THE ELECTROMAGNETIC DECAY RATES OF THE PROCESSES $\psi'(3685) \to \gamma\eta_c(2980)$, $\gamma\eta'_c(3590)$ AND $\psi(3095) \to \gamma\eta_c(2980)$ IN THE HARMONIC OSCILLATOR QUARK MODEL

By M. Ayub

Department of Physics, Gomal University*

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The harmonic oscillator quark model of the $c\bar{c}$ system is employed to calculate the decay widths of the transitions $\psi'(3685) \to \gamma\eta_c(2980)$, $\psi'(3685) \to \gamma\eta'_c(3590)$ and $\psi(3095) \to \gamma\eta_c(2980)$. Taking the harmonic oscillator parameter for the $c\bar{c}$ system $\alpha'^2 \simeq 0.27$ GeV^2 and the magnetic moment of the c-type quark smaller by the amount m_u/m_c than the u-type quark, we get reasonable widths for these processes.

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1. Introduction

The non-relativistic quark model has been used extensively to study the different properties of hadrons. We recall in particular the early successful computation of the radiative and pionic widths of baryons [1], the magnetic moments and the electromagnetic mass differences of hadrons [2] and several other properties of hadrons [3]. Since the discovery of the particles ψ and ψ', the properties of the charmonium have been the subject of intense experimental and theoretical work [4]. Of particular interest has been the question of the existence of the pseudoscalar partners of ψ and ψ'. Potential models have also been used for understanding the charmonium spectrum and a number of calculations have been made to estimate the different decay rates of the charmonium [5].

In previous papers [6] we have applied the non-relativistic harmonic oscillator (HO) quark model to a number of the radiative and pionic decays of mesons and reasonable agreement with experiment has been obtained. The processes which have been dealt with in Ref. [6] include the radiative widths of the low-lying positive parity $L = 1$ "old" mesons[1], the widths of the transitions $\psi' \to \gamma\chi_J$'s, χ_J's $\to \gamma\psi$, $\psi \to \gamma f$ and the two body pionic widths

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* Address: Department of Physics, Gomal University, Dera Ismail Khan, Pakistan.

[1] "Old" mesons refer to those which can be interpreted as the bound states of the ordinary quarks (u, d, s) and the antiquarks ($\bar{u}$, $\bar{d}$, $\bar{s}$).

of the members of the $J^{PC} = 1^{++}$ multiplet. In case of the radiative transitions, we have three parameters, namely the HO parameter for the ordinary $q\bar{q}$ system i.e. α^2, the HO parameter for the $c\bar{c}$ system i.e. α'^2 and the magnetic moments of the quarks. We take the gyromagnetic ratios of quarks as 1 so that the magnetic moment equals charge/2 × mass. The magnetic moments of baryons give the magnetic moment of the u-type quark $\mu_u = 2.79e/2m_p$ [7] which also implies $m_u = 0.336$ GeV. We take $\mu_q = \mu_u$ (where q = u, d, s) and the magnetic moment of the charmed quark to be smaller than that of the u-type quark by the amount m_u/m_c. We take $m_u = 0.336$ GeV, $m_c = 2$ GeV [2] and the charge of the charm quark $q^{(c)} = 2/3$. As all the radiative decays dealt with in the previous papers [6] could be reasonably described with the values $\alpha^2 \simeq 0.055$ GeV^2, $\alpha'^2 \simeq 0.27$ GeV^2, $\mu_q = \mu_u$ and $\mu_c = 0.336\mu_u/2$, we continue to use these values of the parameters in the present calculations. We take $\eta_c(2980)$ and $\eta_c'(3590)$ as the pseudoscalar partners of ψ and ψ' respectively. As the contributions to the matrix elements due to the $(u\bar{u}+d\bar{d})$ and $s\bar{s}$ contents of all these states are expected to be negligible, we take them to be pure $c\bar{c}$ states in the present calculations.

2. *Decay widths*

The $c\bar{c}$ system is assumed to be described by the Hamiltonian

$$H = \frac{1}{2M}(p_1^2+p_2^2)+\frac{\kappa}{2}(r_1-r_2)^2 \tag{1}$$

in obvious notation [8]. By introducing the centre of mass and relative coordinates the Hamiltonian can be reduced to a form which describes the internal motion of $c\bar{c}$ system which in turn can be solved easily [9]. The physical spin of the meson is obtained by coupling the total quarks spin state with the orbital angular momentum of the spatial wave functions. Total wave functions for the $c\bar{c}$ states are obtained by combining the spin and the spatial wave functions. The photon emission (along the z-axis) is assumed to proceed through the interaction

$$\mathscr{H} = \sum_j \mu_j q^{(j)} \sqrt{4\pi}\exp(ikz^{(j)})/\sqrt{k_0}\,[kS_+^{(j)} - 1/g(p_x^{(j)}+ip_y^{(j)})]. \tag{2}$$

The decay width of a particle with spin J is given as

$$\Gamma = \frac{1}{(2\pi)^2}\frac{2}{2J+1}\frac{E_1E_\gamma k}{E_c}\int\sum_{m'}|d^J_{m'm}(\theta)A_{m'}|^2 d\Omega^\gamma(\theta,\phi). \tag{3}$$

In (3) k is the c.m. momentum of the photon and $d\Omega^\gamma(\theta,\phi)$ is the solid angle for the photon. E_c, E_1 and E_γ are the c.m. energy of the decaying particle, final state hadron and the photon respectively. $d^J_{m'm}(\theta)$ are the rotation matrix elements corresponding to the angular mo-

[2] The non-relativistic treatment of the new particles (the ψ-sector) gives the mass of the charmed quark in the range 1.3–2 GeV.

mentum J and $A_{m'}$ are the transition matrix elements where m' refers to the decaying particle. The transition matrix elements in the three processes are found as

$$A_{-1} = \langle \eta_c J = 0 | \mathscr{H} | \psi' J_z = -1 \rangle \simeq 0.5 q^{(c)} \mu_c \frac{k^3}{\alpha'^2} \exp\left(\frac{-k^2}{8\alpha'^2}\right),$$

$$A_{-1} = \langle \eta_c' J = 0 | \mathscr{H} | \psi' J_z = -1 \rangle \simeq \sqrt{8\pi}\, q^{(c)} \mu_c k \exp\left(\frac{-k^2}{8\alpha'^2}\right),$$

$$A_{-1} = \langle \eta_c J = 0 | \mathscr{H} | \psi J_z = -1 \rangle \simeq \sqrt{8\pi}\, q^{(c)} \mu_c k \exp\left(\frac{-k^2}{8\alpha'^2}\right).$$

The hyper-geometric functions involved in the radial integrals appearing in the matrix element for the process $\psi' \to \gamma\eta_c$ can be approximated to a unity. With the values of the parameters mentioned in the introduction, we get $\Gamma(\psi' \to \gamma\eta_c) \simeq 3$ keV, $\Gamma(\psi' \to \gamma\eta_c') \simeq 0.7$ keV and $\Gamma(\psi \to \gamma\eta_c) \simeq 1.5$ keV. These values are to be compared with the experimental values $\Gamma(\psi' \to \gamma\eta_c) = 0.3$–$1.7$ keV [10], $\Gamma(\psi' \to \gamma\eta_c') = 0.4$–$3$ keV [10] and $\Gamma(\psi \to \gamma\eta_c) = 0.5$–$0.7$ keV [11].

3. Conclusion

In this paper we have applied the non-relativistic HO quark model to the newly discovered radiative transitions of ψ and ψ'. The results obtained are as good as the quark model results for the radiative transitions of other mesons. On the whole, the quark model results are reasonable in the sense that experimentally large or small widths are predicted to be large or small respectively. Obviously, this model results depend upon our assumptions about the HO parameters, masses and charges of quarks (the present results depend upon α'^2, m_c and $q^{(c)}$) and also on the belief that the magnetic moment of the charmed quark is smaller than those of the ordinary quarks. Further, these results are flexible because the parameters involved in the calculations are adjustable and maximum experimental information is desirable to settle the values of these parameters once and for all. With the presently available experimental information, it is difficult to argue insistently in favour of the specific values for the parameters like quark masses, quark magnetic moments or, for that matter any quantity related to quarks. Any way, it is encouraging to note that the naive HO quark model gives predictions for quite a large number of meson radiative transitions which are in reasonably good agreement with experiment.

That the harmonic oscillator potential gives such apparently reasonable results is also encouraging from the point of view of the developments in the quark model which seek permanent confinement of quarks inside a hadron. Such a confinement can be achieved by a choice of static $q\bar{q}$ interaction of the form $V(r) = r^{\varepsilon}$ and the most favoured one seems to be the one which rises linearly. But as noted by Jackson [12], the order of appearance of successive levels with increasing energy is insensitive to the details of the interaction, provided it is confining. It has also been shown explicitly by Gromes et al. [13] that the linear potential can be approximated by an appropriate oscillator potential for the energy levels for the S-states (the Schrodinger equation with a linear potential can be solved analytically only for $L = 0$ i.e. S-states). The sum of linear and Coulomb potential (QCD inspired) can still be approximated by an appropriate HO potential, as long as the spectrum is essentially determined by the linear part. Another point which goes in favour of this approximation (of the linear potential with the harmonic oscillator potential) is the fact that the contribution (from the overlap integral of the wave functions) comes from the region where the linear and harmonic oscillator potentials do not differ too much, especially for the low-lying levels. In conclusion, the present calculations also favour the identification of $\eta_c(2980)$ and $\eta_c'(3590)$ as the pseudoscalar partners of ψ and ψ' respectively.

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