ENERGY-ENERGY CORRELATIONS*

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(*Received October 8, 1984*)

The status of the theory of energy weighted cross sections (in particular energy-energy correlations) is reviewed. For e^-e^+ annihilations the results are presented in a form which is equally applicable at low energies (where only the virtual photon channel plays a role) and at high energies (where the Z^0 channel is also important). Energy-energy correlations for the $p\bar{p}$ collider are also discussed.

PACS numbers: 12.35.Eq, 13.65.+i, 13.85.Hd, 14.80.Er

1. Introduction

Energy weighted cross sections, in particular energy-energy correlations (EEC) have been suggested a few years ago as possible tests of QCD in e^-e^+ annihilation [1]. The advantages of using EEC as a test of QCD have been mentioned in the pioneering papers:

1. It is calculable and characteristic of QCD.
2. It is proportional to the QCD coupling constant squared $\alpha_s(W)$ (as contrasted to $R = \sigma_{\text{tot}}^{\text{hadronic}}\ \sigma$ pointlike).
3. A complicated event by event analysis of data is not required.
4. Fragmentation corrections are easily calculated.
5. A large part of fragmentation corrections drops out from the properly defined asymmetry.

In the past few years (angle integrated) EEC (or energy weighted angular correlation) has been measured by several experimental groups [2–4]. It is one of the best possibilities of an experimental determination of α_s. While a large part of the original motivation is still valid, there is a serious uncertainty in connection with fragmentation. At present energies ($\approx$ 30 GeV) the fragmentation correction is large, model dependent and also affects the asymmetry. To illustrate the size of uncertainty I quote in Table I the α_s values of the last two publications. Both determinations have taken into account $O(\alpha_s^2)$ corrections (in the $\overline{\text{MS}}$ scheme). The unpublished data of several other PETRA groups [4] show similar dependence of the α_s value on the fragmentation scheme used.

* Presented at the XXIV Cracow School of Theoretical Physics, Zakopane, June 6–June 20, 1984.

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TABLE I

Measurements of α_s at $W \approx 34$ GeV from e^+e^- energy-energy correlation data assuming independent fragmentation (IF) and string fragmentation (L) for hadronization. $O(\alpha_s^2)$ corrections have been taken into account in both cases. $\Lambda_{\overline{\text{MS}}}^{(5)}$ refers to the $\overline{\text{MS}}$ scheme and $N_f = 5$

	α_s(IF)	α_s(L)	$\Lambda_{\overline{\text{MS}}}^{(5)}$(IF)	$\Lambda_{\overline{\text{MS}}}^{(5)}$(L)
MARK J	0.12 ± 0.02	0.14 ± 0.02	90^{+110}_{-70} MeV	200^{+200}_{-100} MeV
CELLO	$0.12-0.15$ $\pm 10\%$	0.19 $\pm 10\%$	90^{+110}_{-70} MeV	900 ± 400 MeV

There are two possibilities to improve the situation. Either one has to try to understand fragmentation better, or one has to minimize the effects of fragmentation. The simplest way to minimize fragmentation is to increase the energy W. While the perturbative result drops proportional to $1/\ln W$, the fragmentation correction is believed to decrease faster, like $1/W$. Therefore at the Z^0 peak (i.e. $W \approx 90$ GeV) one expects a better situation. EEC for energies comparable to the mass of the Z^0 have been calculated in Refs. [5, 6].

In the following I shall review what is known on EEC (or more generally energy weighted cross sections, where possible), giving the results in a form, which is equally applicable at low and high energies. EEC at $\chi \approx 0°$ and $\chi \approx 180°$ will not be discussed, also radiative corrections are omitted in this review. Due to the uncertainties of fragmentation corrections a detailed comparison with experiment will also be omitted. For these topics I refer to the original papers.

2. Definitions and basic properties of energy weighted cross sections

The definition of the normalized l-fold energy weighted cross section may be given in terms of calorimeter measurements:

l calorimeters with opening angles $d\Omega_1, d\Omega_2, \ldots, d\Omega_l$ are placed at l directions (with unit vectors $r_1, \ldots r_l$) from the interaction point (see Fig. 1). In the A-th hadronic annihilation event the calorimeters measure the energies $dE_{A_1}, \ldots, dE_{A_l}$ respectively. The average over all events of the product of calorimeter energies (divided by W and the product of opening angles) defines the normalized l-fold energy weighted cross section:

$$\frac{1}{\sigma_{\text{tot}}} \frac{d^l\Sigma}{d\Omega_1 \ldots d\Omega_l} = \frac{1}{N} \sum_{A=1}^{N} \frac{dE_{A_1}}{W d\Omega_1} \cdots \frac{dE_{A_l}}{W d\Omega_l}. \tag{1}$$

The normalization condition

$$\int \frac{d^l\Sigma}{d\Omega_1 \ldots d\Omega_l} d\Omega_l = \frac{d^{l-1}\Sigma}{d\Omega_1 \ldots d\Omega_{l-1}} \tag{2}$$

follows immediately. Eq. (2) is valid assuming "transparent" calorimeters, i.e. when e.g. $\Omega_1 = \Omega_l$ the same energy is counted twice in the definition, (as if defected simultaneously in the 1st and l-th calorimeters).

The energy dE_{Ai} is obviously a sum of the energies of the particles incident into the i-th calorimeter in the A-th event. In terms of particle energies we have

$$\frac{1}{\sigma_{tot}} \frac{d^l\Sigma'}{d\Omega_1 \dots d\Omega_l} = \frac{1}{d\Omega_1 \dots d\Omega_l} \frac{1}{N} \sum_{A=1}^{N} \sum_{p} \frac{E_{Ap_1} \dots E_{Ap_l}}{W^l}, \tag{3}$$

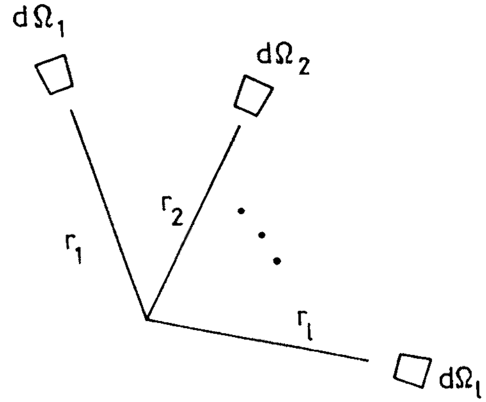

Fig. 1. Kinematics of energy weighted cross sections

where the second sum goes over all l-plets of particles which are detected in the calorimeters. Each l-plet is counted only once, but a single particle may contribute to several l-plets. Particle types are not distinguished, it is only the energy, what is relevant.

The number of events which contain the same l-plet of particles is given in terms of exclusive n particle cross sections as

$$N_0 \sum_{n=2}^{\infty} \int \prod_{a=1}^{n} E_a^{-1} d^3p_a \frac{d^n\sigma}{E_1^{-1}d^3p_1 \dots E_n^{-1}d^3p_n} S_n \Delta\Omega_1 \dots \Delta\Omega_l$$

$$\sum_{i_1,\dots,i_l=1}^{n} \delta(\bar{\Omega}_{i_1}-\Omega_1) \dots \delta(\bar{\Omega}_{i_l}-\Omega_l)\delta(E_{i_1}-E_1) \dots \delta(E_{i_l}-E_l), \tag{4}$$

where S_n is a symmetrization factor, taking into account phase-space reduction for identical particles, $d^3p_a = p_a^2 d\bar{\Omega}_a$ and N_0 is the integrated luminosity. Multiplying by the missing factors and summing up over all l-plets, we get the definition in terms of exclusive cross--sections:

$$\frac{d^l\Sigma}{d\Omega_1 \dots d\Omega_l} = \sum_{n=2}^{\infty} \int \prod_{a=1}^{n} E_a^{-1} d^3p_a \frac{d^n\sigma}{E_1^{-1}d^3p_1 \dots E_n^{-1}d^3p_n} S_n$$

$$\times \sum_{i_1,\dots,i_l=1}^{n} \frac{E_{i_1} \dots E_{i_l}}{W^l} \delta(\bar{\Omega}_{i_1}-\Omega_1) \dots \delta(\bar{\Omega}_{i_l}-\Omega_l). \tag{5}$$

For $l = 0$ we get the total cross section, $l = 1$ is the antenna or energy pattern, $l = 2$ is EEC, $l = 3$ is triple energy correlation. An important property of $\dfrac{d^l\Sigma}{d\Omega_1 \dots d\Omega_l}$ is symmetry under exchange of any two Ω_i's.

The fully differential energy weighted cross sections are of course not measured in experiment. Experiment determines the normalized angle integrated EEC, which is defined as

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma}{d\cos\chi} = \frac{1}{\sigma_{\text{tot}}} \int \frac{d^2\Sigma}{d\Omega_1 d\Omega_2} \delta(\cos\chi - \cos\theta_1 \cos\theta_2 - \sin\theta_1 \sin\theta_2 \cos(\phi_1 - \phi_2)). \tag{6}$$

In terms of particle energies we have

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma}{d\cos\chi} = \frac{2}{\Delta\cos\chi} \frac{1}{N} \sum_{A=1}^{N} \sum_{p} \frac{E_{Aa}E_{Ab}}{W^2}, \tag{7}$$

where the second sum is over all pairs of particles with a relative angle χ. Each pair is counted only once. The normalization condition is

$$\frac{1}{\sigma_{\text{tot}}} \int \frac{d\Sigma}{d\cos\chi} d\cos\chi = 1, \tag{8}$$

this is assured by the explicit factor 2 in Eq. (7), (which should be absent for the diagonal cases $\chi = 0$ and $E_{Aa} = E_{Ab}$).

3. *Calculation of the energy weighted cross sections in e^-e^+ annihilation*

We perform the calculation to lowest order in the electro-weak interaction and — by introducing appropriate structure functions — exactly in the strong interactions. We have to calculate the graphs shown on Fig. 2. The squared amplitude $|T|^2$ is given by

$$|T|^2 \propto \sum_f |\langle f_+|J_{\gamma\mu}|0\rangle \frac{1}{W^2} \langle 0|j^\mu_\gamma|e^-e^+\rangle$$

$$+\langle f_+|J_{\text{weak}\,\mu}|0\rangle \frac{1}{W^2 - M_Z^2 + iM_Z\Gamma_Z} \langle 0|j^\mu_{\text{weak}}|e^-e^+\rangle|^2, \tag{9}$$

where $j^\mu_{\gamma(\text{weak})}$, $(J^\mu_{\gamma(\text{weak})})$ is the lepton (hadron) electromagnetic (weak) current and f is an arbitrary outgoing hadronic final state. $|T|^2$ is assumed to contain already the necessary energy factors, too, therefore it refers to any of the energy weighted cross sections. We rewrite $|T|^2$ as

$$|T^2 \propto \sum_f |a_1 v^\mu \langle f_+|V_\mu|0\rangle + a_2 v^\mu \langle f_+|A_\mu|0\rangle + a_3 a^\mu \langle f_+|V_\mu|0\rangle + a_4 a^\mu \langle f_+|A_\mu|0\rangle|^2, \tag{10}$$

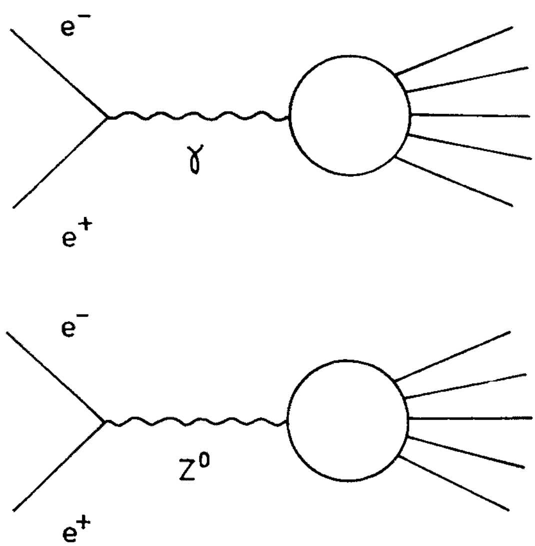

Fig. 2. $e^-e^+ \to$ hadrons graphs, to lowest order in the electro-weak interaction

where v^μ (a^μ) is the matrix element of the vector (axial-vector) leptonic current, V_μ (A_μ) is the hadronic vector (axial-vector) current. All the coupling constants as well as the γ and Z^0 propagators are included in $a_1, \ldots, a_4$. For hadron production through a quark-antiquark pair of flavour f the standard model gives

$$a_{1f} = \frac{e^2}{W^2} Q_f + \frac{g_V G_{Vf}}{W^2 - M_Z^2 + iM_Z\Gamma_Z}, \quad a_{2f} = \frac{g_V G_{Af}}{W^2 - M_Z^2 + iM_Z\Gamma_Z},$$

$$a_{3f} = \frac{g_A G_{Vf}}{W^2 - M_Z^2 + iM_Z\Gamma_Z}, \quad a_{4f} = \frac{g_A G_{Af}}{W^2 - M_Z^2 + iM_Z\Gamma_Z}, \tag{11}$$

where

$$g_A = e/(4 \sin\theta_W \cos\theta_W), \quad g_V = (1 - 4\sin^2\theta_W) g_A,$$

Q_f, G_{Vf}, G_{Af} are the eigenvalues of the matrices Q,

$$G_V = g_A(W^0 - 4\sin^2\theta_W Q), \quad G_A = g_A W^0,$$

with

$$W^0 = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}, \quad Q = \begin{pmatrix} \frac{2}{3} & & & \\ & \frac{2}{3} & & \\ & & -\frac{1}{3} & \\ & & & -\frac{1}{3} \end{pmatrix} \tag{12}$$

(for two generations).

Since particle types are not detected, the final state is effectively invariant under charge conjugation; therefore

$$\sum_f \langle 0|V^\mu|f_+\rangle \langle f_+|A^\nu|0\rangle = 0. \tag{13}$$

With the notation

$$\bar{V}^{\mu\nu} = \sum_f \langle 0|V^\mu|f_+\rangle \langle f_+|V^\nu|0\rangle,$$

$$\bar{A}^{\mu\nu} = \sum_f \langle 0|A^\mu|f_+\rangle \langle f_+|A^\nu|0\rangle, \tag{14}$$

neglecting final state interactions, TCP invariance yields

$$\bar{V}^{\mu\nu} = \bar{V}^{\nu\mu}, \qquad \bar{A}^{\mu\nu} = \bar{A}^{\nu\mu}. \tag{15}$$

For massless quarks, i.e. assuming chiral symmetry, we also have

$$\bar{V}^{\mu\nu} = \bar{A}^{\mu\nu}. \tag{16}$$

This relation will be valid for energies much higher than any quark mass thresholds. The necessary phase space integrations and polarization sums effect only final state particle variables, therefore they may be peformed on $\bar{V}^{\mu\nu}$, $\bar{A}^{\mu\nu}$; the final results are denoted by $V^{\mu\nu}$, $A^{\mu\nu}$. These tensors must then be contracted with the leptonic tensors. Since $V^{\mu\nu}$, $A^{\mu\nu}$ is symmetric, only the symmetric parts of the leptonic tensors enter, which are given by

$$v^i v^{k*} = (\delta^{ik} - \delta^{i3}\delta^{k3})(1 + \vec{s}_\perp \cdot \vec{s}_\perp' - s_L \cdot s_L') - s_\perp'^i s_\perp^k - s_\perp'^k s_\perp^i,$$

$$a^i a^{k*} = (\delta^{ik} - \delta^{i3}\delta^{k3})(1 - \vec{s}_\perp \cdot \vec{s}_\perp' - s_L \cdot s_L') + s_\perp'^i s_\perp^k + s_\perp'^k s_\perp^i,$$

$$v^i a^{k*} = (\delta^{ik} - \delta^{i3}\delta^{k3})(s_L' - s_L) + i(s_\perp^1 s_\perp'^1 - s_\perp^2 s_\perp'^2)(\delta^{i1}\delta^{k2} + \delta^{i2}\delta^{k1}), \tag{17}$$

where we have neglected the lepton masses and the direction of the e^- is chosen to be the third axis, while e^+ moves opposite to it. $\vec{s}_\perp$, s_L ($\vec{s}_\perp'$, s_L') refer to the e^-(e^+) polarization vectors.

Only the space-space parts of the leptonic tensors are non-zero. Of course V^{ik} (A^{ik}) are different for the various energy weighted cross sections. Defining appropriate rotation invariant structure functions, we get the following decompositions:

$l = 0$
$$V^{ik} = A(W)\delta^{ik}, \tag{18}$$

$l = 1$
$$V^{ik} = A(W)\delta^{ik} + B(W) r_1^i r_1^k,$$

$l = 2$
$$V^{ik} = \mathscr{A}(W, \chi) \cdot (2\delta^{ik} - r_1^i r_1^k - r_2^i r_2^k)$$
$$+ \mathscr{B}(W, \chi) \cdot (\delta^{ik} r_1 \cdot r_2 - \tfrac{1}{2}(r_1^i r_2^k + r_2^i r_1^k)) + \mathscr{C}(W, \chi) \cdot \delta^{ik}; \qquad \cos\chi = r_1 \cdot r_2,$$

$l \geqslant 3$
$$V^{ik} = A^1 r_1^i r_1^i + A^2 \tfrac{1}{2}(r_1^i r_2^k + r_2^i r_1^k) + A^3 \tfrac{1}{2}(r_1^i r_3^k + r_3^i r_1^k)$$
$$+ A^4 r_2^i r_2^k + A^5 \tfrac{1}{2}(r_2^i r_3^k + r_3^i r_2^k) + A^6 r_3^i r_3^k.$$

In case of $l \geqq 3$ the structure functions A^1, ... A^6 depend on W and the scalar products $r_i \cdot r_k$ of the unit vectors. As a consequence of the symmetry of the energy weighted cross sections under $r_i \leftrightarrow r_k$ we obtain constraints for the A^{l}'s. E.g. for $l = 3$ we get [15]

$$\begin{aligned}
A_4(r_1\cdot r_2, r_1\cdot r_3, r_2\cdot r_3) &= A_1(r_1\cdot r_2, r_2\cdot r_3, r_1\cdot r_3),\\
A_6(r_1\cdot r_2, r_1\cdot r_3, r_2\cdot r_3) &= A_1(r_2\cdot r_3, r_1\cdot r_3, r_1\cdot r_2),\\
A_3(r_1\cdot r_2, r_1\cdot r_3, r_2\cdot r_3) &= A_2(r_1\cdot r_3, r_1\cdot r_2, r_2\cdot r_3),\\
A_5(r_1\cdot r_2, r_1\cdot r_3, r_2\cdot r_3) &= A_2(r_2\cdot r_3, r_1\cdot r_3, r_1\cdot r_2),\\
A_1(r_1\cdot r_2, r_1\cdot r_3, r_2\cdot r_3) &= A_1(r_1\cdot r_3, r_1\cdot r_2, r_2\cdot r_3),\\
A_2(r_1\cdot r_2, r_1\cdot r_3, r_2\cdot r_3) &= A_2(r_1\cdot r_2, r_2\cdot r_3, r_1\cdot r_3).
\end{aligned} \tag{19}$$

A^{ik} has similar invariant decompositions, with structure functions which are identical to the V^{ik} structure functions only in the chiral symmetric, i.e. zero quark mass case.

For the *zero quark mass case* all energy weighted cross sections will have the general, final form

$$\frac{d^l\Sigma}{d\Omega_1 \ldots d\Omega_l} = W^2 V^{ik} \sum_f [v_i v_k^*(|a_{1f}|^2 + |a_{2f}|^2) + a_i a_k^*(|a_{3f}|^2 + |a_{4f}|^2) + v_i a_k^*(a_{1f} a_{3f}^* + a_{2f} a_{4f}^*) + a_i v_k^*(a_{1f}^* a_{3f} + a_{2f}^* a_{4f})]. \tag{20}$$

V^{ik} does not depend on quark flavour. Inserting the invariant decompositions of V^{ik}, we get e.g.

$$\sigma_{\rm tot} = W^2 2 \sum_f [(1 - s_L s_L') \cdot \sum_{i=1}^{4} |a_{if}|^2 + 2(s_L' - s_L) \cdot {\rm Re}\,(a_{3f} a_{1f}^* + a_{4f} a_{2f}^*)] A(W), \tag{21}$$

where

$$A(W) = \frac{1}{8\pi}[1 + \alpha_s(W)/\pi + \ldots] \text{ in QCD.}$$

In the zero quark mass case the following theorem is true. *When integrated over at least one azimuthal angle, the normalized energy weighted cross sections are independent of initial state polarization and weak interaction parameters* (*i.e.* Z^0 *mass and width and coupling constants*). More precisely, the azimuthal angle mentioned above is an angle, which rotates the system of unit vectors $r_1, \ldots, r_l$ (which determine the calorimeter positions) rigidly around the third axis (i.e. the e^- direction.) I.e. before integration one has to introduce new variables: $2l$–3 internal angles and 3 external angles. The internal angles determine the relative positions of the unit vectors, while the external angles determine the position of the whole system of unit vectors in the coordinate frame used. One of the external angles is the azimuthal angle mentioned above.

The proof of the theorem is easy[1], the important point is that the structure functions obviously do not depend on the azimuthal angle. The significance of the theorem is that all the results obtained for angle integrated energy weighted cross sections at low energies

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[1] For $l = 2$ we shall outline it in the next Section.

(i.e. neglecting the Z^0 graphs of Fig. 2) may be easily extended to higher energies [6]. One has simply to multiply by σ_{tot} (Z^0 included)/σ_{tot} (Z^0 excluded). Since σ_{tot} (Z^0 included) is well known, this is a trivial exercise. I wish to emphasize that the validity of this statement is based on general arguments, therefore it applies to both perturbative and non-perturbative contributions. E.g. the higher order correction results, the $\chi \approx 0°$ or $\chi \approx 180°$ results, as well as fragmentation corrections may be immediately extended to the high energy case. Finite quark mass effects as well as radiative corrections modify this simple prescription.

4. *The energy-energy correlation*

First I sketch the proof of the theorem of the previous section for $l = 2$, i.e. for the EEC. The ECC (in the zero quark mass case) is given by Eq. (20) as a function of the polar and azimuthal angles $\theta_1, \phi_1, \theta_2, \phi_2$, which characterize the directions r_1, r_2. The relative position of the unit vectors is characterized by χ, which is determined as $\cos\chi = r_1 \cdot r_2$. The angle χ may be introduced e.g. in place of ϕ_2. We get:

$$\frac{d^2\Sigma}{d\cos\theta_1 d\cos\theta_2 d\phi_1 d\cos\chi} = \frac{1}{\Delta(\theta_1, \theta_2, \chi)}\left(\left.\frac{d^2\Sigma}{d\Omega_1 d\Omega_2}\right|_{\phi_2=\phi_1+\alpha} + \left.\frac{d^2\Sigma}{d\Omega_1 d\Omega_2}\right|_{\phi_2=\phi_1-\alpha}\right), \tag{22}$$

where

$$\alpha = \arccos\left(\frac{\cos\chi - \cos\theta_1\cos\theta_2}{\sin\theta_1\sin\theta_2}\right),$$

$$\Delta(\theta_1, \theta_2, \chi) = (1+2\cos\theta_1\cos\theta_2\cos\chi - \cos^2\theta_1 - \cos^2\theta_2 - \cos^2\chi)^{1/2}. \tag{23}$$

For a given χ, ϕ_1 may be chosen arbitrarily, while θ_1 and θ_2 are constrained by the requirement that $\Delta(\theta_1, \theta_2, \chi)$ should be real. This yields the restriction

$$\cos\theta_2 \in (\cos(\chi+\theta_1), \cos(\chi-\theta_1)). \tag{24}$$

Alternatively, one may introduce the angle ω of the spherical triangle with the sides θ_1, θ_2, χ (see Fig. 3). In this case

$$\frac{d^2\Sigma}{d\cos\theta_1 d\omega d\cos\chi d\phi_1} = \Delta(\theta_1, \theta_2, \chi)\frac{d^2\Sigma}{d\cos\theta_1 d\cos\theta_2 d\cos\chi d\phi_1} \tag{25}$$

and $0 \leqslant \omega \leqslant \pi$.

In both cases the azimuthal angle which figures in the theorem is ϕ_1, which is unconstrained. We write down a single term from Eq. (22) in the case of EEC in detail:

$$\begin{aligned} V_{ik}v^i v^{k*} &= (2\mathscr{A} + \cos\chi\mathscr{B} + \mathscr{C})2(1 - s_L s'_L) \\ &-\mathscr{A}[(2-\cos^2\theta_1 - \cos^2\theta_2)(1+\vec{s}_\perp \cdot \vec{s}'_\perp - s_L s'_L) \\ &-2r_1 \cdot \vec{s}'_\perp r_1 \cdot \vec{s}_\perp - 2r_2 \cdot \vec{s}'_\perp r_2 \cdot \vec{s}_\perp] \\ &-\mathscr{B}[(\cos\chi - \cos\theta_1\cos\theta_2)(1+\vec{s}_\perp \cdot \vec{s}'_\perp - s_L s'_L) - \vec{s}'_\perp \cdot r_1 \vec{s}_\perp \cdot r_2 - \vec{s}'_\perp \cdot r_2 \vec{s}_\perp \cdot r_1]. \end{aligned} \tag{26}$$

Since $\mathscr{A}$, $\mathscr{B}$, $\mathscr{C}$ depend only on χ, ϕ_1 dependence appears only in the coefficients. Thus the ϕ_1 integration may be performed easily. The complete result is

$$\frac{1}{\sigma_{\text{tot}}} \frac{d\Sigma}{d\cos\chi d\cos\theta_1 d\cos\theta_2} = \tfrac{3}{8}\{\mathscr{A}(2+\cos^2\theta_1+\cos^2\theta_2)$$

$$+\mathscr{B}(\cos\chi+\cos\theta_1\cos\theta_2)+2\mathscr{C}\}\frac{2}{\Delta(\chi_1,\theta_1,\theta_2)}\frac{1}{8\pi A(W)}, \tag{27}$$

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\Sigma}{d\cos\chi} = \pi\{4\mathscr{A}+2\cos\chi\mathscr{B}+3\mathscr{C}\}\frac{1}{8\pi A(W)}, \tag{28}$$

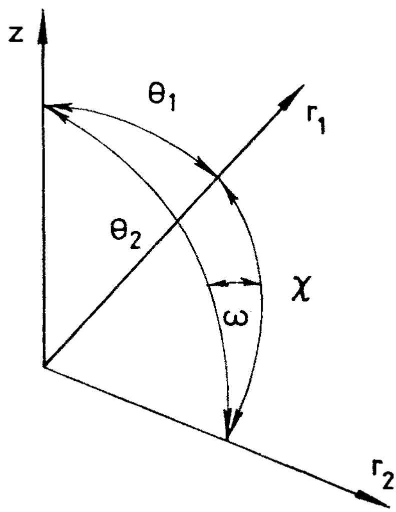

Fig. 3. Kinematics of energy-energy correlations

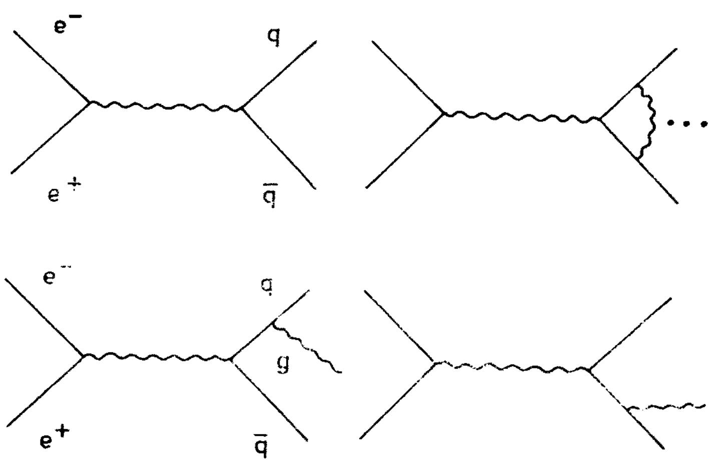

Fig. 4. Lowest order QCD graphs, which determine energy-energy correlations in e^-e^+ annihilation

where the denominator $8\pi A(W)$ comes from σ_{tot} $(8\pi A(W) = 1+\alpha_s(W)/\pi+ \ldots)$ and $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ are the EEC structure functions[2]. Eqs. (27), (28) are valid foı arbitrary initial polarization and do not depend on weak interaction parameters — as stated by the theorem of Sect. 3.

The structure functions $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ may be determined in QCD from the graphs of Fig. 4. The graphs with the $q\bar{q}$ final state contribute only at $\chi = 0°$ (self correlation) and $\chi = 180°$. (Their contribution is proportional to a δ function.) Since we are interested only in angles χ which satisfy $\delta < \chi < 180° - \delta$ with $\delta \approx 30°$, only the $q\bar{q}g$ final states are relevant. The result is

$$\mathcal{A} = \frac{\alpha_s(W)}{12\pi^2}\frac{1}{1-\xi}\left(\frac{3-4\xi}{\xi^5}\ln(1-\xi)+\frac{3}{\xi^4}-\frac{5}{2\xi^3}-\frac{1}{\xi^2}\right),$$

$$\mathcal{B} = \frac{\alpha_s(W)}{12\pi^2}\frac{1}{1-\xi}\left(\frac{4(3-\xi)(1-\xi)}{\xi^5}\ln(1-\xi)+\frac{12}{\xi^4}-\frac{10}{\xi^3}\right),$$

$$\mathcal{C} = 0, \tag{29}$$

where $\alpha_s(W)$ is the running strong coupling at energy W and $\xi = \frac{1}{2}(1-\cos\chi)$. These results are singular when $\chi \to 0°$ or $\chi \to 180°$. The fact that the results are singular is an indication that perturbation theory breaks down in these limits and one has to sum up an infinite series of graphs. We shall not persue this subject further, only remark that this is the reason why Eqs. (29) yield a good approximation only in the region $\chi \in (\delta, 180° - \delta)$ with $\delta \approx 30°$.

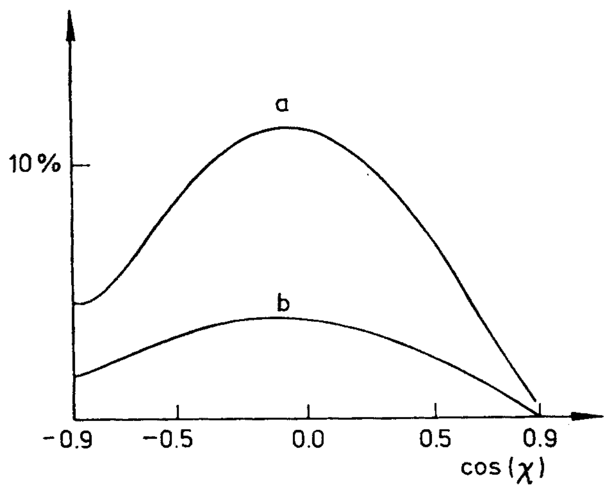

Fig. 5. Ratios of the massive quark and zero mass quark contributions to $\frac{d\Sigma}{d\cos\chi}$ for a single flavour; *a*) shows the vector-vector contribution ratio, *b*) the axial vector — axial vector contribution ratio. The quark mass is 35/90 (in units of W)

[2] The structure functions $\mathcal{A}$, $\mathcal{B}$, $\mathcal{C}$ here are the traditional ones and differ by a constant factor from the structure functions defined in Sect. 3.

The results Eqs. (27), (28), (29) are modified, when quark masses are taken into account [7]. In particular, when $W \approx M_Z$ u, d, s, c and b quarks may be taken to be massless, while the mass of the t quark is not negligible. In the massive quark case $V^{\mu\nu} \neq A^{\mu\nu}$, both depend on the quark mass, thus Eq. (20) is modified to

$$\frac{d'\Sigma}{d\Omega_1 \ldots d\Omega_l} = W^2 \sum_f \{V_f^{ik} \cdot [v_i v_k^* |a_{1\,f}|^2 + a_i a_k^* |a_{3\,f}|^2 + v_i a_k^* a_{1\,f} a_{3\,f}^* + a_i v_k^* a_{1\,f}^* a_{3\,f}] + A_f^{ik}[v_i v_k^* |a_{2\,f}|^* + a_i a_k^* |a_{4\,f}|^* + v_i a_k^* a_{2\,f} a_{4\,f}^* + a_i v_k^* a_{2\,f}^* a_{4\,f}]\}, \tag{30}$$

where the sum goes over quark flavour.

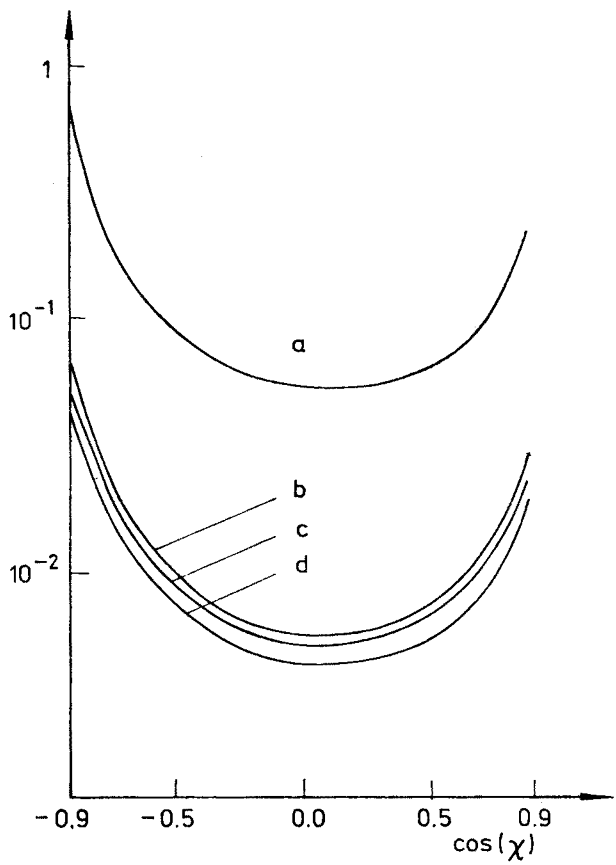

Fig. 6. *a*) $\frac{1}{\sigma_{tot}} \frac{d\Sigma}{d\cos\chi}$ for zero quark masses at $W = 77$ GeV, *b*, *c*, *d* show $\frac{1}{\sigma_{tot}} \frac{d\Sigma}{d\cos\chi}\Big|_{m_t=0} - \frac{1}{\sigma_{tot}} \times \frac{d\Sigma}{d\cos\chi}\Big|_{m_t\neq 0}$ for top quark masses $m_t = 30, 35, 40$ GeV, at $W = 70$ GeV

The expressions of $V^{\mu\nu}$ and $A^{\mu\nu}$ for one massive quark are calculated easily, the final result is obtained by a simple one dimensional numerical integral. The contribution of the massive quark to $\frac{d\Sigma}{d\cos\chi}$ changes much (Fi. 5). However, in $\frac{1}{\sigma_{tot}}\frac{d\Sigma}{d\cos\chi}$ both the

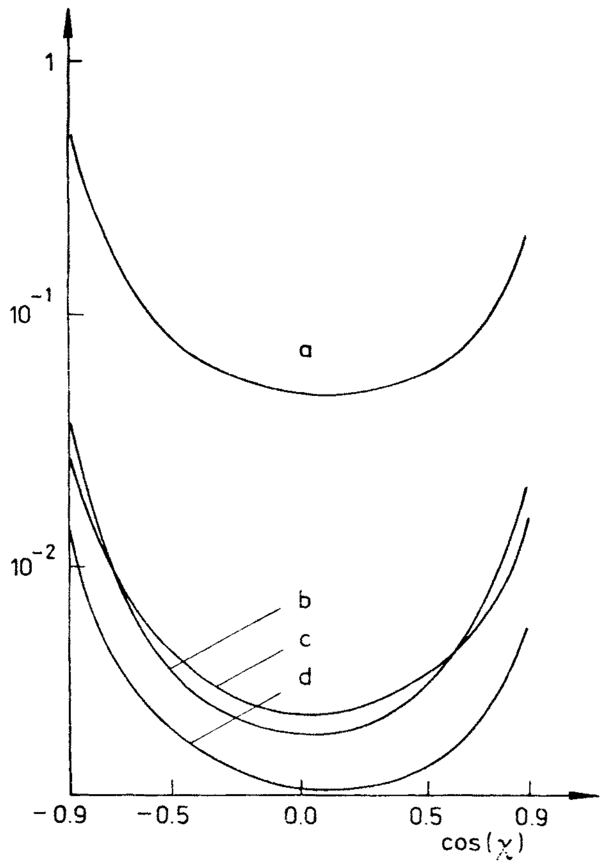

Fig. 7. *a*) $\frac{1}{\sigma_{tot}}\frac{d\Sigma}{d\cos\chi}$ for zero quark masses at $W = M_Z$, *b*, *c*, *d* show $\left.\frac{1}{\sigma_{tot}}\frac{d\Sigma}{d\cos\chi}\right|_{m_t=0} - \frac{1}{\sigma_{tot}}$ $\times \left.\frac{d\Sigma}{d\cos\chi}\right|_{m_t\neq 0}$ for top quark masses $m_t = 20, 30, 40$ GeV at $W = M_Z$

numerator and denumerator is a sum for several quarks, therefore the change is much smaller, see Figs. 6, 7. In particular for $W = M_Z$ the change is at most 10% (6%) for top quark masses chosen in the range 20 GeV$-M_Z/2$ and $-0.9 < \cos\chi < 0.9$ $(-0.5 < \cos\chi < 0.5)$.

5. Higher order corrections for EEC

Higher order corrections for EEC have been calculated for unpolarized initial state, 1γ annihilation and $\frac{d\Sigma}{d\cos\chi}$ only [8, 9]. By the theorem of Sect. 3 these results are easily extended to higher energies and arbitrary initial polarization. In the following I give only a brief review, for more details I refer to the original papers. To $O(\alpha_s^2)$

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\Sigma}{d\cos\chi} = \left(1-\frac{\alpha_s}{\pi}\right)\left(\frac{\alpha_s}{\pi}g^{(1)}(\chi)+\left(\frac{\alpha_s}{\pi}\right)^2 g^{(2)}(\chi)\right), \tag{31}$$

where $g^{(1)}(\chi)$ and $g^{(2)}(\chi)$ are the functions of Ref. [8]. $\alpha_s(W)$ is given by the formulae

$$\frac{\alpha_s(W)}{\pi} = \frac{1}{b_0 \ln(W/\Lambda)} - \frac{b_1 \ln(\ln(W^2/\Lambda^2))}{2b_0^3 \ln^2(W/\Lambda)},$$

$$b_0 = \tfrac{1}{6}(11C_A - 2N_f), \quad b_1 = \tfrac{1}{6}(17C_A^2 - 5C_AN_f - 3C_fN_f), \tag{32}$$

$C_F = \frac{4}{3}$, $C_A = 3$ N_f = no of flavours. $g^{(1)}(\chi)$ is the well known lowest order result, $g^{(2)}(\chi)$ was evaluated from the graphs of the type shown in Figs. 4, 8. All the squared matrix elements are known from the literature. To calculate the EEC one nas to multiply by the appropriate energy factors and integrate over phase space. Thus

$$g^{(2)}(\chi) = \int (d4) \sum_{i,j} M^{(4)} \frac{E_iE_j}{W^2}\delta(\hat{p}_i\cdot\hat{p}_j - \cos\chi)$$

$$+ \int (d3) \sum_{i,j} M^{(3)} \frac{E_iE_j}{W^2}\delta(\hat{p}_i\cdot\hat{p}_j - \cos\chi), \tag{33}$$

where $(dN) = N$ body phase space, $M^{(N)}$ = "squared" matrix element ($M^{(3)}$ is the interference term of the basic $q\bar{q}$ graphs of Fig. 4 with the loop corrected graphs), and $\hat{p}_i$ is the unit vector pointing in the direction of the momentum p_i. Both integrals in Eq. (33) are infrared divergent. The method of cancellation is given in Ref. [10]. To regularize, the integrals are peformed in 4–2ε dimensions. Then $M^{(4)}$ is expanded into partial fractions, so that each term is only singular, when one of the $s_{ij} = (p_i+p_j)^2 \to 0$ (i.e. p_i, p_j collinear).

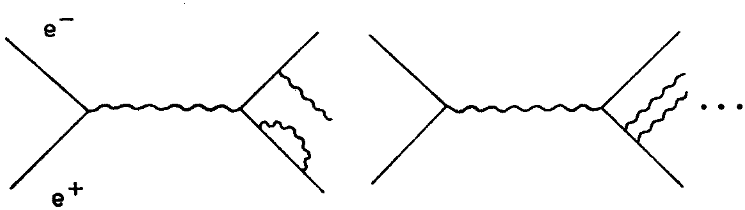

Fig. 8. A few graphs, which determine the $O(\alpha_s^2)$ corrections to the energy-energy correlations in e⁻e⁺ annihilation

The singularities are then isolated and combined with a quasi 3 body phase space and energy weighting. Thus

$$g^{(2)}(\chi) = \{\int (d4)\Sigma M^{(4)} - \int (d\bar{4})\overline{\Sigma M}^{(4)}\} + \{\int (d3)\Sigma M^{(3)} + \int (d\bar{4})\overline{\Sigma M}^{(4)}\}. \tag{34}$$

Both combinations are separately finite in the $\varepsilon \to 0$ limit. The integrals are evaluated numerically. A simple analytic approximation of the results is given in [8].

A few remarks are in order. a) Comparison with experiment shows that the $O(\alpha_s^2)$ corrections reduce the fitted value of α_s by about 20%. b) The corrections calculated in Refs. [8, 9] are calculated in the $\overline{\text{MS}}$ scheme. Of course, there is a scheme dependence of the calculated EEC. c) The conventional choice for the scale of α_s is the total center of mass energy W. However, it is possible to choose a χ dependent scale $\overline{W}(\chi)$ [8], so that a substantial part of the correction is absorbed into $\alpha_s(\overline{W}(\chi))$. Of course the fitted value of α_s increases. It turns out that the prediction is very close to the PMS prediction [11]. Presumably it is also close to the prediction of the Grunberg prescription [12]. Ref. [8] concludes that the available data are not sensitive enough to the small differences in χ dependence of the various effective couplings.

6. *Fragmentation corrections*

In a perturbative QCD calculation the final state is composed of quarks and gluons, which are assumed to hadronize. As is well known, the description of this last process is not yet solved on the basis of QCD. There are two types of models: a) independent fragmentation [3] and b) string fragmentation models [14]. The $q\bar{q}$ fragmentation is quite similar in the two models, the first major difference appears in $q\bar{q}g$ fragmentation. The string kinematics in a $q\bar{q}g$ event shifts the particles from the original parton directions towards the regions between quarks and gluons, thus making a 3-jet event look more 2 jet-like. Therefore, fitting the same data the value of α_s will be higher in the string model than in independent fragmentation models.

The best method to take into account fragmentation is to perform a complete Monte Carlo calculation. In order to get a feeling about the effect of fragmentation I recall the original calculation of Ref. [1]. With the help of our theorem the result will be presented in a form, which is readily applicable to the high energy case.

First we calculate the fragmentation to the antenna (energy) pattern. The calculation for EEC may be performed similarly. Suppose a quark (antiquark) of momentum $\vec{p}$ produces dn hadrons in d^3h. We put

$$dn = \frac{d^3h}{h^0} f_1(\vec{h}; \vec{p}). \tag{35}$$

f_1 is normalized as

$$\int \frac{d^3h}{h^0} h^\mu f_1(\vec{h}; \vec{p}) = p^\mu. \tag{36}$$

The antenna pattern cross section is then given by

$$\Delta\Sigma \equiv \frac{d^1\Sigma}{d\Omega}\cdot\Delta\Omega = \int d\Omega_p \frac{d\sigma}{d\Omega_p}\int_\Delta \frac{d^3h}{h^0}\,\frac{h^0}{W}(f_1(\vec{h};\vec{p})+f_1(\vec{h};\,-\vec{p})), \tag{37}$$

where the integration over d^3h is in a cone with opening angle $\Delta\Omega$ and axis direction Ω. Defining

$$F_1(\eta) \equiv \frac{2}{W}\int h^2 dh f_1(\vec{h};\vec{p}), \qquad \cos\eta = \hat{\vec{p}}\cdot\hat{\vec{h}}, \tag{38}$$

$$\int d\Omega F_1(\eta) = 1,$$

we may write

$$\frac{d^1\Sigma}{d\Omega} = \tfrac{1}{2}\int d\Omega_p \frac{d\sigma}{d\Omega_p}\,[F_1(\eta)+F_1(\pi-\eta)]. \tag{39}$$

The cross section for $q\bar{q}$ production (through the 1γ channel) is

$$\frac{d\sigma}{d\Omega_p} = \frac{\alpha^2}{2W^2}\sum_f 3Q_f^2\sin^2\xi, \tag{40}$$

where the e^- is perpendicularly polarized (in the direction of a unit vector $\vec{b}$), while the e^+ is polarized in the $-\vec{b}$ direction, and $\cos\xi = \vec{b}\cdot\hat{\vec{p}}$. To perform the $d\Omega_p$ integration we choose the z axis to be the direction of $\vec{h}$. With the notation of Fig. 9 the azimuthal angle is α, thus

$$\overline{\sin^2\xi} = \frac{\int \sin^2\xi d\alpha}{2\pi} = \sin^2\psi+\tfrac{1}{2}\sin^2\eta\cdot(3\cos^2\psi-1) \tag{41}$$

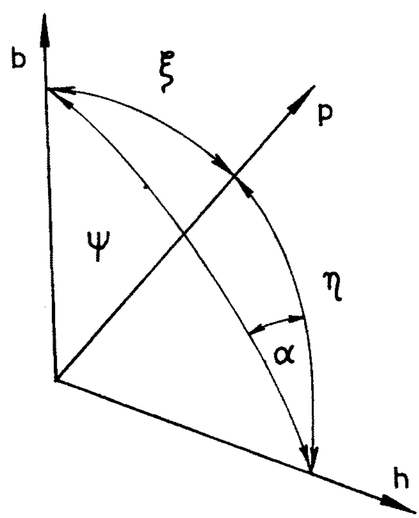

Fig. 9. Kinematics of quark fragmentation in e⁻e⁺ annihilation

and

$$\frac{d\Sigma}{d\Omega}=\frac{\alpha^2}{2W^2}\sum_f 3Q_f^2\int d\Omega_p(\sin^2\psi+\tfrac{1}{2}\sin^2\eta\cdot(3\cos^2\psi-1)(F_1(\eta)+F_1(\pi-\eta)). \quad (42)$$

Since the integrand depends only on η, we may perform the integration over $d\Omega$ instead of $d\Omega_p$. Thus — after normalization and integration over the azimuthal angle — we get

$$\frac{1}{\sigma_{\text{tot}}}\frac{d\Sigma}{d\cos\theta}=\tfrac{3}{8}((1+\cos^2\theta)+\tfrac{1}{2}\langle\sin^2\eta\rangle\cdot(2-3\cos^2\theta)), \quad (43)$$

where

$$\langle\sin^2\eta\rangle=\int d\Omega\sin^2\eta F_1(\eta) \quad (44)$$

and θ is the polar angle in the original frame (i.e. when the e^- direction is the third axis). As a consequence of the theorem in Sect. 3, this result is valid for arbitrary initial polarization and high energies, too.

Until now, the calculation is completely model independent, the form of $f_1(\vec{h},\vec{p})$ is not yet specified. It is sufficient to make some general assumptions on $f_1(\vec{h},\vec{p})$:

a) Scaling i.e. $f_1(\vec{h},\vec{p})=f_1(z,h_\perp)$, where $z=\dfrac{2h_{\|}}{W}$.

b) $h_\perp$ dependence is exponentially decreasing.

c) No backward production.

Under these assumptions it is possible to prove that $\langle\sin^2\eta\rangle\approx\dfrac{\pi C}{2}\dfrac{\langle h_\perp\rangle}{W}$, where C is a constant. This means that the fragmentation correction is $O\left(\dfrac{1}{W}\right)$.

Next we discuss fragmentation correction to EEC. Via fragmentation the $q\bar{q}$ final state also contributes to EEC at $\chi\neq 0°$, $\chi\neq 180°$. This is actually the largest correction, since $q\bar{q}g$ fragmentation correction is already proportional to $\alpha_s(W)$. Let us denote the number of hadron pairs produced by a quark (or antiquark) by d^2n. We put

$$d^2n=\frac{d^3h}{h^0}\frac{d^3h'}{h'^0}f_2(\vec{h},\vec{h}';\vec{p}). \quad (45)$$

The normalization of f_2 is:

$$\int\frac{d^3h'}{h'^0}h'^\mu f_2(\vec{h},\vec{h}';\vec{p})=(p^\mu-h^\mu)f_1(\vec{h};\vec{p}). \quad (46)$$

The contribution of the $q\bar{q}$ final state to EEC is:

$$\frac{d^2\Sigma^{q\bar{q}}}{d\Omega d\Omega'}=\int d\Omega_p\frac{d\sigma}{d\Omega_p}\int\frac{h^2dh}{h^0}\frac{h'^2dh'}{h'^0}\frac{h^0}{W}\frac{h'^0}{W}$$

$$\{[f_1(\vec{h};\vec{p})f_1(\vec{h}';-\vec{p})+f_2(\vec{h},\vec{h}';\vec{p})+h^0\delta(\vec{h}-\vec{h}')f_1(\vec{h},\vec{p})]+[\vec{p}\leftrightarrow-\vec{p}]\}, \quad (47)$$

where the self correlation is also included. Defining

$$F_2(\eta, \eta', \chi) = \left(\frac{2}{W}\right)^2 \int h^2 dh h'^2 dh' [f_2(\vec{h}, \vec{h}'; \vec{p}) + h^0 \delta(\vec{h}-\vec{h}') f_1(\vec{h}; \vec{p})], \tag{48}$$

$$\cos\eta = \hat{\vec{p}}\cdot\hat{\vec{h}}, \quad \cos\eta' = \hat{\vec{p}}\cdot\hat{\vec{h}}', \quad \cos\chi = \hat{\vec{h}}\cdot\hat{\vec{h}}',$$

with the normalization

$$\int d\Omega' F_2(\eta, \eta', \chi) = F_1(\eta), \tag{49}$$

we have

$$\frac{d^2\Sigma^{q\bar{q}}}{d\Omega d\Omega'} = \tfrac{1}{4}\int d\Omega_p \frac{d\sigma}{d\Omega_p}[F_1(\eta)F_1(\pi-\eta') + F_1(\pi-\eta)F_1(\eta')$$

$$+ F_2(\eta, \eta', \chi) + F_2(\pi-\eta, \pi-\eta', \chi)]. \tag{50}$$

Assuming a specific function for F_2 and F_1 the integral may be evaluated numerically. To determine the energy dependence a few general assumptions on f_2 (scaling, limited transverse momentum) are sufficient. It may be established that

a. When both $\eta, \eta' \gg \frac{1}{W}$, $F_2 \propto \frac{1}{W^2}$.

b. F_2 is strongly peaked, when either η or η' is small.

It follows that the largest contribution to the integral (50) is obtained when either Ω or Ω' is collinear or anticollinear to Ω_p. Keeping only these regions one gets

$$\frac{d^2\Sigma^{q\bar{q}}}{d\Omega d\Omega'} \approx \tfrac{1}{2}(F_1(\chi) + F_1(\pi-\chi))\left(\frac{d\sigma}{d\Omega} + \frac{d\sigma}{d\Omega'}\right)$$

$$\approx \frac{C}{4\pi}\frac{\langle h_\perp\rangle}{W}\sin^{-3}\chi\left(\frac{d\sigma}{d\Omega} + \frac{d\sigma}{d\Omega'}\right). \tag{51}$$

With some more work it is possible to show that the corrections to this result are $O\left(\frac{1}{W^2}\right)$ [1]. For the normalized angle integrated EEC one gets:

$$\frac{1}{\sigma_{tot}}\frac{d\Sigma^{q\bar{q}}}{d\cos\chi} \approx \frac{C}{2}\frac{\langle h_\perp\rangle}{W}\sin^{-3}\chi. \tag{52}$$

This is the fragmentation correction to EEC for arbitrary initial state polarization and high energies, too. Defining the EEC asymmetry as

$$A(\chi) = \frac{1}{\sigma_{tot}}\left(\frac{d\Sigma}{d\cos\chi}(\pi-\chi) - \frac{d\Sigma}{d\cos\chi}(\chi)\right), \tag{53}$$

it is clear that in the above approximation the $q\bar{q}$ contribution drops out. This is why the asymmetry is considered to be a better quantity than the EEC itself. Of course $q\bar{q}g$ fragmentation does contribute to $A(\chi)$ even in the leading approximation. It is the model dependence of this contribution, which results in the widely differing values of α_s quoted in Table I.

7. *Triple energy correlations*

While in experiment only the normalized angle integrated EEC is measured it is worthwhile to look for further independent tests of QCD. The next member of the hierarchy of energy weighted cross sections is the triple energy correlation $\frac{d^3\Sigma}{d\Omega_1\, d\Omega_2\, d\Omega_3}$. This is certainly not very practical since it depends on 6 angles. Integrating out over all external angles we get $\frac{d\Sigma}{d\chi_1\, d\chi_2\, d\chi_3}$, where the angles are defined through $r_1 \cdot r_2 = \cos\chi_1$, $r_1 \cdot r_3 = \cos\chi_2$, $r_2 \cdot r_3 = \cos\chi_3$. In terms of the structure functions of Sect. 3 we get[3]

$$\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma}{d\chi_1 d\chi_2 d\chi_3} = \frac{128\pi^3}{3(\sin\chi_1 \sin\chi_2 \sin\chi_3)^{-1}}\frac{1}{\Delta(\chi_1, \chi_2, \chi_3)}$$
$$\{A_1 + A_2\cos\chi_1 + A_3\cos\chi_2 + A_4 + A_5\cos\chi_3 + A_6\}, \tag{54}$$

where

$$\Delta(\chi_1, \chi_2, \chi_3) = 1 + 2\cos\chi_1\cos\chi_2\cos\chi_3 - \cos^2\chi_1 - \cos^2\chi_2 - \cos^2\chi_3)^{1/2}.$$

Probably $\frac{d\Sigma}{d\chi_1\, d\chi_2\, d\chi_3}$ is not practical either, thus one may integrate also over the angle χ_3. (The integration region is determined by the requirement that $\Delta(\chi_1, \chi_2, \chi_3)$ should be real.) Alternatively one may define the "planar" triple energy correlation, which is triple energy correlation measured with planar calorimeter positions. Since three jets give the dominant contribution, this is equal (to a good approximation) to $\int d\chi_3 \frac{d\Sigma}{d\chi_1\, d\chi_2\, d\chi_3}$.

The lowest order QCD result may be easily obtained:

$$\frac{1}{\sigma_{\rm tot}}\frac{d\Sigma_{\rm planar}}{d\chi_1 d\chi_2} = \frac{64\alpha_s(W)}{3\pi}\frac{E_1^2E_2^2E_3^2}{W^6}$$
$$\left\{\frac{E_1^2+E_2^2}{(W-2E_1)(W-2E_2)} + \frac{E_1^2+E_3^2}{(W-2E_1)(W-2E_3)} + \frac{E_2^2+E_3^2}{(W-2E_2)(W-2E_3)}\right\}, \tag{55}$$

---

[3] $\sigma_{\rm tot}$ here is the lowest order QCD result. Inclusion of higher order corrections of $\sigma_{\rm tot}$ is achieved by multiplying (54) by $\left(1 - \frac{\alpha_s}{\pi} + \ldots\right)$.

where

$$E_1 = W \frac{-\sin(\chi_1+\chi_2)}{\sin\chi_1+\sin\chi_2-\sin(\chi_1+\chi_2)},$$

$$E_2 = \frac{-\sin\chi_2}{\sin(\chi_1+\chi_2)} E_1, \qquad E_3 = \frac{\sin\chi_1}{\sin\chi_2} E_2. \tag{56}$$

The lowest order result Eq. (55) is a good approximation, if $\delta < \chi_i < 180° - \delta$ ($i = 1, 2, 3$, $\chi_3 = 2\pi - \chi_1 - \chi_2$), with $\delta \approx 30°$, provided fragmentation corrections are also added. The axonometric view of the function (55) is shown on Fig. 10 for $\alpha_s = 0.13$.

The "planar" triple energy correlation is not yet measured experimentally. The motivation for studying it is [15]:

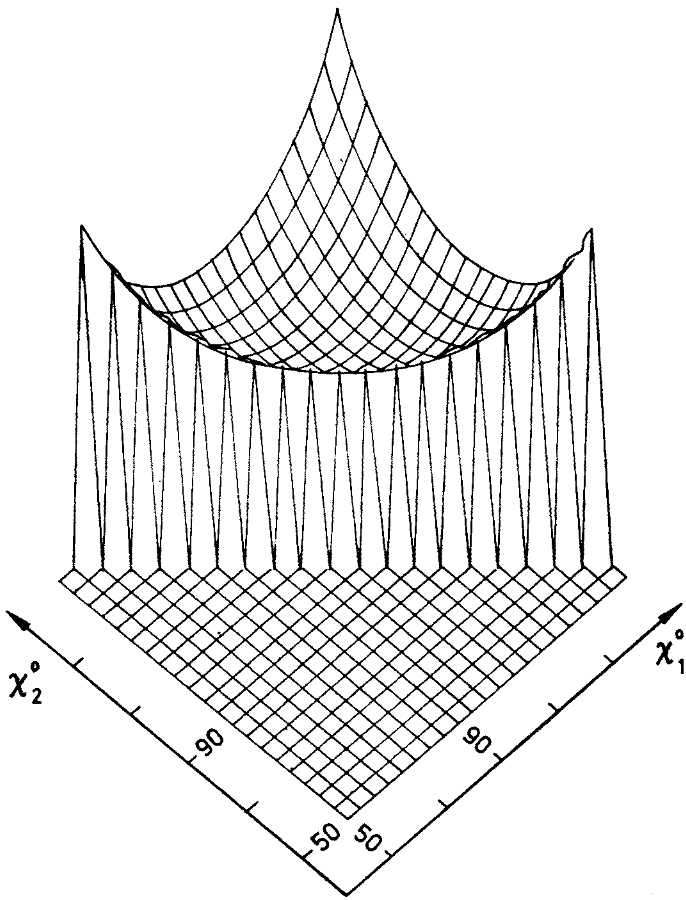

Fig. 10. Axonometric view of the normalized, planar triple energy correlation function for $\delta = 30°$. The equal maxima are at $\chi_1 = 60°$, $\chi_2 = 150°$; $\chi_1 = 150°$, $\chi_2 = 60°$; $\chi_1 = \chi_2 = 150°$; while the minimum is at $\chi_1 = \chi_2 = 120°$. For $\chi_1 + \chi_2 < 180° + \delta$ (a region, where the QCD result is not applicable) the function has been arbitrarily set equal to zero

a. It is a further independent test of QCD.

b. It takes into account 3 jets in a more natural (symmetric) way than EEC.

c. The fragmentation correction may decrease faster with increasing energy than for EEC. (Preliminary results indicate $O\left(\frac{1}{W^2}\right)$ [16].)

8. *EEC at the $p\bar{p}$ collider*

It is quite natural to try to investigate energy weighted cross sections, in particular EEC at the $p\bar{p}$ collider. In this case of course QCD is applicable at high $p_\perp$ only, furthermore the initial state is much more complicated than in the e^+e^- case. The type of QCD graphs, which are relevant are shown on Fig. 11. They are all the 2 parton → 2 parton

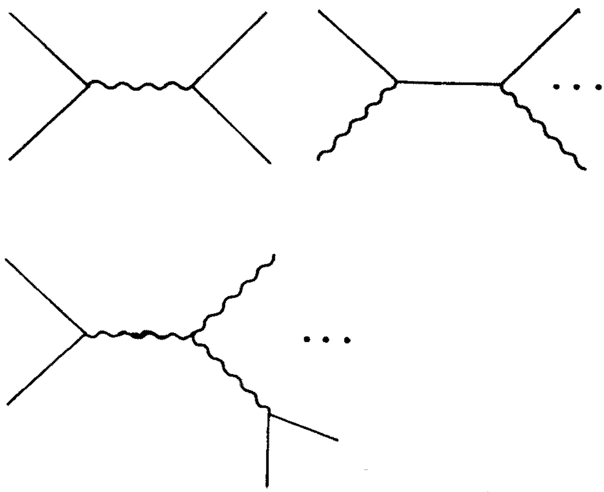

Fig. 11. The types of QCD graphs, which are relevant for $p\bar{p}$ collisions

and 2 parton → 3 parton graphs. The cross-sections for these processes are known from the literature [17].

Comparing with the e^+e^- annihilation there are two more major differences. 1) The $p\bar{p}$ CM (center of mass) frame is different from the parton CM frames. 2) The singularity structure of the subprocess cross sections is different. A final state gluon may be collinear with initial state partons, too. Of course, these singularities are at low $p_\perp$, this phase space region is excluded anyway, however, we can not normalize with the total cross section.

There are several possibilities to define EEC. Probably the simplest way is to define EEC in the $p\bar{p}$ CM frame, integrating out over all angles (in an appropriate region), keeping fixed only the relative angle of the calorimeter directions [18]. In the colliding parton CM frame the two jet production contributes to the EEC only at 0° and 180°. Transforming back to the $p\bar{p}$ CM frame, however, we get a contribution for all angles. Thus the lowest order contribution is two jet production, i.e. $O(\alpha_s^2)$. Since the $p\bar{p}$ CM frame angle is determined by the Lorentz transformation, which in turn is determined by the parton distributions, the $p\bar{p}$ CM frame EEC is actually more sensitive to the parton distributions, than any details of the hard scattering subprocesses. The $O(\alpha_s^3)$ corrections are first sensitive to the 3 parton final state. The complete $O(\alpha_s^3)$ contribution is at present not calculable, since the one loop corrections to the $2 \to 2$ subprocesses are not yet known. The $2 \to 3$ subprocess contributions alone yield a divergent answer due to uncompensated soft gluon singularities.

To define EEC-s, which are calculable and sensitive to the 3 parton final states one may choose between two possibilities. 1) EEC is defined in the $p\bar{p}$ frame, all angles except the relative azimuthal angle are integrated out [19]; 2) EEC is defined in the CM frame of the colliding partons [20]. To minimize parton distribution dependences it is convenient to take cross section ratios. In the following we discuss briefly only the first choice. The (preliminary) quantitative conclusions for the second choice are very similar.

In the first case the definition of the normalized EEC reads:

$$\frac{1}{\sigma'}\frac{d\Sigma'}{d\phi} = \frac{\int_{E_{\mathrm{T\,min}}}^{\sqrt{s}} dE_{\mathrm{T}} \frac{d^2\Sigma'}{dE_{\mathrm{T}}d\phi}}{\int_{E_{\mathrm{T\,min}}}^{\sqrt{s}} dE_{\mathrm{T}} \frac{d\sigma}{dE_{\mathrm{T}}}} = \frac{1}{N}\sum_{A=1}^{N}\frac{1}{\Delta\phi}\sum \frac{2E_{\mathrm{T}a}^{A}E_{\mathrm{T}b}^{A}}{(E_{\mathrm{T}}^{A})^2}, \tag{57}$$

where the first sum on the right hand side is over all accepted events A with total transverse energy $E_{\mathrm{T}}^{A} = \sum_a E_{\mathrm{T}a}^{A} \geqslant E_{\mathrm{T\,min}}$. The second sum is over hadron pairs (a, b) whose transverse momenta have relative angles ϕ to $\phi + d\phi$. The accepted events are those in which at least one jet has a certain trigger transverse energy, which is larger than $E_{\mathrm{T\,min}} = 2E_{\mathrm{T\,trigger}}$. It is also required that the detected partons (jets) lie in a central pseudorapidity range $|\eta| < \eta_0 = 2.5$. The allowed azimuthal angle difference range is $30° < \phi < 150°$. (Note that for $\phi = 0°$, $\phi = 180°$ the lowest order QCD calculation is divergent.)

The main question is, how sensitive is this definition of normalized EEC to a) parton distribution parametrization, b) choice of the argument of α_s. Since these ambiguities affect both the numerator and denumerator, it is expected that a large part of them cancel. In fact Fig. 12 shows that the parton distribution [21] dependence is about 10%. In the QCD calculation the numerator comes from 3 parton production, i.e. it is $\propto \alpha_s^3(Q)$, where Q is chosen to be the maximum of the transverse energies of the three partons. The denumerator is determined by two parton production, i.e. it is $\propto \alpha_s^2(Q)$, where Q is the transverse energy. Thus the ratio is proportional to $\alpha_s(\langle Q \rangle)$, where $\langle Q \rangle$ is some effective (or average) value. An experimental determination could measure this effective α_s and test the QCD prediction in $p\bar{p}$ annihilation.

9. *Conclusion*

Energy weighted cross sections (mainly EEC) have been studied in great detail for e^-e^+ annihilations both theoretically and experimentally. As a test of QCD and a possible method of α_s determination, EEC is still much affected by the uncertainties of jet fragmentation at the energies of available experiments ($\approx$ 30 GeV). For energies near the Z^0 peak these uncertainties will be much reduced, thus EEC will be useful both for α_s determination and testing QCD. In this energy region triple energy correlation may become also measurable and serve for the same purposes as an independent possibility. Though the process is much more complicated, EEC may be useful to test QCD and measure an effective α_s at the $p\bar{p}$ collider, too.

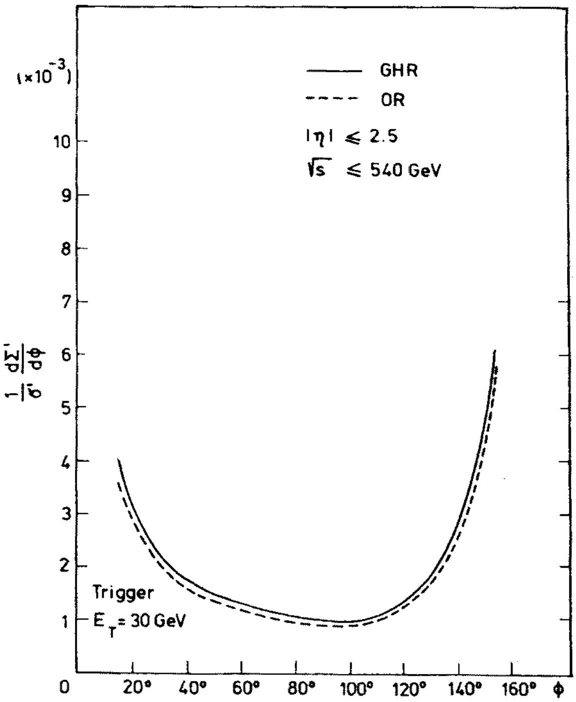

Fig. 12. Normalized, angle integrated EEC as a function of azimuthal angle difference ϕ. The figure is taken from Ref. [19]

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