REDISTRIBUTION OF THE MUON AND NUCLEUS POLARIZATIONS DUE TO THE HYPERFINE INTERACTION

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The redistribution of the muon and nuclear polarizations in 1s orbit of the muonic atoms due to the hyperfine interaction is calculated for the arbitrary nuclear spin. The results due to Hambro and Mukhopadhyay (1977) are corrected and generalized.

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In the absence of any initial nuclear orientation the hyperfine interaction on the K shell of the muonic atoms leads to the additional depolarization of muons (Überall 1959) as well as to the induced nuclear polarization (Shmushkevich 1959). Shmushkevich's results, however, are not complete and need the reinterpretation because of the assumed incorrect condition on the mean value of the spin operator $|\langle s\rangle| \leqq s$. In fact this condition is valid only for $s \leqq \frac{1}{2}$. The similar mistake is contained in the contributions by Hambro and Mukhopadhyay (1975 and 1977). As was shown by Werle (1966, § 32) for arbitary s

$$\mathrm{Tr}\, \varrho^2 \leqslant 1 \Rightarrow |\langle s\rangle| \leqslant s\sqrt{\tfrac{2}{3}(s+1)}\,. \tag{1}$$

If the nuclear target is initially polarized the hyperfine interaction can repolarize the muons. In fact the hyperfine interaction leads to the redistribution of the initial muon and nuclear polarizations. This effect has been considered for the arbitrary nuclear spin j by Hambro and Mukhopadhyay (1977). However, the results of Hambro and Mukhopadhyay are valid only for $j = \frac{1}{2}$ what we would like to show in the present note. Moreover, we generalize these results by taking into account the nuclear orientations of arbitrary rank.

The hf interaction can be presented in the form $H = \Sigma\omega_F P_F$, where P_F is the projector on the subspace with the definite value of the total spin $F = j\pm\frac{1}{2}$ ($\Sigma P_F = \mathrm{id}$ and $P_E \circ P_F = \delta_{EF}P_F$) and $\{\omega_F\}$ are some real coupling constants. Using the shorthand notation $\varrho_{EF} \equiv P_E \circ \varrho \circ P_F$ for the spin density operator ϱ of the muonic atom on the K-shell, it is

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easy to see that

$$\frac{d\varrho}{dt} = i[H, \varrho] \Rightarrow \varrho_{EF}(t) = \varrho_{EF} \exp i(\omega_E - \omega_F)t. \tag{2}$$

Physically the most relevant is the time-averaged spin density operator. For the muonic atoms it is a good *approximation* that for $E \neq F$ the time averaged $\varrho_{EF}(t)$ vanishes, and effectively

$$\varrho \overset{\text{hf}}{\rightarrow} \Sigma P_F \circ \varrho \circ P_F. \tag{3}$$

Let $[j]$ denote the $(2j+1)$-dimensional irreducible SO(3) module. Then one can define the rank L Wigner tensor operator $\mathscr{W}_j^L$ by means of the relation

$$\langle\beta|\mathscr{W}_j^L(v)|\alpha\rangle \equiv \langle\beta|\alpha \otimes v\rangle \quad \text{for} \quad \forall v \in [L], \quad \alpha \in [j] \quad \text{and} \quad \beta \in [j]^*. \tag{4}$$

The right-hand side of this definition is the Clebsch-Gordan coefficient. Then we define the rank-L *spin-j* tensor operator $S_j^L \equiv c_j^L \mathscr{W}_j^L$ through the recurrence relation

$$C_{A0B0}^{L0} S_j^L = S_j^A \otimes S_j^B \circ P_{AB}^L, \tag{5}$$

where P_{AB}^L is the projector on $[L] \subset [A] \otimes [B]$. From (5) it follows that

$$C_{A0B0}^{L0} c_j^L = (-)^{L+2j} \hat{L}\hat{j} c_j^A c_j^B \begin{Bmatrix} A & B & L \\ j & j & j \end{Bmatrix}, \tag{6}$$

which gives

$$c_j^L = 2^{-L}\hat{j}^{-1} \left\{ \frac{(2j+1+L)!}{(2j-L)!} \right\}^{1/2}. \tag{7}$$

The above convention (5) differs from the one employed in our previous paper (Oziewicz 1985). The exact relation is

$$c^L(\text{previously}) = 2^{L/2} L! \, \{(2L)!\}^{1/2} \, c^L(\text{presently}).$$

We note also that c_j^L coefficient (7) is closely related to $\beta_L(j)$ introduced by Zemach (1965) and Csonka et al. (1966) using different approach,

$$(2L+1)! \, \beta_L(j) = 2^L (L! c_j^L)^2 (2j+1). \tag{8}$$

Using spin-tensor operators (5) we define rank L spin operator of the muonic atom

$$S_{ef}^L \equiv S_{1/2}^e \otimes S_j^f \circ P_{ef}^L. \tag{9}$$

Let us denote $\langle S_{ef}^L \rangle_0 \equiv \operatorname{Tr}(S_{ef}^L \circ \varrho)$ and

$$\langle S_{ef}^L \rangle \equiv \operatorname{Tr}(S_{ef}^L \circ \Sigma P_F \circ \varrho \circ P_F). \tag{10}$$

Here ϱ is the spin-density operator of the muonic atom on the K-shell *before* the hf interaction was switched on. Then according to the discussion in (Oziewicz 1985, § 8) it is easy to derive the main formula of this note (one should insert (108) into the first equation (111) in (Oziewicz 1985)

$$\langle S^L_{0L}\rangle = \left(1-\frac{L(L+1)}{(2j+1)^2}\right)\langle S^L_{0L}\rangle_0+\frac{L+1}{2L+1}\sqrt{L(2L-1)}\left(1-\frac{L^2}{(2j+1)^2}\right)\langle S^L_{1L-1}\rangle_0$$

$$+\frac{4L}{2L+1}\,\frac{\sqrt{(L+1)(2L+3)}}{(2j+1)^2}\langle S^L_{1L+1}\rangle_0. \tag{11}$$

In order to discuss this result and to compare with one presented by Hambro and Mukhopadhyay (1977) one should point out that S^1_{01} is the spin operator of the nucleus and S^1_{10} is the spin operator of the muon. Obviously for the total spin $\mathscr{F}\equiv S^1_{10}+S^1_{01}$ we have

$$\langle\mathscr{F}\rangle = \langle\mathscr{F}\rangle_0. \tag{12}$$

The last equation follows from the fact that $[\mathscr{F}, P_F]=0$ and has nothing to do with the hf interaction suggested incorrectly by Hambro and Mukhopadhyay (1977).

Formula (11) gives the arbitrary rank L final nuclear polarization due to hf interaction. Combining (11) with (12) we get the corresponding expression for the final muon polarization

$$\langle S^1_{10}\rangle = \tfrac{1}{3}\left(1+\frac{2}{(2j+1)^2}\right)\langle S^1_{10}\rangle_0+\frac{2}{(2j+1)^2}\langle S^1_{01}\rangle_0-\tfrac{4}{3}\cdot\frac{\sqrt{10}}{(2j+1)^2}\langle S^1_{12}\rangle_0. \tag{13}$$

The above formulae (11) and (13) describe the redistribution of the muon and nucleus polarizations due to the hf interaction. These formulae are correcting and generalizing the results of Hambro and Mukhopadhyay (1977). In particularly two statements by Hambro and Mukhopadhyay (1977) should be corrected.

1. In the absence of any initial nuclear polarization (for nuclei with spin j) the nuclei are polarized by $\frac{2}{3}\left(1-\frac{1}{(2j+1)^2}\right)$, (i.e. by $\frac{16}{27}$ for $j=1$) of the polarization the muons had when they entered the 1s orbit. This follows from formula (11) with $L=1$.
2. In the limit $j\to\infty$ formula (11) gives

$$\langle S^L_{0L}\rangle = \langle S^L_{0L}\rangle_0+\frac{L+1}{2L+1}\sqrt{L(2L-1)}\,\langle S^L_{1L-1}\rangle_0\,.$$

This means that also in this limit the nuclear polarization *does change.*

For the calculations of the muon and nucleus polarizations in *each* of the hyperfine states it is convenient to use the model independent relations (Oziewicz 1985, formulae (112)–(113))

$$P_F\circ S^1_{01}\circ P_F = \left(1-\frac{2(F-j)}{2j+1}\right)\mathscr{F}\circ P_F,$$

$$P_F\circ S^1_{10}\circ P_F = \frac{2(F-j)}{2j+1}\mathscr{F}\circ P_F. \tag{14}$$

These relations are compatible with the Shmushkevich's result (1959). In the absence of any initial nuclear polarization one can get (Überall 1959)

$$\langle \mathscr{F} \circ P_F \rangle = \left\{ \tfrac{1}{2} + \tfrac{1}{3}(F-j)\left(2j+1+\frac{2}{2j+1} \right) \right\} \langle S^1_{10} \rangle_0. \tag{15}$$

Using the results of (Oziewicz 1985, § 7–8) one can easily generalize (15) for the arbitrary initial nuclear polarization.

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