MAGNETIC BRANS-DICKE-BIANCHI TYPE-I COSMOLOGICAL MODELS

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We derive new cosmological solutions of the Brans-Dicke theory of gravitation. We consider some magnetic Bianchi type-I models both in the vacuum as well as in the stiff matter case. It is shown that the primordial magnetic field can alter the character of the initial singularity. The BDT-scalar field determines the behaviour of the given solutions in a somewhat more complicated form.

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1. *Introduction*

The issue of a primordial magnetic field in the Universe presents an interesting problem in cosmology. Hoyle (1958) was the first to suggest the possibility that a primordial magnetic field may exist as a basic constituent of the Universe like radiation and matter. From several lines of observational evidence, such as analysis of the Faraday rotation of the polarization of radio waves emitted by extragalactic radio sources, it appears that the Universe might have a large-scale magnetic field of strength approximately 10^{-8} G, in all likelihood of primordial origin (Kawabata 1969; Sofue et al. 1969; Reinhardt and Thiel 1970; Reinhardt 1972; Kolobov et al. 1976; Hurwitz 1979; Shapiro and Wassermann 1981; Melchiorri et al. 1982). However, no firm conclusion seems possible yet.

The idea of a Universe with a homogeneous primordial magnetic field was proved to be very successful on the basis of the flat anisotropic Bianchi type-I model (Zel'dovich 1965, 1969; Doroshkevich 1965; Shikin 1966, 1967; Thorne 1965, 1967; Jacobs 1969a, b, 1977; Vajk and Eltgroth 1970; Zel'dovich and Novikov 1975, 1983; Tupper 1977; Dunn and Tupper 1980; Lorenz 1980a, 1981, 1982; Fargion 1980; Hacyan 1984) in the genera theory of relativity (GRT). The dynamical role of a primordial electromagnetic field in more general, non-flat, GRT-Bianchi types I–IX models has been recently reviewed by Ruban (1982). More recent interest in any realistic cosmological magnetic field comes from Grand Unified Theories (GUT), which predict and require the existence of heavy

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monopoles (Craigie et al. 1982; Carrigan and Trower 1983). It has been shown by Fargion (1983) that the combined presence of a realistic cosmological magnetic field and a magnetic monopole density is not consistent with any relevant observable monopole flux in case of a GRT-Bianchi type-I model.

Among the various modifications of the GRT the BDT (Brans-Dicke theory) is treated most seriously (Brans and Dicke 1961; Weinberg 1972). In order to study the influence of the corresponding BDT-scalar field on the structure of a cosmological magnetic field an investigation of the BDT-Bianchi type-I model filled with radiation matter was made by Chakravarti and De (1983). However, it has been shown by us in a recently published paper (Lorenz-Petzold 1984b) that the corresponding solutions are wrong and the correct solutions were given. In this paper we consider the triaxial BDT-Bianchi type-I model filled with "stiff" matter. The possible relevance of the equation of state $p = \varepsilon$, where p and ε are, respectively, the pressure and density of matter, as regards the matter content of the Universe in its early stages has been discussed by a number of authors, since it was first proposed by Zel'dovich (1961, 1970). We refer to the paper of Barrow (1978). We also obtain the BDT-Bianchi type-I magnetic vacuum solution as a limiting case.

2. *Field equations and solutions*

We consider the general Bianchi type-I model with metric

$$ds^2 = -dt^2 + R_i^2(dx^i)^2, \quad i = 1, 2, 3. \tag{1}$$

The BDT-field equations to be solved are

$$\dot{H}_i + 3HH_i + H_i(\ln\phi)^{\cdot} = [k\varepsilon(1+\omega(2-\gamma)) + E_{ii}]\phi^{-1}, \tag{2}$$

$$H_1H_2 + H_1H_3 + H_2H_3 + 3H(\ln\phi)^{\cdot} - (\omega/2)(\ln\phi)^{\cdot 2} = [\varepsilon + E]\phi^{-1}, \tag{3}$$

$$(R^3\dot{\phi})^{\cdot} = k\varepsilon(4-3\gamma)R^3, \quad R^3 = R_1R_2R_3, \tag{4}$$

where $R_i = R_i(t)$ are the cosmic scale-functions, $H_i = \dot{R}_i/R_i$ are the Hubble parameters, $3H = \Sigma H_i$, $2E = \Sigma E_{\mu\nu}$, $k = 1/(3+2\omega)$, ω the BDT-coupling parameter, $\phi = \phi(t)$ the BDT-scalar field (corresponding to the variable gravitational "constant": $\phi \sim G^{-1}$), $E_{\mu\nu}$ the components of the magnetic stress-energy tensor and $(\,)^{\cdot} = d/dt$. The magnetic field is given by

$$E_{00} = E_{11} = E_{22} = -E_{33} = (f/R_1R_2)^2, \quad f = \text{const.}, \tag{5}$$

with field energy density $\varrho = (f/R_1R_2)^2$ (see Lorenz 1980b for details).

The perfect fluid matter is characterized by the equation of state

$$p = (\gamma-1)\varepsilon, \quad 1 \leqslant \gamma \leqslant 2, \tag{6}$$

with energy density $\varepsilon = m/R^{3\gamma}$, $m =$ const. The radiation case $\gamma = 4/3$ has been already discussed by us (Lorenz-Petzold, 1984b). We now consider the "stiff" matter case $\gamma = 2$ as well as the vacuum case $m = 0$. Introducing the new variables g_i and η by $g_1 = R_1R_3\phi$,

$g_2 = R_2R_3\phi$ and $dt = R_1 d\eta$, we obtain the following decoupled field equations (see Lorenz--Petzold 1983, 1984a for details of our new method of reduction of the BDT-field equations):

$$g_2'' = 0, \quad (\ln g_1)'' + (\ln g_1)'(\ln g_2)' = 0, \tag{7}$$

$$(g_2(\ln \phi)')' + 2km(\phi/g_2) = 0, \tag{8}$$

$$(\ln (y\phi))'^2 - 4(\ln g_1)'(\ln g_2)' + (3+2\omega)(\ln \phi)'^2 + 4(m+f^2y)\phi g_2^{-2} = 0, \tag{9}$$

where $y = R_3^2$ and $(\,)' = d/d\eta$. Equations (7) and (8) can be easily solved to give

$$g_2 = a_2\eta + b_2, \quad g_1 = (a_2\eta + b_2)^{a_1/a_2}, \quad a_2 \neq 0,$$

$$g_1 = \exp(a_1\eta + b_1), \quad a_2 = 0,$$

$$\phi = -mk\tau^2 + c\tau + d, \tag{10}$$

where $a_i, b_i, c, d =$ const. and $g_2 d\tau = \phi d\eta$. The scalar field ϕ can be reexpressed as $\phi = \phi(\eta)$.

By setting

$$A^2 = 4a_1a_2 - (3+2\omega)c^2 - 4md, \quad d\eta = g_2 d\xi, \quad (\,)^{\cdot} = d/d\xi, \tag{11}$$

we obtain from (9) and (10) the simple differential equation

$$\dot{z}^2 = A^2z^2 - 4f^2z^3 \tag{12}$$

with the general solution

$$R_3^2 = B[[\exp(-A\xi) + f^2B/A^2]^2\phi \exp(A\xi)]^{-1}, \quad B = \text{const.} \tag{13}$$

The solutions are completed by

$$R_1R_3 = g_1/\phi, \quad R_2R_3 = g_2/\phi, \tag{14}$$

$$a_2(\xi - \xi_0) = \ln(a_2\eta + b_2), \quad a_2 \neq 0: \quad b_2(\xi - \xi_0) = \eta, \quad a_2 = 0, \tag{15}$$

(*i*) $m = 0$:

$$\phi = \phi_0 \exp((c_2/b_2)\eta), \quad a_2 = 0,$$

$$\phi = \phi_0(a_2\eta + b_2)^{c_2/a_2}, \quad a_2 \neq 0, \tag{16}$$

(*ii*) $m \neq 0$:

$$\phi = [\ln (a_2\eta + b_2)^{c/2a_2\sqrt{d}}]^{-2}, \quad a_2 \neq 0, \quad \Delta = 0,$$

$$\left.\begin{aligned} \phi &= \phi_0[\cos[(\sqrt{\Delta}/2b_2)(\eta + d_2)]]^{-2}, \ a_2 = 0, \\ \phi &= \phi_0[\cos[\ln (a_2\eta + b_2)^{\sqrt{\Delta}/2a_2}]]^{-2}, \ a_2 \neq 0, \end{aligned}\right\} \Delta > 0,$$

$$\left.\begin{aligned} \phi &= \phi_0[\cosh[(\sqrt{-\Delta}/2b_2)(\eta + d_2)]]^{-2}, \ a_2 = 0, \\ \phi &= \phi_0[\cosh[\ln (a_2\eta + b_2)^{\sqrt{-\Delta}/2a_2}]]^{-2}, \ a_2 \neq 0, \end{aligned}\right\} \Delta < 0, \tag{17}$$

where

$$\phi_0 = -(1/4m)\Delta(3+2\omega), \quad \Delta = -k[(3+2\omega)c^2+4md], \tag{18}$$

and $c_2, d_2, \xi_0 = \text{const.}$

3. Conclusion

Thus we have obtained a rich spectrum of new magnetic BDT-Bianchi type-I cosmological solutions. By taking appropriate limits we rediscover a lot of known solutions. The GRT-limit $\omega \to \infty$, $c = 0$, $a_2 \neq 0$, i.e. $\phi = \text{const.}$, has been first given by Jacobs (1969a, b) in a somewhat different form (both for the "stiff" matter case as well as for the pure magnetic case). Doroshkevich (1965) discovered this solution in the axisymmetric case (leaving the time dependence in the form of an integral). By setting $B = 2A$ we obtain the GRT-solution (in general with both an electric and a magnetic field) first given by Ruban (1971) (see also Ruban 1977, 1978, 1982). The case $B = A^2/f^2$ turns out to be identical with the solution given by us previously (Lorenz 1980a). The pure magnetic solution was first given by Rosen (1962, 1964) and was rediscovered for the general case by Jacobs (1969a, b) and for the case of axial symmetry by Shikin (1966) (see also Vajk and Eltgroth (1970) and Kramer et al. (1980). By setting $m = f = 0$ we obtain the Kasner (1921) solution in a somewhat different form.

The BDT-Bianchi type-I vacuum solutions ($m = f = 0$) defined by the scalar fields (16) can be transformed into the BDT-Kasner solution first given by Ruban and Finkelstein (1972, 1975) (for details see Lorenz-Petzold, 1984a). The $m \neq 0, f = 0$ case may comprise the "stiff" matter solutions first given by Ruban and Finkelstein (1975), Johri and Goswami (1980) and Singh et al. (1983) (see also Mohanty et al. (1982)). However, this has not been checked explicitly. In this connection we mention also the "stiff" matter Bianchi type-I solutions first given by Nariai (1972) and Raychaudhuri (1975, 1979) on the basis of the conformally transformed version of the BDT given by Dicke (1962). The Bianchi type-I model has been also considered by Matzner et al. (1973).

In the investigation of cosmological effects near the initial singularity, there are no longer any grounds for disregarding the possible existence of a vector (magnetic) field in addition to the scalar field ϕ (see Belinskii and Khalatnikov 1972; 1976). It follows from our Eq. (13)–(18) that the primordial magnetic field ($f \neq 0$) influences together with the scalar field ϕ the character of the initial singularity (pancake). We note that our solutions with $a_2 = 0$ have no analogues in the GRT.

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