Vol. B16 (1985) ACTA PHYSICA POLONICA No 2

NOTE ON THE CLASSICAL HIGGS MODEL

By P. Kosiński

Institute of Physics, University of Łódź*

and P. Maślanka

Institute of Mathematics, University of Łódź**

(Received July 4, 1984)

It is shown that there exist no static, finite energy "plane" solutions to the Yang-Mills-Higgs equations.

PACS numbers: 11.15.–q, 11.15.Kc

In this paper we will consider the static solutions to the Yang-Mills-Higgs theory. Let G be any compact matrix Lie group and g the corresponding Lie algebra. The Higgs field takes its values in a vector space L in which G acts as a transformation group. We assume that G acts linearly on L and denote by ϱ the corresponding representation. Then the action density takes the form

$$\mathscr{A}(A, \phi) = -\tfrac{1}{4}(F_{\mu\nu}, F^{\mu\nu}) + \tfrac{1}{2}(\nabla_A\phi_\mu, \nabla_A\phi^\mu) - \frac{\lambda}{8}(|\phi|^2-1)^2.$$

Here $F_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + [A_\mu(x), A_\nu(x)]$, $A_\mu(x)$ is the gauge potential which takes its values in g and $\nabla_A\phi_\mu = \partial_\mu\phi + \varrho(A_\mu)\phi$; $(\cdot\,,\cdot)$ denotes the inner product in g resp. L (the relative normalization of $(\cdot\,,\cdot)$ on g and on L is irrelevant).

The field configuration $\{A, \phi\}$ is static if A_μ and ϕ do not depend of time and $A_0 = 0$. In this case we may define the action per unit time; it simply equals $-E$, where E is the total energy of the configuration

$$E = \int \left[\tfrac{1}{4}(F_{ij}, F_{ij}) + \tfrac{1}{2}(\nabla_A\phi_i, \nabla_A\phi_i) + \frac{\lambda}{8}(|\phi|^2-1)^2\right] d^3\vec{x}.$$

We are interested in the static solutions to the field equations which correspond to the finite total energy E. From the Noether theorem it follows that for any static solution the

* Address: Instytut Fizyki, Uniwersytet Łódzki, Nowotki 149/153, 90-236 Łódź, Poland.

** Address: Instytut Matematyki, Uniwersytet Łódzki, Banacha 22, 90-238 Łódź, Poland.

stress tensor

$$T_{ij} = (F_{ik}, F_{jk}) - \frac{1}{4}\delta_{ij}(F_{kl}, F_{kl}) + (\nabla_A\phi_i, \nabla_A\phi_j) - \frac{1}{2}\delta_{ij}(\nabla_A\phi_k, \nabla_A\phi_k) - \frac{\lambda}{8}\delta_{ij}(|\phi|^2-1)^2$$

is divergenceless

$$\partial_j T_{ij} = 0.$$

This, together with the finiteness of E implies [1] that $T_{ij}(\vec{x})$ is integrable and

$$\int d^3\vec{x}\, T_{ij}(\vec{x}) = 0.$$

Putting in the above equation $i = j = 1$ and $i = j = 2$ and adding together we obtain

$$\int d^3\vec{x}(F_{12}, F_{12}) = \int d^3\vec{x}\left[(\nabla_A\phi_3, \nabla_A\phi_3) + \frac{\lambda}{4}(|\phi|^2-1)^2\right].$$

Assume now that the "magnetic" field has vanishing third component, i.e. $F_{12} \equiv 0$. Then $|\phi| \equiv 1$ and it can be proved (cf. Ref. [1], pp. 32–33) that up to a gauge transformation $A \equiv 0$, $\phi \equiv 1$, i.e. the solution is trivial.

The above result does not seem to be very interesting in the case of nonabelian gauge group; it simply states that the monopoles cannot be "plane". However, let us take $G = U(1)$. As it is well known [2] the abelian Higgs model does admit static solutions in the form of an infinite solenoid stabilized by the nonelectromagnetic attractive force originating from the Higgs potential. The energy per unit length is finite (the total energy is infinite, of course). The Higgs field defines the homotopy class $[\phi] \in \Pi_1(S_1) = Z$ and the magnetic flux is proportional to the integer N characterizing the class $[\phi]$. Such solutions are topologically stable. The Nielsen-Olesen vortex solution was used as a theoretical base for the string model of hadrons. Consider the finite energy solution. If we exclude the monopoles it follows that the magnetic flux tube should be closed. If we assume further that the curvature of the tube is large as compared to its thickness we can use the Nielsen-Olesen solution as a good approximation. In the limit of infinitely large coupling the tube becomes infinitely thin and topologically stable. We obtain the string model.

However, the result stated above shows clearly that the finite energy solutions, if exist, are very complicated. For example, the simplest solution in the form of toroidal solenoid is excluded. It is not difficult to understand why this is the case: the interaction between the magnetic field and the current tends to decrease the radius of tori. Consequently, the solenoid cannot be stabilized by the nonelectromagnetic attractive forces. It seems to us that such an argument is general and there are no static finite energy solutions to the abelian Higgs model in three dimensions.

We are grateful to Professor W. Tybor for helpful discussions and critical remarks We also would like to thank Dr. B. Nowak for his interest in this work.

REFERENCES

[1] A. Jaffe, C. Taubes, *Vortices and Monopoles*, Birkhauser, Boston 1980.
[2] H. Nielsen, P. Olesen, *Nucl. Phys.* **B61**, 45 (1973).