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ALPHA-NUCLEUS DIFFERENTIAL CROSS SECTION AT 4.45 GeV/c/NUCLEON

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The absolute differential cross sections of 17.9 GeV/c α-particles scattering on nuclear targets have been measured over a four momentum transfer range of $0.009 < |t| < 0.22$ (GeV/c)^2 using a magnetic spectrometer ALPHA. The nuclear targets comprised liquid H and He and solid C, Al and Cu. The experimental data are compared to results of the Glauber model calculations and analyzed in the diffraction cone range. Nuclear differential cross sections for $\alpha A \rightarrow \alpha A$ elastic scattering are extracted from the data for all targets under consideration. The corresponding total and elastic cross sections and other diffraction cone parameters have been evaluated.

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1. *Introduction*

The approach to the description of diffraction scattering processes within the framework of multiple scattering theory has been proposed by Glauber [1] and Sitenko [2]. In spite of some significant simplifications the approach yielded an unexpectedly good qualitative description of differential cross-section for the hadron-nucleus scattering. An important element of testing the universality of the Glauber model is an attempt to apply the model to the problem of nucleus-nucleus scattering. Not much has been achieved up to now in this direction. In particular, the amount of data on nucleus-nucleus scattering at incident momenta in the GeV/nucleon region is insufficient [3], e.g. no measurements

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yielding the shape of the diffraction cone are available. The paper of Czyż and Maximon [4] was the first theoretical attempt to apply the idea of the model to nucleus-nucleus scattering.

The aim of this work is to obtain farther information on nucleus-nucleus scattering from the analysis of differential cross-sections for α-particle scattering on a series of nuclei from hydrogen to copper.

Solid C, Al and Cu [5] targets and cryogenic H and He [6] targets were used in the experiment.

An external beam of α-particles with a 4.45 GeV/c momentum per incident nucleon was used. The momentum value is convenient from the point of view of a further analysis of experimental data since the momentum dependence of cross sections and other scattering parameters for nucleon-nucleon collisions is rather weak and, moreover, the corresponding values for pp and np scattering are very approximate to each other over this momentum range.

A brief presentation of technical details of a magnetic spectrometer ALPHA is given in Section 2. Section 3 is devoted to experimental results of measurements of differential cross sections including elastic and quasi-elastic (without pion production) scattering processes.

Section 4 contains some information on parameters and approximations used in numerical $d\sigma/dt(t)$ calculations performed within the framework of a slightly modified classical Glauber model.

The comparison of the calculated results with the experimental data enabled us to extract the nuclear elastic cross sections for αA collisions. The experimental results for αp and αα collisions are compared with those obtained in other experiments (Section 5).

The analysis of the data concerning the diffraction cone region is presented in Section 6.

In the last Section of this paper the summary of the obtained results is given as well as the discussion of differences between the calculated and measured $d\sigma/dt$ values.

2. *Experiment*

An external beam of (17.90 ± 0.13) GeV/c α-particles from the Dubna synchrophasotron was used in the experiment. The measurements were carried out by means of the magnetic spectrometer ALPHA shown schematically in Fig. 1.

The spectrometer operated on-line with a BESM-4 computer, thus enabling an on-line control of the data acquisition from multiwire proportional chambers and scintillation counters working in digital and amplitude modes. This system selected out interactions between doubly charged ($Z = 2$) beam particles and target nuclei, accompanied by the emission of a single secondary particle ($Z = 2$) at an angle $\theta > \theta_0$ with respect to the primary particle.

The momenta, p, of scattered particles and their scattering angles, θ, were measured with errors $\Delta p \simeq 8 * 10^{-3} * p$ and $\Delta\theta \simeq 0.75$ mr, respectively (multiple Coulomb scattering within the target being neglected). The above precision of the spectrometer measurements

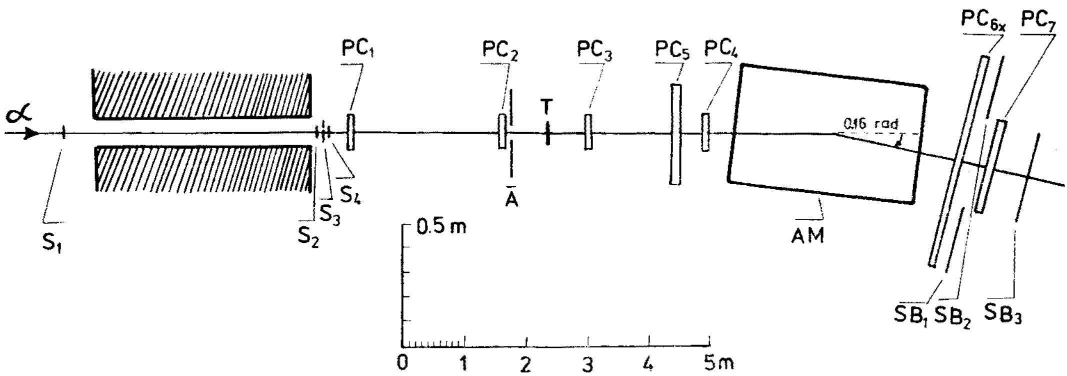


Fig. 1. Schematic diagram of the spectrometer. S — scintillation counters (S_3 and SB with *dE/dx* information readout), A — anticoincidence scintillation counter, PC — proportional chambers, T — cryogenic or solid target, AM — analyzing magnet

allowed one to distinguish between α-particle scattering and interactions with pion production. However, pure elastic scattering and quasi-elastic scattering, with excitation or target nucleus fragmentation, were indistinguishable.

Details concerning the ALPHA spectrometer and its operation on-line with the computer are given by Ableev et al. [7], whereas details on a cryogenic target filled with liquid hydrogen or helium can be found in [8]. The thicknesses of targets (see Table I) did not exceed 2 g/cm² in order to reduce the contamination of events due to alphas scattered two or more times within the target.

TABLE I

Thickness values of the different targets

	h [g/cm²]
H	0.847 ± 0.001
He	1.494 ± 0.002
C	2.00 ± 0.01
Al	1.46 ± 0.01
Cu	1.37 ± 0.01

3. *Experimental results*

The absolute differential cross sections discussed in this work for α-particle scattering on natural solid C, Al and Cu targets are published in paper [5] and those obtained for H and He targets in paper [6].

The methods of classifying registered events and correction procedures are presented in papers [7]. The number of elastic scattering events was estimated by the difference of the effects of "full" and "empty" targets.

TABLE II

Alpha-nucleus differential cross sections

$-t$ (GeV/c)^2	$\frac{d\sigma^{\rm Expt}}{dt}$ mb (GeV/c)$^{-2}$				
	αp	αα	αC	αAl	αCu
0.0088	—	—	14420 ± 310	22270 ± 700	25300 ± 1400
0.0113	—	—	10660 ± 240	14580 ± 500	12500 ± 1000
0.0142	—	4130 ± 190	7890 ± 170	8850 ± 340	5760 ± 600
0.0173	—	3015 ± 150	5380 ± 120	5020 ± 250	3720 ± 440
0.0208	588 ± 21	2555 ± 120	3670 ± 90	2830 ± 180	3090 ± 400
0.0245	528 ± 18	1855 ± 90	2470 ± 70	1650 ± 130	2350 ± 250
0.0286	426 ± 14	1525 ± 80	1680 ± 50	1140 ± 100	3280 ± 220
0.0330	363 ± 12	1160 ± 70	1080 ± 40	847 ± 80	2850 ± 180
0.0377	306 ± 10	874 ± 35	707 ± 32	892 ± 70	2310 ± 150
0.0427	266 ± 9	591 ± 26	514 ± 26	936 ± 60	1740 ± 120
0.0481	243 ± 8	474 ± 21	428 ± 22	940 ± 60	1230 ± 100
0.0537	194 ± 7	322 ± 17	332 ± 18	839 ± 48	725 ± 80
0.0597	162 ± 6	248 ± 15	291 ± 17	768 ± 46	550 ± 70
0.0660	131.0 ± 4.8	165 ± 12	281 ± 17	471 ± 38	580 ± 70
0.0726	102.3 ± 4.1	129 ± 11	286 ± 17	526 ± 44	370 ± 60
0.0795	82.1 ± 3.5	94 ± 10	244 ± 17	300 ± 32	300 ± 60
0.0867	66.3 ± 2.9	72 ± 8	231 ± 14	284 ± 31	481 ± 60
0.0942	48.4 ± 2.2	59 ± 7	204 ± 27	210 ± 27	561 ± 60
0.1021	38.4 ± 1.9	54 ± 7	180 ± 12	162 ± 24	355 ± 50
0.1103	27.6 ± 1.6	40 ± 6	167 ± 12	190 ± 23	304 ± 42
0.1187	19.6 ± 1.2	30 ± 5	109 ± 10	146 ± 20	160 ± 35
0.1275	17.6 ± 1.1	31.5 ± 4.8	98 ± 9	190 ± 21	253 ± 38
0.1366	10.8 ± 0.9	30.8 ± 4.7	82 ± 8	191 ± 23	129 ± 32
0.1461	8.9 ± 0.8	16.3 ± 3.7	111 ± 10	156 ± 22	209 ± 37
0.1558	5.1 ± 0.6	19.3 ± 4.3	68 ± 9	106 ± 19	157 ± 34
0.1658	3.7 ± 0.5	31.9 ± 4.9	56 ± 7	137 ± 20	154 ± 29
0.1762	3.1 ± 0.5	10.1 ± 3.4	57 ± 8	65 ± 14	—
0.1869	2.2 ± 0.5	18.0 ± 5.0	—	—	—
0.1979	1.44 ± 0.47	11.0 ± 5.0	—	—	—
0.2092	1.31 ± 0.47	7.1 ± 3.6	—	—	—
0.2208	0.80 ± 0.32	—	—	—	—

The obtained cross sections $d\sigma^{\rm Expt}/dt$ were corrected for the geometrical efficiency of the spectrometer, the efficiency of the detectors and absorption downstream the beam in the spectrometer material, the contamination of single-charged particles fragmentation deuterons in the elastic peak, the contamination of inelastic interactions with slow pion production, spectrometer resolution and double scattering within the target.

The results of measuring the $d\sigma^{\rm Expt}/dt$ are given in Table II and in Figs. 2, 3 and 4. The errors of these values contain both statistical errors and errors corresponding to uncertainties resulting from a discrete character of the coordinates mesured by the proportional chambers.

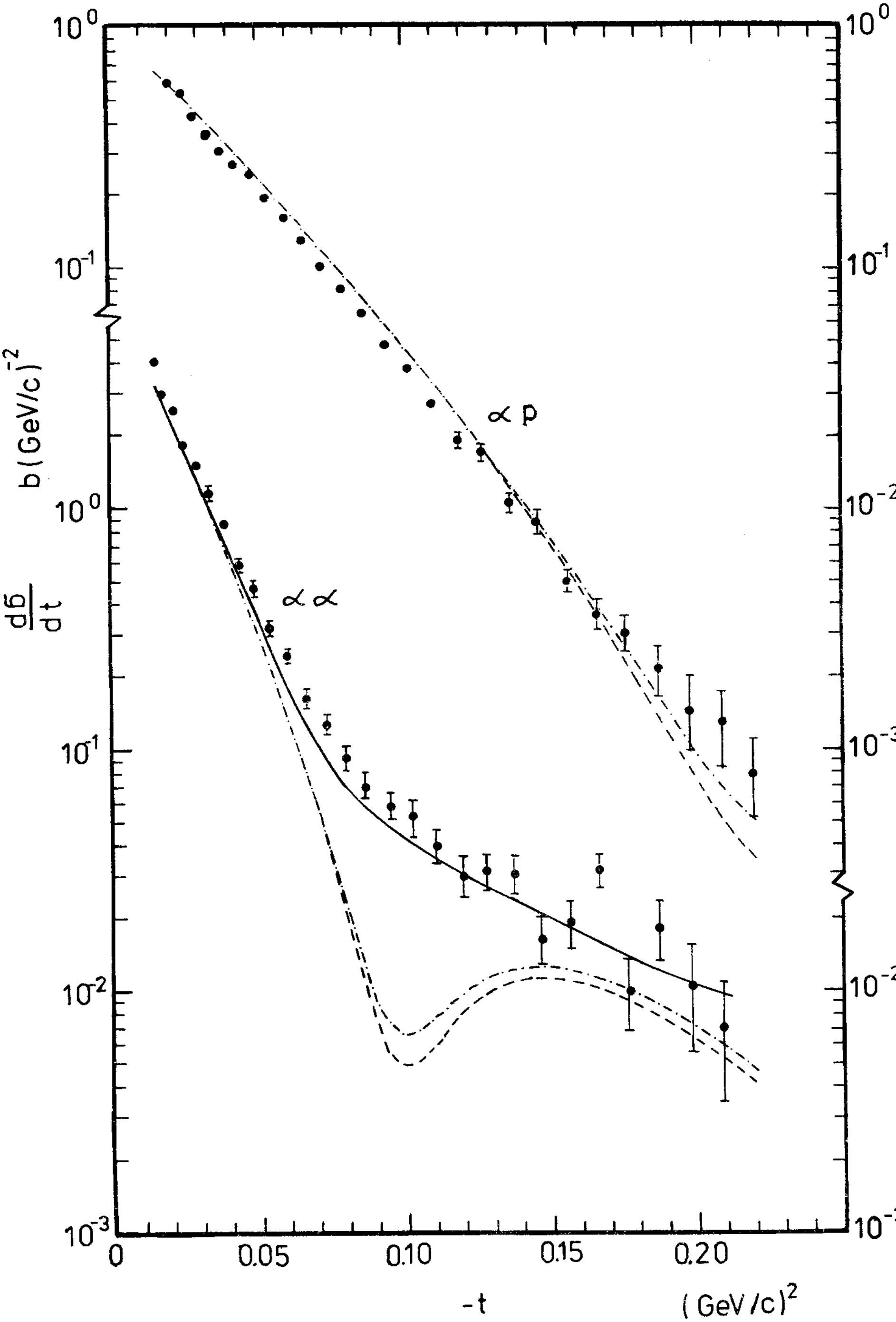


Fig. 2. The differential cross sections for αp and $\alpha\alpha$ scattering. The dashed line represents the elastic scattering calculated in the Glauber model, and the dash-dotted line represents the optical model calculations with MOP [12]. The solid line for $\alpha\alpha$ scattering is the sum of elastic and quasi-elastic scattering cross sections calculated in a closure approximation

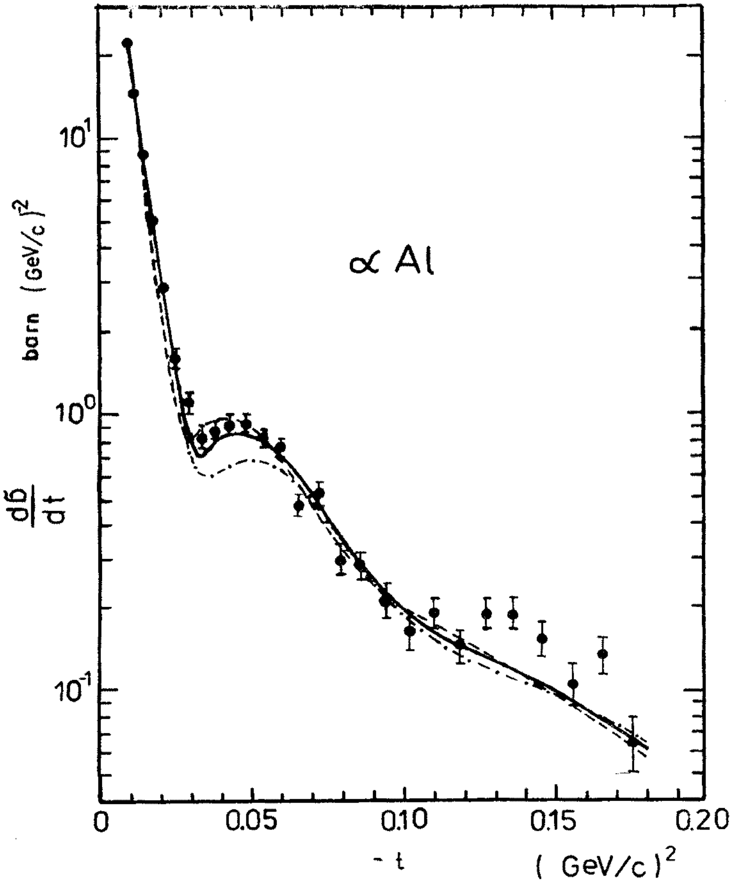


Fig. 3

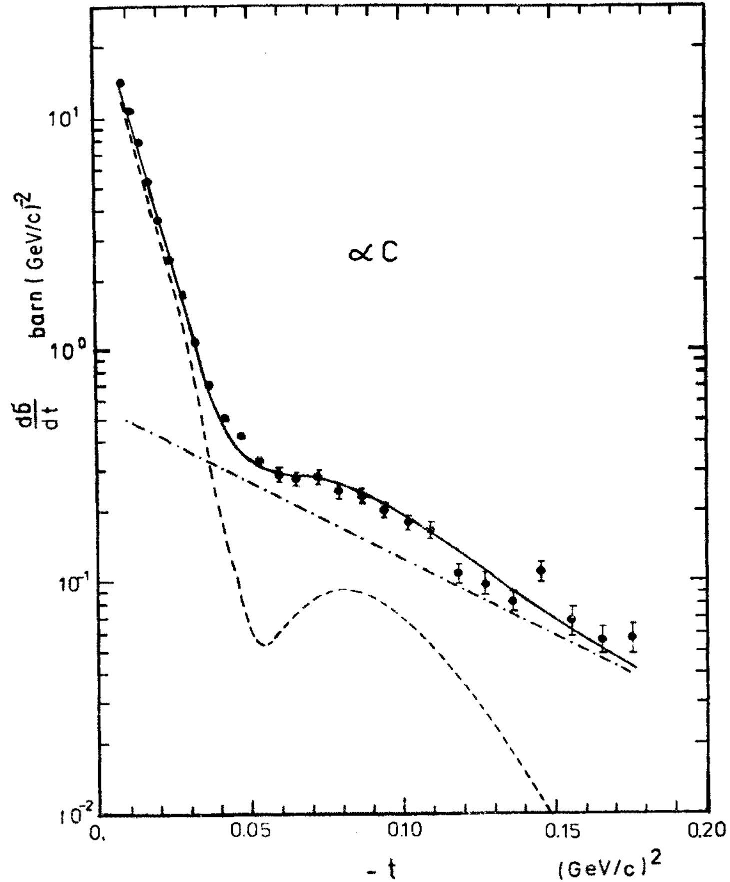


Fig. 4a

Fig. 3. The differential cross sections for the sum of elastic and quasi-elastic αAl scattering at 17.9 GeV/c. The three curves represent different approximations in the framework of the Glauber model: dashed line — rigid α-particle approximation [17], dash-dotted line — the optical approximation in mass number [14], solid line — Swarm projectile model [18]

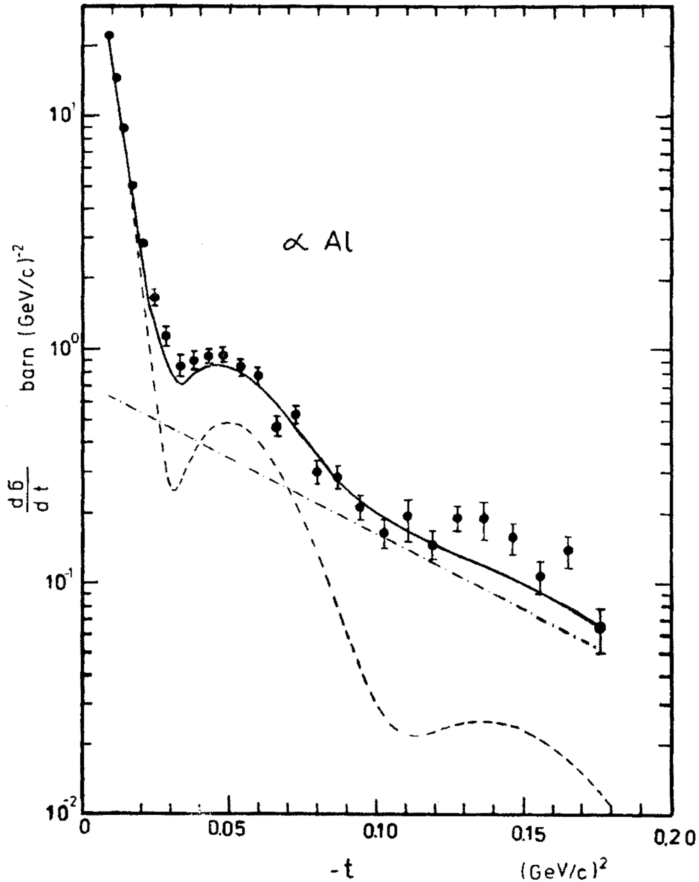


Fig. 4b*

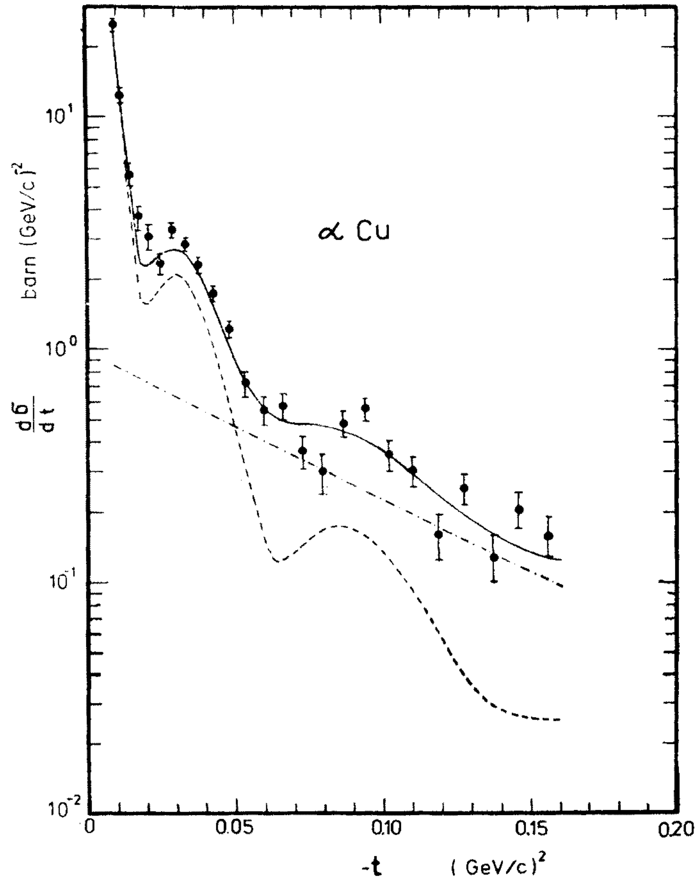


Fig. 4c

Fig. 4. The differential cross sections for a) αC, b) αAl and c) αCu scattering at 17.9 GeV/c. The dash-line represents the elastic scattering cross section calculated by the Swarm projectile model [18] including Coulomb interaction, the dash-dotted line is the quasi-elastic scattering and the solid line is the resulting cross section

The systematic errors of $d\sigma^{Expt}/dt$ normalization do not exceed 3%. They contain errors in determining:
— flux of an electronically selected beam of primary α-particles across the target,
— target thickness,
— corrections for spectrometer efficiency and absorption during the beam transport,
— contamination of inelastic interactions and singly-charget particles.
The error due to the inaccuracy of incident momentum is not included into this normalization error (see Section 4).

4. *Calculation methods*

The differential cross sections $d\sigma/dt$ for elastic scattering of α-particles on hydrogen nuclei and for the sum of elastic and quasi-elastic scattering of α-particles on He, C, Al and Cu nuclei were calculated within the framework of the Glauber model.

The nucleon-nucleon interaction amplitude, averaged over spin and isospin values, was assumed to be of the form: $f(t) = \frac{k}{4\pi} \cdot \sigma_{NN}(i+\varrho_{NN}) \exp(-b_{NN} \cdot t/2)$, where σ_{NN}, ϱ_{NN}, b_{NN} are respectively the total cross section, the ratio of the real to the imaginary part of the forward scattering amplitude and the slope of the diffraction cone for NN scattering.

The above parameters derived by interpolation from experimental data [9] are: $\sigma_{NN} = 41.5$ mb, $\varrho_{NN} = -0.35$ and $b_{NN} = 7.6\,(\text{GeV}/c)^{-2}$.

At an α momentum of about 4.5 GeV/c/nucleon the parameter values are rather insensitive to the momentum value, and they are approximate to each other for pp and pn scattering. Thus averaging over isospin values is reasonable, and it has no significant influence on the calculation results.

The nucleon density distribution in the ground state of an α-particle was assumed to be of the gaussian form: $\varrho(r) = (\pi R^2)^{-3/2} \exp(-r^2/R^2)$, $R = 1.37$ fm [10], whereas for C, Al and Cu target nuclei the Fermi shape of the distribution was used: $\varrho(r) = \varrho_0/(1+\exp[(r-R)/c])$.

The parameters of the latter distribution are given in Table III [11].

The dash curves in Fig. 2 show the $d\sigma/dt(t)$ for elastic scattering αp and αα, calculated within the framework of a simple version of the Glauber model without taking Coulomb interactions into account. In order to estimate the eikonal and Coulomb effects, model calculations have been also carried out within the framework of the model with a micro-

TABLE III

The Fermi density distribution parameters

	^{12}C	^{27}Al	^{64}Cu
R (fm)	2.30	3.00	4.15
c (fm)	0.42	0.52	0.55

scopic optical potential (MOP) [12] and with Coulomb interactions taken into account. The dashed-dotted lines in Fig. 2 show the results of the calculation.

The MOP approach does not include eikonal approximation, which is typical for the standard approach within the Glauber model. Fig. 2 shows that the dash-dotted and the dashed lines do not differ too strongly beyond the first diffraction minimum. One can conclude therefore that the non-eikonal and Coulomb effects do not play an important role in αp and $\alpha\alpha$ elastic scattering, especially for t values before the first minimum. It seems also reasonable to assume that the dependence of the sum of $d\sigma/dt$ for elastic and quasi--elastic scattering on noneikonal effects is weak. The above sum of $d\sigma/dt$ was measured for target nuclei heavier then hydrogen.

In order to carry out calculations for nucleus-nucleus scattering, it was necessary to introduce some additional assumptions [13] not needed in the case of pα scattering, namely:

a) a closure approximation of a system of wave function for final states of the target--nucleus excited by the projectile α-particle (this approximation is good because of the closure of the final states system $|A\rangle$ in the configuration space of target nuclei),

b) distributions of nucleons in interacting nuclei are not intercorrelated,

c) an optical approximation for target nucleus in mass number A is valid [14].

The Coulomb effects in the scattering process were taken into account by adding Coulomb phase shifts to purely nuclear scattering ones [10].

The correction for center of mass motion is taken into consideration as a multiplicative factor, viz. $\exp(q^2R^2/4A)$, to the scattering amplitude [15]. This correction is not required for the target nucleus since it has no influence on the value of $d\sigma/dt = d\sigma^{el}/dt + d\sigma^{qel}/dt$ for hadron-nucleus and nucleus-nucleus scattering due to summation of the contributions of all possible excited states of the target [16].

For $\alpha\alpha$ scattering the calculation of the sum $d\sigma/dt$ using the conventional Glauber method [1, 2] and the closure approximation turns out to be feasible. The results are shown in Fig. 2 (solid line).

Cross sections for nucleus-nucleus scattering could not be calculated because of a rather large number of slowly converging terms in the Glauber series in the case of intermediate and heavy nuclei. In order to avoid these difficulties, some simplified methods were developed to calculate the amplitudes for elastic αA scattering based on "accurate" relations of the Glauber model:

1° Rigid α-particle approximation [17].

2° Optical approximation in mass number $A_\alpha (A_\alpha \to \infty)$ of the projectile particle; this approach leads to an explicit expression for phase shifts under additional assumption $r_{NN}^2 \ll R_\alpha^2,\ R_A^2$ [14].

3° Expansion of the αA scattering amplitude into a series with respect to effects of screening of nucleons in the α-particle (so called "swarm projectile model", or the shadowing effect series method) [18].

In principle the approximation 3°, in contrast to 1° and 2°, makes it possible to calculate the scattering amplitude with any desired precision. In our case the shadowing effects for nucleon pairs only were taken into account. The corrections for higher order shadowing effects were estimated to be 2–4% depending on the target nucleus.

For calculating the differential cross section of quasi elastic nucleus-nucleus scattering the analytical expression:

$$d\sigma^{qel}/dt = (d\sigma^{qel}/dt)_{t=0} * \exp(b^{qel} * t)$$

was taken. The parameters $(d\sigma^{qel})_{t=0}$, b^{qel}, given in Table IV, were calculated according to the optical approximation 2° (first order correction $\sim 1/2A$ being taken into account).

TABLE IV

The parameters of quasi-elastic differential cross-section

		^{12}C	^{27}Al	^{64}Cu
$\frac{d\sigma^{qel}}{dt}(0)$	b (GeV/c)$^{-2}$	0.585	0.735	0.972
b^{qel}	(GeV/c)$^{-2}$	15.3	15.1	14.4

5. Comparison of results of model calculations with experimental data

Fig. 3 compares results of the calculation based on three approaches 1°, 2°, 3° with experimental data for αAl scattering. It is seen that the consistence of the model calculations with the experimental results is best for the approach based on the swarm projectile model (3°). The rigid α-particle approximation (1°), which does not take into account the effects of virtual α-particle excitation in the scattering process, predicts too a sharp t: dependence of the αA scattering cross sections in the diffraction cone region (dashed line). The optical approximation (2°) (dash-dotted line) yields too low values of $d\sigma/dt$ in the region of the second diffraction maximum. Similar conclusions were drawn when comparing calculation results with the experimental data for αC and αCu scattering.

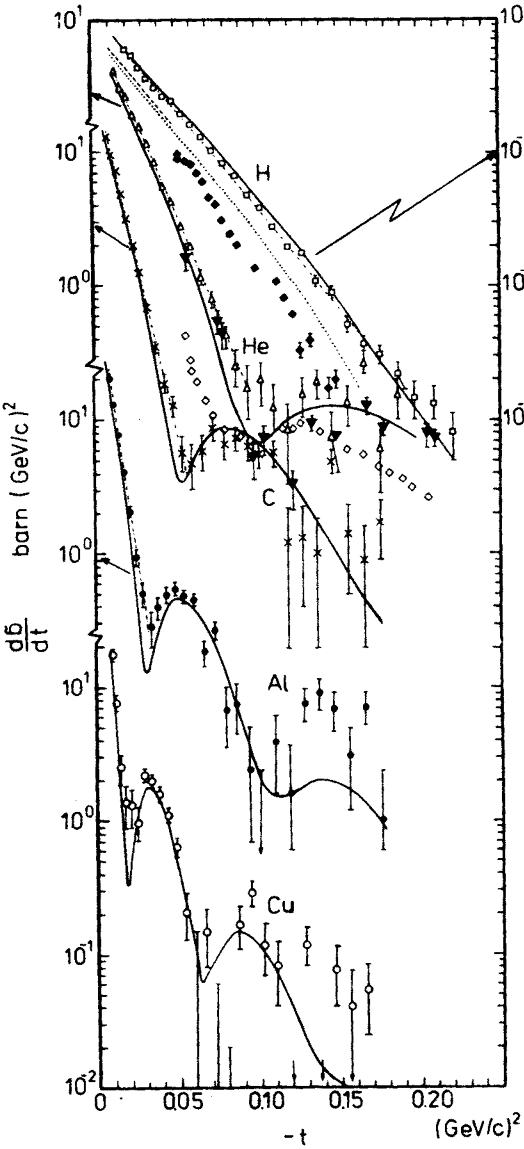
Fig. 4 shows the differential cross sections for αA scattering on ^{12}C, ^{27}Al and ^{64}Cu nuclei (Fig. 4a, b, c, respectively). The dashed line corresponds to $d\sigma^{el}/dt$ versus t dependence calculated using the Swarm projectile model-3° including Coulomb interaction, the dash-dotted line to that calculated with the $d\sigma^{qel}/dt$ formula, and the solid lines show the resulting cross sections for the sum of elastic and quasielastic processes. It is seen that the Glauber approach extended to nucleus-nucleus scattering leads to a satisfactory consistency of the calculated cross sections and their t-dependence with those obtained experimentally. Malecki et al. [19] came to the same conclusion analyzing αα scattering data [3, 6]. Due to a rather high precision of the measurements and especially to low systematic errors of the experimental $d\sigma^{Expt}/dt$ data, the presented analysis allows one to conclude that discrepancies between the theoretical calculations and the experimental data were observed. The differences observed not only in the neighborhood of diffraxion extrema, but also in the region of the diffraction cone do not exceed 25%. The differences are, however, sufficiently small to support the statement that the contributions to the total $d\sigma/dt$ due to quasi elastic and Coulomb processes in α-nucleus scattering were properly taken into account in our calculations. The validity of this statement is obvious for low $|t|$ values for which relative contributions due to these effects are insignificant. In the region of the whole diffraction cone these contributions are also sufficiently well defined for our purpose (their uncertainties are surely not larger than one standard deviation of $d\sigma^{\text{Expt}}/dt$).

Within such an approximation nuclear elastic cross section $d\sigma^{\text{el}}/dt_i = d\sigma^{\text{Expt}}/dt_i - d\sigma^{\text{qel}}/dt_i - \Delta_i^{\text{c}}$ can be extracted from the measured differential cross section $d\sigma^{\text{Expt}}/dt$. We have used here $d\sigma^{\text{qel}}/dt$ from the previously given expression (see Section 4) and the Coulomb correction was evaluated in the optical approximation, viz. $\Delta_i^{\text{c}} = (d\sigma^{\text{el}}/dt_i)_{\text{with Coulomb}} - (d\sigma^{\text{el}}/dt_i)_{\text{without Coulomb}}$. The values of $d\sigma^{\text{el}}/dt_i$ for all targets series, obtained by the above procedure, are displayed in Fig. 5 together with the corresponding theoretical curves. The method of calculation of the theoretical cross section is described in Sec. 6. Only the statistical errors are depicted in Fig. 5.

So far elastic cross section for incident α-particles with a momentum of ~ 4.5 GeV/c per nucleon was not measured. However, taking into account a weak momentum dependence of $d\sigma^{\text{el}}/dt(t)$, one can compare ourdata for αp and αα scattering with corresponding cross sections measured in other experiments [20] at different values of incident momentum in the GeV region. The values of $d\sigma^{\text{el}}/dt_i$ for pα scattering for incident proton momenta 1.73, 1.97 and 2.5 GeV/$c$per nucleon precisely measured by Courant et al. [21] are in agreement with our results within two standard deviations. The results obtained in Ref. [21] lead however to a smaller value of the slope of the diffraction peak and $(d\sigma^{\text{el}}/dt)_{t=0}$. This fact can be attributed to different values of the incident momentum used in the two experiments.

In the case of αα scattering and at an intermediate energy the available data are scarce and come from the work by Berger et al. [3]. In their experiment the incident α particle momentum was 4.32 GeV/c and 5.07 GeV/c. The corresponding values of $d\sigma^{\text{el}}/dt_i$ are depicted in Fig. 5 by full triangles. Because of different values of the parameters — σ^{NN}, b^{NN} and ϱ^{NN} in this and our experiment one could expect sizable differences between the cross sections, but in this case the agreement turns out to be surprisingly good. Our αα data were analysed by Malecki, Picozza, and Satta [19]. The authors calculations are in good agreement with experimental data. It is worthwhile to note that the results of our own calculation and those of Malecki et al. agree with each other within the accuracy of diagram (Fig. 1) shown in [19].

On the other hand, the measured cross sections are rather different from those obtained at much higher energies. In Fig. 5 the dotted line corresponds to results for pα scattering of 45 GeV/c protons, measured at Batavia [22]. Somewhat higher ($\sim 7\%$) of this line lie the data from SPS [23] (the dashed line in Fig. 5). CERN-ISR results [24] are also shown in the same figure. The values of $d\sigma^{\text{el}}/dt$ obtained for αp scattering at $\sqrt{s} = 89$ GeV are denoted by the full rhomb and for αα scattering with $\sqrt{s} = 126$ GeV by the open rhomb. The slopes of the dotted and dashed lines are close to the value obtained in this analysis, but they give us the values of the differential cross section lower by a factor of ~ 1.5. The ISR data [24] are ~ 2.5 and 5 times lower than ours for pα and αα scattering, respectively. They do not follow a smooth curve and are in a disagreement with results of other CERN experiment [25] for $|t| > 0.23$ (GeV/c)^2.



□ – αp; Δ – αα; × – αC; ● – αAl; ○ – αCu – this experiment; ········ – P_p = 45 GeV/c, Batavia pα data [22]; – – – – – P_p = 100 GeV/c, CERN-SPS pα data [23]; ◆ – S = 89 GeV, CERN-ISR pα data [24]; ▼ – P = 4.32 and 5.07 GeV/c, Saclay αα data [3]; ◇ – P = 126 GeV, CERN-ISR αα data [24]

Fig. 5. The nuclear differential cross sections for αp, αα, αC, αAl and αCu elastic scattering at 17.9 GeV/*c*

Results of recent Proriol computation [26] for αα scattering, at ISR energy are also in disagreement with these data [24]. A recent paper of The Axial Field Spectrometer Collaboration [27] presents results of new high precision measurements of differential cross sections for αα scattering (not shown in Fig. 5). The results concerning the region of the diffraction cone and of the first minimum are inconsistent with results of [24].

For αC, αAl and αCu scattering no data are available for $p > 1$ GeV/c per nucleon for a comparison with our results.

6. *Diffraction cone*

It is seen from Fig. 5 that up to the first diffraction minimum all theoretical curves exhibit a nonlinear behaviour with a positive derivative. The experimental values of $d\sigma^{el}/dt(t)$ are well approximated by an exponential distribution.

For all the targets under consideration we have carried out a least square fitting procedure using $d\sigma^{el}/dt = (d\sigma/dt)_{t=0} * \exp(-bt)$ and taking into account all points for which $|bt_i| < 5.8$. In Table V we present the values of the parameters: $(d\sigma/dt)_{t=0}$, b and the values of χ^2 per one degree of freedom together with CL for each fit.

The above values have been subsequently used to transform the elastic differential cross sections for all the targets into the coordinate system introduced by Singh and Roy [28]: $d\sigma_j/dt/(d\sigma_j/dt)_{t=0}(b_j t)$, where j is a nuclear target index.

The corresponding points are displayed in Fig. 6 (except a few points for H and He targets which would overlap with other ones). All 57 points, for which $|b_j t_i| < 5.8$, have been used to obtain a single exponential fit with the result: $\chi^2/(N-2) = 0.95$ and CL $= 0.55$. Thus the single exponential curve seems to be good enough for nucleus-nucleus scattering to reproduce $d\sigma^{el}/dt(t)$ in the diffraction cone region (it might be a consequence of the large value of $\varrho^{NN} \approx -0.35$).

For $|bt| < 2.1$ Singh and Roy [28] have estimated the upper limit of the imaginary part of the scattering amplitude disregarding spin effects. This estimate is presented by the solid line in Fig. 6.

TABLE V

The parameters of the exponential fit to $\frac{d\sigma^{el}}{dt_i}$ in the diffraction cone region

Terget	N	$\left(\frac{d\sigma}{dt}\right)_{t=0}$ b (GeV/c)$^{-2}$	b (GeV/c)$^{-2}$	$\frac{\chi^2}{N-2}$	CL
H	22	1.158 ± 0.021	33.6 ± 0.28	1.5	0.09
He	15	10.34 ± 0.38	67.2 ± 1.0	0.74	0.70
C	10	39.0 ± 1.0	122.8 ± 1.4	0.90	0.53
Al	7	110.5 ± 7.0	190.8 ± 4.7	0.52	0.75
Cu	3	396 ± 153	353 ± 39	0.14	0.70

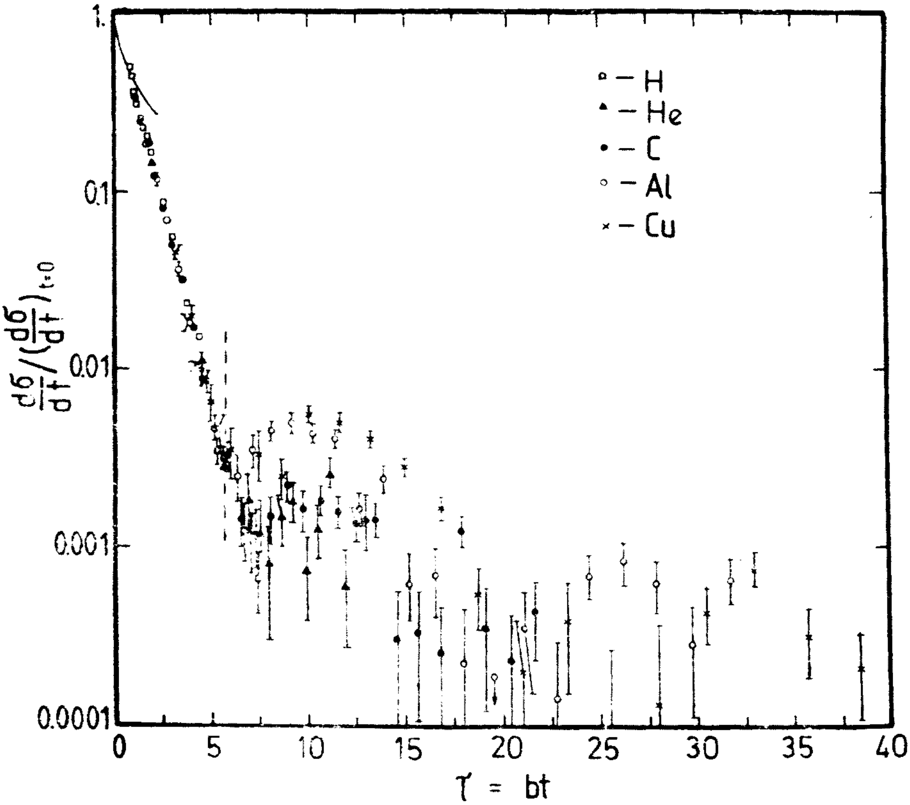


Fig. 6. The nuclear elastic differential cross sections for all targets in one coordinate system $(d\sigma/dt)/(d\sigma/dt)_{t=0}(bt)$

From the fitted values of $(d\sigma/dt)_{t=0}$ and b (cf. Table V) we have calculated: the total cross section — σ_{tot}, the elastic cross section — σ_{el}, the radius of the interaction region between the α-particle and the target nucleus — $R_{\alpha A}$ and ratio of the cross sections — k in the following approximation

$$\sigma_{tot} = \sqrt{16\pi/(1+\varrho^2)\cdot(d\sigma^{el}/dt)_{t=0}}, \qquad \sigma_{el} = (d\sigma^{el}/dt)_{t=0}/b,$$

$$R_{\alpha A} = 2\sqrt{b}, \qquad k = \sigma_{el}/\sigma_{tot}.$$

The numerical values of these parameters for all considered target nuclei are presented in Table VI. For each entry we give a statistical error (upper row) and a total error (lower row) which embodies, the statistical error, the normalization error (3%) and the systematic error caused by the error in the adopted incident momentum of the α-particle. The latter has been estimated from the errors in the value of $d\sigma/dt_i$ for each point contributing to the fitting procedure

$$\Delta(d\sigma/dt)_i = 2bt_i d\sigma/dt_i \Delta p/p, \quad (\Delta p/p = 0.008).$$

TABLE VI

The diffraction cone parameters

	$\left(\frac{d\sigma}{dt}\right)_{t=0}$ b (GeV/c)$^{-2}$	b (GeV/c)$^{-2}$	σ_{tot} barn	σ_{el} barn	$k = \frac{\sigma_{el}}{\sigma_{tot}}$	$R_{\alpha A}$ fm
H	$1.158 \pm^{0.021}_{0.041}$	$33.6 \pm^{0.028}_{0.5}$	$0.1450 \pm^{0.0013}_{0.0026}$	$0.0344 \pm^{0.0004}_{0.0026}$	$0.237 \pm^{0.0010}_{0.005}$	$2.289 \pm^{0.010}_{0.018}$
He	$10.34 \pm^{0.38}_{0.47}$	$67.2 \pm^{1.0}_{1.4}$	$0.443 \pm^{0.008}_{0.010}$	$0.154 \pm^{0.0037}_{0.006}$	$0.348 \pm^{0.0026}_{0.008}$	$3.235 \pm^{0.024}_{0.033}$
C	$39.9 \pm^{1.0}_{1.6}$	$122.8 \pm^{1.4}_{1.7}$	$0.884 \pm^{0.011}_{0.018}$	$0.325 \pm^{0.005}_{0.013}$	$0.368 \pm^{0.0018}_{0.008}$	$4.373 \pm^{0.025}_{0.031}$
Al	$111 \pm^{7}_{8}$	$191 \pm^{4.7}_{6.0}$	$1.47 \pm^{0.047}_{0.05}$	$0.579 \pm^{0.023}_{0.031}$	$0.394 \pm^{0.0044}_{0.010}$	$5.45 \pm^{0.07}_{0.08}$
Cu	$396 \pm^{150}_{150}$	$353 \pm^{39}_{39}$	$2.8 \pm^{0.5}_{0.5}$	$1.12 \pm^{0.31}_{0.31}$	$0.404 \pm^{0.036}_{0.037}$	$7.42 \pm^{0.41}_{0.41}$

The total error given in Table VI does not include possible uncertainties resulting from the procedure of the separation of the elastic cross section described in Section 5. It is rather difficult to estimate exactly the size of these uncertainties, but we expect it would be smaller than the error denoted as the total.

To evaluate σ_{tot} for α-H and α-He scattering, we take $\varrho_{\alpha p} = -0.28$ and $\varrho_{\alpha\alpha} = -0.18$ calculated from the Glauber model. The value of $\varrho_{\alpha p}$ is in good accord with that obtained at 5 GeV/c [29], viz. $\varrho_{\alpha p} = -0.30 \pm 0.12$. For the remaining targets we take: $\varrho_{\alpha C} = \varrho_{\alpha Al} = \varrho_{\alpha Cu} = 0$. The errors in σ_{tot} (given in Table VI) do not comprise errors in the adopted values for ϱ.

The measured quantities b, σ_{tot}, σ_{el} for αp, αα, and αC scattering are close to the values obtained by Jaros et al. [30] for an incident momentum of 2.9 GeV/c per nucleon.

The difference of the b values for αp scattering obtained in this paper and measured for incident momenta of 5 GeV/c [29] and 45–400 GeV/c [22] can be probably explained by its momentum dependence.

The values of parameter b obtained by Bruton et al. [31] at 18.6 GeV/c is markedly different from that obtained in this work and in the above papers.

7. *Summary and conclusions*

We have investigated α-particle scattering on H, He, C, Al and Cu nuclei for momentum transfer over a range of $0.009 < |t| < 0.22$ (GeV/c)^2.

From our measurements for the first time the sum of the differential cross sections for elastic and quasielastic scattering and for an incident momentum of 4.45 GeV/c per nucleon has been obtained (see Table II and Figs. 2, 3, 4).

We have carried out Glauber type calculations using three common approximations. The experimental values of $d\sigma^{\mathrm{Expt}}/dt$ show the best agreement for the swarm projectile model [18]. The comparison with Glauber model made possible to extract the nuclear differential cross section for elastic scattering of α particles on He, C, Al and Cu nuclei (cf. Fig. 5). For the three last targets our measurements constitute the only available data in the medium and high energy range. For all targets nuclei linear function in t approximates surprisingly well to $\ln (d\sigma/dt(t_i))$ within the diffraction cone.

Outside the diffraction cone $(d\sigma/dt)/d\sigma/dt)_{t=0}$ vs bt differs from one target to another (cf. Fig. 6) but this may be a consequence of an insufficient knowledge of the t-dependence of $d\sigma^{\mathrm{qel}}/dt$. However, the position of the first diffraction minimum is shifted towards $t = 0$ and the height of the second maximum increases with increasing target mass number. This behaviour can be caused by the change of the shape of nuclear density, from light to heavy target nuclei, even if geometrical scaling takes place.

The measured quantities $d\sigma^{\mathrm{Expt}}/dt$ (cf. Table II), b and σ_{tot} (cf. Table VI) for αp and αα scattering show no special and unreasonable deviations from the corresponding values measured at different incident momenta which are available in the literature. One exception has to be mentioned and this is the slope of the diffraction cone for pα scattering for a 18.6 GeV/c incident proton momentum [31].

As seen from Table VI, the ratio k of the cross sections increases from 0.237 for αp to 0.404 for αCu scattering what seems to indicate that this ratio may approach a value of 1/2 for a heavy nucleus.

Owing to small systematic errors in the measured cross sections $d\sigma/dt$, one can examine the accuracy of the Glauber model. The result of the model calculations differs from the experimental data by no more than 25% not only in the regions of diffraction extrema but, also in the diffraction cone region.

In our opinion, for the incident energies considered in this paper the corrections to the Glauber model most probably originate from two sources: a) the model disregards the excited states which may be formed in the intermediate states between two successive multiple scattering (inelastic screening [32]), b) the model does not take into account short-range correlations which can be of a multiquark type [33].

In conclusion we wish to point out that for αp scattering the theoretical cross sections show a tendency to lie above the experimental values whereas for all the remaining targets an opposite tendency is observed. This question has still to be resolved.

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