AN OUTLINE OF A PROGRAMME INVESTIGATING PARTICLE CREATION BY A BLACK HOLE WITH THE HELP OF WELL--KNOWN QUANTUM-FIELD EFFECTS IN FLAT SPACETIME

BY R. M. NUGAYEV

Kazan State Pedagogical Institute, Kazan 420021, USSR

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A programme investigating particle creation in a black hole by the application of flat--spacetime, quantum-field results is carried out. The utilization of the Casimir-effect results and those of accelerated mirrors reveals that a black hole should produce blackbody radiation that exactly coincides with Hawking's. An important difference between the vacuum stress-tensors of scalar and electromagnetic fields is found. The blackbody spectrum of Hawking radiation is due to the interaction of the radiation with a "cavity" formed by the potential barrier of the gravitational field. The consideration of the potential-barrier finite conductivity makes it possible to eliminate the pathology of the vacuum stress-tensor on the horizon and to reveal that the blackbody radiation should be created in the whole region $[3M, \infty]$.

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1. Introduction; the insufficiency of existing accounts

The generalizations of quantum-field theory to Riemannian spacetimes have begun to be applied seriously to various cosmological and astrophysical problems. In consequence the processes of creation of particles have been predicted in a variety of general relativistic situations (see, for example, De Witt, 1975, and references cited therein). But the careful study of gravitational problems has puzzled the researchers with paradoxes and questions which have so far been successfully evaded in other applications of quantum field theory. A major stumbling block appears to be the inability to extract from the mathematical formalism of Riemannian-spacetime generalizations any correlate of the intuitive notion of particles. Indeed, according to some scholarly notions, elementary particles are certain representations of the Poincare group. But there is no global Killing vector at all in curved spacetime generic situation. Hence quantum field theory cannot ultimately be based on the Poincare group. What is needed is a theory that takes into account the full covariance of Einstein's view of spacetime as a Riemannian manifold.

In particular, it was discovered that, because of quantum-field effects, a black hole, formed by gravitational collapse, has to emit particles-thermal blackbody radiation (Hawking, 1975). However, the fact that quantum field theory in curved spacetimes does not yet exist, at least as a coherent discipline, induces severe ambiguities in the description of important aspects of this effect.

First, it has never been clear where the Hawking radiation is being created. At least three schools of thought have emerged (Fulling, 1977). According to one group of thinkers, thermal radiation must originate even in the collapsing matter.

Secondly, it has never been distinctly known what happens near the horizon of the evaporating black hole. The clarification of the situation depends on the knowledge of the energy-momentum tensor of the quantum field in the vicinity of the horizon. But this quantity is formally divergent, and the meaningful component must be extracted by a regularization procedure. Such procedures always involve ambiguities which must be resolved by the application of additional criteria, e.g. of "physical reasonableness". Different procedures hinge on different assumptions, and it is difficult to find compelling justification for any of them. Moreover, the published literature on particle creation by a black hole is mathematically too sophisticated without providing any easy physical insight as to what is involved. And what is the physical explanation as to why a distant stationary observer will find thermal radiation with a temperature that is inversely proportional to hole mass?

In view of the above, it seems reasonable not to increase the number of treatises based on "more physically significant definitions", but to look instead for a way (i.e. method) of reducing black-hole evaporation effect to better understood effects observed in the laboratory. This should enable us to give either experimental, or less ambiguous theoretical answers to the questions put above. At least part of the task can be fulfilled by the reduction of the effect of particle creation in the gravitational field of a black hole to quantum-field effects in flat spacetime. This programme can be carried out by looking for means of approximating the gravitational field of the black hole by something more convenient.

This paper sketches the programme of reduction of the black-hole-evaporation effect to the Casimir effect and to the effect of particle creation by accelerated mirrors. The influence of a spherically-symmetric gravitational field on the propagation of massless waves is taken into account with the help of a spherical conducting barrier. The first ideal model of evaporation process was constructed with the help of the Casimir effect (Nugayev and Bashkov, 1979). As is well-known, the gravitational field of a black hole acts as a barrier to propagation of massless waves. Viewing the peak of the barrier ($r = 3M$; $c = G = 1$) as the surface of a reflecting sphere permitted us to apply to a black hole the results of various calculations of the Casimir effect. It appeared that the flow of negative Casimir energy should cause the area of the horizon to shrink at a rate consistent with the energy flux observed at future infinity. But the model was too primitive since it provided only qualitative agreement with Hawking's result.

Hence, the second stage of the programme had to be realized. It consisted in the construction of a more sophisticated model capable of demonstrating that the mere existence of a spherical barrier and of the horizon is sufficient to compel the black hole to produce thermal radiation of a temperature that exactly coincides with the result of Hawking (Nugayev, 1982). That was done by means of reducing the black-hole-evaporation effect to the effect of particle creation by accelerated mirrors. The connection between these effects made it possible to find an important difference between the vacuum expectation values of the energy-momentum tensors of scalar and electromagnetic fields in the vicinity of the horizon (Nugayev, 1983). But even the second model appears to be too primitive to provide a satisfactory description of the particle-creation process since the vacuum stress--tensor diverges in the reference frame of a freely falling observer as $r \to 2M$. The pathology of the second model is due to the assumption of ideal conductivity, which is obviously not the case for the spherical potential barrier of a black hole. The purpose of this paper is to carry out the next stage of the programme and to take into account the potential--barrier-finite-conductivity term. It helps to reveal that particles are created in the $[3M, \infty]$ region. A complete plan of the reductionist programme is drawn up which can show the way of solving of the above questions. The identity of the reduction method to that of Hawking is established and an astonishing coincidence of his and of our results is explained.

2. *The potential barrier*

The possibility of application of quantum-field results to the case of the black hole is based on the following fact.

Considering the behaviour of massless integer spin waves in the gravitational field of a nonrotating black hole Price (1972) discovered that the curvature of spacetime creates an effective potential barrier penetrable for high-frequency waves and impenetrable for waves with low frequency. In particular, the Klein-Gordon equation in spherical coordinates with ψ having an angular dependence of a spherical harmonic can be reduced to the equation

$$\psi_{,tt} - \psi_{,r^*r^*} + F_e(r)\psi = 0. \tag{2.1}$$

Here the comma denotes differentiation with respect to time t and Regge-Wheeler "tortoise" coordinate $r^* = r + 2M \ln (r/2M - 1) + const$ ($c = G = 1$); M is the black hole's mass, r is the space coordinate of the Schwarzschild frame of reference $\{t, r, \theta, \varphi\}$.

The equation has the same form as the Schrödinger equation in one dimension for a particle of energy W^2 in the potential $F_e(r) = (1 - 2M/r) \times l(l+1)/r^2$ .The useful and interesting property of the curvature potential $F_e(r)$ consists in that it is a localized barrier for massless waves. The numerical calculations (Price, 1972; Page, 1975, Sanchez, 1978) indicate that the barrier is so well-localized near $r = 1.5\ R_g$ ($R_g = 2M$) that for the propagation of scalar and electromagnetic waves we can consider the regions quite near the horizon and far away from it as "flat". Almost all the scattering takes place in the small region near the peak of the potential barrier.

Fabbri (1975) evaluated the absorption cross section for the absorption of electromagnetic waves by a Schwarzschild black hole. Having imposed purely ingoing (on the horizon) and outgoing (far away from it) boundary conditions, he estimated the transmission coefficients T_l of the barrier first. For frequencies less than

$$W_c = 2/\sqrt{27}\, R_g; \qquad T_l \approx \left[\frac{(l+1)!\,(l-1)!}{(2l)!\,(2l+1)!!}\right]^2 (WR_g)^{2l+2}.$$

For $W > W_c$

$$T_l = 0 \quad \text{for} \quad l > l_c,$$

$$T_l = 1 \quad \text{for} \quad l < l_c.$$

When the frequency W is smaller than the critical frequency W_c turning points exist for all partial waves, that is, for all values of l. When $W > W_c$, turning points exist only for high l waves; more precisely, they exist if l is greater than the critical parameter l_c given by $l_c(l_c+1) = 27W^2M^2$.

$$\sigma_{\text{abs.}} = \frac{\pi}{W^2} \sum_{l=1}^{\infty} (2l+1)T_l.$$

In the high-frequency limit $\sigma^{(\infty)}_{\text{abs.}} = \frac{27}{4}\pi R_g^2$. In the low-frequency limit $\sigma_{\text{abs.}} = \frac{4}{3}\pi(R_g)^4 W^2$. Consequently, $\sigma_{\text{abs.}} \approx \sigma^{(\infty)}_{\text{abs.}}$ under $W > W_c$, and $\sigma_{\text{abs.}} \ll \sigma^{(\infty)}_{\text{abs.}}$ at $W \ll W_c$. The black hole is almost unable to absorb electromagnetic waves at $W \ll W_c$. Hence W_c (or something not very different from it) is the cutoff frequency for the absorption of electromagnetic waves by a Schwarzschild black hole.

The cases of scalar fields (Starobinsky, 1973) and gravitational fields (not affecting the background metric) (Fackerell, 1971) are analogous to the electromagnetic case. In particular, Fackerell found that the transmission coefficient of gravitational radiation for $l \geq 2$ vanishes at $W \to 0$ as W^{2l+2}.

Thus, the potential barrier of the black hole acts as a real conductor which conducts well at low frequencies, but as the frequencies increase, its conductivity diminishes. That is why for the purpose of investigating the propagation of massless waves in Schwarzschild background we can replace the gravitational field of the black hole by a real conducting shell with properties described below. The radius of the shell is $R = 1.5\ R_g$.

It appears that the mere existence of the conducting shell near the horizon is sufficient to compel the black hole to emit black-body radiation of a temperature inversely proportional to M. The purpose of the remaining parts is to show that the evaporation can be described by two effects familiar from quantum electrodynamics: the Casimir effect (the rough estimate) and the effect of particle creation by accelerating mirrors (a more precise account).

3. *Particle creation by a black hole as a consequence of the Casimir effect*

3A. Vacuum fluctuations of the electromagnetic field give rise to an attractive force between a pair of neutral parallel, flat conducting plates (Casimir, 1948, 1949). When one quantizes the field subject to the appropriate boundary conditions on the plates and calculate the vacuum energy with a wavelength cutoff, one finds that as the separation between the plates changes, the vacuum energy per unit area changes by a finite, cutoff--independent amount. So, in spite of the formal divergence of vacuum energy, a change in the configuration of the plates causes a finite shift in the vacuum-state energy, described by the expression

$$\Delta E = -\pi^2 \hbar c A / 720 d^3. \tag{3.1}$$

Here A denotes the area of each plate, and d is a finite separation between them.

It should be specially pointed out that the Casimir energy is of pure vacuum origin. No real particles are involved, only virtual ones. But the experiments (Deryagin, Abrikosova, Lifshitz, 1956; Sparnaay, 1958) encourage us to take it seriously.

For the electromagnetic field[1] the stress-tensor of the vacuum between the plates was calculated by De Witt (1975):

$$\langle T^{\mu\nu} \rangle_{\text{vac}} = T^{\mu\nu}_{(-)} + T^{\mu\nu}_{(+)} = \frac{\pi^2 \hbar c^2}{720 d^4} \operatorname{diag}(-1, 1, 1, 3)$$

$$+ \frac{3\Lambda^4 \hbar c^2}{\pi^2} \operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}), \tag{3.2}$$

where Λ is the frequency cutoff that cuts off the high-frequency waves.

$T^{\mu\nu}_{(-)}$ can be interpreted as corresponding to a gas with rather "bizarre" (De Witt) properties: negative energy and pressure in the direction perpendicular to the plate surfaces, but positive pressure in the other two directions. The "gas" satisfies the thermodynamical law $dE = TdS - pdV$. Hence if one slowly ($dS = 0$) pulls the conductors apart, the work done against the tension shows up exactly as an increase in the vacuum energy.

The cutoff-dependent term $T^{\mu\nu}_{(+)}$ has the same form as the stress tensor of an ordinary photon gas. This term is identical with the expression for the uncluttered vacuum, so it is said not to have any connection with the effect observed in the laboratory. Being universal, it is usually discarded.

The works of Boyer (1968, 1970) offer a method for calculating the vacuum energy inside the uncharged sphere made from a physically realizable conductor. (A real conductor conducts well at low frequencies, but as the frequencies increase its conductivity diminishes considerably.)

Let us approximate a sphere of radius d by two parallel plates of area πd^2 at a distance d apart. With the help of (3.1) and (3.2) we can obtain

$$\Delta E = -\pi^3 \hbar c / 720 d + 3\hbar c \Lambda^4 d^3 / \pi, \tag{3.3}$$

where the second part is a correction for finite conductivity of the plates. The approximation is justified by the exact calculations of Boyer (1968) and Davies (1972) performed independently. Having computed the vacuum energy of a sphere with ideal conductivity, they demonstrated that ΔE *exactly* coincides in magnitude with the cutoff-independent part of Eq. (3.3). Only the sign changes. So, for finite conductivity

$$\Delta E = \pi^3 \hbar c / 720 d - 3\hbar c \Lambda^4 d^3 / \pi.$$

[1] The expression for the stress tensor of vacuum in the case of massless scalar field differs from that for the electromagnetic field only by the factor (1/2).

Let us now turn to the case of a Schwarzschild black hole for an application of the obtained results.

3B. Considering the peak of the potential barrier ($r = 1.5\ R_g$) as the surface of a Casimir conducting sphere, we can calculate the energy of the vacuum between the barrier and the horizon[2]. Yet, to choose a correct sign we have to consider the process of black-hole formation. At the very beginning the surface of a collapsing star R_1 and the peak of the potential barrier R_2 can be approximated by Casimir spheres. First the negativity of the vacuum energy between the barrier and the star is determined by the comparability of R_1 and R_2 (Boyer, 1968). Then the negative flow is strengthened by the formation of the horizon. It should be pointed out that the negativity of the finite amount of vacuum energy inside the sphere with $R = 3M$ does not violate the positive-energy conditions (R. Penrose and S. Hawking) crucial to the very existence of black holes because of a certain amount of particles created by time-dependent gravitational field. The resulting positive flow depends on the details of the collapse. At late times (essential for Hawking radiation) the particles will disperse.

Thus, we must utilize (3.3) with $d = 1.5\ R_g$:

$$\Delta E \cong -1.3 \frac{\pi^3 \hbar c}{3R_g \cdot 720} + 26 \frac{\hbar c}{\pi R_g \cdot 729};$$

the energy density

$$\varepsilon = -\frac{2\hbar c}{720 R_g^4} + \frac{1.2\hbar c}{729 R_g^4}. \tag{3.4}$$

The positive cutoff-dependent term $\varepsilon_{(+)}$ cannot be discarded in accordance with usual practice, for it depends now on $\Lambda = \omega_c \cong \sqrt{2}/\sqrt{27} R_g \sim d^{-1}$, i.e. on the distance between the plates. Both parts of the vacuum energy T^{00} have the same order of magnitude and the same dependence on R_g. The first part $\varepsilon_{(-)}$ represents the flux of negative virtual energy flowing into the horizon of the black hole and diminishing its mass according to the law $dM/dt = -const./M^2$. It can be shown that the negative energy flux should cause the area of the horizon to shrink at a rate consistent with the energy flux observed at infinity (Sciama, Candelas, Deutsch, 1980). That is why the cutoff-dependent part $\varepsilon_{(+)}$, having the same form as the stress-tensor of an ordinary photon gas, must be considered as representing the flux of real particles "created" per unit of the barrier's surface.

It is quite reasonable to admit that, because of the interaction with "walls", the thermodynamic equilibrium installs itself inside the cavity formed by the surfaces of the barrier and of the collapsing star. Consequently, the radiation there should be blackbody radiation. For a distant stationary observer at I^+ the surface of the collapsing star reaches the horizon during the infinite time interval. So the blackbody spectrum of Hawking radiation at future infinity is, perhaps, due to the peculiarities of interaction of the "walls" with radiation in the black-hole-formation process.

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[2] The region under the horizon is inaccessible to a distant observer.

According to the Stefan-Boltzmann law, $c\varepsilon = \sigma T^4$, hence

$$T \cong \frac{\hbar c}{\pi \cdot k \cdot R_g} \tag{3.5}$$

This expression coincides in the order of magnitude with the result of Hawking (1975). The qualitative agreement of (3.5) with Hawking's formulae should not surprise us since $\Lambda = \sqrt{\frac{2}{27}}\frac{1}{R_g}$ is not an exact cutoff frequency just as $d = 1.5\ R_g$ is not an exact radius of the peak of the barrier at all.

Secondly, the real potential barrier owes by a considerable depth, consisting of various strata, each with a cutoff frequency of its own.

Thirdly, the uniformity of vacuum-energy distribution between parallel flat plates (Ford, 1974) is obviously not the case inside the spherical potential barrier.

All these indicate that the Casimir model of black-hole evaporation is too rough to reproduce the important features of Hawking radiation. That is why a more precise account of this process must hinge on a model that is less dependent on these features of the potential barrier. The model should produce a more adequate account of the situation in the region between the peak of the barrier and the horizon. This can be done by applying accelerated--mirror results.

4. *Utilizing the accelerated mirrors*

4A. Both for scalar and for electromagnetic fields the vacuum stresses induced by uniform acceleration of a perfect plane conductor correspond to the absence from the vacuum of blackbody radiation (Candelas and Deutsch, 1977). Calculating the physically significant or renormalized stress energy tensor $\langle T^{\mu\nu}(x)\rangle$ as the difference between the vacuum expectation value of the stress energy tensor $\langle T^{\mu\nu}\rangle_{\text{vac}}$ and the value it would have with the plane conductor at rest, one finds that in the $\xi/b \to \infty$ limit (far from conductor) for scalar field

$$\langle T^{\mu}_{\nu}\rangle \sim -\frac{1}{2\pi^2\xi^4}\int_0^\infty \frac{\omega^3 d\omega}{e^{2\pi\omega}-1}\,\text{diag}\,(-1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}). \tag{4.1}$$

For electromagnetic field

$$\langle T^{\mu}_{\nu}\rangle \sim -\frac{1}{\pi^2\xi^4}\int_0^\infty \frac{d\omega(\omega^3+\omega)}{e^{2\pi\omega}-1}\,\text{diag}\,(-1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}). \tag{4.2}$$

Here accelerated (Rindler) coordinates are introduced:

$$t = \xi\ \text{sh}\ \tau, \quad x = \xi\ \text{ch}\ \tau, \quad d\varrho^2 = -\xi^2 d\tau^2 + d\xi^2 + dy^2 + dz^2.$$

In this system of reference the curves $\xi = \text{const}$, $(y, z) = \text{const}$ are worldlines of constant proper acceleration ξ^{-1}. The surface $\xi = b$ represents the trajectory of the barrier.

The asymptotic forms (4.1)–(4.2) are independent of Neumann or Dirichlet boundary conditions on the barrier. They are also independent of the acceleration b^{-1} of the barrier. They depend on the acceleration of the local Killing trajectory only.

The temperature of the blackbody radiation (in the frame of an observer with proper acceleration ξ^{-1}) is $T = (2\pi\xi)^{-1}$. It means that if thermal radiation of temperature $(2\pi\xi)^{-1}$ was added, the resulting state would be indistinguishable, under $\xi/b \to \infty$, from the usual Minkowski vacuum (Unruh, 1976; Candelas and Raine, 1976).

For treating the more important case of an accelerating plane barrier made from a physically realizable conductor we have to consider the 2-dimensional results first. In the case of a barrier at rest the vacuum tensor reduces to that of uncluttered Minkowski space. Because of the conformal invariance of the 2-dimensional theory one expects the same to be true for an accelerated barrier, and so it is (De Witt, 1975). Yet here a problem occurs: if we insert an oscillating factor exp $(i\omega/\Lambda)$ into the expression for vacuum energy, we find that

$$\langle T^{\mu\nu}\rangle_{\text{vac}} = \xi^{-2}\,\frac{\Lambda^2}{2\pi}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}. \tag{4.3}$$

But this means that the vacuum stress tends to zero as $\xi \to \infty$, something it does not do in the unaccelerated case. The cause of this phenomenon is that ω has the significance of a local particle frequency only at $\xi = 0$ (on the surface of the conductor). Anywhere else the local frequency is $\xi^{-1}\omega$. The cutoff Λ in (4.3) refers not to a local frequency but to a Doppler shifted one. If we agree to use a Λ that varies with position in such a way as to give always the same local cutoff frequency, then (4.3) should be replaced by

$$\langle T^{\mu\nu}\rangle_{\text{vac}} = \frac{\Lambda^2}{2\pi}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}. \tag{4.4}$$

Nevertheless, calculations performed with the help of covariant point-separation technique (Fulling and Davies, 1975) disclose that (4.4) is only a finite-conductivity correction. The complete 2-dimensional stress-tensor expression is

$$\langle T^{\mu\nu}\rangle_{\text{vac}} = T^{\mu\nu}_{(-)} + T^{\mu\nu}_{(+)} = -\frac{1}{24\pi\xi^2}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix} + \frac{\Lambda^2}{2\pi}\begin{pmatrix}1 & 0\\ 0 & 1\end{pmatrix}. \tag{4.5}$$

Let us compare (4.1) and $T^{\mu\nu}_{(-)}$ of (4.5). While passing to four dimensions the ξ^{-2} term transforms to ξ^{-4}. The dimension arguments provide also the transformation of Λ^2 term to Λ^4. Hence in the 4-dimensional case we obtain the following finite-conductivity correction:

$$T^{\mu\nu}_{(+)} \cong \frac{3\Lambda^4}{\pi^2}\,\text{diag}\,(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}), \tag{4.6}$$

where the coefficient behind Λ^4 and the bracket-expression are justified by comparison with the equation for the finite-conductivity correction of a single unaccelerated conductor. (This equation coincides with $T^{\mu\nu}_{(+)}$ of Eq. (3.2), obtained as a result of explicit calculations.)

Therefore, the full expression for the vacuum stress-tensor of scalar field induced by the acceleration of a physically realizable plane conductor is:

$$\langle T^{\mu\nu}\rangle_{\text{vac}} \cong -\frac{1}{2\pi^2\xi^4}\int_0^\infty \frac{\omega^3 d\omega}{e^{2\pi\omega}-1}\,\text{diag}\,(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3})+\frac{3\Lambda^4}{\pi^2}\,\text{diag}\,(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}). \tag{4.7}$$

The equation (4.7) is analogous to that of the Casimir effect in the sense that in both cases the finite-conductivity corrections are independent of the state of motion of the conductors. (4.7) is independent of the proper acceleration of the conductor. It depends on the local Killing acceleration only.

Consider a particle which is at rest in the gravitational field of a Schwarzschild black hole. Its 4-velocity $u^\alpha \equiv dx^\alpha/d\tau = \left(\left(1-\frac{2M}{r}\right), 0, 0, 0\right)$. The proper acceleration of the particle is $a^\alpha \equiv Du^\alpha/d\tau = du^\alpha/d\tau + \Gamma^\alpha_{\beta\gamma}u^\beta u^\gamma = \Gamma^\alpha_{tt}u^t u^t(\alpha, \beta, \gamma = t, r, \theta, \varphi)$. The single non-vanishing component of Γ^α_{tt} is $\Gamma^r_{tt} = (M/r^2)\left(1-\frac{2M}{r}\right)$. Hence

$$a^\alpha = \left(0, \frac{M}{r^2}, 0, 0\right), \quad |a| \equiv (g_{\alpha\beta}a^\alpha a^\beta)^{1/2} = \left(1-\frac{2M}{r}\right)^{-1/2}\frac{M}{r^2}. \tag{4.8a}$$

A distant stationary observer will measure

$$d^\alpha \equiv \frac{Du^\alpha}{d\tau}\frac{d\tau}{dt} = a^\alpha\left(1-\frac{2M}{r}\right)^{1/2};$$

$$|d| \equiv (g_{\alpha\beta}d^\alpha d^\beta)^{1/2} = \frac{M}{r^2}. \tag{4.8b}$$

The potential barrier (localized near $r = 3M$) has a nonzero proper acceleration $b^{-1} \sim M^{-1}$. Now we can utilize accelerated-mirror results.

4B. According to the Strong (or Einstein's) Principle of Equivalence (Thorne, Lightman and Lee, 1973), we can replace a set of observers that are at rest ($r = r_0$) in the gravitational field of a black hole by a set of observers that move with proper accelerations $a = \left(1-\frac{2M}{r_0}\right)^{-\frac{1}{2}}\frac{M}{r_0^2}$ in flat spacetime. (The horizon with all its trapped-surface properties is, of course, left unchanged.) The observer that rests on the peak of the potential barrier in the gravitational field of a Schwarzschild black hole is equivalent to an observer on the surface of a real conducting shell that expands with proper acceleration $b^{-1} \sim M^{-1}$ in flat spacetime. The success of the approximation of Casimir sphere by two parallel plates permits us, for the calculation of $\langle T^{\mu\nu}\rangle_{\text{vac}}$, to exchange the expanding sphere by two plane conductors with equal proper accelerations.

It should be pointed out that this plate-approximation is valid only for sufficiently distant regions: for the vicinity of the horizon and for future infinity J^+. Indeed,
(i) for $r \leqslant 3M$ the turning points of waves with various frequencies fill in all the interval $[2M, 3M]$ (Fabbri, 1975). But for an observer in the vicinity of the horizon ($r \approx 2M$)

the replacement of short-tailed potential barrier at $r \approx 3M$ by a plane conductor should not distort the image considerably.

(*ii*) According to § 3, the mass of the hole must diminish owing to the initial flow of negative Casimir energy:

$$\frac{dM}{dt} = -\frac{A}{M^2}; \text{ consequently, } M \to 0 \text{ and } \frac{\xi}{b} \sim M^{-2} \to \infty.$$

So, the asymptotic forms (4.1) and (4.2) can be utilized at late times only. The replacement of the long-tailed part of the spherical potential barrier by a real conducting plane should not spoil the frequency picture only if the observer is placed at the future infinity J^+.

Now we can investigate the vacuum stress-tensor of scalar and electromagnetic fields near the horizon.

(a) An observer who is at rest ($r = r_0$) in the gravitational field near the horizon will discover a negative flux of blackbody radiation with temperature

$$T = (2\pi\xi)^{-1} = \frac{1}{2\pi}\frac{M}{r_0^2}\left(1 - \frac{2M}{r_0}\right)^{-1/2}.$$

The ratio of observed energy $\hbar\omega$ to energy $\hbar\omega_0$ (the gravitational blue shift) of the photon emitted at J^+ is

$$\frac{\omega}{\omega_0} = (g_{tt})^{-1/2} = \left(1 - \frac{2M}{r_0}\right)^{-1/2}.$$

Along the light ray $\frac{W}{T} = \text{const}$ (Misner, Thorne, Wheeler, 1973). Hence $\frac{W_0}{T_0} = \frac{W}{T}$, from which $T_0 = \frac{W}{W_0}T$.

A distant stationary observer at future infinity J^+ will find that the vacuum stress in the vicinity of the horizon corresponds to absence from the vacuum of radiation with temperature $T_0 = (2\pi)^{-1}(M/r_0^2)$.

According to (4.8), $\frac{M}{r_0^2}$ is the magnitude of acceleration of a particle at rest in the gravitational field of a Schwarzschild black hole. It tends to so-called "surface gravity" κ when the particle is infinitesimally close to the event horizon (Bardeen, Carter, Hawking, 1973). Strictly speaking, the result follows from the fact that the local Killing acceleration $\xi^{-1} = |a| = \sqrt{a_\alpha a^\alpha}$ ($a^\alpha = u^\alpha_{;\beta}u^\beta$, $u^\beta = u^t K^\beta$, K^β — time translating Killing vector) tends to $-K_{\sigma;\beta}h^\alpha K^\beta = \kappa$ (where $h^\alpha K_\alpha = -1$) at the horizon of a Schwarzschild black hole. κ represents the extent to which the time coordinate t is not an affine parameter along the generators of the horizon. Along the horizon κ is constant. For a nonrotating black hole

$$\kappa = \frac{\mathfrak{G}M}{R_g^2} = \frac{c^4}{4\mathfrak{G}M}.$$

The invariance of the distribution function $N = h^{-4}(J_\nu/\nu^3)$ along the world line of a photon (Misner at al., 1973, part 2, ch. 22) guarantees that the radiation (or the lack of it) which is blackbody in one local Lorentz frame would remain blackbody for any other local Lorentz observer. That is why an observer at J^+ discovers that the vacuum stress in the vicinity of the horizon corresponds to the absence from the vacuum of *blackbody* radiation with $T = \frac{\kappa}{2\pi}$. Owing to Candelas, Deutsch and Sciama (1981), we can conclude that the negative flow through the horizon shows up as the positive flux of particles at J^+ with $T = \frac{\kappa}{2\pi}$. This result exactly coincides with that of Hawking. Thus we can now explain the origin of "the striking formal resemblance between the black-hole-formation process and the accelerating mirror system" (Davies, Fulling, Unruh, 1976).

(b) Now we can evaluate the vacuum stress tensor for massless fields in the vicinity of the horizon and far away from it. Using the expressions $\omega_0 = \omega\kappa$, $\kappa = (4M)^{-1}$ and Eq. (4.8a) we obtain (for scalar field):

$$\langle T^{\mu\nu}\rangle_{\text{vac}} = T^{\mu\nu}_{(-)} + T^{\mu\nu}_{(+)} = -\frac{M^4(4M)^4}{\pi^2\left(1-\frac{2M}{r}\right)^2 r^8}\int_0^\infty \frac{\omega_0^3 d\omega_0}{e^{8\pi M\omega_0}-1}$$

$$\times \operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}) + \frac{3\Lambda^4}{\pi^2}\operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}), \tag{4.9a}$$

where Λ is to be evaluated in $r \to 2M$ and $(r/M) \to \infty$ limits separately.

For electromagnetic field

$$\langle T^{\mu\nu}\rangle_{\text{vac}} \cong -\frac{M^4(4M)^4}{2\pi^2\left(1-\frac{2M}{r}\right)r^2}\int_0^\infty \frac{\omega_0 d\omega_0(\omega_0^2+\kappa^2)}{e^{2\pi\omega_0/\kappa}-1}\operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3})$$

$$+ \frac{3\Lambda^4}{2\pi^2}\operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}). \tag{4.9b}$$

In the latter case the spectrum of the lack of blackbody radiation is not Planckian, but is precisely thermal.

In the vicinity of the horizon $\Lambda = \omega_c\left(1-\frac{2M}{r}\right)^{-\frac{1}{2}}$, hence (for scalar field):

$$\langle T^{\mu\nu}\rangle_{\text{vac}} \underset{r\sim 2M}{\cong} -\frac{1}{\pi^2\left(1-\frac{2M}{r}\right)^2}\int_0^\infty \frac{\omega_0^3 d\omega_0}{e^{8\pi M\omega_0}-1}\operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3})$$

$$+ \frac{3\omega_c^4}{\pi^2\left(1-\frac{2M}{r}\right)^2}\operatorname{diag}(1, \tfrac{1}{3}, \tfrac{1}{3}, \tfrac{1}{3}). \tag{4.10}$$

In the electromagnetic case

$$\langle T^{\mu\nu}\rangle_{\text{vac}} \underset{r-2M}{\cong} -\frac{1}{2\pi^2\left(1-\dfrac{2M}{r}\right)^2}\int_0^\infty \frac{\omega_0 d\omega_0(\omega_0^2+\kappa^2)}{e^{2\pi\omega_0/\kappa}-1}\operatorname{diag}\left(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}\right)$$

$$+\frac{3\omega_c^4}{2\pi^2\left(1-\dfrac{2M}{r}\right)^2}\operatorname{diag}\left(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}\right). \tag{4.11}$$

The first parts of both (4.10) and (4.11) exactly coincide with the results of strict calculations (Candelas, 1980), obtained by the usual methods of quantum-field theory in curved spacetimes. These results were obtained for Boulware vacuum $|B\rangle$, defined by requiring normal modes to be positive frequency with respect to the Killing vector $\dfrac{\partial}{\partial t}$ with respect to which the exterior region is static. This vacuum corresponds to the familiar concept of an empty state for large radii. It is considered pathological at the horizon in the sense that the expectation value of the stress tensor $T^{\mu\nu}_{(-)}$ evaluated in a freely falling frame diverges as $r \to 2M$. But the account of the finite-conductivity correction term $T^{\mu\nu}_{(+)}$ puts things right. We might like to think of $T^{\mu\nu}_{(-)} = \langle B|T^{\mu\nu}|B\rangle$ as only the pure vacuum polarization part of $\langle T^{\mu\nu}\rangle_{\text{vac}}$. The "radiation part" $T^{\mu\nu}_{(+)}$ also becomes infinite on the horizon, making the sum of the "pure vacuum polarization part" and the "radiation part" finite.

In the case of a uniformly accelerated perfect conductor Candelas and Deutsch (1977, 1978) found that for a massless field of spin s

$$\langle T^{\mu\nu}\rangle_{\text{vac}} \sim -\frac{h(s)}{2\pi^2\xi^4}\int_0^\infty \frac{\omega d\omega(\omega^2+s^2)}{\exp(2\pi\omega)-(-1)^{2s}}\operatorname{diag}\left(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}\right), \tag{4.12}$$

where $h(s)$ is the number of helicity states.

The "very close resemblance" of black-hole results to those for an accelerated plane conductor led P. Candelas to conjecture that for a black hole

$$\langle B|T^{\mu\nu}|B\rangle \underset{r\sim 2M}{\sim} -\frac{h(s)}{2\pi^2\left(1-\dfrac{2M}{r}\right)^2}\int_0^\infty \frac{d\omega\omega(\omega^2+\kappa s^2)}{e^{2\pi\omega/\kappa}-(-1)^s}\operatorname{diag}\left(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}\right). \tag{4.13}$$

He verified this conjecture by direct calculation for the $s = 1$ case. But, with the help of (4.8a) and $\omega_0 = \omega\kappa$, we can easily transform (4.12) into (4.13). Taking into account the finite-conductivity correction terms leads to

$$\langle B|T^{\mu\nu}|B\rangle \underset{r\sim 2M}{\cong} -\frac{h(s)}{2\pi^2\left(1-\frac{2M}{r}\right)^2}\int_0^\infty \frac{d\omega_0\omega_0(\omega_0^2+\kappa s^2)}{e^{2\pi\omega_0/\kappa}-(-1)^{2s}}\,\mathrm{diag}\,(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3})$$

$$+\frac{3h(s)\omega_c^4}{2\pi^2\left(1-\frac{2M}{r}\right)^2}\,\mathrm{diag}\,(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}). \tag{4.14}$$

Far from the horizon ($r \gg 2M$), unlike the $r \sim 2M$ case, the vacuum-polarization term $T^{\mu\nu}_{(-)}$ is negligible, and what is left is the "radiation part" $T^{\mu\nu}_{(+)}$:

$$\langle T^{\mu\nu}\rangle_{\mathrm{vac}} \underset{r\gg 2M}{\cong} \frac{3h(s)}{2\pi^2}\left(\tfrac{2}{27}\right)^2\frac{1}{M^4}\,\mathrm{diag}\,(1,\tfrac{1}{3},\tfrac{1}{3},\tfrac{1}{3}). \tag{4.15}$$

The vacuum stress-tensor at infinity corresponds to that of ordinary photon gas. Hence $T^{tt}_{(-)}$ represents the energy density depressed below that of the Minkowski vacuum that flows down the horizon and diminishes its surface. This flow appears to an observer at J^+ as a positive flux of particles created by the gravitational field of a Schwarzschild black hole. The investigation of the Casimir effect indicates that the particles involved in the region between the horizon and the peak of the potential barrier are virtual ones. Consequently the real particles must be created outside the region $[2M, 3M]$. This conclusion is supported by the following considerations revealing the mechanism of the particle-creation process.

If a conducting barrier is present in vacuum, the reflection of vacuum fluctuations gives rise to a cloud of virtual negative energy in the vicinity of the conductor surface. The case of the (nonuniformly) accelerated barrier differs from the stationary one in that the reflected wave is amplified to give a significant flux of particles. The flux is observable far from the barrier, where the virtual cloud is absent. Now the case of the nonrotating black hole differs from the one mentioned above in that the role of the real accelerated conductor is played by the gravitational potential barrier that "pushes" the partial waves in the direction of the distant observer at future infinity J^+. The "conductor" is forced to move with nonuniform ($M \neq 0$) acceleration by the flux of negative virtual energy that flows into the hole and sets the whole cutoff mechanism in motion. But while considering the evaporation process we must keep in mind that the replacement of the potential barrier by a real thin conducting shell is justified if we are interested in the vacuum stresses in the vicinity of the horizon only. When considering the process of particle creation we must take into account the fact that the potential barrier has a long tail of its own — a long mantle that falls from the peak at $r = 3M$ up to spatial infinity $r = \infty$. Hence each partial wave with fixed (w, l) has a barrier of its own (i.e. the turning point) located somewhere in the $[3M, \infty]$ region.

Consequently, none of the real particles are created under the horizon. Nor are they created "even in the collapsing matter", nor in the vicinity of $r = 2M$. They are created by the tail of the potential barrier in all the region $[3M, \infty]$.

To answer the question why the radiation appearing at J^+ is the blackbody radiation let us look at the region between the peak of the potential barrier and the horizon. Consider the Casimir effect for a sphere placed around the horizon of a black hole. All the action of the gravitational field on the propagation of the radiation inside the sphere reduces to supplying the virtual frequencies multiplied by the $(1-2M/r)^{-\frac{1}{2}}$ coefficients. According to Ford (1975), they are the wavelengths of the order of d (the distance between the Casimir plates) that give the main impact to the Casimir energy. In the hole case these wavelengths correspond to unstable photons moving around the horizon at $r = 3M$ with impact parameters that slightly differ from the critical one. These particles give the main impact to the Hawking energy (see the next section). We can consider the interaction of these waves with the surface of the sphere without taking the horizon into account. The interaction of impact-parameter photons with the surface of the sphere leads to the installation of thermodynamical equilibrium. Hence the radiation (or the lack of it) inside the cavity must be the blackbody radiation. No real particles are involved in the region inside the cavity, only virtual ones. But the existence of the horizon and (especially) of the potential barrier forces us to take them seriously. The initial flow of negative Casimir energy into the hole forces the barrier to shrink ($r_0 \to r_1$) releasing the main-impact photons at $r = 3M$. After penetrating through the $r = 3M$ barrier each virtual (w, l) wave appears to the observer at J^+ as a real one created by accelerated barrier at the turning point.

5. *Comparison with the Hawking method*

The identity of the proposed method to that of Hawking can be revealed in a more direct way with the help of results obtained by Davies (1975). He indicated that Hawking's conclusion on the existence of blackbody radiation hinges on the existence of the event horizon in the Schwarzschild system, which divides the solutions of the massless wave equation into two classes. The solutions of the first class manage to propagate from J^- through the center of the collapsing object and out to J^+. Those solutions which are trapped by the formation of the horizon and do not reach J^+ form the second class. It is sudden variation in the Fourier transform on J^- due to this division that is responsible for particle production.

Davies noticed that a similar situation would arise in the Rindler case. Almost identical properties may be ascribed to the Rindler system by equipping the space with a reflecting wall. Its purpose is to turn incoming waves into outgoing ones just in the same manner as incoming waves are changed into outgoing waves by passage through the centre of a collapsing object in the black hole system. He showed that a straightforward application of Hawking's argument to the flat-spacetime system leads to a Fourier transform which is essentially identical to that of the black hole case.

The mathematical technique of Davies's paper will be of special importance to us; that is why we shall consider it more thoroughly.

Rindler coordinates may be defined for the two-dimensional Minkowski space by

$$z = (x^2 - t^2)^{1/2}, \quad 0 < z < \infty,$$
$$b = \tanh^{-1}(t/x), \; -\infty < b < \infty, \tag{5.1}$$

with lines of constant z corresponding to the world lines of observers undergoing uniform acceleration of z^{-1}. The two asymptotes ($z = 0$, $v = -\infty$) and ($z = 0$, $v = +\infty$) therefore behave as event horizons. They were referred to by Davies as the past and the future horizons respectively.

The surfaces $v =$ const are the Cauchy surfaces for the region covered by Rindler coordinates, so we may express the field at a general point (z, v) in terms of a complete set of incoming solutions of the massless covariant wave equation. In particular we may consider the decomposition

$$\phi(z, v) = \sum_{\omega} (a_\omega f_\omega + a_\omega^+ \bar{f}_\omega) \tag{5.2}$$

on a surface with large negative v. In a quantum theory a_ω, a_ω^+ may be interpreted as annihilation and creation operators for incoming scalar particles.

In distant regions sizeable contributions to the field disturbance will arise from two sources. First the incoming disturbances discussed above will cross the surface $v = \infty$. This surface is a future event horizon for the accelerating observers, so that the region beyond it is analogous to a "black hole". There will also be outgoing disturbances caused by reflection at the wall. They appear to a distant observer to have crossed the past event horizon.

In a region with v large and positive (near the point 0) we may therefore decompose the field as follows:

$$\phi(z, v) = \sum_{\omega} (b_\omega g_\omega + b_\omega^+ \bar{g}_\omega + c_\omega h_\omega + c_\omega^+ \bar{h}_\omega), \tag{5.3}$$

where g_ω represent solutions which are reflected by the wall and pass out to large z while h_ω represent solutions which cross the horizon into the "black hole". The operators b and c are the respective annihilation and creation operators for particles of these types.

If we write

$$g_\omega = \int_0^\infty (\alpha_{\omega\omega'} f_{\omega'} + \beta_{\omega\omega'} \bar{f}_{\omega'}) d\omega' \tag{5.4}$$

then $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ are in general nonzero. Consequently the vacuum state for incoming particles will not be the vacuum state for an observer near 0. He will instead see the production of particles with an expectation value $d\omega \int_0^\infty |\beta_{\omega\omega'}|^2 d\omega'$ for the range ω to $\omega + d\omega$.

Having determined the form of g_ω in the region of large negative v by adopting the argument used by Hawking in the Schwarzschild case, Davies calculated $\alpha_{\omega\omega'}$ and $\beta_{\omega\omega'}$ by taking a Fourier transform of g_ω. The result turned out to be the same as that given by Hawking:

$$|\alpha_{\omega\omega'}| = \frac{|\Gamma(1 - i\omega)|}{(\omega\omega')^{1/2}}; \quad |\beta_{\omega\omega'}| = \frac{e^{-\pi\omega}|\Gamma(1 - i\omega)|}{(\omega\omega')^{1/2}}. \tag{5.5}$$

If (5.5) is substituted into the expression for the amount of particle production, this expression diverges logarithmically. This means a steady rate of particle production over an infinite period of time. This may be demonstrated by constructing wave packets.

Consider an incoming wave packet approaching the wall at an advanced time $\tau > \tau_0$ (τ_0 — some characteristic proper time). All of this packet will cross the future horizon into the "black hole" region. To a Rindler observer such as 0 it will appear to have been completely absorbed by the wall. However, 0 will also see the emission of radiation by the wall, due to the particle production effect, from wave packets which approach the wall for $\tau < \tau_0$, and then reflect to reach 0. For him all of these outgoing wave packets will appear to have crossed the past horizon (in the absence of the wall) in a manner identical to that of the passage of the former incoming packet into the "black hole". The ratio of emission to absorption cross section turns out to be $(e^{2\pi\omega}-1)^{-1}$. This expression has the form of that for a black body radiator of temperature $(2\pi)^{-1}$. It does not depend on the distance between the wall and the origin of coordinates. To convert this expression to geometric units it should be remembered that an observer with coordinates (z, v) would interpret a wave with time dependence $\exp(iwv)$ as having a frequency $\omega/z = \omega x$ (proper acceleration of observer). To such an observer the wall surface would appear to have a temperature of $(acceleration)/2\pi$.

Now we can apply Davies's results to the case of a black hole. The potential barrier localized near $r \cong 3M$ behaves like a wall that turns the incoming waves into outgoing ones. An observer at J^+ is accelerated towards an observer on the peak of the wall with proper acceleration

$$|a| = \left(1 - \frac{2M}{r_0}\right)^{-1/2} \frac{M}{r_0^2}\bigg|_{r_0 \cong 3M} \cong \frac{1}{3\sqrt{3}\,M}. \tag{5.6}$$

Therefore from the point of view of an observer at J^+ the barrier radiates as a blackbody with temperature $a/2\pi \cong (6\sqrt{3}\pi M)^{-1}$, which is in qualitative agreement with Hawking's result. The decay is again due to the fact that $r = 3M$ is not an exact peak of the barrier at all. And, of course, the approximation of a sphere by a plane conductor at $r = 3M$ is too rough.

The agreement of (5.6) with Hawking's results is of no surprise since Davies's method is almost identical to that of Hawking. But the origin of this identity lies in the deep analogy between Rindler and Kruskal coordinates. Indeed, compare the expressions that determine the transition from Minkowski to Rindler coordinates

$$\{\xi = (x^2 - t^2)^{1/2};\ \tau = \tanh^{-1}(t/x)\}$$

with the expressions for the transition from Kruskal coordinates to those of Schwarzschild:

$$\left\{\left(\frac{r}{2M} - 1\right) e^{r/2M} = u^2 - v^2;\ t = 4M \tanh^{-1}(v/u)\right\}.$$

It can obviously be seen that the relation of Rindler coordinates to those of Minkowski is almost the same as the relation of Schwarzschild coordinates to those of Kruskal.

All the mathematics of Hawking's 1975 paper can be reinterpreted as describing the particle creation by a spherical barrier in flat spacetime with horizon. The reader acquainted with that paper can easily affirm that this is the case. As in Hawking's paper, f_i, p_i, q_i are again the solutions of the scalar wave equation that are determined on J^-, on the event horizon and on J^+ respectively. But in considering the solution p_ω, which propagates backwards from J^+ with zero Cauchy data on the event horizon, the following commentary is necessary. A part $p_\omega^{(1)}$ of the solution p_ω will also be scattered by static Schwarzschild field outside the collapsing body and will end up on J^- with the same frequency ω, giving a $\delta(\omega-\omega')$ term in $\alpha_{\omega\omega'}$. The remainder $p_\omega^{(2)}$ of p_ω will now enter the region between the horizon and the potential barrier where it will be partly scattered[3] and partly reflected, eventually emerging at J^+. To an observer near the horizon the wave would seem to have a very large blue-shift. Because its effective frequency is very high, the wave would propagate by geometric optics. As was pointed out by De Witt (1975) the waves crowd together infinitely densely in the region just outside the horizon. This is an expression of gravitational red shift. The nearer to the horizon a wave finds itself the shorter must be its local wavelength in order that it has a predetermined fixed frequency at infinity.

It is these waves that dominate in the flow of created particles at J^+. Hence there exists an effective ray v_0 such that for $v > v_0$ the impact is almost zero. Calculating the form of $p_\omega^{(2)}$ on J^- near $v = v_0$, we can estimate the total particle flux at future infinity. And this can be done with the help of Hawking's arguments and with all the null vectors and other techniques for work near the horizons. All the remaining part of Hawking's 1975 paper is firm and undeniable support of the conclusion that it is the horizon and the potential barrier that create particles with $T = \kappa/2\pi$. It is now quite understandable why Hawking's final result does not depend on the details of the collapse! It is because his mathematics in fact pictures the effect of particle creation by a spherical potential barrier. This can be seen with more clarity from De Witt's 1975 description of black-hole evaporation.

6. *Discussion*

We have provided explanations of why the radiation is the blackbody radiation and where it comes from. Yet our considerations are qualitative ones. The strict answer depends on the future quantitative analysis of the situation in the vicinity of the potential barrier. I believe that the appropriate way to resolve these problems is to realize a programme of reduction of black-hole evaporation to quantum-field effects in flat spacetime. This programme tries to fit the reality by producing a sequence of models that give more and more strict and complete descriptions of the evaporation process. The irrefutable "hard core" of the programme (J. Lakatos) consists in the assertion that "the effect of black-hole evaporation can be understood with the help of quantum-field effects in Minkowski spacetime". Its "positive heuristic" or strategy of research should consist of a set of auxiliary hypotheses which define the important problems and install a sequence of models that describe the

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[3] $r = 3M$ is a critical radius only for stable photon orbits. The region $[2M, 3M]$ is filled with photons with impact parameter that slightly differs from the critical one (Ford, 1982).

process with increasing precision. The "positive heuristic" of our programme consists of the following assertion: "to understand the process of particle creation by a black hole we must replace its gravitational field by a real conductor". The form of the conductor is determined by the particular model.

The first ideal model of the programme — a pair of plate conductors at rest near the horizon of nonrotating black hole — appeared to be too rough to describe important peculiarities of the evaporation process (for instance, the blackbody spectrum of created particles was lacking). Hence the next step should consist of the following:
(2) exact calculation of the vacuum stress-tensor for a sphere made from a physically realizable conductor. Then:
(3) $\langle T^{\mu\nu}\rangle_{vac}$ for an accelerated infinite barrier with finite conductivity;
(4) $\langle T^{\mu\nu}\rangle_{vac}$ for two plates made from a physically realizable conductor;
(5) $\langle T^{\mu\nu}\rangle_{vac}$ for sphere made from an ideal conductor and expanding with acceleration;
(6) The sphere is made from a physically realizable conductor;
(7) Two concentric spheres (one of them made from an ideal conductor) expand with equal proper accelerations. The sphere with $R_1 < R_2$ has all the reflecting properties of the even horizon.
(8) The sphere R_2 is made from a physically realizable conductor.

None of the above flat-spacetime problems is solved completely. But without their resolution it is impossible to proceed on the way of clear physical understanding of the evaporation process.

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