LETTERS TO THE EDITOR

THE A-DEPENDENCE OF STRUCTURE FUNCTIONS FROM PION AND DELTA ISOBAR MODEL

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(Received November 19, 1984)

We analyze the structure functions of nucleons bound in the nuclei of He, Al, Fe and Pb in the framework of the $\Delta-\pi$ model. Our calculations agree well with the experimental data.

PACS numbers: 13.40.Fn, 21.60.–n

After the first EMC experiment [1] and its SLAC confirmation [2] indicated a difference between nucleon structure functions for Fe and D targets a multitude of theoretical calculations of nucleon structure functions inside nucleus appeared. Most of them managed to reproduce the data satisfactorily, and some went even further, predicting (or explaining) the A-dependence of structure functions, which measurement was performed recently at SLAC [3].

In our paper we present the results of calculation of the ratio of the structure functions $F_2^A(x)/F_2^D(x)$ for A = He, Al, Fe, Pb. We use the model with Δ isobar [4] and pion [5] degrees of freedom inside nucleus, in the version proposed by Kubar, Plaut and Szwed [6]. Detailed description of the model can be found in Ref. [6, 7], below we give only the main ingredients of the construction.

The effective nucleon structure function in an isoscalar nucleus with A nucleons is given by:

$$F_2^A(x) = \sum_{a=N,\Delta,\pi} \int_x^A f^a(z) F_2^a(x/z) dz, \tag{1}$$

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where $z = Ap_+^a/p_+^A$ denotes the momentum fraction per nucleon, $f^a(z)$ — the distribution inside nucleus (given by nuclear physics calculations [9]) and $F_2^a(x/z)$ — the structure function of the particle a, respectively ($a = \mathrm{N}, \Delta, \pi$).

We assume:

$$f^{\mathrm{N}}(z) = (1-\langle n_\Delta\rangle) f^{\mathrm{F}}(z), \quad f^{\Delta}(z) = \langle n_\Delta\rangle f^{\mathrm{F}}(z), \tag{2}$$

where $\langle n_\Delta\rangle$ is the average number of Δ isobars per nucleon, $f^{\mathrm{F}}(z)$ is the longitudinal momentum distribution of a free, non-relativistic Fermi gas:

$$f^{\mathrm{F}}(z) = \begin{cases} \frac{3}{4}\left(\frac{m_n}{k_{\mathrm{F}}}\right)^3 \left[\left(\frac{k_{\mathrm{F}}}{m_{\mathrm{n}}}\right)^2 - (z-\eta)^2\right]; & |z-\eta| < \frac{k_{\mathrm{F}}}{m_{\mathrm{n}}}, \\ 0 \qquad\qquad\qquad\qquad\quad ; & |z-\eta| > \frac{k_{\mathrm{F}}}{m_{\mathrm{n}}}, \end{cases} \tag{3}$$

where m_n is the nucleon mass, k_{F} — the Fermi momentum (taken from [8]), $\eta = \int_0^\infty dz z f^{\mathrm{F}}(z)$ — the momentum fraction carried by nucleons and Δ isobars. $f^\pi(z)$ is calculated from the momentum distribution ϱ given in [9]:

$$f^\pi(z) = \frac{\pi}{2} \int_0^{k_{\max}^2} d(k_\perp^2) \left(m_{\mathrm{n}} + \frac{m_\pi^2 + k_\perp^2}{m_{\mathrm{n}} z^2}\right) \varrho(\vec{k}(k_\perp^2, z)). \tag{4}$$

In order to speed up the calculations, the function $f^\pi(z)$ obtained from Eq. (4) is parametrized by an analytical function of z. The parameters k_{F}, $\langle n_\pi\rangle$, $\langle n_\Delta\rangle$ and η_π (fraction of momentum carried by the pions) are given by nuclear physics. The other parameters of distributions $f^a(z)$ are chosen such as to fulfill the sum rules:

$$\int_0^A dz f^a(z) = \langle n_a\rangle, \quad a = N, \Delta,$$

$$\sum_{a=\mathrm{N},\Delta,\pi} \int_0^A dz z f^a(z) = 1. \tag{5}$$

Numerical values of the parameters are given in Table I.

TABLE I

Values of nuclear parameters for different targets

A	n_π	n_Δ	n_π	η_Δ	k_F [fm^{-1}]
D	0.0256	0.005	0.0161	0.0049	—
He	0.0954	0.04	0.0675	0.0373	0.93
Al	0.122	0.04	0.0674	0.0373	1.08
Fe	0.129	0.04	0.0711	0.0372	1.11
Pb	0.145	0.05	0.0796	0.046	1.23

Structure functions $F_2(x)$ are calculated using quark densities which are parametrized as follows:

$$xq(x) = a_1 x^{a_2}(1-x)^{a_3}(1+a_4 x) \tag{6}$$

and a_i can be found in Table II.

TABLE II

Parameters of quark structure functions

Quark	a_1	a_2	a_3	a_4
u proton	1.021	0.37	2.64	2.95
d proton	2.809	0.78	4.35	0.72
sea proton	0.213	0.0	5.66	0.0
val. pion	1.142	0.41	0.95	0.0
sea pion	0.399	0.0	8.4	0.0

The calculations of F_2^A/F_2^D are performed using the convolution formula for both the numerator (as described above) and the denominator. The change introduced in the case of deuterium comes in the nucleon distribution $f(z)$. The Fermi gas formula (3) is replaced by the momentum distribution following from the Paris potential calculation [10]:

$$f(\vec{k}) = |h_0|^2 + |h_2|^2,$$

$$h_0 = \sqrt{\frac{2}{\pi}} \sum_{J=1}^{13} \frac{C_J}{k^2 + m_J^2}, \qquad h_2 = -\sqrt{\frac{2}{\pi}} \sum_{J=1}^{13} \frac{D_J}{k^2 + m_J^2},$$

(where $m_J = \alpha + (J-1)M_0$; the values of C_J, D_J, α and M_0 can be found in [10]) integrated over the transverse momentum. As compared to the approximation where the nucleons inside deuterium are free the ratio F_2^A/F_2^D changes by 5–10% for the intermediate x.

The results are drawn in Fig. 1. Small disagreement with the experimental data for large x can be attributed to the Fermi gas approximation. At low x, however, the apparent discrepancy vanishes when the ratio of cross sections (as published in the SLAC data) is transformed to the ratio of structure functions [11].

In Fig. 2 the same ratio is drawn as a function of A for fixed $x = 0.3$ and 0.58. Predicted points lay along a straight line which, within experimental errors, reproduces the behaviour of the data.

Our calculations are limited to four targets for which the π momentum distributions are given in Ref. [9]. Other targets can be treated along the same lines.

To summarize, we studied in this note the A-dependence of the Δ-π model. Earlier studies have proven that the model describes successfully the x and Q^2 dependences of the ratio F_2^{Fe}/F_2^D. The present analysis was done with no additional assumptions. Thus the main conclusion, that the model based on conventional nuclear physics describes well the EMC

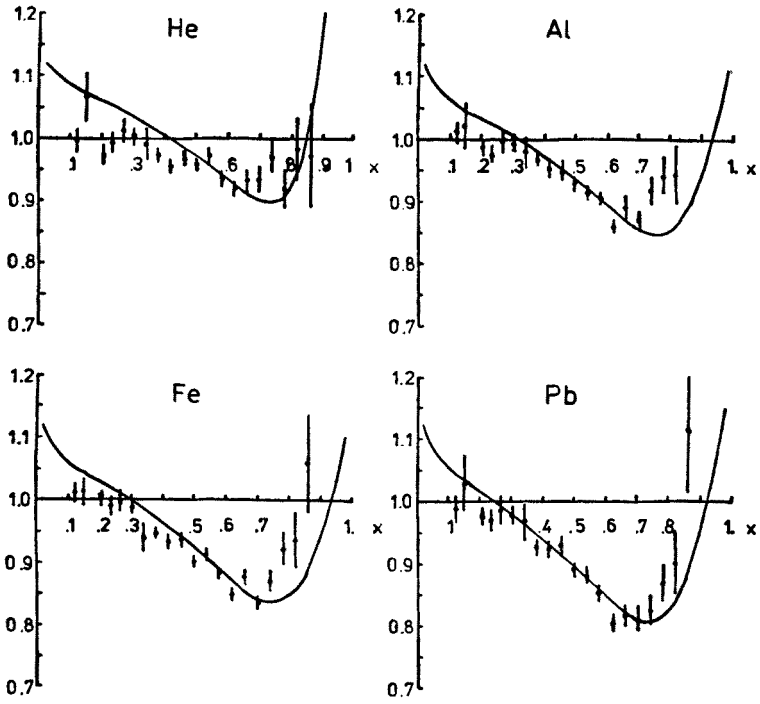

Fig. 1. The ratio $F_2^A(x)/F_2^D(x)$ vs x for A = He, Al, Fe, Pb compared with the SLAC data [3]. The Pb curve is compared with the data for Au

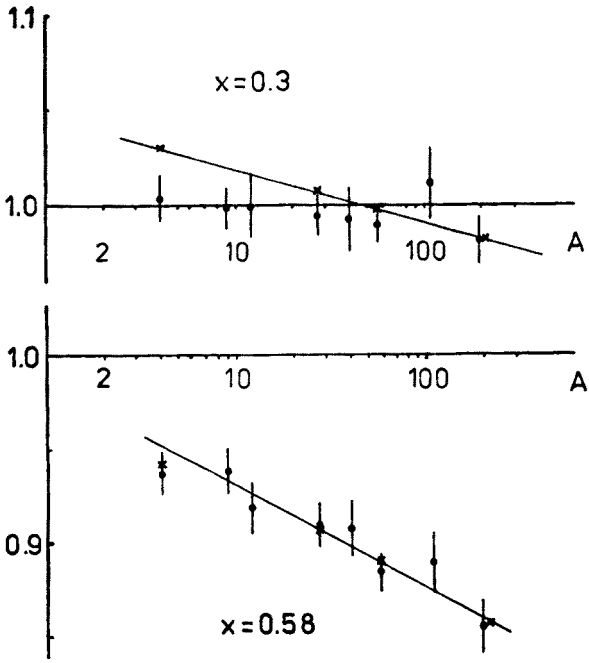

Fig. 2. The A-dependence of the ratio F_2^A/F_2^D for fixed $x = 0.3$ and 0.58. The straight line is a linear fit through the model points (denoted by ×)

effect, remains valid. One can try to refine this approach at very small x, where the convolution formula is not justified, as well as at x close to one, where the simple Fermi gas approximation was used.

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