$$K(\xi, \boldsymbol{P}) = \frac{\left(\boldsymbol{P} \cdot \xi + \dfrac{m_\alpha}{m+m_\alpha} \xi^2\right) g_t(|\xi + \tfrac{1}{2}\boldsymbol{P}|) g_1\left(\left|\boldsymbol{P} + \dfrac{m_\alpha}{m+m_\alpha} \xi\right|\right)}{\left|\boldsymbol{P} + \dfrac{m_\alpha}{m+m_\alpha} \xi\right| \left(\zeta^2 + \dfrac{m+m_\alpha}{2m_\alpha} P^2 + \xi \cdot \boldsymbol{P} + mE\right)}, \tag{29}$$

$$W(\xi, \boldsymbol{P}) = \frac{\left(\boldsymbol{P} \cdot \xi + \dfrac{m_\alpha}{m+m_\alpha} \xi^2\right) g_1\left(\left|\dfrac{m}{m+m_\alpha} \boldsymbol{P} + \dfrac{m_\alpha}{m+m_\alpha} \xi\right|\right) g_1\left(\left|\boldsymbol{P} + \dfrac{m_\alpha}{m+m_\alpha} \xi\right|\right)}{\left|\boldsymbol{P} + \dfrac{m_\alpha}{m+m_\alpha} \xi\right| \left(\xi^2 + \dfrac{m+m_\alpha}{2m_\alpha} P^2 + \xi \cdot \boldsymbol{P} + mE\right)}$$

$$+ \frac{g_1\left(\left|\dfrac{m}{m+m_\alpha} \boldsymbol{P} + \dfrac{m_\alpha}{m+m_\alpha} \xi\right|\right) g_t(|\xi + \tfrac{1}{2}\boldsymbol{P}|)}{\xi^2 + \dfrac{m+m_\alpha}{2m_\alpha} P^2 + \xi \cdot \boldsymbol{P} + mE}$$

$$\times \frac{m}{\pi^2} \int d\xi [\lambda_t^{-1} - F_t(P)]^{-1} K(\xi, \boldsymbol{P}), \tag{30}$$

$$F_t(\ ) = \frac{m}{2m^2} \int d\boldsymbol{q} \frac{g_t^2(q)}{q^2 + \dfrac{2m+m_\alpha}{4m_\alpha} P^2 + mE}, \tag{31}$$

and

$$G_\alpha(P) = -\tfrac{3}{2}\int dq \; \frac{g_1^2(q)}{q^2 + \dfrac{2m+m_\alpha}{4m_\alpha}P^2 + mE}. \tag{32}$$

In equation (27), $\phi(P)$ is used instead of $\phi(\boldsymbol{P})$ since the nucleon-nucleon interaction is considered in its S-state interaction and therefore it depends only on the magnitude of P and not on its direction.

Let us consider the case of the first excited state of the ^{6}Li nucleus. This excited state of the ^{6}Li* nucleus has momenta and parity defined by $J^\pi = 0^+$ and has only $^{31}S_0$ state. The nucleon-nucleon interaction is taken as a single separable potential and can be expressed in momentum space as

$$V_{np}(\boldsymbol{q}, \boldsymbol{q}') = -\frac{\lambda}{2\pi^2} g_s(q) g_s(q'). \tag{33}$$

The nucleon-alpha interaction has been introduced by Mitra [10] in the form

$$V_{N\alpha}(\boldsymbol{q}, \boldsymbol{q}') = -\frac{\lambda_0}{2m_R} g_0(q) g_0(q')$$

$$-\frac{2\pi}{m_R} g_1(q) g_1(q') \sum_M Y_1^{*M}(\hat{q}) \left[1 + \frac{\boldsymbol{L}\cdot\boldsymbol{\sigma}}{t'}\right] Y_1^M(\hat{q}'). \tag{34}$$

The nucleon-alpha interaction as given by equation (34) consists of two terms. The first term on the right hand side represents the S-wave, while the second term stands for the P-wave. Since in our present case of the first excited state of ^{6}Li* ($J^\pi = 0^+$) nucleus we are concerned only with the S-state, then the nucleon-alpha potential is given by

$$V_{N\alpha}(\boldsymbol{q}, \boldsymbol{q}') = -\frac{\lambda_0}{2m_R} g_0(q) g_0(q'). \tag{35}$$

These expressions for the nucleon-nucleon and nucleon-alpha interactions as given by equations (33) and (35), respectively, are introduced in equation (9) result in the following integral equations

$$[\lambda^{-1} - F_s(P)]\phi(P) = \frac{m}{\pi^2}\int d\xi K(\xi, \boldsymbol{P})\phi(\xi), \tag{36}$$

$$[\lambda_0^{-1} - G_s(P)]\phi(P_2) = \tfrac{1}{2}\int d\xi W(\xi, \boldsymbol{P})\phi(\xi). \tag{37}$$

In this case the kernels $K(\xi, \boldsymbol{P})$ and $W(\xi, \boldsymbol{P})$ in equations (36) and (37) have the expressions

$$K(\xi, \boldsymbol{P}) = \frac{g_s(|\xi + \frac{1}{2}\boldsymbol{P}|) g_0\left(\left|\boldsymbol{P} + \dfrac{m_\alpha}{m+m_\alpha}\xi\right|\right)}{\xi^2 + \dfrac{m+m_\alpha}{2m_\alpha}P^2 + \xi\cdot\boldsymbol{P} + mE}, \tag{38}$$

and

$$W(\xi, \boldsymbol{P}) = \frac{g_0\left(\left|\boldsymbol{P} + \frac{m_\alpha}{m+m_\alpha}\xi\right|\right) g_0\left(\left|\frac{m}{m+m_\alpha}\boldsymbol{P} + \frac{m_\alpha}{m+m_\alpha}\xi\right|\right)}{\left(\xi^2 + \frac{m+m_\alpha}{2m_\alpha}P^2 + \xi\cdot\boldsymbol{P} + mE\right)}$$

$$+ \frac{g_s(|\xi + \tfrac{1}{2}\boldsymbol{P}|) g_0\left(\left|\frac{m}{m+m_\alpha}\boldsymbol{P} + \frac{m_\alpha}{m+m_\alpha}\xi\right|\right)}{\left(\xi^2 + \frac{m+m_\alpha}{2m_\alpha}P^2 + \xi\cdot\boldsymbol{P} + mE\right)}$$

$$\times \frac{m}{\pi^2}\int d\xi[\lambda^{-1} - F_s(P)]^{-1} K(\xi, \boldsymbol{P}). \tag{39}$$

Also, $F_s(P)$ and $G_s(P)$ present in equations (36) and (37) have the following integral forms

$$F_s(P) = \frac{m}{2\pi^2}\int d\boldsymbol{q}\ \frac{g_s^2(q)}{q^2 + \frac{2m+m_\alpha}{4m_\alpha}P^2 + mE} \tag{40}$$

and

$$G_s(P) = -\tfrac{1}{2}\int d\boldsymbol{q}\ \frac{g_0^2(q_2)}{q^2 + \frac{2m+m_\alpha}{4m_\alpha}P^2 + mE}. \tag{41}$$

The coupled sets of integral equations given by equations (27), (28) and (36), (37) have to be solved in order to calculate the different form factors and ^{6}Li nucleus binding energies based on a three-body problem.

3. Numerical calculations and results

We have considered the ^{6}Li nucleus in two different states. The triplet ground state and the singlet first excited state. In the previous section, sets of couplet integral equations have been obtained for these two different cases of the ^{6}Li nucleus studied as a three-body problem. In the following, we introduce numerical calculations of the different form factors. In the present calculations, a Gaussian potential form is used for the two-body interactions. This Gaussian potential form is introduced by equation (3). Thus, for the triplet ground state of the ^{6}Li nucleus, the set of the coupled three-body integral equations is given by equations (27) and (28). In this case, the form factor is given by equation (31) as

$$F_t(P) = \frac{2mA^2}{\pi}\int dq\ \frac{q^2 e^{-bq^2}}{q^2 + Z}, \tag{42}$$

where

$$b = 1/2a^2h^2 \text{ and } Z = \frac{2m+m_\alpha}{4m_\alpha} P^2 + mE.$$

This form factor for the nucleon-nucleon interaction is calculated for different values of the ^{6}Li energies. The different parameters of the Gaussian nucleon-nucleon interaction is used as determined by Osman [21]. The results of these calculations are shown in Fig. 1. Calculations for the $^1S_0$ state have the same shape shown in Fig. 1. This is clear from the mathematical formulations presented in Section 2, by comparing the two form factors given by equations (31) and (40) which are the same in the case of using a Gaussian type potential. For the first excited state of the ^{6}Li nucleus at calculated value for the energy of $E = 0.563$ MeV, the agreement between the singlet and triplet states occurs only for values $k \geqslant 3$ fm^{-1}. It has been found from these calculations that the value of the form factor of the singlet state is about 3/2 its value for the triplet state at $k = 0$.

Now, for the nucleon-alpha interactions, we can calculate the form factors using the integral given by equation (32). This integral for the Gaussian type form given by equation (3), can be written as

$$G_\alpha(P) = -6\pi A^2 e^{-bQ_2P^2} \int dq \frac{q^2 e^{-(Q_1q^2+PqQ_3W)}}{q^2+Z}, \tag{43}$$

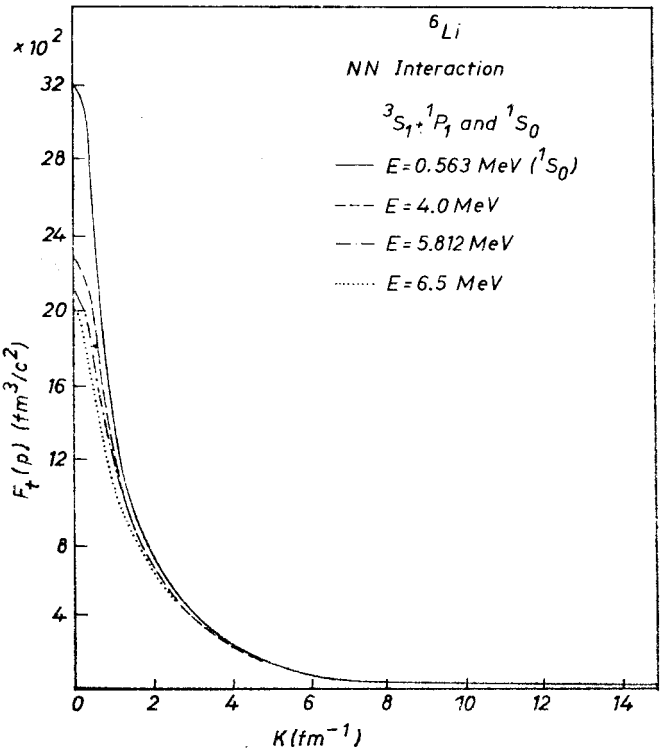

Fig. 1. The form factor $F_t(k)$ for nucleon-nucleon interaction for different ^{6}Li energies

where

$$Q_1 = \left(\frac{m_\alpha}{m+m_\alpha}\right)^2, \quad Q_2 = \left(\frac{2m+m_\alpha}{2(m+m_\alpha)}\right)^2, \quad Q_3 = \frac{m_\alpha(2m+m_\alpha)}{(m+m_\alpha)^2}, \quad W = \cos\theta_{qp}.$$

The form factors $G_\alpha(P)$ given by equation (43) are calculated for different values of the ^{6}Li energies and the results are given in Fig. 2 for the triplet state and in Figure 3 for the singlet state. At the value $k = 0$, it is noticed that the value of the form factor for the singlet state (Fig. 3) is only one third of its value for the triplet state (Fig. 2). A very interesting form for the Gaussian potential is that suggested by Leung and Park [19] and applied by Osman [22] is

$$f_G(q) = \alpha(q_c^2 - q^2)(a + bq^2)\exp(-dq^2). \tag{44}$$

This potential is represented as a repulsive potential surrounded by an attractive potential. It is zero at $q_c = q$ which corresponds to an energy value given by $E_c = 83\, q_c^2$ and the value of E_c is taken by Tabakin [13] to be equal to 240 MeV. Or in other words, the value of the crossing point of the zero potential is $q_c = 1.70046\ \text{fm}^{-1}/c$. The form factor $G_\alpha(P)$ is calculated for the potential form given by equation (44), for different values of the ^{6}Li energies and the results are shown in Fig. 4. It is noticed from Fig. 4 that the form factor $G_\alpha(P)$ vanishes at a value of $k = 17.56\ \text{fm}^{-1}$, and then it reappears with increasing k but its values of the form factor are negligible for large values of k. We also notice that the value of $G_\alpha(P)$ changes rapidly for the region of $k = 0 - 17.56\ \text{fm}^{-1}$.

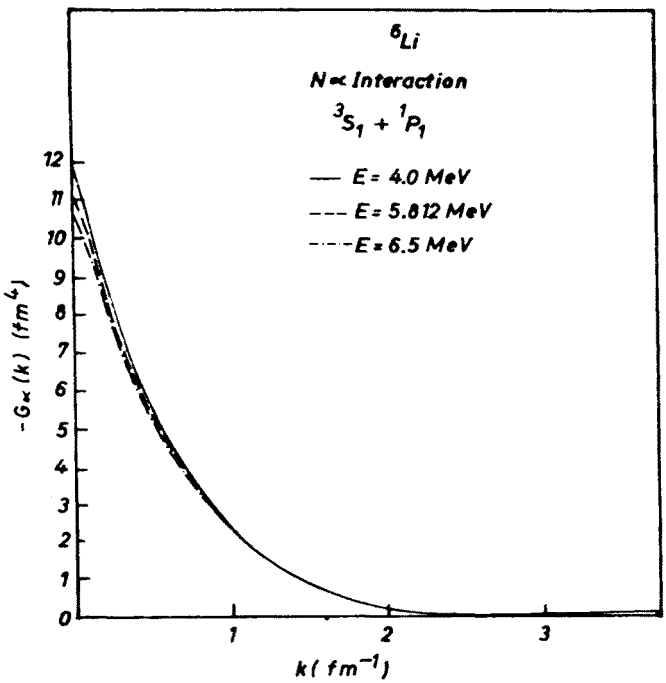

Fig. 2. The form factor $G_\alpha(k)$ for nucleon-alpha interaction for different ^{6}Li energies

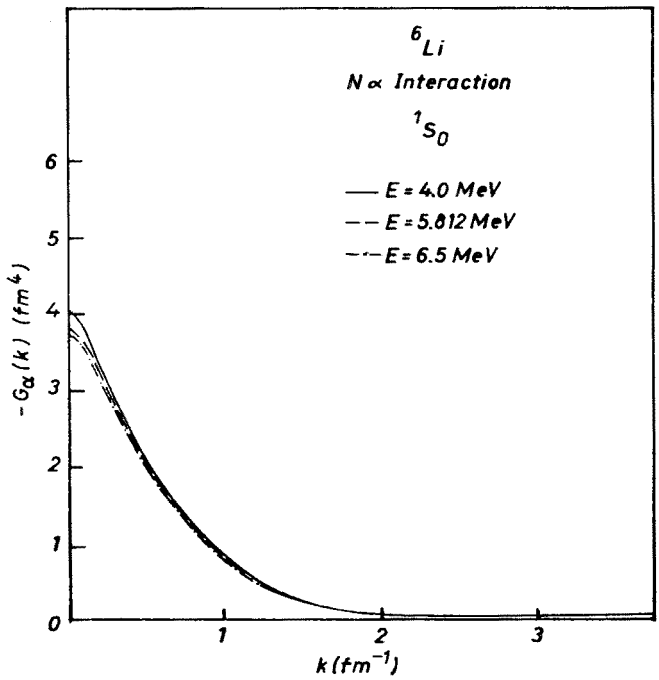

Fig. 3. The form factor $G_\alpha(k)$ for nucleon-alpha interaction for different ^{6}Li energies

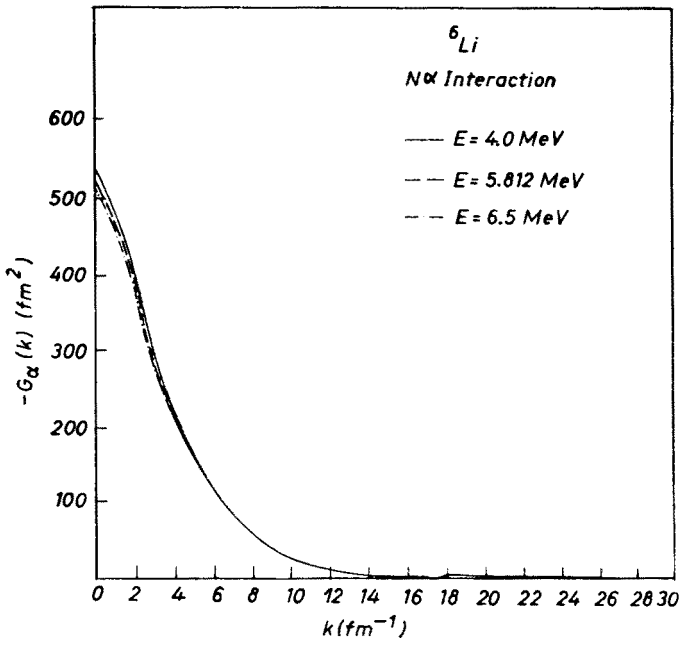

Fig. 4. The form factor $G_\alpha(k)$ for nucleon-alpha interaction (Tabakin form) for different ^{6}Li energies

Since the nucleon-nucleon interaction is the dominant part in the ^{6}Li nucleus, the effective range can be calculated using the formula [23]:

$$r_0 = \frac{2}{K^2}\left(K - \frac{1}{a}\right), \tag{45}$$

where a is the scattering length and K is the wave number and is given by [24]

$$K = \left(\frac{M\varepsilon}{h^2}\right)^{1/2}, \tag{46}$$

M is the nucleon mass and ε is the binding energy. The scattering length a has been calculated by Tabakin [13] and has been found to have the values $a_t = 5.396$ fm and $a_s = -23.68$ fm for the triplet $^3S_1$ and the singlet $^1S_0$ S-states, respectively. Introducing these calculated values of the scattering length into equation (45), the effective range can be calculated as a function of the wave number k for both the triplet and singlet S-states. The results of the calculations are shown in Fig. 5. The experimental value [25] of the ^{6}Li binding energy, which is 4.53 MeV, corresponds to a wave number $k = 0.33$ fm^{-1}. This value from figure 5 extracts the effective ranges for the triplet and singlet S-states as $r_{0t} = 2.66$ fm and $r_{0s} = 5.83$ fm, respectively. The values of the scattering length and effective range for the triplet and singlet S-states obtained and calculated here are compared

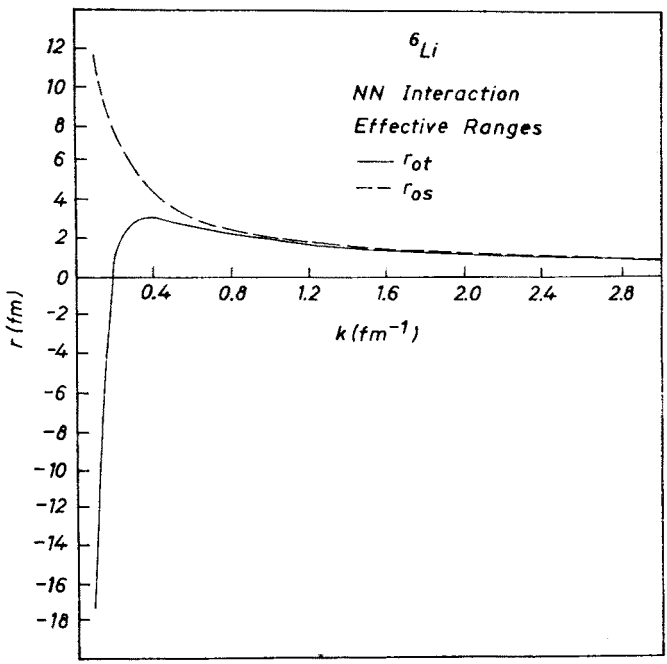

Fig. 5. The effective-range $r(k)$ for nucleon-nucleon interaction in the ^{6}Li nucleus for the triplet and singlet cases

with other previously obtained values from low-energy nucleon-nucleon scattering experiments and from different data analysis. All these values are given in Table I.

The value of the ^{6}Li binding energy is evaluated by solving the ^{6}Li three-body integral equations (27), (28) and (36), (37). These three body integral equations represent an eigenvalue problem, which can be solved in the normal way. To solve this problem, the value of the binding energy is taken as a parameter. In this method, a value is suggested for the binding energy, and then the value of the quantity λ is calculated by using the standard method. The obtained value of the parameter λ is called the three-body value of λ. This

TABLE I

Effective-range and scattering length for nucleon-nucleon interactions

References	a_s (fm)	a_t (fm)	r_t (fm)	r_s (fm)
Tabakin [13]	-23.68	5.396	1.737	2.7
Preston [24]	-23.71 ± 0.07	5.38 ± 0.03	1.71 ± 0.03	2.4 ± 0.3
Lehman et al. [25]		5.4	1.747	
*Noyes [26]	-23.678	5.396	1.726	2.51
*Shull et al. [27]	-23.5	5.2	1.6	2.7
*Wilson [28]		5.425 ± 0.004	1.749 ± 0.008	
Blatt and Weisskopf [29]	-23.7	5.39	1.7	1.5–3.5
Segre [30]	-23.677 ± 0.029	5.4 ± 0.011	1.732 ± 0.014	2.46 ± 0.12
Bohr and Mottelson [31]	-23.7	5.39	1.703	2.7
Lomon and Wilson [32]	-23.719 ± 0.013	5.414 ± 0.005	1.75 ± 0.005	2.76 ± 0.05

* Refers to the experimental values.

value must at the same time match the two-body value of λ obtained from the two-body equations. If there is no matching between the obtained three-body and the two-body values of λ, then the input suggested parameter of the ^{6}Li binding energy value should be adjusted until we get an accurate matching. Using this method, the ^{6}Li binding energy is calculated as a three-body problem for both the ground (1^+) and the first excited (0^+) states. Also, the Coulomb corrections are added and taken into account [3, 17]. We have obtained the following values of Coulomb-corrected binding energies for the ^{6}Li nucleus

(*i*) for ^{6}Li ground state (triplet),

$$E(1^+) = 5.163 \text{ MeV};$$

(*ii*) for ^{6}Li* first excited state (singlet),

$$E(0^+) = 0.428 \text{ MeV}.$$

These values are close to the previously obtained values.

4. *Discussion and conclusions*

In the present work, we study the ^{6}L nucleus. The ^{6}Li nucleus is considered as composed of an alpha particle, a proton and a neutron. With this construction, the ^{6}Li nucleus is studied as a three-body problem. The different two-body interactions are taken in the separable form with a Gaussian dependence. The three-body problem is solved by means of a set of coupled three-body integral equations, which are solved numerically to yield the value of the ^{6}Li binding energy. The ^{6}Li binding energies are calculated for the ^{6}Li (1^+) ground state and also for the ^{6}Li* (0^+) excited state. The obtained values of the binding energies are corrected to include the Coulomb corrections. The theoretically calculated values of the binding energies are in good agreement with the experimental values. They are also comparable to the previously obtained values. Furthermore the different form factors, effective ranges and scattering lengths are calculated. We can conclude that our present construction and model of the structure of ^{6}Li nucleus, successfully describes the static properties of the nucleus. The three-body study of the nucleus provides accurate theoretical calculations for the nuclear structure of light nuclei.

Editorial note. This article was proofread by the editors only, not be the authors.

REFERENCES

[1] A. Ghovanlou, D. R. Lehman, *Phys. Rev.* **C9**, 1730 (1974); M. Rai, D. E. Lehman, A. Ghovanlou, *Phys. Lett.* **59B**, 327 (1975).

[2] W. E. Meyerhof, T. A. Tombrello, *Nucl. Phys.* **A109**, 1 (1968).

[3] T. Lauritsen, F. Ajzenberg-Selove, *Nucl. Phys.* **78**, 1 (1966).

[4] D. S. Chuu, C. S. Han, D. L. Lin, *Phys. Rev.* **C7**, 1329 (1973).

[5] J. L. Gammel, B. J. Hill, R. M. Thaler, *Phys. Rev.* **119**, 267 (1960).

[6] P. H. Wackman, N. Austern, *Nucl. Phys.* **30**, 529 (1962); B. Barsella, L. Lovitch, S. Rosati, *Nucl. Phys.* **A117**, 638 (1968).

[7] S. Cohen, D. Kurath, *Nucl. Phys.* **73**, 1 (1965).

[8] P. M. Fishbane, J. V. Noble, *Phys. Rev.* **171**, 1150 (1968).

[9] D. R. Thompson, Y. C. Tang, *Phys. Rev.* **179**, 971 (1969).

[10] H. Hebach, P. Henneberg, H. Kümmel, *Phys. Lett.* **24B**, 134 (1967); P. E. Shanley, *Phys. Rev. Lett.* **21**, 627 (1968); *Phys. Rev.* **187**, 1328 (1969); V. A. Alessandrini, D. Avalos, L. Epele, H. Fanchiotti, C. A. García Canal, M. A. Gregorio, *Phys. Lett.* **29B**, 83 (1969); V. A. Alessandrini, D. R. Avalos, L. N. Epele, H. Fanchiotti, *Nucl. Phys.* **A150**, 273 (1970); M. S. Shah, A. N. Mitra, *Phys. Rev.* **C1**, 35 (1970); A. N. Mitra, *Nucl. Phys.* **32**, 529 (1962); A. N. Mitra, V. S. Bhasin, B. S. Bhakar, *Nucl. Phys.* **38**, 316 (1962); M. G. Fuda, *Nucl. Phys.* **A116**, 83 (1968); M. G. Fuda, *Phys. Rev.* **C7**, 1365 (1973); S. Adya, *Phys. Rev.* **166**, 991 (1968); S. K. Adhikari, *Phys. Rev.* **C14**, 782 (1976), B. Charnomordic, C. Fayard, G. H. Lamot, *Phys. Rev.* **C15**, 864 (1977); A. Osman, *Phys. Rev.* **C19**, 1127 (1979); D. R. Harrington, *Phys. Rev.* **139**, B691 (1965); V. F. Kharchenko, N. M. Petrov, S. A. Storozhenko, *Nucl. Phys.* **A106**, 464 (1968); A. N. Mitra, S. P. Pandya, *Nucl. Phys.* **20**, 455 (1958); Y. Yamaguchi, *Phys. Rev.* **95**, 1628 (1954); Y. Yamaguchi, *Phys. Rev.* **95**, 1635 (1954).

[11] L. D. Faddeev, *Zh. Eksp. Teor. Fiz.* **39**, 1459 (1960) [English translation: *Sov. Phys. JETP* **12**, 1014 (1961)]; C. Lovelace, *Phys. Rev.* **135**, B1225 (1964).

[12] B. S. Bhakar, *Nucl. Phys.* **46**, 572 (1963); B. S. Bhakar, A. N. Mitra, *Phys. Rev. Lett.* **14**, 143 (1965); V. K. Gupta, B. S. Bhakar, A. N. Mitra, *Phys. Rev.* **153**, 1114 (1967); A. N. Mitra, G. L. Schrenk,

V. S. Bhasin, *Ann. Phys.* (NY) **40**, 357 (1966); G. L. Schrenk, A. N. Mitra, *Phys. Rev. Lett.* **19**, 530 (1967).
[13] F. Tabakin, *Phys. Rev.* **174**, 1208 (1968).
[14] R. D. Amado, *Phys. Rev.* **132**, 485 (1963).
[15] A. Osman, *Nuovo Cimento* **48A**, 121 (1978).
[16] W. F. Ford, *Nucl. Phys.* **37**, 119 (1962).
[17] A. Osman, *Phys. Rev.* **C4**, 302 (1971).
[18] D. D. Brayshaw, *Phys. Rev.* **D11**, 2583 (1975).
[19] A. N. Mitra, V. S. Bhasin, *Phys. Rev.* **131**, 1265 (1963); L. Schulman, *Phys. Rev.* **156**, 1129 (1967); C. C. H. Leung, S. C. Park, *Phys. Rev.* **187**, 1239 (1969).
[20] F. Tabakin, *Ann. Phys.* (NY) **30**, 51 (1964).
[21] A. Osman, *Nucl. Phys.* **A153**, 524 (1970).
[22] A. Osman, *Phys. Rev.* **C17**, 341 (1978).
[23] H. A. Enge, *Introduction to Nuclear Physics*, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, Palo Alto, London, New York, Dallas, Atlanta, Barrington, Illinois 1968, p. 67.
[24] M. A. Preston, *Physics of The Nucleus*, Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, Palo Alto, London 1965, p. 26.
[25] D. R. Lehman, M. Rai, A. Ghovanlou, *Phys. Rev.* **C17**, 744 (1978).
[26] H. P. Noyes, *Phys. Rev.* **130**, 2025 (1963); *Nucl. Phys.* **74**, 508 (1965).
[27] C. G. Shull, E. O. Wollan, G. A. Morton, W. L. Davidson, *Phys. Rev.* **73**, 842 (1948).
[28] R. Wilson, *Comments on Nucl. Part. Phys.* **2**, 141 (1968); H. P. Noyes, *Annu. Rev. Nucl. Sci.* **22**, 465 (1972).
[29] J. M. Blatt, V. F. Weisskopf, *Theoretical Nuclear Physics*, John Wiley and Sons, Inc., 1952, p. 71.
[30] E. Segre, *Nuclei and Particles*, W. A. Benjamin, Inc., New York, Amsterdam 1965, p. 386.
[31] A. Bohr, B. R. Mottelson, *Nuclear Structure*, Vol. I, W. A. Benjamin, Inc., New York, Amsterdam 1969, p. 241.
[32] E. Lomon, R. Wilson, *Phys. Rev.* **C9**, 1329 (1974).