ON THE ABSORPTIVE POTENTIAL AND THE MEAN FREE PATH OF A NUCLEON IN NUCLEAR MATTER*

By J. Dąbrowski and W. Piechocki

Institute for Nuclear Studies, Warsaw**

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A simple expression for the nuclear absorptive potential W in nuclear matter in terms of free NN cross section is derived from the Brueckner theory and tested against "exact" results. It is a modification of the semiclassical expression, which takes into account the Pauli blocking and dispersive effects. It is used to calculate W and the nucleon mean free path λ in nuclear matter for nucleon energies up to $\sim$ 200 MeV. Results obtained with non-locality corrections agree reasonably well with experiment. The dependence of W and λ on the temperature of nuclear matter is also discussed.

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1. Introduction

The absorption of a nucleon in the nuclear medium plays an important role in several nuclear phenomena. It may be characterized by the imaginary part W of the nuclear optical potential or by the directly related quantity, the nucleon mean free path λ. The magnitude of λ is decisive for choosing an appropriate approximation in describing a given phenomenon. A long λ suggests an independent particle approximation, whereas for a short λ the fluid limit is approached.

We shall restrict ourselves to the case of nuclear matter, i.e., of an homogeneous nuclear medium with equal number of neutrons and protons. (We shall not consider the internal structure of nucleons, or more precisely, we shall disregard any modification of this structure in nuclear matter.) There exist numerous calculations of W in nuclear matter (see, e.g., [1], [2], and [3] and references therein), which start from the NN interaction and lead to results for both the real and imaginary parts, V and W, of the optical potential. Calculations of this type are complicated since they involve the whole machinery of the many-body theory. In the case of V, this appears unavoidable. If however, one is

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** Address: Instytut Problemów Jądrowych, Hoża 69, 00-681 Warszawa, Poland.

interested only in W, one may follow a much simpler procedure which goes back to the early paper by Lane and Wandel [4], and relate W directly to the NN cross section.

To explain this simple procedure of determining W, let us consider a nucleon "0" (a neutron) with momentum $\boldsymbol{k}_0$ (in units of $\hbar$), moving through nuclear matter of density ϱ. The probability per unit time w of changing its momentum is given by

$$w = \varrho\langle v\bar{\sigma}\rangle_{\mathrm{Av}} = \frac{2\hbar}{M}\,\varrho\langle k\bar{\sigma}\rangle_{\mathrm{Av}}, \tag{1.1}$$

where $\bar{\sigma}$ is the total average NN cross section,

$$\bar{\sigma} = \tfrac{1}{2}(\sigma_{\mathrm{nn}}+\sigma_{\mathrm{np}}), \tag{1.2}$$

and v is the relative velocity between the projectile nucleon "0" and the target nucleon "1" of nuclear matter (with momentum $\boldsymbol{k}_1$),

$$v = |\hbar\boldsymbol{k}_0/M - \hbar\boldsymbol{k}_1/M| = 2\hbar k/M, \tag{1.3}$$

where

$$\boldsymbol{k} = (\boldsymbol{k}_0 - \boldsymbol{k}_1)/2 \tag{1.4}$$

is the "0"–"1" relative momentum. The averaging over nucleon momenta (i.e., over $\boldsymbol{k}_1$) in nuclear matter is denoted by $\langle\rangle_{\mathrm{Av}}$.

The lifetime of the $\boldsymbol{k}_0$ state of nucleon "0" is $\tau = 1/w$, and the mean free path

$$\lambda = v_0\tau = v_0/\varrho\langle v\bar{\sigma}\rangle_{\mathrm{Av}} = k_0/2\varrho\langle k\bar{\sigma}\rangle_{\mathrm{Av}}, \tag{1.5}$$

where $v_0 = \hbar k_0/M$ is the velocity of the "0" nucleon.

The absorptive potential W is connected with w by the relation

$$W = -\tfrac{1}{2}\hbar w = -\frac{\hbar^2}{M}\,\varrho\langle k\bar{\sigma}\rangle_{\mathrm{Av}}, \tag{1.6}$$

which follows directly from the time dependent Schroedinger equation for the motion of the "0" nucleon in the optical potential $V+iW$ (see, e.g., p. 213 of [5]).

Semiclassical expressions (1.6) and (1.5) for W and λ are oversimplified and lead to erroneous results. For instance, for $k_0 = 1.8\ \mathrm{fm}^{-1}$ (which corresponds to the energy of about 25 MeV of the incoming nucleon) expression (1.6) gives $W \cong -50$ MeV, whereas the empirical value of W at this energy is about -5 MeV. The point is that expressions (1.6) and (1.5) disregard important many-body effects: the Pauli blocking and dispersive effects (momentum dependence of V).

It is very easy to incorporate both these effects by a simple modification of expression (1.6).

Let us denote by $\boldsymbol{k}_0'$ and $\boldsymbol{k}_1'$ the final momenta of the nucleons "0" and "1" after their scattering, and introduce the exclusion principle operator Q which vanishes whenever k_0' or k_1' (or both) are smaller than the Fermi momentum k_{F} of nuclear matter (otherwise

$Q = 1$). To take into account the Pauli blocking in (1.6), we simply replace $\langle k\bar{\sigma}\rangle_{\mathrm{Av}}$ by $\langle Qk\bar{\sigma}\rangle_{\mathrm{Av}}$.

For the nucleon single particle (s.p.) energies in the final states (i.e., states above the Fermi sea), we assume the effective mass approximation,

$$e(k'_{\mathrm{N}}) = \hbar^2 k'^2_{\mathrm{N}}/2M^* + \mathrm{const}, \tag{1.7}$$

which is the simplest way of representing the dispersive effects. Now, the probability w is proportional to the density of final states, which in turn is proportional to the effective mass M^*. Consequently, to take into account the dispersive effects, we multiply expression (1.6) by M^*/M.

Our modified expression for W is then:

$$W = -\frac{\hbar^2}{M}(M^*/M)\varrho\langle Qk\bar{\sigma}\rangle_{\mathrm{Av}}. \tag{1.8}$$

The scattering in nuclear matter of the projectile nucleon "0" on the target nucleon "1" obeys the energy conservation:

$$e(k_0)+e(k_1) = e(k'_0)+e(k'_1), \tag{1.9}$$

where the function $e(k_{\mathrm{N}})$, which determines the s.p. energy of a nucleon with momentum k_{N}, in general is different for states in the Fermi sea ($k_{\mathrm{N}} = k_1$) and for the states above the Fermi sea ($k_{\mathrm{N}} = k_0,\ k'_0,\ k'_1$) and consequently

$$k' < k, \tag{1.10}$$

where k' is the final relative momentum. (Only if the same form of e, Eq. (1.7) with a constant M^*, is assumed also for the states in the Fermi sea, one gets the equality $k = k'$, typical for the free NN scattering.)

As we shall see in the next Section, expression (1.8) can be derived from the Brueckner theory. Thus corrections to this expression, in principle, are calculable. Still, it is difficult to estimate precisely the accuracy of expression (1.8). In the present paper, we restrict ourselves to a comparison of the results for W obtained with the help of expression (1.8) with the results of J.-P. Jeukenne et al. [6] who calculated W by applying the Brueckner theory. The result of this comparison is encouraging. Needless to say, however, that the accuracy of the scheme applied in [6] is not quite certain either. Actually we believe that expression (1.8) is quite reliable. It is intuitively simple, its accuracy is probably not worse than that of the more sophisticated approaches, and because of its simplicity no additional approximations — often hidden in the more sophisticated approaches — are required.

Expression (1.8) with $M^* = M$ is the original expression used by Lane and Wandel [4]. It takes into account the Pauli blocking but ignores the dispersive effects. Expression (1.8) with a momentum dependent effective mass was used in [7]. However, within the approximations applied there, relation (1.10) was replaced by $k = k'$. In this way one of the consequences of the dispersive effects was disregarded. Recently, expression (1.8) in the case of $k = k'$ was discussed by Köhler [8].

The purpose of the present paper is to justify expression (1.8), and to utilize its simplicity in discussing the absorptive optical potential and the nucleon mean free path in nuclear matter, and their dependence on the nucleon energy, and on the density and temperature of nuclear matter.

Before comparing our results with experiment, we take into account the necessary non-locality corrections, pointed out by Fantoni, Friman and Pandharipande [9], and by Negele and Yazaki [10] (see also [11]). This correction is particularly important in calculating the nucleon mean free path.

The paper is organized as follows. In Section 2, we derive the simple expression for the absorptive potential, Eq. (1.8), from the Brueckner theory. In Section 3, we adjust phenomenologically the parameters of the s.p. energies in nuclear matter for states in and above the Fermi sea, and apply our s.p. energies in the energy conservation equation for NN collisions in nuclear matter. In Section 4, we describe the simple parametrization of the NN cross section, used in our calculations. In Section 5, we test expression (1.8) against the "exact" results of J.-P. Jeukenne et al. [6]. In Section 6, we define the equivalent local absorptive potential and derive the expression for the nucleon mean free path in nuclear matter. In Section 7, we present our results for the absorptive potential and for the mean free path, and compare them with experiment. In Section 8, we discuss the temperature dependence of the absorptive potential and of the mean free path. Section 9 contains our conclusions and final comments. In Appendix A, the effect of the anisotropy of the NN cross section on the expression for W is described. In Appendix B, we derive the expression for the angle averaged exclusion principle operator for finite temperatures.

2. *The absorptive potential W*

We start with the Brueckner theory expression for the s.p. energy of the "0" nucleon in nuclear matter (see, e.g., [3]):

$$\tilde{e}(k_0) = \hbar^2 k_0^2/2M + \mathscr{V}(k_0), \tag{2.1}$$

where the s.p. (optical) potential

$$\mathscr{V}(k_0) = V(k_0) + iW(k_0) = \int^{<k_F} \frac{d\boldsymbol{k}_1}{(2\pi)^3} \langle \boldsymbol{k} | \tfrac{1}{4} \operatorname{Tr}_{\sigma\tau} \mathscr{K}(1-P_{01}) | \boldsymbol{k} \rangle$$

$$= \tfrac{1}{2} \sum_T (2T+1) \sum_{Sm_s} \int^{<k_F} \frac{d\boldsymbol{k}_1}{(2\pi)^3} \langle \boldsymbol{k} S m_s T | \mathscr{K} | \boldsymbol{k} S m_s T \rangle, \tag{2.2}$$

where P_{01} is the exchange operator. Since $\mathscr{V}$ does not depend on the spin and isospin quantum numbers of the "0" nucleon, we may extend the summation over the spin-isospin states of nucleon "1" to those of nucleon "0" and introduce a factor 1/4. This leads to the appearance of $\frac{1}{4}\operatorname{Tr}_{\sigma\tau}$. This trace in the spin-isospin space of the two nucleons is performed n the represen tation of total spin S, its z-component m_s, and the total isospin T (notice ihat the matrix elements of $\mathscr{K}$ do not depend on T_3).

The reaction matrix $\mathscr{K}$ is defined by

$$\mathscr{K} = v+v\frac{Q}{\alpha+i\eta}\mathscr{K}, \tag{2.3}$$

where v is the NN potential, Q is the exclusion principle operator (a projection operator onto states above the Fermi sea), and

$$\alpha = e(k_1)+e(k_0)-e(k_1')-e(k_0'), \tag{2.4}$$

where

$$e(k_N) = \text{Re}\,\{\tilde{e}(k_N)\} = \varepsilon(k_N)+V(k_N), \tag{2.5}$$

where $\varepsilon(k_N) = \hbar^2 k_N^2/2M$, and k_0' and k_1' are momenta in the intermediate states. Singularities in Q/α, i.e., real energy conserving transitions $N_0N_1 \to N_0'N_1'$ are expected to occur. The infinitesimal parameter $+i\eta$ guarantees that only outgoing waves appear in states degenerate with our initial state (ground state of nuclear matter + nucleon "0" with momentum $\boldsymbol{k}_0$). This means, we consider the decay of our initial state, i.e., the absorption of the nucleon "0" in nuclear matter.

From Eq. (2.3), we get for the hermitian conjugate matrix $\mathscr{K}^+$ the equation:

$$\mathscr{K}^+ = v+\mathscr{K}^+\frac{Q}{\alpha-i\eta}v, \tag{2.6}$$

from which we have

$$v = \left[1+\mathscr{K}^+\frac{Q}{\alpha-i\eta}\right]^{-1}. \tag{2.7}$$

By inserting this expression for v into Eq. (2.3), we obtain the identity

$$\mathscr{K}-\mathscr{K}^+ = \mathscr{K}^+\left[\frac{Q}{\alpha+i\eta}-\frac{Q}{\alpha-i\eta}\right]\mathscr{K} = -2i\pi\mathscr{K}^+Q\delta(\alpha)\mathscr{K}, \tag{2.8}$$

which leads immediately to the optical theorem:

$$-2\,\text{Im}\,\langle \boldsymbol{k}Sm_sT|\mathscr{K}|\boldsymbol{k}Sm_sT\rangle = (2\pi)^{-2}\int d\boldsymbol{k}'Q(\boldsymbol{K},\boldsymbol{k}')\delta(\alpha)$$
$$\times\sum_{m'_s}|\langle \boldsymbol{k}'Sm_s'T|\mathscr{K}|\boldsymbol{k}Sm_sT\rangle|^2. \tag{2.9}$$

Here $\boldsymbol{K}$ is the conserved total momentum,

$$\boldsymbol{K} = \boldsymbol{k}_0+\boldsymbol{k}_1 = \boldsymbol{k}_0'+\boldsymbol{k}_1', \tag{2.10}$$

and

$$\boldsymbol{k}' = (\boldsymbol{k}_0'-\boldsymbol{k}_1')/2 \tag{2.11}$$

is the relative momentum in the intermediate state. By $Q(\boldsymbol{K}, \boldsymbol{k}')$, we denote the exclusion principle operator in the total and relative momentum representation,

$$Q(\boldsymbol{K}, \boldsymbol{k}') = [1-n_0(k'_0)]\,[1-n_0(k'_1)]$$
$$= [1-n_0(|\tfrac{1}{2}\boldsymbol{K}+\boldsymbol{k}'|)]\,[1-n_0(|\tfrac{1}{2}\boldsymbol{K}-\boldsymbol{k}'|)], \tag{2.12}$$

where n_0 is the s.p. distribution function of the ground state,

$$n_0(k_N) = \theta(k_F-k_N). \tag{2.12'}$$

We approximate $Q(\boldsymbol{K}, \boldsymbol{k}')$ by its angle average:

$$Q(\boldsymbol{K}, \boldsymbol{k}') \cong Q(K, k') = (4\pi)^{-1} \int d\hat{K} Q(\boldsymbol{K}, \boldsymbol{k}'). \tag{2.13}$$

The explicit expression for $Q(K, k')$ is given in Eqs (B.5a, b) of Appendix B. Notice that without approximation (2.13) Q would introduce into the $\mathscr{K}$ matrix equation a coupling of different orbital angular momenta [12], [13].

In the effective mass approximation, explained later,

$$\alpha = -\hbar^2 k'^2/M\nu + \text{terms indep. of } k', \tag{2.14}$$

where $\nu = M^*/M$ is the ratio of the effective to the real nucleon mass for states above the Fermi sea. With this form of α, we can perform the k' integration in (2.9), and obtain

$$\operatorname{Im} \langle \boldsymbol{k}Sm_sT|\mathscr{K}|\boldsymbol{k}Sm_sT\rangle = -\tfrac{1}{4}\,\nu \frac{Mk'}{(2\pi\hbar)^2}\, Q(K, k')$$
$$\times \sum_{m'_s} \int d\hat{k}' |\langle \boldsymbol{k}'Sm'_sT|\mathscr{K}|\boldsymbol{k}Sm_sT\rangle|^2, \tag{2.15}$$

where k' is determined by the energy conservation equation $\alpha = 0$.

Let us introduce the total NN cross section in nuclear matter in the isotopic spin state T:

$$\sigma^{\mathrm{NM}}(T) = \tfrac{1}{4} \sum_{Sm_s} \sum_{m'_s} \frac{k'}{k} \left(\frac{M}{4\pi\hbar^2}\right)^2 \int d\hat{k}' |\langle \boldsymbol{k}'Sm'_sT|\mathscr{K}|\boldsymbol{k}Sm_sT\rangle|^2, \tag{2.16}$$

and the np and nn total cross sections in nuclear matter:

$$\sigma_{\mathrm{np}}^{\mathrm{NM}} = \sigma^{\mathrm{NM}}(T=1)+\sigma^{\mathrm{NM}}(T=0), \tag{2.17}$$
$$\sigma_{\mathrm{nn}}^{\mathrm{NM}} = 2\sigma^{\mathrm{NM}}(T=1). \tag{2.18}$$

Because of the antisymmetrization, the nn scattering amplitude is equal 2 times the $T = 1$ scattering amplitude, and the nn differential cross section is equal 4 times the $T = 1$ differential cross section. However, because the two neutrons are indistinguishable, the total nn cross section is obtained from the nn differential cross section by integration over all directions of $\hat{k}'$ and division by 2. Consequently, the factor $4/2 = 2$ appears in (2.18).

If we put in Eq. (2.3) $Q = 1$ and $\alpha = \alpha_0$ (where α_0 is defined as α, Eq. (2.4), with all the s.p. energies e replaced by pure kinetic energies ε), then Eq. (2.3) becomes an equation for the free NN scattering matrix $\mathscr{K}_0$, and (2.17–18) become expressions for total (elastic) np and nn cross sections, σ_{np} and σ_{nn}, for isolated np and nn pairs.

Now, we may write Eq. (2.15) as

$$\mathrm{Im} \sum_{Sm_s} \langle kSm_sT|\mathscr{K}|kSm_sT\rangle = -4\hbar^2 \nu \frac{k}{M} Q(K, k')\sigma^{\mathrm{NM}}(T). \tag{2.19}$$

This expression inserted into Eq. (2.2) leads to the result:

$$W(k_0) = -2\hbar^2 \nu \int^{<k_F} \frac{dk_1}{(2\pi)^3} Q(K, k') \frac{k}{M} \sum_T (2T+1)\sigma^{\mathrm{NM}}(T)$$

$$= -4\hbar^2 \nu \int^{<k_F} \frac{dk_1}{(2\pi)^3} Q(K, k') \frac{k}{M} \bar{\sigma}^{\mathrm{NM}}, \tag{2.20}$$

where

$$\bar{\sigma}^{\mathrm{NM}} = (\sigma_{np}^{\mathrm{NM}} + \sigma_{nn}^{\mathrm{NM}})/2. \tag{2.21}$$

Now, we make the crucial approximation:

$$\bar{\sigma}^{\mathrm{NM}} \cong \bar{\sigma} = (\sigma_{np} + \sigma_{nn})/2, \tag{2.22}$$

which enables us to express W through the experimental cross sections σ_{np} and σ_{nn}:

$$W(k_0) = -4\hbar^2 \nu \int^{<k_F} \frac{dk_1}{(2\pi)^3} Q(K, k') \frac{k}{M} \bar{\sigma} = -\frac{\hbar^2}{M} \nu \varrho \langle Q(K, k') k \bar{\sigma} \rangle_{\mathrm{Av}}. \tag{2.23}$$

This expression is identical with expression (1.8) for W, discussed in Section 1. It differs from the semiclassical expression, Eq. (1.6), by the presence of the exclusion principle operator Q, and the effective mass factor ν which together with the energy conservation equation $\alpha = 0$ takes care of the dispersive effects.

Approximation (2.22) amounts to replacing $\mathscr{K}$ by $\mathscr{K}_0$ in (2.20). In principle, one could calculate corrections to this approximation, using the equation

$$\mathscr{K} = \mathscr{K}_0 + \mathscr{K}_0[Q/(\alpha + i\eta) - 1/(\alpha_0 + i\eta)]\mathscr{K}. \tag{2.24}$$

Attempts in this direction were made in [7]. Still, it is hard to present a fully convincing justification of approximation (2.22), especially at low energies which we consider. For this reason the first approach to the problem of W [4], in which approximation (2.22) was applied, was called the "frivolous model". In the present paper, in Section 5, we shall test approximation (2.22) by comparing our results for W with the "exact" results of Jeukenne et al. [6].

3. The s.p. energies and the energy conservation in the NN collisions in nuclear matter

The energy conservation equation,

$$\alpha = e(k_0)+e(k_1)-e(k'_0)-e(k'_1) = 0, \tag{3.1}$$

and the total momentum $\boldsymbol{K}$ conservation, Eq. (2.10), enable us to determine the final relative momentum k', Eq. (2.11), in the NN scattering in nuclear matter in terms of the initial momenta.

For the s.p. energy $e(k_N)$, Eq. (2.5), we assume the effective mass approximation:

$$e(k_N) = \begin{cases} \varepsilon(k_N)/\mu+D & \text{for} \quad k_N < k_F, \\ \varepsilon(k_N)/\nu+C & \text{for} \quad k_N > k_F, \end{cases} \tag{3.2}$$

where the ratio of the effective to the real nucleon mass, M^*/M, is denoted by μ for $k_N < k_F$ and by ν for $k_N > k_F$.

We determine μ and D from two conditions:

$$\tfrac{1}{2}\langle \varepsilon(k_N)+e(k_N)\rangle_{Av} = E/A, \tag{3.3}$$

$$e(k_F) = \partial E/\partial A = E/A+\tfrac{1}{3}k_F d(E/A)/dk_F, \tag{3.4}$$

where E/A is the energy per nucleon of nuclear matter. Eq. (3.3) is the Brueckner theory expression for E/A, and Eq. (3.4) represents the assumption that $e(k_F)$ is equal to the nucleon separation energy.

We assume the validity of the two conditions with the "empirical" value of E/A which is a function of the density ϱ, or of the Fermi momentum k_F ($\varrho = 2k_F^3/3\pi^2$). In the limit of very small density, the interaction energy vanishes and E/A approaches $3\varepsilon(k_F)/5$. Furthermore, we know that E/A attains its minimal value of ε_{vol} (the volume energy of nuclear matter) at the equilibrium density $\varrho = \varrho_0$ ($k_F = k_{F0}$). To satisfy these requirements, we make the Ansatz:

$$E/A \equiv f(k_F) = \tfrac{3}{5}\varepsilon(k_F)+b(k_F/k_{F0})^3+c(k_F/k_{F0})^4, \tag{3.5}$$

with

$$b = 4\varepsilon_{vol}-\tfrac{6}{5}\varepsilon(k_{F0}), \qquad c = \tfrac{3}{5}\varepsilon(k_{F0})-3\varepsilon_{vol}. \tag{3.6}$$

In the present paper, we use the values: $k_{F0} = 1.35\ \mathrm{fm}^{-1}$ and $\varepsilon_{vol} = -15.8$ MeV. The resulting values of b and c are: $b = -108.5$ MeV and $c = 70.1$ MeV.

Notice that the nuclear compressibility obtained with expression (3.5), $k_{F0}(d^2f/dk_F)_0 = 6\varepsilon(k_{F0})/5-12\varepsilon_{vol} = 240$ MeV, agrees nicely with empirical estimates (see, e.g., the review by Blaizot [14]). If we replaced the last term in (3.5) by a term proportional to a higher power of k_F/k_{F0}, the resulting compressibility would be too big (compare [15]).

With E/A given by Eq. (3.5), Eqs (3.3) and (3.4) lead to the following results for D and μ:

$$D = 2b(k_F/k_{F0})^3+\tfrac{3}{2}c(k_F/k_{F0})^4, \tag{3.7}$$

$$\mu = 1/[1+(\mu_0^{-1}-1)(k_F/k_{F0})^2], \tag{3.8}$$

where

$$\mu_0 = \mu(k_{F0}) = 1/[1+\tfrac{5}{6}\, c/\varepsilon(k_{F0})]. \tag{3.9}$$

Values of D and μ at $k_F = k_{F0}$ are: $D_0 = -112$ MeV, $\mu_0 = 0.4$.

The constant C of the s.p. potential for $k_N > k_F$ is fixed by the assumed continuity of $e(k_N)$ at $k_N = k_F$:

$$C = (1/\mu - 1/\nu)\varepsilon(k_F) + D. \tag{3.10}$$

To estimate ν, we notice that Eq. (3.2) implies the following dependence of $V = e - \varepsilon$ on e:

$$V = (1-\nu)e + \nu C. \tag{3.11}$$

On the other hand, the optical model analysis of nucleon-nucleus scattering for energies below 100 MeV suggests that the real part of the optical model potential at the center of nuclei, i.e. V, can be represented (see, e.g., p. 237 of [5]) by

$$V = 0.3e - 52 \text{ MeV}. \tag{3.12}$$

Comparing (3.12) and (3.11), we conclude that $\nu_0 = \nu(k_{F0}) = 0.7$. With this value of ν_0, we get for $\nu_0 C_0 = -49$ MeV ($C_0 = -70$ MeV) which is reasonably close to the value of -52 MeV of Eq. (3.12).

For the dependence of ν on ϱ, we use the form [15]

$$\nu = 1/[1+(\nu_0^{-1}-1)\varrho/\varrho_0]. \tag{3.13}$$

With $e(k_N)$ given by Eq. (3.2), energy conservation equation (3.1) and the total momentum conservation give (notice that $k_1 < k_F$, and k_0, k_0', and k_1' are bigger than k_F):

$$k' = \{k^2 - \tfrac{1}{2}(\nu/\mu - 1)(k_F^2 - k_1^2)\}^{1/2}. \tag{3.14}$$

Contrary to the case of free NN scattering, $k' < k$, except for the case of $k_1 = k_F$, in which $k' = k$. Obviously $k' \equiv k$ if $\nu = \mu$, i.e., if the s.p. energies are purely quadratic functions of nuclear momenta.

For $k' < k$ the Pauli principle blocks more of the final nucleon states and thus reduces $|W|$ more strongly than in the case of $k' = k$. To illustrate it, let us consider the case of $k_1 = 0$, and $k_0 = 1.8\, k_F$ ($e = 26$ MeV) for $k_F = k_{F0}$. The final nucleon momenta are

$$\left.\begin{matrix} k_0' \\ k_1' \end{matrix}\right\} = (\tfrac{1}{4}K^2 + k'^2 \pm Kk'x)^{1/2}, \tag{3.14'}$$

where $x = \hat{K}\hat{k}'$, $K = k_0$ and k' is determined by Eq. (3.14) with $k_1 = 0$. The results obtained for k_0' and k_1' as functions of x are shown as solid curves in Fig. 1 which also contains the corresponding results (broken curves) obtained under the assumption that $\mu = \nu$ ($k' = k$). The Pauli principle requires that both k_0' and k_1' are bigger than k_F. Thus only the ranges of x values indicated by the corresponding arrows are allowed, and we notice that the range in the case of $\mu < \nu$ (solid arrow) is smaller than the range in the case of $\mu = \nu$ (broken arrow). Notice that any form of $e(k_N)$ with an effective mass $M^*(k_N) = \hbar^2 k_N/(\partial e/\partial k_N)$ increasing with increasing k_N would lead to a similar situation.

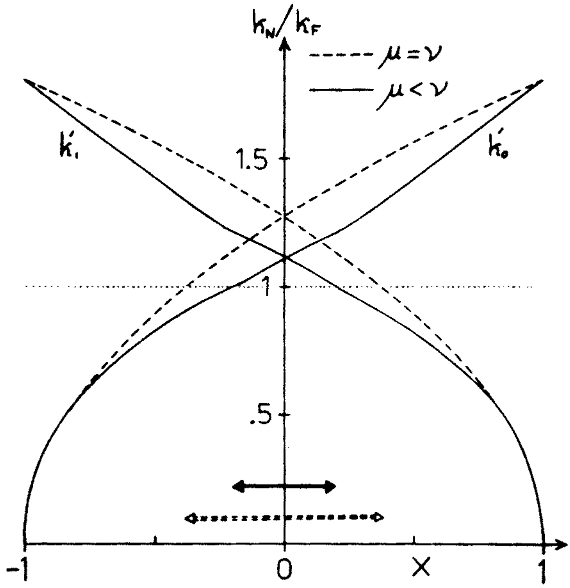

Fig. 1. Final nucleon momenta as functions of $x = \hat{K}\hat{k}'$ for $k_1 = 0$ and $k_0 = 1.8\ k_F$ at $k_F = 1.35\ \mathrm{fm}^{-1}$

To calculate $W(k_0)$, Eq. (2.23), we have to perform the $\boldsymbol{k}_1$ integration. The cross section $\bar{\sigma}$ is a function of k. The arguments k' and K of the Q operator are determined by k and $k_1 : k'$ by expression (3.14), and K by

$$K = 2\{\tfrac{1}{2}(k_1^2+k_0^2)-k^2\}^{1/2}. \tag{3.15}$$

Thus the $\boldsymbol{k}_1$ integration involves a function of k_1 and k, and may be performed according to the formula:

$$\int d\boldsymbol{k}_1 F(k_1, k) = \frac{8\pi}{k_0}\int_0^{k_F} dk_1 k_1 \int_{|k_0-k_1|/2}^{(k_0+k_1)/2} dk k F(k_1, k). \tag{3.16}$$

4. *The NN cross section*

For the total elastic cross sections, σ_{nn} and σ_{np}, we use the parametrization [17]

$$\sigma_{nn} = (1.063/\beta^2-2.992/\beta+4.29)\ \mathrm{fm}^2,$$
$$\sigma_{np} = (3.41/\beta^2-8.22/\beta+8.22)\ \mathrm{fm}^2, \tag{4.1}$$

where β is the relative NN velocity in units of c,

$$\beta = v/c = (2E_L/Mc^2)^{1/2} = 2\hbar k/Mc, \tag{4.2}$$

where the laboratory energy E_L is the kinetic energy of the scattered nucleon in the rest frame of the target nucleon.

We apply parametrization (4.1) for laboratory energies $E_L > 20$ MeV, and for $E_L < 20$ MeV the effective range approximation,

$$\sigma_{nn} = 2\pi/[k^2+(1/a_{snn}-r_{snn}k^2/2)^2],$$
$$\sigma_{np} = \pi\{1/[k^2+(1/a_{snp}-r_{snp}k^2/2)^2]+3/[k^2+(1/a_{tnp}-r_{tnp}k^2/2)^2]\}, \quad (4.3)$$

with the following values (in fm) of the respective singlet (s) and triplet (t) scattering lengths (a) and effective ranges (r): $a_{snn} = -16.1$, $r_{snn} = 3.2$, $a_{snp} = -23.714$, $r_{snp} = 2.704$, $a_{tnp} = 5.4$, and $r_{tnp} = 1.73$.

A comparison with elastic NN scattering data listed in [18] and [19] shows that parametrization (4.1) is correct up to $E_L \sim 400$ MeV, i.e., well above the threshold for π production (300 MeV).

We shall calculate W, Eq. (2.23), for $e \lesssim 200$ MeV, i.e., for $k_0 \lesssim 3$ fm^{-1} (see Eq. (3.2)). For $k_0 = 3$ fm^{-1}, the maximum relative momentum needed in calculating W is (see Eq. (3.16)) $k_{max} = (k_0+k_F)/2 = 2.2$ fm^{-1}. The corresponding laboratory energy $E_L^{max} = 2\hbar^2k_{max}^2/M \simeq 400$ MeV. The corresponding average quantities are: $k_{Av} = 0.5(k_0^2 + 0.6k_F^2)^{1/2} = 1.6$ fm^{-1}, $E_L^{Av} = 212$ MeV. Both E_L^{max} and E_L^{Av} lie within the range of validity of parametrization (4.1).

5. *Test of the approximation*

To test our approximate expression (2.23) for $W(k_0)$ (with Eq. (3.14) for k'), we apply it to the case of the Reid hard core NN interaction [20], and compare our results with those of Jeukenne, Lejeune, and Mahaux [6] who calculated W using Brueckner theory and the Reid hard core interaction. We shall refer to the results of [6] as to the "exact" results.

In our test calculation, we adopt for the Fermi momentum the value $k_F = k_{F0} = 1.4$ fm^{-1} used in [6]. The corresponding values of the parameters of the s.p. energy are: $\mu_0 = 0.4$, $D_0 = -116$ MeV, $\nu_0 = 0.7$, and $\nu_0 C_0 = -52$ MeV.

The σ_{nn} and σ_{np} cross sections have been obtained from the nuclear bar phase shifts of the Reid hard core potential, given in [20] for $20 < E_L < 350$ MeV. Similarly as in the "exact" calculations of [6], all S and D partial waves with $J \leqslant 2$ and P waves were included. For $E_L < 20$ MeV, expressions (4.3) were applied with $a_{snn} = a_{snp} = -16.7$ fm, $r_{snn} = r_{snp} = 2.87$ fm, $a_{tnp} = 5.397$ fm, and $r_{tnp} = 1.724$ fm, which are the values of the scattering parameters of the Reid hard core potential (actually, the shape parameters P_s and P_t given in [20] were also included). To stay within the range of $E_L < 350$ MeV, considered in [20], we keep $k_0 \lesssim 2.7$ fm^{-1}, i.e., $k_0/k_F \lesssim 1.9$ (see Section 4).

The results of our approximation for W, Eqs. (2.23) and (3.14), together with the "exact" results of [6], are shown in Fig. 2. The agreement between our approximation and the "exact" results is very satisfactory, except for the region of $k_0/k_F \gtrsim 1.9$. However, according to the results of [6] for V, the effective mass is slowly increasing with increasing nucleon momentum k_0, and for $k_0/k_F \cong 2$ the value of ν_0 is about 0.8, whereas in our approximation we use a constant value of $\nu_0 = 0.7$. This appears to be the main reason for the difference between our approximation and the "exact" results for $k_0/k_F \gtrsim 1.9$. Let us also mention that in the region of $k_0/k_F \gtrsim 1.9$, we relied partly on an extrapolation of the phase shifts given in [20].

In calculating the width Γ_Σ of the Σ hyperon in nuclear matter in [21], [22], the additional approximation $V(k_1) \cong \langle V \rangle_{Av}$ was used for the s.p. potential of nucleons in the Fermi sea. Results obtained for W with this additional approximation are also shown in Fig. 2. They approach zero with $k_0 \to k_F$ much too fast, because the additional approximation introduces a gap in the s.p. energy spectrum at the Fermi surface. For calculating $W(k_0)$

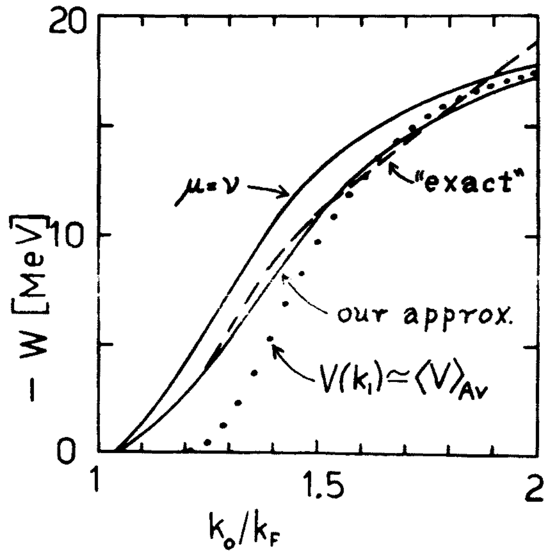

Fig. 2. Our results for W for the Reid hard core potential (at $k_{F0} = 1.4\ \mathrm{fm}^{-1}$), compared with the "exact" results of [6]

at k_0 close to k_F, the additional approximation is certainly unjustified. On the other hand, in the problem of Γ_Σ, this approximation might work better because of the large momentum release in the $\Sigma N \to \Lambda N$ process.

If we put $\mu = \nu$ and $C = D$ in (3.2) (i.e., extrapolate empirical relation (3.11) to states below the Fermi surface), we obtain further simplification of our procedure. In this case $k' = k$, and the value of C is irrelevant for calculating $W(k_0)$. Results for such simplified calculation (with $\nu = 0.7$) are also shown in Fig. 2. They overestimate $|W|$ by about 10–20%. But within this accuracy, the $\mu = \nu$ approximation has the advantage of great simplicity.

A similar test of approximate expressions for V and W has been presented recently by Köhler [8]. Obviously, the problem of calculating both V and W is much more difficult, and to achieve an agreement with the "exact" results, Köhler suggests an approximation (M) which appears computationally complicated.

6. *The equivalent local absorptive potential and the mean free path*

To compare our results with phenomenological absorptive optical potentials, which usually are assumed to be local, we introduce an equivalent local potential W_L. The necessity of introducing W_L was pointed out by Fantoni, Friman and Pandharipande [9], and by Negele and Yazaki [10] (see also [11]).

The local potential W_L is defined as equivalent to W in the sense that it gives the same result for the mean free path λ. Now, as it was explained in Section 1, λ is equal to the lifetime $\tau = 1/w$ of the nucleon state with momentum $\boldsymbol{k}_0$ times the nucleon velocity. The important point is that it is the group velocity

$$v_0^g = \hbar^{-1}\partial e(k_0)/\partial k_0 = \hbar k_0/M^* = v_0/\nu, \tag{6.1}$$

which in a dispersive medium like nuclear matter differs from the phase velocity v_0. Thus, with the help of (1.6), we get

$$\lambda = v_0^g/w = -\frac{\hbar}{2}\, v_0/\nu W = -\frac{\hbar^2 k_0}{2M}\Big/\nu W. \tag{6.2}$$

In the case of a local optical potential $V_L + iW_L$, we have $M^* = M$ and $v_0^g = v_0$, and

$$\lambda = -\frac{\hbar^2 k_0}{2M}\Big/ W_L. \tag{6.3}$$

By comparing (6.2) and (6.3), we get the relation:

$$W_L = \nu W. \tag{6.4}$$

This relation may also be derived in different ways [9], [10] (see also [23]).

7. *Results for W, W_L, and λ*

The results for $W(k_0)$ at the equilibrium density of nuclear matter ($k_F = k_{F0} = 1.35\ \text{fm}^{-1}$) are shown in Fig. 3. To visualize the factors which affect the magnitude of W, we show in Fig. 3 not only the results of our approximation for W (Eqs. (2.23) and (3.14) with all the parameters determined in Section 3), but also three other curves. By putting $Q = 1$ and $\nu = 1$ in Eq. (2.23), i.e., by applying expression (1.6) for W, we get the upper curve. The curve $\mu = \nu = 1$ differs from the upper curve by the presence of the exclusion principle operator Q (this is the approximation of Lane and Wandel [4]). Obviously the mere effect of Pauli blocking is the most important factor in drastically reducing $|W|$, especially at low momenta k_0. By putting $\mu = \nu = 0.7$ (and $C = D$ in (3.2)) we have the case of the same form of $e(k_N)$, quadratic in k_N, for $k_N > k_F$ and $k_N < k_F$, in which $k' = k$. The corresponding values of W differ from $W(\mu = \nu = 1)$ exactly by the factor $\nu = 0.7$. At higher momenta k_0, this factor represents the main effect of the momentum dependence of V. A further reduction in $|W|$ is introduced in our approximation in which $k' < k$, and consequently the final nucleons are slowed down, and the role of the exclusion principle in reducing $|W|$ is increased. As is seen from the two lowest curves in Fig. 3, this additional reduction is particularly important at low momenta k_0.

The depth of the phenomenological local optical potential is determined in nucleon--nucleus scattering experiments as a function of the projectile energy. Hence, to be able to compare our results with the empirical ones, we present our calculated values of W_L, Eq. (6.4), as functions of the energy e. In the case of neutron scattering, the connection between the projectile neutron momentum k_0 in nuclear matter and the neutron energy

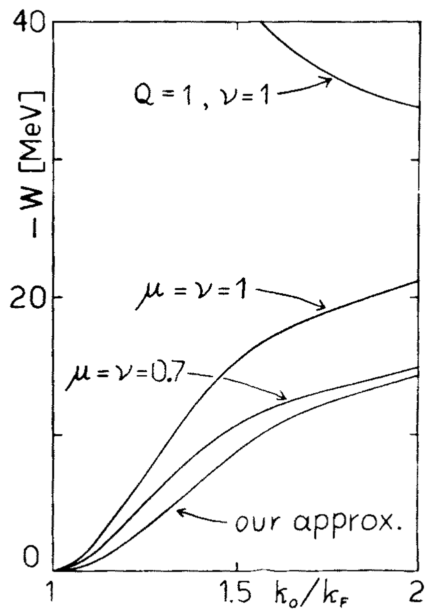

Fig. 3. Results for $W(k_0)$ at $k_F = k_{F0} = 1.35\ \text{fm}^{-1}$

$e(k_0)$ (which is equal to the kinetic energy of the projectile neutron before it enters the target nucleus) is given by Eq. (3.2) (for $k_0 = k_N > k_F$), and we have

$$k_0 = \hbar^{-1}[2M\nu(e-C)]^{1/2}. \tag{7.1}$$

Our results for W_L at $\varrho = \varrho_0$ are shown as the solid curve in Fig. 4 which also contains empirical estimates of the central depth of the imaginary optical potential. The dotted curve is from the compilation of Bohr and Mottelson (p. 237 of [5]), and crosses are from recent proton scattering on ^{208}Pb and ^{40}Ca (Nadasen et al. [24]). The proton energy e_p used in [24] is connected with e by:

$$e_p = e + U_C - \tfrac{1}{4}\frac{N-Z}{A}U_1, \tag{7.2}$$

where U_C is the average Coulomb potential, and U_1 is the symmetry potential. For a uniform charge distribution within the sharp radius $R = r_0 A^{1/3}$, we have

$U_C = 1.2(e^2/r_0)Z/A^{1/3}$. With $r_0 = 1.12$ fm, we get $U_C = 11$ MeV for ^{40}Ca and 25 MeV for ^{208}Pb. In the case of ^{208}Pb the symmetry potential term, $-0.25[(N-Z)/A]U_1 = -0.05\, U_1$, has to be considered. At low energies $U_1 \sim 100$ MeV, and we have $e_p = e + (25-5)$ MeV. Although U_1 decreases with increasing energy, we disregard it, and find it sufficiently accurate in Fig. 4 to place the results of [24] for p$-$Pb at energies $e = e_p - 20$ MeV. Similarly for p$-$Ca, we use $e = e_p - 10$ MeV. Thus, we use the same energy corrections as those used in [9].

We see in Fig. 4 that our calculated W_L agrees with empirical results, favouring slightly the newer results of Nadasen et al. [24]. Needless to say that without the correction for non-locality we could not achieve this agreement: calculated values of W, shown as the dashed curve in Fig. 4, stay well above the empirical points.

The dependence of $W_L(e)$ on the density of nuclear matter ϱ is shown in Fig. 5 in which the numbers at each of the $W_L(e)$ curves are equal to ϱ/ϱ_0. At low energies $|W_L|$ decreases with increasing ϱ, because the role of the exclusion principle increases with

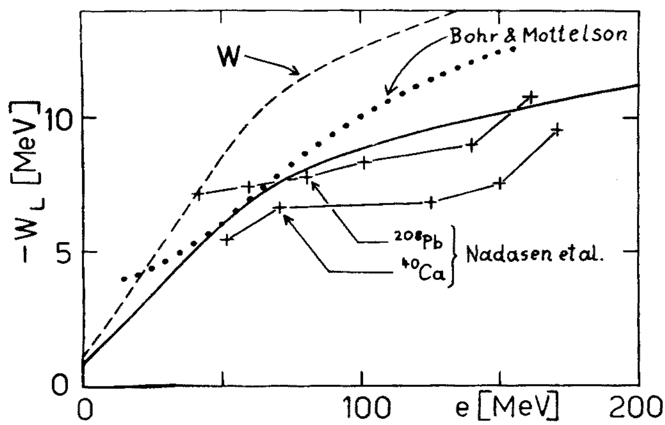

Fig. 4. Values of W_L calculated at $\varrho = \varrho_0$ (solid curve) compared with empirical estimates

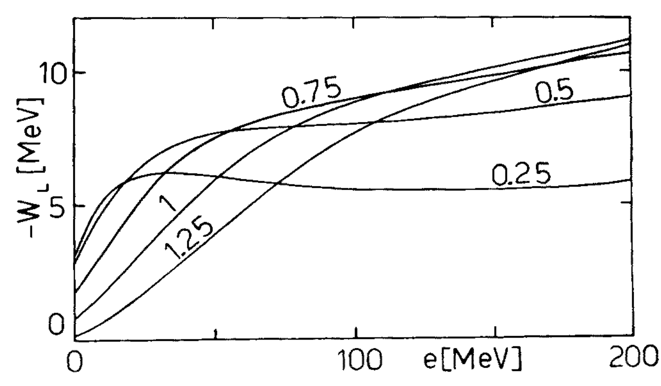

Fig. 5. $W_L(e)$ calculated at different densities ϱ. Numbers at each curve indicate the value of ϱ/ϱ_0

increasing ϱ. (Notice that also v decreases with increasing ϱ, Eq. (3.13)). At higher energies (and not too high densities), $Q \cong 1$, $k \cong k_0/2$, and the most important factor determining W_L, Eqs. (2.23) and (6.4), is $v^2\varrho$, and consequently $|W_L|$ is increasing with increasing ϱ (for $\varrho \lesssim \varrho_0$).

Another way of visualizing this density and energy dependence of W_L is presented in Fig. 6. For two fixed energies, $e = 25$ and 150 MeV, we show the dependence of W_L on ϱ. We notice that $W_L(\varrho = \varrho_0) > W_L(\varrho = \varrho_0/2)$ et $e = 150$ MeV, and $W_L(\varrho = \varrho_0) < W_L(\varrho = \varrho_0/2)$ at $e = 25$ MeV. This is in agreement with the phenomenological analyses of the optical potential, which suggest the absorptive potential which is proportional to ϱ at higher energies, and is peaked at nuclear surface at lower energies.

Calculated values of the mean free path λ, Eq. (6.3), for $\varrho = \varrho_0$, are shown as the solid curve in Fig. 7 which also contains empirically determined values of λ. The dotted curve was obtained from the corresponding dotted curve in Fig. 4 by applying Eq. (6.3) with k_0 determined by Eq. (7.1). In the same way the crosses in Fig. 7 were obtained from the corresponding crosses in Fig. 4. Thus the dotted curve and the crosses represent values of λ determined from phenomenological optical potentials.

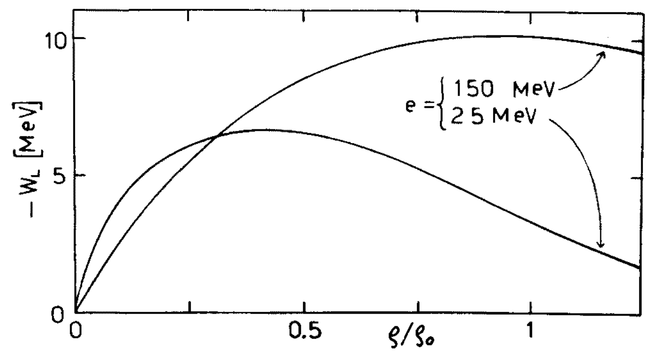

Fig. 6. The dependence of W_L on ϱ at fixed energies e

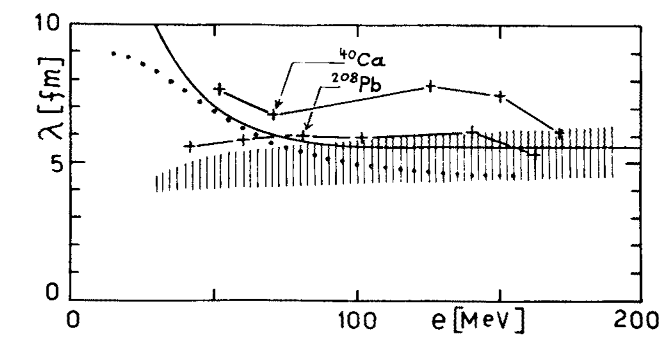

Fig. 7. The calculated mean free path of a nucleon in nuclear matter at $\varrho = \varrho_0$ (solid curve) compared with empirical estimates

Let us notice that the precise way of obtaining λ from a phenomenological local energy dependent optical potential $V_L + iW_L$, consists in solving the Schroedinger equation

$$\left(-\frac{\hbar^2}{2M}\Delta + V_L + iW_L\right)\psi = e\psi. \tag{7.3}$$

By inserting the plane wave solution

$$\psi = \exp\left[i(k_0 + i/2\lambda)z\right], \tag{7.4}$$

we get for λ expression (6.3), and for k_0:

$$k_0^2 - \tfrac{1}{4}\lambda^2 = \frac{2M}{\hbar^2}(e - V_L) = \frac{2Mv}{\hbar^2}(e - C), \tag{7.5}$$

where in the last step expression (3.11) for the energy dependence of V_L was used. By solving Eqs. (7.5) and (6.3) for k_0 we get

$$k_0 = \hbar^{-1}[2Mv(e-C)]^{1/2}[1 + \tfrac{1}{2}y^2/(1+y^2)^{1/2}]^{1/2}, \tag{7.6}$$

where

$$y = -W_L/v(e-C). \tag{7.7}$$

For $y^2 \ll 1$, i.e., for $k_0 \gg \lambda^{-1}$, expression (7.6) goes over into expression (7.1). (E.g., for $e = 0$ and $|W_L| < 5$ MeV, we have $y^2 < 1/100$.)

The shaded band in Fig. 7 denotes the range of λ values determined by Nadasen et al. [24] from p — ^{40}Ca reaction cross section. In drawing this band, we used the energy correction $e = e_p - 10$ MeV.

We see that for $e \gtrsim 50$ MeV the calculated mean free path is within the range of empirical estimates. Possible reasons for the discrepancy at lower energies will be mentioned in Section 9.

8. *The temperature dependence of W and λ*

We want to discuss the dependence of W and λ on the temperature T (in units of MeV) of nuclear matter in the range of low temperatures. Among the factors which contribute to the temperature dependence of W and λ, we consider only the change in the occupation of the s.p. states. This means, we use for small temperatures T of nuclear matter the same s.p. energies we used for the ground state ($T = 0$). The point is that the change in the s.p. energies due to finite temperatures is quadratic in T and may be neglected at low temperatures (see, e.g., p. 19 of [25]).

Furthermore, in discussing finite temperatures, we use what we previously called the $\mu = v$ approximation, i.e., we assume that the s.p. energies are given for all values of k_N by

$$e(k_N) = \varepsilon(k_N)/v + C, \tag{8.1}$$

where v and C are constants fixed at $T = 0$.

The standard procedure (see, e.g., Chapt. X of Tolman [26]) of determining the distribution $n(k_N)$, i.e., the probability that the $\boldsymbol{k}_N$ state is occupied, consists in maximizing the thermodynamical probability under the constraint of constant number of nucleons, and constant energy,

$$\delta E = \int d\boldsymbol{k}_N g(k_N) e(k_N) \delta n(k_N), \tag{8.2}$$

where $g(k_N)$ is the density of the s.p. states in the $\boldsymbol{k}_N$ space, i.e., $g(k_N)\Delta\boldsymbol{k}_N$ is the number of the s.p. states in $\Delta\boldsymbol{k}_N$ ($g(k_N) = 4\Omega/(2\pi)^3$, Ω = volume of the periodicity box). The result is:

$$n(k_N) = 1/\{1+\exp[(\varepsilon(k_N)-\tilde{\varepsilon}_F)/\nu T]\} \tag{8.3}$$

where the constant $\tilde{\varepsilon}_F$, determined from the condition $A = \int d\boldsymbol{k}_N gn$, is

$$\tilde{\varepsilon}_F = \varepsilon_F \left\{1 - \frac{\pi^2}{12}\left(\frac{\nu T}{\varepsilon_F}\right)^2 + \ldots\right\}, \tag{8.4}$$

where $\varepsilon_F = \varepsilon(k_F)$.

To determine the thermal excitation energy of nuclear matter, $\delta_T E = E(T)-E(0)$, at a low temperature T, we use the expression

$$\delta_T E = \int d\boldsymbol{k}_N g(k_N) e(k_N) \delta_T n(k_N) \tag{8.5}$$

for the small change in the energy associated with the small change in the distribution,

$$\delta_T n(k_N) = n(k_N) - n_0(k_N), \tag{8.6}$$

where $n_0(k_N)$, Eq. (2.12′) is the distribution for $T = 0$. (Notice that in expression (8.5), which leads to the resulting $\delta_T E \sim T^2$, only the s.p. energies e at $T = 0$ appear!)

After inserting into (8.5) expression (8.1) for e, we obtain (details of the calculation — with obvious modifications — are the same as in [26]):

$$\delta_T E/A = \left(\frac{\pi}{2}\right)^2 \frac{\nu}{\varepsilon_F} T^2. \tag{8.7}$$

In the case of a finite temperature T, expression (2.23) for W takes the form

$$W_T(k_0) = -4\hbar^2 \nu \int \frac{d\boldsymbol{k}_1}{(2\pi)^3} n(k_1) Q_T(K, k) \frac{k}{M} \bar{\sigma}, \tag{8.8}$$

where $Q_T(K, k)$ is the angle averaged exclusion principle operator at temperature T (see Appendix B). Notice that $k' = k$ in the $\mu = \nu$ approximation used here. In applying Eq. (8.8), we use for $n(k_1)$ expression (8.3) with $\tilde{\varepsilon}_F \cong \varepsilon_F$.

For the mean free path of a nucleon in nuclear matter of temperature T, we have (see Eq. (6.3)):

$$\lambda_T = -\frac{\hbar^2 k_0}{2M}\Big/ \nu W_T. \tag{8.9}$$

To visualize how the finite temperature of nuclear matter affects the mean free path, we present our results in the form of the ratio of λ_T/λ_0, where λ_0 is the mean free path at zero temperature. In Fig. 8, the ratio λ_T/λ_0 for $\varrho = \varrho_0(k_F = k_{F0} = 1.35\,\text{fm}^{-1})$ is shown at three fixed values of T as functions of the energy e and k_0 (the connection between e and k_0 is given in Eq. (8.1). In Fig. 9, the ratio λ_T/λ_0 for $\varrho = \varrho_0$ is shown at four fixed values of e as function of T and the corresponding thermal excitation energy per nucleon $\delta_T E/A$, Eq. (8.7).

For $k_0 \to k_F$, we have $\lambda_T/\lambda_0 \to 0$, because $\lambda_0 \to \infty$, and λ_T remains finite. In spite of this big reduction of λ_T at $k_0 \cong k_F$, λ_T has a maximum at $k_0 = k_F$, and at low energies is a decreasing function of energy. And the minimum value of λ_T is reached at energies where the reduction is already small (for the low temperature range considered). For

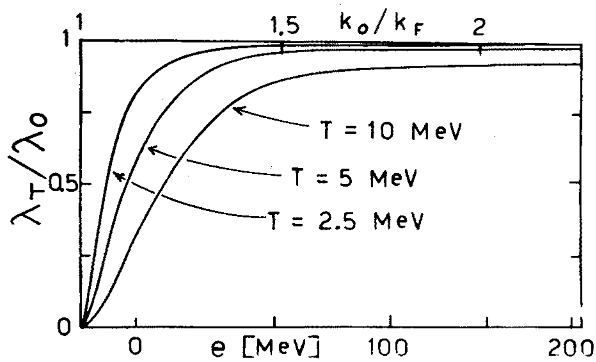

Fig. 8. The ratio λ_T/λ_0 for $\varrho = \varrho_0$ as function of the energy e (lower scale) and the corresponding nucleon momentum k_0 in nuclear matter (upper scale) at $T = 2.5$, 5, and 10 MeV

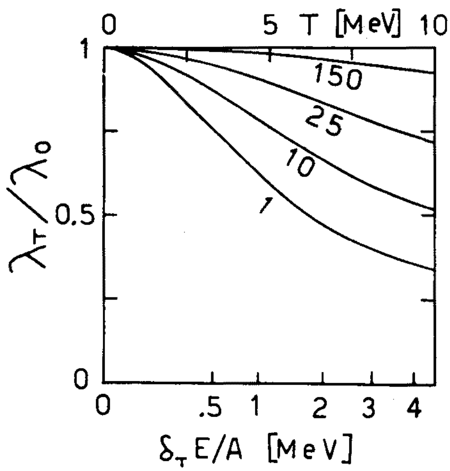

Fig. 9. The ratio λ_T/λ_0 for $\varrho = \varrho_0$ as function of T (upper scale) and the corresponding thermal excitation energy per nucleon (lower scale) at fixed values of e. Numbers at each curve indicate the value of e in MeV

instance, for $T = 10$ MeV, we have $\lambda_T = 10.8$ fm at $k_0 = k_F$, and the minimum value of $\lambda_T = 4.8$ fm is reached at k_0/k_F ($e = 80$ MeV).

We conclude that for energies $e \gtrsim 50$ MeV, where $\lambda \cong 5$ fm, thermal excitations $\delta_T E/A$ up to ~ 5 MeV/nucleon ($T \sim 10$ MeV) shorten λ insignificantly. For smaller energies e, in spite of a significant shortening of λ caused by thermal excitation, λ_T is increasing with decreasing energies e.

These conclusions concerning the effect of the temperature T on λ_T/λ_0 agree with the results obtained by Collins and Griffin [27]. However, these authors apply the Fermi gas model ($M^* = M$), i.e., disregard the dispersive effects. In this way, they miss one factor v present in our Eq. (2.23) for W, and another factor v present in our Eq. (6.2) for λ. Consequently, their values of λ are about two times ($v^2 \sim 1/2$) shorter than ours.

To see how the dispersive effects ($M^* < M$) affect the T dependence of λ_T, let us consider λ_T at a fixed value of the nucleon momentum k_0 in nuclear matter. In the present $\mu = v$ approximation, $v^2\lambda_T$ is then a function of the product vT, since both $n(k_1)$ and Q_T depend only on this product. Thus the ratio λ_T/λ_0 is a function of the product vT. This means that λ_T/λ_0 calculated with $M^* = M$ (i.e., with $v = 1$) at the temperature T has the same value as λ_T/λ_0 calculated with $M^* = vM$ (with $v < 1$) at the corresponding lower temperature vT. Consequently, the effect of the temperature on λ, i.e., the shortening of λ with increasing T (at a fixed value of k_0), is reduced by the introduction of the effective mass $M^* < M$. (The situation at a fixed value of e is more complicated: it depends on the assumed relation between e and k_0 for $M^* = M$. Notice that putting $v = 1$ in (8.1) would contradict empirical relation (3.12).)

A similar scaling law applies to the dependence of λ_T/λ_0 on the thermal excitation energy. Relation (8.7) implies that λ_T/λ_0 at a fixed value of k_0 depends only on the product $v\delta_T E/A$.

9. *Conclusions and final comments*

We have calculated the imaginary part of the optical potential W (and of the equivalent local potential W_L) and the mean free path λ of a nucleon in nuclear matter using simple expressions which involve free NN cross section and phenomenologically adjusted s.p. energies. Our results reproduce quite well the empirical values of W_L and λ. (Although, we should be aware of possible ambiguities in these empirical values (see [11]).)

There is a considerable spread in the empirical values of W_L and λ shown in our Figs 4 and 7. Still it appears that as $k_0 \to k_F$, our calculated W_L approaches zero too fast, and λ increases too fast. One reason for it is the neglect of nucleon-nucleon correlations in the ground state of nuclear matter (see, e.g. [28]). Because of them, the nucleon momentum distribution $n(k_N)$ deviates from the step function $n_0(k_N) = \theta(k_F - k_N)$, and at $k_0 = k_F$ the Pauli blocking is not complete. Thus with a diffused distribution $n(k_N)$, similarly as in the case of nonzero temperature, $W(k_0 = k_F) \neq 0$. Consequently, also the mean free path λ does not increase infinitely as $k_0 \to k_F$.

Another reason for our results being less reliable at very low energies is our simplified treatment of the effective mass M^*. Without going into a more sophisticated discussion of the effective mass (see [3] and [29]), we notice that an increase in $\nu = M^*/M$ at $k_0 = k_F$ in our expressions (2.23), (6.4), and (6.3) would lead to an increase in $|W_L|$ and a decrease in λ at $k_0 = k_F$. (Also at higher energies, we expect M^* to increase, which might affect our results at $e \sim 200$ MeV.)

In the present paper, we considered only elastic NN scattering. As we saw in Section 4, at $e \sim 200$ MeV our expression (2.23) for W involves $\bar{\sigma}$ at E_L partly already above the threshold for pion production. To calculate W (and λ) for higher energies e, we could simply apply semiclassical expression (1.6) with $\bar{\sigma}$ equal to the total (elastic + reaction) cross section, arguing that at high energies the Pauli blocking is not important. However, when a pion is produced the two colliding nucleons are slowed down, and the Pauli blocking might be relevant in reducing the contribution to W of nonelastic scattering for energies not too far above the threshold. A quantitative estimate of this Pauli blocking is complicated because of the additional pion in the final state. Obviously, the same remarks apply whenever a new channel is opened.

In our calculation of W and λ, we neglected the angular distribution of the NN cross section. Of course, one might easily take into account the anisotropy of the cross section (see Appendix A). Calculations of this type performed in [27] suggest that for $e \lesssim 200$ MeV the isotropic approximation introduces only a small error of about 5%.

In Section 8, we have calculated the mean free path λ_T of a nucleon in nuclear matter at small thermal excitations. At the temperatures considered ($T \leqslant 10$ MeV) λ_T is only slightly reduced by the thermal excitation, except for nucleons with momenta $k_0 \cong k_F$, whose mean free path still remains long. The restriction to low temperatures simplified the calculations because we could use the $T = 0$ s.p. energies. Notice also that considering higher temperatures would require NN cross section at correspondingly higher energies.

Our general conclusion is that one may get a reasonably accurate estimate of the absorption rate of a nucleon in nuclear matter by applying the simple modification, Eq. (1.8), of the semiclassical expression, Eq. (1.1).

APPENDIX A

The effect of the anisotropy of the NN cross section

By approximating the exclusion principle operator $Q(\mathbf{K}, \mathbf{k}')$ by its angle average $Q(K, k')$, Eq. (2.13), we neglect the angular distribution of the NN cross section. Without approximation (2.13), we would have (see Eq. (2.15))

$$\tfrac{1}{2} \sum_T (2T+1) \sum_{Sm_s} \operatorname{Im} \langle \mathbf{k}Sm_sT|\mathscr{K}|\mathbf{k}Sm_sT\rangle = -\nu \frac{Mk'}{(2\pi\hbar)^2} \tfrac{1}{2} \sum_T (2T+1)$$

$$\times \tfrac{1}{4} \sum_{Sm_sm_s'} \int d\hat{k}' Q(\mathbf{K}, \mathbf{k}') \, |\langle \mathbf{k}'Sm_s'T|\mathscr{K}|\mathbf{k}Sm_sT\rangle|^2. \tag{A.1}$$

Let us introduce the differential NN cross section (in the CM system) in nuclear matter in the isotopic spin state T:

$$d\sigma^{\mathrm{NM}}(T)/d\hat{k}' = \tfrac{1}{4}\sum_{Sm_sm_s'}\frac{k'}{k}\left(\frac{M}{4\pi\hbar^2}\right)^2 \langle \boldsymbol{k}'Sm_s'T|\mathscr{K}|\boldsymbol{k}Sm_sT\rangle|^2, \tag{A.2}$$

and the corresponding nn and np cross sections:

$$d\sigma_{\mathrm{nn}}^{\mathrm{NM}}/d\hat{k}' = 4d\sigma^{\mathrm{NM}}(T=1)/d\hat{k}', \tag{A.3}$$

$$d\sigma_{\mathrm{np}}^{\mathrm{NM}}/d\hat{k}' = \tfrac{1}{4}\sum_{Sm_sm_s'}\frac{k'}{k}\left(\frac{M}{4\pi\hbar^2}\right)^2\left|\sum_T \langle \boldsymbol{k}'Sm_s'T|\mathscr{K}|\boldsymbol{k}Sm_sT\rangle\right|^2$$

$$= \sum_T d\sigma^{\mathrm{NM}}(T)/d\hat{k}' + X, \tag{A.4}$$

where X is the $T=0$–$T=1$ interference term:

$$X = \tfrac{1}{4}\sum_{Sm_sm_s'}\frac{k'}{k}\left(\frac{M}{4\pi\hbar^2}\right)^2 \langle \boldsymbol{k}'Sm_s'T=1|\mathscr{K}|\boldsymbol{k}Sm_sT=1\rangle^*$$

$$\times\langle \boldsymbol{k}'Sm_s'T=0|\mathscr{K}|\boldsymbol{k}Sm_sT=0\rangle + \text{c.c.}\,. \tag{A.5}$$

Notice that if in Eqs (A.3–4) we substitute $\mathscr{K}_0$ for $\mathscr{K}$, these equations become expressions for differential nn and np cross sections (in the CM system).

With the help of the above cross sections, we may write Eq. (A.1) as

$$\tfrac{1}{2}\sum_T(2T+1)\sum_{Sm_s}\mathrm{Im}\langle \boldsymbol{k}Sm_sT|\mathscr{K}|\boldsymbol{k}Sm_sT\rangle$$

$$= -4\nu\hbar^2\frac{k}{M}\int d\hat{k}'Q(\boldsymbol{K},\boldsymbol{k}')\,\tfrac{1}{2}\sum_T(2T+1)d\sigma^{\mathrm{NM}}(T)/d\hat{k}'$$

$$= -4\nu\hbar^2\frac{k}{M}\int d\hat{k}'Q(\boldsymbol{K},\boldsymbol{k}')\,\{d\bar{\sigma}^{\mathrm{NM}}/dk' - \tfrac{1}{2}X\}, \tag{A.6}$$

where

$$d\bar{\sigma}^{\mathrm{NM}}/d\hat{k}' = \tfrac{1}{2}\,[d\sigma_{\mathrm{np}}^{\mathrm{NM}}/d\hat{k}' + \tfrac{1}{2}\,d\sigma_{\mathrm{nn}}^{\mathrm{NM}}/d\hat{k}'] \tag{A.7}$$

is the average NN differential CM cross section in nuclear matter. The factor 1/2 at $d\sigma_{nn}^{NM}/d\hat{k}'$ reflects the indistinguishability of the two neutrons. For the same reason, the total nn cross section

$$\sigma_{\mathrm{nn}}^{\mathrm{NM}} = \tfrac{1}{2}\int d\hat{k}'d\sigma_{\mathrm{nn}}^{\mathrm{NM}}/d\hat{k}', \tag{A.8}$$

and we have (see Eq. (2.11):

$$\int d\hat{k}'(d\bar{\sigma}^{\mathrm{NM}}/d\hat{k}') = \bar{\sigma}^{\mathrm{NM}}. \tag{A.9}$$

Since $Q(\boldsymbol{K}, \boldsymbol{k}')$ is an even, and X an odd function of $\hat{k}'$, the $\hat{k}'$ integration of the QX term in (A.6) vanishes. Thus for W, we get from Eqs (2.2) and (A.6):

$$W(k_0) = -4\hbar^2 v \int^{<k_F} \frac{d\boldsymbol{k}_1}{(2\pi)^3} \frac{k}{M} \int d\hat{k}' Q(\boldsymbol{K}, \boldsymbol{k}') d\bar{\sigma}^{\text{NM}}/d\hat{k}'. \tag{A.10}$$

APPENDIX B

The operator Q_T

The operator $Q_T(\boldsymbol{K}, \boldsymbol{k})$ is defined by (see Eq. (2.12)):

$$\begin{aligned} Q_T(\boldsymbol{K}, \boldsymbol{k}) &= [1-n(|\tfrac{1}{2}\boldsymbol{K}+\boldsymbol{k}|)]\,[1-n(|\tfrac{1}{2}\boldsymbol{K}-\boldsymbol{k}|)] \\ &= 1-[1+\exp \gamma\kappa(x)]^{-1}-[1+\exp \gamma\kappa(-x)]^{-1} \\ &\quad +\{[1+\exp \gamma\kappa(x)]\,[1+\exp \gamma\kappa(-x)]\}^{-1}, \end{aligned} \tag{B.1}$$

where

$$\gamma = \hbar^2/2MvT, \qquad \kappa(x) = K^2/4+k^2+Kkx-k_F^2, \tag{B.2}$$

and $x = \hat{k}\hat{K}$.

The angle averaged operator $Q_T(K, k)$ is (see Eq. (2.13)):

$$Q_T(K, k) = \tfrac{1}{2} \int_{-1}^{1} dx Q_T(\boldsymbol{K}, \boldsymbol{k}) = \int_{0}^{1} dx Q_T(\boldsymbol{K}, \boldsymbol{k}), \tag{B.3}$$

where in the last step we use the invariance of $Q_T(\boldsymbol{K}, \boldsymbol{k})$ uder changing of $\boldsymbol{k}$ into $-\boldsymbol{k}$ (or x into $-x$).

After performing the elementary x-integration, we obtain:

$$\begin{aligned} Q_T(K, k) &= \{\exp 2\gamma\kappa(0)/[1-\exp 2\gamma\kappa(0)]\} \\ &\quad \{1+(\gamma k K)^{-1} \ln ([1+\exp \gamma\kappa(-1)]/[1+\exp \gamma\kappa(1)])\} \\ &= \frac{1}{2\gamma Kk}[1+\operatorname{cth} \gamma\kappa(0)] \ln \left\{ \frac{\operatorname{cth} \frac{1}{2}\gamma\kappa(0) \operatorname{cth} \frac{1}{2}\gamma Kk+1}{\operatorname{cth} \frac{1}{2}\gamma\kappa(0) \operatorname{cth} \frac{1}{2}\gamma Kk-1} \right\}. \end{aligned} \tag{B.4}$$

In the limit of $T \to 0$, one may easily obtain from expression (B.4) the well known result for $Q_{T=0} \equiv Q$:

for $K/2 < k_F$:

$$\begin{aligned} Q(K, k) &= 0 && \text{for} \quad k < (k_F^2-K^2/4)^{1/2}, \\ &= 1 && \text{for} \quad k > K/2+k_F, \\ &= \kappa(0)/kK && \text{otherwise}, \end{aligned} \tag{B.5a}$$

for $K/2 > k_F$:

$$\begin{aligned} Q(K, k) &= 1 && \text{for} \quad |k-K/2| > k_F, \\ &= \kappa(0)/kK && \text{otherwise}. \end{aligned} \tag{B.5b}$$

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