LOW-ENERGY BAGGED QCD: QUARK-MESON INTERPLAY IN TWO PHASES*

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In present-day attempts to understand low-energy hadron physics one is addressed to various model assumptions in low-energy quantum chromodynamics (QCD). In this context, in the first part of the lectures we want to illustrate a natural appearance of a sort of two-phase description. A bag containing quarks calls for pions inhabitating the bag exterior. In the second part of the lectures we review a more recent (opposite in a sense) approach. A pion field may be viewed as a classical solitonic background field and the nucleon as a defect (soliton) in the Skyrme solitonic field. We intend to emphasise both the essentially novel features and the limitations of such an approach.

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1. Introduction

Half a century ago Yukawa [1] predicted pions as a nuclear glue, the mediator of the strong force binding nucleons in nuclei. Since then our view on both fundamental interactions and elementary constituents of matter has undergone substantial changes: the true mediators of fundamental forces are governed by a local gauge symmetry principle, while hadrons have been established to be composed of more elementary constituents, quarks[1]. Also, the gauge glue has been represented by vector particles called gluons, which seemingly left the "one-of-many" role to the pion. However, contrary to these expectations, the pion, as will be demonstrated below, keeps a distinguished place in the new theoretical framework of Quantum Chromodynamics. More specifically, taking the role of the chiral field, the pion has become a subject of intensive theoretical study.

Quantum Chromodynamics (QCD), the theory of coloured quarks and gluons, represents a desired gauge theory of strong interactions. Yet, all quantitative predictions of QCD are restricted to a short distance (a large momentum transfer region) where the property of asymptotic freedom enables one to perform a perturbative calculation. QCD alone does not provide us with quantitative predictions about hadron structure or interactions at the hadronic (~ 1 GeV) mass scales, where one is faced with the richest phenomenology. There are two most important ingredients of low-energy QCD:

(*i*) confinement,

(*ii*) chiral symmetry (χS)

being of a non-perturbative nature and not yet fully understood.

A successful phenomenological approach to account for problem (*i*) is based on the picture of a bag (or a potential) confining quarks and gluons in a restricted region of space. The original MIT bag model [3], being successful in predicting the static properties of low--energy hadrons, fails to account for the momentum-transfer dependence and does not contain the condition of partial conservation of the axial current (PCAC). Since I participated in curing some of these problems, I will report in more detail on the topic of treating the recoil by the bag-boosting method, even for a pion-surrounded bag.

In fact, there is growing evidence for the necessity of implementing the simple (e.g. MIT bag) quark models with the pionic degree of freedom. In order to restore chiral symmetry (i.e. to account for problem (*ii*)), the pion field is represented by the chiral field, and is treated on an equal footing with quarks. The latter may be viewed as sources of the chiral fields of chiral-bag models [4] where pions are created at the surface of the bag [5], or cloudy-bag models [6] where pions are also allowed inside the bag.

The starting point of the models mentioned above are chirally invariant Lagrangians whose bosonic part is represented by the σ model [7]. In a non-linear realisation of χS, pions appear as Goldstone bosons, and there are soft-pion theorems [8] which are successful in describing the low-energy pion-nucleon interaction. From the point of view of QCD, it would be desirable to derive such an effective theory. In the Feynman-path-integral

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[1] In fact, a predecessor of quark models in which the pion was viewed as a quark-antiquark bound state was an early observation of Fermi and Yang [2] that the pion could be considered as a bound state of the nucleon-antinucleon pair.

anguage, this means integrating out the quark and gluon degrees of freedom. In this context, the old Skyrme model [9] has been rediscovered; this model represents a stabilised version of the non-linear σ model, allowing for stable topological solitons (Skyrmeons). Its revival was initiated by Witten's [10] conjecture that such a model in which Skyrmeons are nucleons might result in the large-number-of-colour (N_c) limit of QCD. There was some success of the above scheme in reproducing the static properties of light baryons [11], followed by the rapidly growing literature on the subject. In the following we intend, first, to present two extreme pictures (MIT bag and Skyrme) and, then, to find a link between them. The new physical features of such a marriage can be most transparently illustrated on an exactly solvable model in 1+1 dimensions, which might provide us with new insights into the physics relevant to such two phases. Concerning the pion field, one must distinguish the background pion field, which turns out to carry the baryon number, from the fluctuating pion field of the traditional low-energy physics.

With a rather "conservative" reconcilement of the general principles (causality, locality and renormalizability) with the coloured-quark and gluon picture, one arrives at the Lagrangian of the candidate theory of strong interactions

$$\mathscr{L}_{\rm QCD} = -\tfrac{1}{4}\,{\rm tr}\, G_{\mu\nu}G^{\mu\nu} + \sum_f \bar{q}_f(x)\,(i\gamma_\mu \overleftrightarrow{D}^\mu - m_f)q_f(x), \tag{1.1}$$

where $\overleftrightarrow{D}^\mu q = \left(\overleftrightarrow{\partial}^\mu + ig\frac{\lambda^\alpha}{2}G^\mu_\alpha\right) q$ and $\overleftrightarrow{\partial}^\mu$ symbolises the symmetrised derivative $\frac{1}{2}(\vec{\partial}^\mu - \overleftarrow{\partial}^\mu)$ acting on quark fields to which vector gluons are universally coupled. It is convenient to decompose the quark fields q into the chiral fields $q_{\rm L}$ and $q_{\rm R}$ (since vector couplings preserve chirality):

$$q = \tfrac{1}{2}(1-\gamma_5)q + \tfrac{1}{2}(1+\gamma_5)q \equiv q_{\rm L} + q_{\rm R},$$

where flavour ($f = 1, \ldots, N$) and colour ($i = 1, 2, 3$) indices are contained implicitly ($q \equiv q^i_f$).

The Lagrangian (1.1) has some symmetries that should match the observed symmetries of the strong interaction:

(a) discrete C, P and T symmetries,

(b) approximate chiral symmetry (χS), $U(N)_{\rm L} \otimes U(N)_{\rm R}$, represented by independent rotations among left- and right-handed *massless* quarks separately:

$$q_{\rm L} \to q'_{\rm L} = Uq_{\rm L}, \quad q_{\rm R} \to q'_{\rm R} = Vq_{\rm R},$$
$$U = e^{i\theta_{\rm L}}, \quad V = e^{i\theta_{\rm R}}. \tag{1.2}$$

This global invariance may be recast in an equivalent form

$$U(N)_{\rm L}\otimes U(N)_{\rm R} = SU(N)_{\rm L}\otimes SU(N)_{\rm R}\otimes U(1)_{\rm V}\otimes U(1)_{\rm A}, \tag{1.3a}$$

where $U(1)_{\rm V}$ corresponds to multiplying $q_{\rm L}$ and $q_{\rm R}$ by a common phase $e^{i\theta}$, leading to conservation of the additive (baryon) quantum number, whereas $U(1)_{\rm A}$ represents the "famous" problem. The latter refers to the symmetry of multiplying the $q_{\rm L}$ fields by a common phase $e^{i\theta}$ and the corresponding $q_{\rm R}$ fields by the opposite phase $e^{-i\theta}$, which has not been observed. Thus, the remaining effective symmetry (when taking into account the complicated structure of the QCD vacuum responsible for $U(1)_A$) is

$$SU(N)_L \otimes SU(N)_R = SU(N)_V \otimes SU(N)_A, \tag{1.3b}$$

(c) There is strong empirical evidence that the above symmetry is spontaneously broken to the subgroup

$$SU(N)_V, \tag{1.3c}$$

which is also the global symmetry of the massless $\mathscr{L}_{QCD}$ and appears to be realised in a Wigner-Weil fashion in the hadron spectrum via accurate SU(2) isospin and approximate SU(3) multiplets. Since the parity doubling in the spectrum has not been observed, the QCD ground state cannot [12] be symmetric under (1.3b). Thus, there is no reason for the vacuum condensates $\langle 0|\bar{q}_L q_R|0\rangle$ to vanish.

A spontaneously broken continuous symmetry calls for Goldstone bosons [13]: instead of seeing the symmetry (1.3b), we see massless pseudoscalars corresponding to generators of (1.3b) which do not leave the vacuum invariant ($Q_5^a|\Omega\rangle \neq 0$). The fact that the particles π, K and η are not exactly massless (approximate Nambu-Goldstone bosons) is attributed to an explicit χSB by small quark masses in $\mathscr{L}_{QCD}$. Performing a perturbation expansion in quark masses [14] results in successful relations between measurable quantities and massless π, K, η in the m_u, m_d, $m_s \to 0$ limit.

There are indications [15] that spontaneous χS breaking (SχSB) may be predicted by $\mathscr{L}_{QCD}$, but this remains a difficult problem of dynamics, similar to the question of QCD confinement. Most likely, SχSB is a consequence of confinement [16]. There is some evidence from lattice calculations [17] that both of these phenomena appear at approximately the same scale, $\Lambda_{\chi SB} \simeq \Lambda_{QCD}$, or at $\Lambda_{\chi SB} \simeq 1$ GeV [18], which is slightly larger than $\Lambda_{QCD} = 0.1$–0.3 GeV. Manohar and Georgi [19] used the possibility of such an intermediate region to construct an effective chiral quark theory which might explain the success of the non-relativistic quark model.

Here we adopt a slightly different approach. We start from a phenomenological bag description of the two-phase world, where $\Lambda_{\chi SB} \simeq \Lambda_{QCD}$ separates the space into an inner and an outer region (Fig. 1).

This is the starting picture of the MIT bag model where the "perturbative vacuum" $|0\rangle$ inside a hadron arises from the "true vacuum" $|\Omega\rangle$ in the presence of quarks and gluons, and differs from the latter by the bag constant, B = energy/volume.

The requirement that there should be no current crossing the spherical bag boundary

$$n^\mu \bar{\psi}(x)\gamma_\mu \psi(x) = 0 \tag{1.4a}$$

is fulfilled by a linear boundary condition

$$-i\gamma_\mu n^\mu \psi_f = \psi_f \tag{1.4b}$$

for each quark flavour f. There is also a quadratic boundary condition

$$in^\mu \partial_\mu(\bar{\psi}_f \psi_f) = 2B \tag{1.5}$$

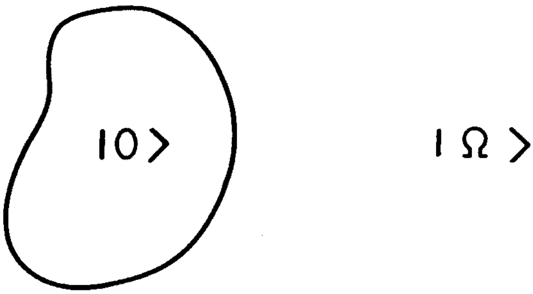

Fig. 1. The inner and outer regions, differing by the bag constant B

which forbids the existence of an empty bag (the bag is under constant pressure $\sim B$). Eqs. (1.1) and (1.2) can be obtained by applying a variational principle [20], together with the equation of motion for quarks within a sphere of radius R,

$$i\partial\!\!\!/ \psi(x) - m\psi(x) = 0. \tag{1.6}$$

The solution of Eq. (1.6) is given by all possible modes (n) of quarks and antiquarks, from which we explicitly write down two classes according to the Dirac quantum number $\kappa = \pm(j+\frac{1}{2})$ for $j = \frac{1}{2}$:

$S_{1/2}(\kappa = -1)$

$$\psi^{(n)}_{1/2,-1,m}(x) = \begin{pmatrix} iu^{(n)}(r)\chi_m \\ v^{(n)}(r)\vec{\sigma}\cdot\hat{r}\chi_m \end{pmatrix} e^{-iE_{n,-1}t}, \tag{1.7a}$$

$P_{1/2}(\kappa = +1)$

$$\psi^{(n)}_{1/2,+1,m}(x) = \begin{pmatrix} -i\tilde{v}^{(n)}(r)\vec{\sigma}\cdot\hat{r}\chi_m \\ \tilde{u}(r)\chi_m \end{pmatrix} e^{-iE_{n,1}t}. \tag{1.7b}$$

The lowest-lying hadronic states are described by the lowest ($n = 1$) mode for the S and P ground states:

$$\psi^{S}_{0}(x) = \begin{pmatrix} iu(r)\chi_m \\ v(r)\vec{\sigma}\cdot\hat{r}\chi_m \end{pmatrix} e^{-iEt}, \tag{1.8a}$$

$$\psi^{P}_{0}(x) = \begin{pmatrix} -i\tilde{v}(r)\vec{\sigma}\cdot\hat{r}\chi_m \\ \tilde{u}(r)\chi_m \end{pmatrix} e^{-i\tilde{E}t}. \tag{1.8b}$$

The radial wave functions are expressed in terms of the spherical Bessel functions as follows:

$$u = \frac{N(\omega)}{\sqrt{4\pi}} j_0(pr), \qquad v = -\frac{N(\omega)}{\sqrt{4\pi}} \left(\frac{\omega - mR}{\omega + mR}\right)^{1/2} j_1(pr), \tag{1.9a}$$

$$\tilde{u} = \frac{N(\tilde{\omega})}{\sqrt{4\pi}} j_0(\tilde{p}r), \qquad \tilde{v} = -\frac{N(\tilde{\omega})}{\sqrt{4\pi}} \left(\frac{\tilde{\omega} + mR}{\tilde{\omega} - mR}\right)^{1/2} j_1(\tilde{p}r). \tag{1.9b}$$

Here

$$p = \frac{1}{R}\,[\omega^2 - m^2R^2]^{1/2}, \tag{1.10}$$

$$N(\omega_\kappa) = \left(\frac{(\omega_\kappa^2 - m^2R^2)^2}{R^3(2\omega_\kappa^2 + 2\kappa\omega_\kappa + mR)\sin^2[(\omega_\kappa^2 - m^2R^2)^{1/2}]}\right)^{1/2}, \tag{1.11}$$

and the frequencies of the modes are determined by the transcendental equation (which follows from Eq. (1.4)),

$$\tan[(\omega_{n\kappa}^2 - m^2R^2)^{1/2}] = \frac{\kappa(\omega_{n\kappa}^2 - m^2R^2)^{1/2}}{\omega_{n\kappa} - \kappa mR + \kappa}. \tag{1.12}$$

So far we have discussed a confining static MIT bag. In the following we shall implement if by freedom to move. Then, a study of low-energy hadronic properties will show the need for introducing also the pseudoscalar meson degrees of freedom.

2. *MIT bag implemented by recoil and PCAC*

2.1. Bag with freedom to move

The MIT bag model [3], based on the mere existence of coloured-quark and gluon field quanta in the restricted spherical cavity region, has been remarkably successful in explaining the static properties and masses of low-lying hadrons, except that the pion mass comes out too large. In the subsequent literature this static model has also been widely used in calculating non-diagonal matrix elements. Since these matrix elements involve a non-vanishing momentum transfer, two new ingredients have to be taken into account.

The first ingredient are *centre-of-mass* (CM) corrections, stemming from the lack of translational invariance of the bag boundary. Attempts [21–23] to handle this spurious motion of the fields as a whole have had partial success when applied to highly relativistic wave functions within a sharp bag boundary (for a review, see Ref. [24]).

In this section we present the second ingredient in detail, namely *recoil corrections.* We shall consider them in the limit of negligible CM effect using the bag-boosting method, which has been developed in recent literature [25–29]. The first [25] of these papers presented the significance of the effect which the pure spinor-rotation part of the boost might introduce for baryon magnetic moments. Although this result was confirmed by another author [26], it was later shown [27–29] that it could be compensated by the *coordinate-transformation* effect which was calculable provided the bag picture was implemented by the assumption of transforming the bag at a point.

The *fundamental* problem that faces us is the boosting of a confined-quark system [30, 31]. For the rigid MIT bag, this problem remains open (neither the static MIT bag solution nor a moving bag represents the momentum (or the Dirac Hamiltonian) eigenstate[2]). This point is worth stressing, since it has been a source of confusion.

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[2] Jaffe [32] has shown that the bag can be arranged to be an eigenstate of the momentum in 1+1 dimensions.

The *practical* problem that faces us is an extension of the MIT bag model which would enable us to calculate physical quantities for low-momentum transfers. Probing the nucleon in the laboratory frame gives rise to a recoil of the nucleon as a whole; the originally spherical bag undergoes an ellipsoidal deformation and the corresponding form factors develop a Lorentz-invariant argument, $q^2 = q_0^2 - \vec{q}^2$. The matrix elements are to be calculated at the time $t = 0$ when the initial and final bag overlap. Obviously, the overlap is optimal if one uses the frame of reference in which both the initial bag and the final bag experience the same Lorentz contraction. This can be achieved in the Breit frame S, the special frame of the probe particle, where this particle carries no energy, $q \to (0, \vec{q}_B)$. In this frame, the genuinely single bag splits in two, the initial and final spherical bag states representing the proper frames $(S^0)_1$ and $(S^0)_2$; in the Breit frame S, $(S^0)_1$ and $(S^0)_2$ are viewed as moving with momenta $\vec{p}_1$ $(= -\vec{q}_B/2)$ and $\vec{p}_2$ $(= \vec{q}_B/2)$, respectively, the pertinent coordinates being related by the Lorentz transformation

$$t^0_{1,2} = \gamma t - \frac{\vec{x} \cdot \vec{p}_{1,2}}{m},$$

$$\vec{x}^0_{1,2} = \vec{x} + \frac{\vec{p}_{1,2}(\vec{x} \cdot \vec{p}_{1,2})}{m(E_{1,2}+m)} - \frac{\gamma}{E_{1,2}} \vec{p}_{1,2} t. \tag{2.1}$$

Let us write down the general decompositions of the matrix elements for the nucleon currents of interest.

The electromagnetic current:

$$\langle \vec{p}_2 | J^\mu(0) | \vec{p}_1 \rangle = \bar{u}(\vec{p}_2) \left[\gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p_1),$$

$$q = p_2 - p_1; \quad u(p) = \left(\frac{E+m}{2E} \right)^{1/2} \begin{pmatrix} \chi \\ \dfrac{\vec{\sigma} \cdot \vec{p}}{E+m} \chi \end{pmatrix}, \; u^\dagger(p) u(p) = 1. \tag{2.2}$$

The axial-vector current:

$$\langle \vec{p}_2 | \underset{\sim}{A}^\mu(0) | \vec{p}_1 \rangle = \bar{u}(\vec{p}_2) \left[\gamma^\mu \gamma_5 G_A(q^2) + \gamma_5 q^\mu H_A(q^2) \right] \frac{\underset{\sim}{\tau}}{2} u(p_1). \tag{2.3}$$

In the Breit frame, we end up with the following matrix elements:

$$\langle \vec{q}/2 | J^0 | -\vec{q}/2 \rangle = \chi^\dagger \frac{m}{E} G_E(\vec{q}^2) \chi, \tag{2.4}$$

expressed by the electric form factor $G_E = F_1 - (\vec{q}^2/4m^2) F_2$;

$$\langle \vec{q}/2 | \vec{J} | -\vec{q}/2 \rangle = \chi^\dagger i \frac{\vec{\sigma} \times \vec{q}}{2E_B} G_M(\vec{q}^2) \chi, \tag{2.5}$$

expressed by the magnetic form factor $G_M = F_1 + F_2$;

$$\langle \vec{q}/2 | \underset{\sim}{A}^0 | -\vec{q}/2 \rangle = 0, \tag{2.6}$$

$$\langle \vec{q}/2 | \vec{\underset{\sim}{A}} | -\vec{q}/2 \rangle = G_A(\vec{q}^2) \chi_2^\dagger \vec{\sigma} \frac{\underset{\sim}{\tau}}{2} \chi_1$$

$$-\vec{q}^2 \frac{1}{2E_B} \left(H_A(\vec{q}^2) + \frac{G_A(\vec{q}^2)}{2(E_B+m)} \right) \chi_2^\dagger (\vec{\sigma} \cdot \hat{q}) \hat{q} \frac{\underset{\sim}{\tau}}{2} \chi_1. \tag{2.7}$$

Let us first consider the contribution from quark currents $\vec{J}^Q(x)$ inside the bag to the matrix element (2.5):

$$\langle \vec{p}_2 | \vec{J}(0) | \vec{p}_1 \rangle = \int_{\text{Bag}} d^3x \vec{J}^0(x) e^{i\vec{q}\cdot\vec{x}}, \quad \vec{q} = \vec{p}_2 - \vec{p}_1. \tag{2.8}$$

In the proper bag frame $(S^0)_{1,2}$, the quark solution for a nucleon bag at rest is given by [3] the lowest order in Eq. (1.7a):

$$\psi_0(x) = \psi_0(r) e^{-iE^Q t}, \quad E^Q = \frac{\omega}{R}. \tag{2.9}$$

Transforming from the proper bag frame $(S^0)_{1,2}$, into the Breit frame S (where $(S^0)_{1,2}$ moves with $\mp\vec{q}/2$) involves the coordinate transformation

$$x_{1,2}^{(0)} \to x = \Lambda_{1,2} x_{1,2}^{(0)}, \tag{2.10}$$

accompanied with the spinor transformation

$$q(x) = S(\Lambda_1) q_0(x_1^{(0)}) \equiv B(-\vec{q}/2) \psi_0(\vec{x}_1^{(0)}) e^{-it_1^{(0)} E_1^Q}, \tag{2.11a}$$

$$\bar{q}(x) = \bar{q}_0(x_2^{(0)}) S^{-1}(\Lambda_2) \equiv e^{it_2^{(0)} E_2^Q} \bar{\psi}(\vec{x}_2^{(0)}) B^{-1}(\vec{q}/2), \tag{2.11b}$$

where

$$B(\vec{p}) = \left(\frac{E+m}{2E} \right)^{1/2} \begin{pmatrix} 1 & \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} \\ \dfrac{\vec{\sigma}\cdot\vec{p}}{E+m} & 1 \end{pmatrix} \tag{2.12}$$

gives the spinor rotation part of the boost.

The requirement of transforming the initial bag into the final bag at a point in space--time ensures a causal description [4] and makes it possible to calculate the coordinate--transformation effect by substituting Eq. (2.1) in Eq. (2.11) at the instant $t = 0$. Then Eq. (2.8) is recast into the following form:

$$\langle \vec{q}/2 | \vec{J}(0) | -\vec{q}/2 \rangle = \int_{\text{Bag}} d^3x e^{i\vec{q}\cdot\vec{x}} e^{-i\frac{\omega}{Rm}\vec{q}\cdot\vec{x}} \bar{\psi}_0(\vec{x}_2^{(0)}) B^{-1}(\vec{q}/2) \vec{\alpha}$$

$$\times B(-\vec{q}/2) \psi_0(\vec{x}_1^{(0)})|_{t=0}. \tag{2.13}$$

It is obvious from expression (2.1) that it is convenient to omit the terms of the second order in the nucleon velocity, because then the expression

$$\vec{x}^{(0)}_{1,2}|_{t=0} = \vec{x}+O(\vec{v}^2) \tag{2.14}$$

implies the spherical boundary to this order. Consequently, Eq. (2.10) becomes a simple integral over a spherical region:

$$\langle\vec{q}/2|\vec{J}(0)|-\vec{q}/2\rangle = 4\pi i(\vec{\sigma}\times\vec{q}) \int_{\text{Bag}} r^2 dr$$

$$\times\left\{(u^2-v^2)j_0(ar)+2v^2\frac{j_1(ar)}{ar}-4muvr\left(1-\frac{\omega}{Rm}\right)\frac{j_1(ar)}{ar}\right\}, \tag{2.15}$$

where $a = (1-\omega/Rm)q$.

In the limit $\vec{q}\to 0$, Eq. (2.15) gives the static MIT value [3],

$$\mu_{\text{P}}(\text{stat}) = -\frac{16\pi m}{3}\int_{\text{Bag}} drr^3uv = 1.902\text{ nm}, \tag{2.16a}$$

the "spin-precession" contribution [25] due to the spinor rotation (2.12),

$$\mu_{\text{P}}(\text{prec}) = 4\pi\int_{\text{Bag}} drr^2\left(u^2-\frac{v^2}{3}\right) = 0.654\text{ nm}, \tag{2.16b}$$

and the "retardation" contribution [27–29] due to the Lorentz transformation of coordinates,

$$\mu_{\text{P}}(\text{ret}) = \frac{16\pi\omega}{3R}\int_{\text{Bag}} drr^3uv = -0.827\text{ nm}. \tag{2.16c}$$

The contributions (2.16b) and (2.16c) practically cancel each other, and there is almost no recoil effect on the magnetic moment of the proton.

Let us for a moment return to the velocity expansion used in Eq. (2.14). As explicitly stated by Guichon [27], it is more than a mere convenience. In fact, the linear boundary condition for quarks

$$i\gamma\cdot n\psi(x) = \psi(x)|_{\partial\text{Bag}} \tag{2.17}$$

can be kept up to the first order in the nucleon velocity $\vec{v} = \vec{q}/2m$. To this order, the "bag-proper-frame" normals $\hat{n}^0_{1,2}$ conform to the Breit-frame normal $\hat{n}^{\text{Breit}}$:

$$\hat{n}^{(0)}_{1,2} = \hat{n}^{\text{Breit}}+O(\vec{v}^2). \tag{2.18}$$

If the terms $O(\vec{v}^2)$ in Eqs. (2.14) and (2.18) are retained, the expansion of $\psi(\vec{x}) = \psi(\vec{x}^{(0)}_{1,2}+\Delta\vec{r}_{1,2})$ leads to a new boundary condition

$$-i(\vec{\gamma}\cdot\hat{n}_{1,2})(\hat{n}_{1,2}\cdot\nabla)\psi(r_{1,2}) = (\hat{n}_{1,2}\cdot\nabla)\psi(r_{1,2})|_{r_{1,2}=R}, \tag{2.19}$$

which is not compatible with Eq. (2.17).

In a way analogous to the evaluation of the proton magnetic moment, one can calculate the contributions coming from quarks to the axial-current matrix element (2.7). These are given by the sum over single-quark contributions:

$$\vec{A}^{Q}(q) = \int_{\text{Bag}} d^3r e^{i\vec{q}\cdot\vec{r}\left(1-\frac{\omega}{mR}\right)}\bar{\psi}_0(x_2^{(0)})B^{-1}(\vec{q}/2)\vec{\gamma}\gamma_5 B(-\vec{q}/2)\psi_0(\vec{x}_1^{(0)}). \tag{2.20}$$

As expected [25], in the $\vec{q} \to 0$ limit there is no effect of the recoil on the axial form factor G_A^Q:

$$G_A^Q(0) = g_A^Q = \tfrac{5}{3}\int_{\text{Bag}} d^3r\left(u^2 - \frac{v^2}{3}\right) = 1.088. \tag{2.21}$$

The recoil gives the momentum-transfer dependence [33] of $G_A^Q(q^2)$ and $H_A^Q(q^2)$:

$$G_A^Q(q^2) = \tfrac{5}{3}\int_{\text{Bag}} d^3r\left[(u^2-v^2)j_0(ar)+2v^2\frac{j_1(ar)}{ar}\right.$$
$$\left. -ruv\left(1-\frac{\omega}{mR}\right)\frac{\vec{q}^{\,2}}{E_B}\frac{j_1(ar)}{ar}\right], \tag{2.22}$$

and similarly

$$H_A^Q(q^2) = -\tfrac{5}{3}\frac{1}{2(E_B+m)}\int_{\text{Bag}} d^3r\left[2v^2\frac{m}{E_B-m}\left(j_0(ar)-3\frac{j_1(ar)}{ar}\right)\right.$$
$$\left. +ruv4m\frac{E_B+m}{E_B}\left(1-\frac{\omega}{mR}\right)\frac{j_1(ar)}{ar}\right]. \tag{2.23}$$

In the $q \to 0$ limit, there is also a non-vanishing value given explicitly by

$$H_A^Q(0) = h_A^Q(\text{prec})+h_A^Q(\text{ret}) = 3.45\ \text{GeV}^{-1}-1.95\ \text{GeV}^{-1} = 1.50\ \text{GeV}^{-1}. \tag{2.24}$$

This value is two orders of magnitude below the experimental value. This calls for introducing the pionic degree of freedom, since only the pion pole has a chance to cure this two-order-of-magnitude discrepancy.

2.2. Pion-surrounded moving bag

The best way to deal with pionic contributions of interest would be to use hybrid chiral-bag models [34, 35]. However, chiral-bag equations are complicated coupled non-linear equations and therefore very difficult to solve. Thus far only one exact solution to these equations has been found, the so-called "hedgehog solution" [4, 34], which still leads to a numerical evaluation of the pion field [36]. Nevertheless, chiral-bag equations simplify considerably if we keep the pion field only in the lowest order in Jaffe's expansion [35] in terms of a small parameter which measures the strength of the classical pion field at the bag surface. This can be a reasonable approximation for values of the radius parameter close to the original MIT value. For quarks, the equations thus obtained are just the original MIT bag equations [3]. The equations for pions

$$\hat{r}\cdot\nabla\underset{\sim}{\phi}_\pi = -\frac{i}{2f_\pi}\bar{q}\gamma_5\underset{\sim}{\tau}q, \qquad r = R, \tag{2.25a}$$

$$\Box\underset{\sim}{\phi}_\pi = 0, \qquad r > R, \tag{2.25b}$$

lead to the pion field of the form

$$\underset{\sim}{\phi}_\pi = -\frac{\omega}{16\pi f_\pi(\omega-1)}\frac{U^\dagger\vec{\sigma}\cdot\hat{r}\underset{\sim}{\tau}U}{r^2}, \qquad r \geqslant R. \tag{2.26}$$

Here U represents the quark spinor-isospinor.

However, one sees that the equations have lost their chiral invariance through the approximation scheme described. We may therefore go one step further in order to approach the world of small but non-vanishing mass of pions. This can be achieved by invoking the PCAC relation $\partial_\mu A^\mu = \mu^2 f_\pi \phi_\pi$ and putting by hand the pion mass $\mu = 0.1396$ GeV. In this way we finally arrive at the MIT bag model with the PCAC incorporated [37].

Let us first formulate the boundary value problem in the rest frame of the nucleon. The pion field created by a single-quark source is to satisfy the Klein-Gordon equation

$$(\Box^{(0)}-\mu^2)\underset{\sim}{\Phi}_0(x^{(0)}) = 0, \qquad r > R, \tag{2.27a}$$

subject to the Neumann boundary condition at the bag surface

$$if_\pi(n^{(0)}\cdot\partial^{(0)})\underset{\sim}{\Phi}_0(x^{(0)}) = \bar{\Psi}_0(x^{(0)})\gamma_5\frac{\underset{\sim}{\tau}}{2}\Psi_0(x^{(0)}), \qquad r = R \tag{2.27b}$$

and the Dirichlet boundary condition at infinity $\Phi_0(r = |\vec{x}^{(0)}| \to \infty) = 0$. Since quarks in the bag are in the same mode, both boundary conditions are time independent, leading to the static solution for the pion field generated by the quark a → quark b transition [37]:

$$\underset{\sim}{\Phi}_0(\vec{r}) = -\frac{\mu^2}{8\pi f_\pi}\frac{\omega}{2(\omega-1)}\frac{1+\varrho}{1+\beta+0.5\beta^2}\frac{e^{\beta-\varrho}}{\varrho^2}U_b^\dagger\vec{\sigma}\cdot\vec{r}\underset{\sim}{\tau}U_a, \tag{2.28}$$

where

$$\varrho = \mu r, \qquad \beta = \mu R, \qquad \omega = 2.0428.$$

We see that the spin-isospin structure of the pion field (2.28) is the same as in the massless case (2.26), but the radial dependence is represented by a more realistic Yukawa-like behaviour.

Now, we consider the pion field of a slowly moving nucleon. The boundary value problem is formulated [33] in the Breit frame of the nucleon at the instant $t = 0$:

$$(\Box - \mu^2)\underset{\sim}{\phi}(x) = 0, \quad (t = 0, r > R_B), \tag{2.29a}$$

$$if_\pi(n \cdot \partial)\underset{\sim}{\phi}(x) = \bar{\psi}(x)\gamma_5 \frac{\underset{\sim}{\tau}}{2} \psi(x), \quad (t = 0, r = R_B), \tag{2.29b}$$

where R_B symbolises the ellipsoidal bag boundary in the Breit frame. In the first order in the velocity expansion (compare Eq. (2.14)), the exterior problem (2.29) for the ellipsoid is reduced to the exterior problem for the sphere. The original Klein-Gordon equation at $t = 0$ reduces to the modified Helmholtz equation

$$(\nabla - \mu^2)\underset{\sim}{\phi}(x) = 0, \quad (t = 0, r > R), \tag{2.30a}$$

subject to a new boundary condition

$$if_\pi \frac{\partial}{\partial r^{(0)}} \underset{\sim}{\phi}(x) = e^{-i\frac{\omega}{Rm}\vec{q}\cdot\vec{x}} \bar{\psi}_0(\vec{x}^{(0)}) B^{-1}(\vec{q}/2)\gamma_5 \frac{\underset{\sim}{\tau}}{2}$$

$$\times B(-\vec{q}/2)\psi(\vec{x}^{(0)}), \quad (t = 0, r = R). \tag{2.30b}$$

Here the point-like emission of the pion at $t = 0$ ensures the time independence and results in a static solution for the pion field

$$\underset{\sim}{\phi}(\vec{r}) = -\frac{\mu^2}{8\pi f_\pi} \frac{\omega}{2(\omega-1)} \left\{ \frac{1+\varrho}{1+\beta+0.5\beta^2} \frac{1}{\varrho^2} U^\dagger(\vec{\sigma}\cdot\hat{r})\underset{\sim}{\tau} U \right.$$

$$+ i\tfrac{2}{3}\left[\frac{\left(\frac{1}{E_B} - \frac{\omega}{m}\right)}{\beta(1+\beta)} \frac{1}{\varrho} + \left(\frac{1}{2E_B} + \frac{\omega}{m}\right) \frac{\beta(3+3\varrho+\varrho^2)}{9+9\beta+4\beta^2+\beta^3} \frac{1}{\varrho^3}\right] U^\dagger\vec{\sigma}\cdot\vec{q}\underset{\sim}{\tau} U$$

$$\left. - i\left(\frac{1}{E_B} + 2\frac{\omega}{m}\right) \frac{\beta(3+3\varrho+\varrho^2)}{9+9\beta+4\beta^2+\beta^3} \frac{1}{\varrho^3} U^\dagger(\vec{q}\cdot\hat{r})(\vec{\sigma}\cdot\hat{r})\underset{\sim}{\tau} U \right\} e^{\beta-\varrho}, \quad r > R. \tag{2.31}$$

In the $q \to 0$ limit, Eq. (2.31) reduces to the preceding solution (2.28), and, furthermore, letting $\mu \to 0$ results in Jaffe's solution (2.26). One should note that it is important to have $\mu \neq 0$ in calculating the matrix elements of $\Delta S = 1$ processes [38] with the pseudoscalar field representing a kaon.

In the following we shall employ solution (2.31) to predict some form factors [33, 38] for which both the pseudoscalar field and the non-vanishing momentum transfer are necessary ingredients.

2.3. Role of boosted pseudoscalar fields in predicting some low-energy quantities

(*i*) Employing the pion field (2.31), we arrive at the pionic axial-vector current in the Breit frame:

$$\vec{\underset{\sim}{A}}^\pi(q) = \int_V d^3r e^{i\vec{q}\cdot\vec{r}} f_\pi \vec{\nabla} \langle N_b | \underset{\sim}{\phi}(r) | N_a \rangle. \tag{2.32}$$

Matching it with the general expression (2.7) gives the pionic contribution to the axial form factor:

$$G_A^\pi(q^2) = \frac{5}{3}\frac{\omega}{2(\omega-1)}\left\{\frac{1+\beta}{1+\beta+\beta^2/2}\frac{j_1(qR)}{qR}\right.$$

$$\left.+\left(\frac{1}{E_B}+\frac{2\omega}{m}\right)\frac{\mu}{\beta}\frac{3+3\beta+\beta^2}{9+9\beta+4\beta^2+\beta^3}j_2(qR)\right\}, \quad q = |\vec{q}|, \tag{2.33}$$

and a similar, though somewhat lengthy expression [33] for the induced pseudoscalar form factor $H_A(q^2)$. The value of the pion contribution to G_A in zero is

$$G_A^\pi(0) = g_A^\pi = g_A^\pi(\text{static}) = 0.477. \tag{2.34a}$$

Adding the quark contribution (2.21) to this value gives

$$g_A = g_A^Q + g_A^\pi = 1.565, \tag{2.34b}$$

a well-known overshoot of the experimental value [39] $g_A^{exp} = 1.260 \pm 0.012$, a common drawback of all hybrid chiral models in which pions are excluded from the bag.

The recoil is necessary in order to match the prediction for the induced pseudoscalar with the existing measurements [40] of $H_A(q^2)$ in the muon capture $\mu^- p \to \nu_\mu n$, which occurs at $q^2 = 0.88 m_\mu^2 \simeq 0.01$ GeV^2 for muons in the 1s orbit in hydrogen. Numerical values for this point are

$$G_A(0.01) = G_A^Q(0.01) + G_A^\pi(0.01) = 1.088 + 0.479 \tag{2.35a}$$

and

$$H_A(0.01) = H_A^Q(0.01) + H_A^\pi(0.01) = (1.49 + 97.19)\ \text{GeV}^{-1}, \tag{2.35b}$$

as compared with the experimental value [39]

$$H_A^{exp}(q^2 = 0.01\ \text{GeV}^2) = (120 \pm 20)\ \text{GeV}^{-1}. \tag{2.36}$$

Furthermore, we may provide a "consistency check" on our model quantities $G_A(q^2)$ and $H_A(q^2)$. Starting from the pion pole dominance of the induced pseudoscalar and invoking the Goldberger-Treiman relation, one obtains at $q^2 = 0$ the relation

$$h_A = \frac{2mg_A}{\mu^2}. \tag{2.37}$$

Eq. (2.37) represents the consistency relation at $q^2 = 0$ in the sense that the model quantities $h_A = H_A(0)$ and $g_A = G_A(0)$ should satisfy it. For non-vanishing q^2 we may replace the PCAC-based relation (2.37) by either

$$H_A^{PCAC}(\vec{q}^{\,2}) \simeq \frac{2mg_A}{\mu^2 + q^2} \tag{2.38a}$$

or

$$H_A^{PCAC}(\vec{q}^2) \simeq \frac{2mG_A(\vec{q}^2)}{\mu^2+\vec{q}^2}, \tag{2.38b}$$

which should be a good approximation for small $\vec{q}^2$ ($\vec{q}^2 \lesssim \mu^2$, for example). Thus, using the values given by (2.34b) and (2.35a), we obtain (with $m = 0.938$ GeV and $\mu = 0.1396$ GeV)

$$H_A^{PCAC}(0) = 150.56\ \text{GeV}^{-1},$$
$$H_A^{PCAC}(0.01) = \begin{cases} 99.52\ \text{GeV}^{-1} & \text{(by 2.38a)} \\ 99.33\ \text{GeV}^{-1} & \text{(by (2.38b)).} \end{cases} \tag{2.39}$$

The values given by Eq. (2.39) are to be compared with the model values $h_A = h_A^Q + h_A = 149.44$ GeV^{-1} and $H_A(0.01) = 98.68$ GeV^{-1}, leading to the difference

$$\Delta H_A = H_A^{PCAC} - H_A = \begin{cases} 1.12\ \text{GeV}^{-1}, & \text{for}\quad \vec{q}^2 = 0 \\ \left.\begin{matrix} 0.84\ \text{GeV}^{-1} \\ \text{or} \\ 0.65\ \text{GeV}^{-1} \end{matrix}\right\} & \text{for}\quad \vec{q}^2 = 0.01 \quad \begin{matrix} \text{(by (2.38a))} \\ \\ \text{(by (2.38b))} \end{matrix} \end{cases} \tag{2.40}$$

Since the contributions from the boost to the H_A form factor are $\simeq -7$ GeV^{-1} for $\vec{q}^2 = 0$ and $\simeq -3$ GeV^{-1} for $\vec{q}^2 = 0.01$, the inclusion of recoil corrections improves the fulfilment of the consistency relation.

For completeness, we mention recent Refs. [41] and [42] which treat nuclear axial form factors from a different point of view.

(*ii*) Solution (2.31) may also represent a K-meson field created in the strange-non-strange quark transition, which has its counterpart in the matrix elements at the baryon level:

$$\langle B'|V^\mu|B\rangle = \bar{u}(B')\left[f_1\gamma^\mu + i\frac{f_2}{M+M'}\sigma^{\mu\nu}q_\nu - \frac{f_3}{M+M'}q^\mu\right]u(B), \tag{2.41a}$$

$$\langle B'|A^\mu|B\rangle = \bar{u}(B')\left[g_1\gamma^\mu + i\frac{g_2}{M+M'}\sigma^{\mu\nu}q_\nu - \frac{g_3}{M+M'}q^\mu\right]\gamma_5 u(B). \tag{2.41b}$$

Actually, in an attempt [38] to study the recoil effects on the $\Delta S = 1$ form factors in Eq. (2.28) (parallel to those [33] of the $\Delta S = 0$ axial form factors), Eeg and Lie-Svendsen independently solved the problem of K-meson contributions using the boosting method. The results which they obtained are rather encouraging[3]: a previously calculated [43] decrease in f_1 and increase in g_1 from the naive SU(3) symmetric value, appear to be compensated by recoil effects and static pseudoscalar field effects, respectively, leading to agreement with experiment [44]. In addition, the induced scalar form factor f_3 and the

[3] For some details, see the lecture at this school by H. Høgaasen.

"weak electric" form factor g_3 appear to be proportional to the mass difference of the final and initial baryon, indicating that the exposed "boosting model" gives a reliable description of recoil effects for momentum transfers appearing in $\Delta S = 1$ semileptonic weak decays.

To conclude, the MIT bag implemented by the pseudoscalar field and the recoil allows for an *effective* description of low-energy quantities. It is desirable to find a link between the ingredients established in such an effective description, and the basic QCD theory.

3. Restoring chiral symmetry and two phases

3.1. From the σ model to the Skyrme model

A confining feature of QCD can be accounted for phenomenologically by a bag boundary hypothesis, but this in turn violates chiral symmetry. Reflecting on the wall changes the chirality (handedness) of the quark. Similarly, the mass term which mixes L- and R-handed particles in the quark Lagrangian

$$\mathscr{L}_q = \frac{i}{2}\,\bar{\psi}\gamma\cdot\overleftrightarrow{\partial}\psi - m\bar{\psi}\psi \tag{3.1}$$

represents a χSB term. A very simple χS restoration in Eq. (3.1) occurs in a σ-model [7, 24, 45–47] fashion by making the replacement

$$\begin{aligned} \bar{\psi}\psi &\to \bar{\psi}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)\psi, \\ m &\to g \text{ (an effective coupling)}, \end{aligned} \tag{3.2a}$$

where ψ undergoes a global transformation

$$\psi \to \psi' = \psi + i\,\frac{\underset{\sim}{\tau}}{2}\cdot\underset{\sim}{\beta}\gamma_5\psi, \tag{3.2b}$$

and introducing the quaternion of fields $(\sigma, \underset{\sim}{\pi})$ through the Lagrangian

$$\tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2 - V(\sigma, \pi). \tag{3.3a}$$

Consequently, the transformation (3.2b) is accompanied by

$$\begin{aligned} \sigma &\to \sigma' = \sigma - \underset{\sim}{\beta}\cdot\underset{\sim}{\pi}, \\ \underset{\sim}{\pi} &\to \underset{\sim}{\pi}' = \underset{\sim}{\pi} + \underset{\sim}{\beta}\sigma. \end{aligned} \tag{3.3b}$$

In this way, we end up with the most general renormalisable χS Lagrangian

$$\mathscr{L} = \frac{i}{2}\,\bar{\psi}\gamma\cdot\overleftrightarrow{\partial}\psi + g\bar{\psi}(\sigma + i\underset{\sim}{\tau}\cdot\underset{\sim}{\pi}\gamma_5)\psi + \tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{2}(\sigma_\mu\underset{\sim}{\pi})^2 - \frac{\lambda^2}{4}\,[(b^2+\pi^2) - f_\pi^2]^2, \tag{3.4}$$

leading to a conserved Nöther axial current

$$\underset{\sim}{A}^{\mu} = \bar{\psi}\gamma^{\mu}\gamma_5 \frac{\underset{\sim}{\tau}}{2} \psi - \underset{\sim}{\pi}\partial^{\mu}\sigma + \sigma\partial^{\mu}\underset{\sim}{\pi}. \tag{3.5}$$

The second row in Eq. (3.4) corresponds to the linear σ model with the symmetry-breaking pion-mass term omitted [24, 47]. In addition, the unobserved σ field is eliminated by the so-called soft-mode constraint on the fields

$$\sigma^2 + \pi^2 = f_\pi^2, \tag{3.6}$$

which leads to the non-linear σ model [7]

$$\mathscr{L}_2 = \tfrac{1}{2}(\partial_\mu\sigma)^2 + \tfrac{1}{2}(\partial_\mu\underset{\sim}{\pi})^2, \quad \sigma^2 + \underset{\sim}{\pi}^2 = f_\pi^2, \quad f_\pi \simeq 93 \text{ MeV}. \tag{3.7}$$

Equation (3.7) can be written in terms of the $(\frac{1}{2}, \frac{1}{2})$ representation of SU(2) ⊗ SU(2) given by 2×2 matrices, $U(x) = \frac{1}{f_\pi}[\sigma(x) + i\underset{\sim}{\tau} \cdot \underset{\sim}{\pi}(x)]$, instead of the (0, 1) vector representation by the quaternion $\begin{pmatrix}\sigma \\ \underset{\sim}{\pi}\end{pmatrix}$ given above. This form of $\mathscr{L}_2$ is

$$\mathscr{L}_2 = \frac{f_\pi^2}{4} \operatorname{Tr}[(\partial_\mu U)(\partial^\mu U^\dagger)]. \tag{3.8}$$

$$U^\dagger U = UU^\dagger = I.$$

Besides having the SU(2) ⊗ SU(2) symmetry (1.3b), $\mathscr{L}_2$ supports a *topological structure*, given by a third homotopy group $\pi_3(\mathrm{SU}(2)) = \pi_3(S^3) = Z$. By Derrick's theorem, the related topologically stable solitons are not energetically stable unless higher-order derivative terms are added to $\mathscr{L}_2$, the simplest choice being proposed by Skyrme [9]:

$$\mathscr{L}_4 = -\frac{\varepsilon^2}{2} \operatorname{Tr}[(\partial_\mu U)U^\dagger, (\partial_\nu U)U^\dagger]^2. \tag{3.9}$$

Such a "quartic" term, being even in time and space derivatives, ensures T and P invariance. This term is unique in the sense that it leads to a Hamiltonian which is positive and of the second order in time derivatives.

The Euler-Lagrange equations for the Skyrme Lagrangian

$$\mathscr{L}_{\mathrm{Sk}} = \mathscr{L}_2 + \mathscr{L}_4 \tag{3.10}$$

can only be handled under the assumption of the spherically symmetric configuration [9]

$$U(\vec{x}) = e^{i\vec{\tau}\cdot\hat{r}\theta(r)} = \cos\theta(r) + i\vec{\tau}\cdot\hat{r}\sin\theta(r), \tag{3.11}$$

which is the hedgehog ansatz known from chiral-bag models [4, 36]. It should be mentioned that for $\theta(0) = \pi$ and $\theta(\infty) = 0$ there is a soliton solution with baryon number 1. More generally, solitons with the topological charge N correspond to the boundary condition $\theta(0) = N\pi$ and have the spectrum in the GeV region, approximated by the quantum rotator for small N [48]

$$E_N \simeq \frac{E_1}{2} N(N+1).$$

3.2. Interpolating quarks and chiral solitons

The Skyrme picture is extreme in the sense that there are no quark degrees of freedom. A pertinent proposal [9] for baryons as topological solitons has been confirmed in essential aspects quite recently [49, 10], and it may be called the Skyrme-Witten approach to baryons. A subsequent evaluation [11, 50] of the static properties of the nucleon and the delta baryon (i.e. for two light-quark flavours in the quark picture) indicates the relevance of such an effective boson theory description for *long-distance* properties of baryons. An extension [10, 51, 52] from the original SU(2)×SU(2) Skyrme model to the SU(3)×SU(3) chiral symmetric model involves proper inclusion of anomalies[4].

One might expect the importance of quark degrees of freedom for *short-distance* effects. Accordingly, a scenario having a peaceful coexistence of quarks and chiral fields appears naturally: chiral fields are binding quarks while quarks are keeping the Skyrmion from collapsing. Thus we are faced with the quark-meson interplay in two phases which can be described by various models. The PCAC-implemented MIT bag from the preceding section can be considered as one of the simplest possibilities. However, here we want to incorporate chiral symmetry and to distinguish between two classes of such models, based on the σ-model Lagrangian (3.4). The distinction will become transparent in Fig. 2, which shows the "Mexican-hat" potential (the last term in Eq. (3.4)).

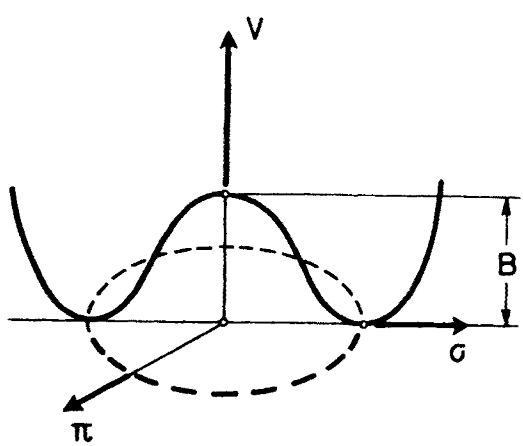

Fig. 2. The "Mexican-hat" potential, where B indicates the difference in energy of the vacua sitting at the top and on the rim of the hat

[4] For anomalies and some subtleties associated with the so-called Wess-Zumino term [10, 53], we refer the reader to the lectures given at this school by J. L. Petersen.

The first possibility is the "non-bag" chiral quark soliton model of Refs. [54] and [55]. This model allows the chiral fields to permeate all the space, whereby the physical vacuum remains on the rim of the hat. The choice $\sigma = f_\pi$, $\pi = 0$ leads to the Goldstone (χSB) mode with pion quantum numbers. Chiral fields in interaction with quark fields (which are their source) produce bound states. The spectrum of quark orbitals may be obtained as a function of winding number (see the figures of orbitals given by Kahana and Ripka [55]). There is some problem in the desired winding number — baryon number identification [56]. Since the classical coupled field equations are solved numerically for the "hedgehog" ansatz, meson fields are in the "Skyrmion" configuration. In addition, filling the positive quark orbitals adds to this configuration an additional baryon number and induces a problem in matching the baryon number of the object as a whole. Still, such solitonic models and their 1+1-dimensional kink pendant [57] are of considerable heuristic value.

Now we turn our attention to the two-phase picture of the bag type. The rim of the hat (Fig. 2) represents the vacuum in the outer region containing chiral fields. The vacuum of the chirally invariant inner region ($\sigma = \underset{\sim}{\pi} = 0$) is represented by the top of the hat, the difference B in Fig. 2 representing the bag-volume energy. The Lagrangian which interpolates between the bagged QCD and the Skyrme picture is given by

$$\mathscr{L} = \left(\frac{i}{2}\bar{\psi}\gamma\overleftrightarrow{\partial}\psi - B\right)\theta_V + g\bar{\psi}U_5\psi\Delta_S + \mathscr{L}_{Sk}\theta_{\bar{V}},$$

$$U_5 = e^{i\vec{\tau}\cdot\hat{r}\theta(r)\gamma_5}. \tag{3.12}$$

Let us note that the general structure of Eq. (3.12) resembles that of hybrid chiral-bag models mentioned in Section 2.

In the next section we shall illustrate the importance of the topology contained in $\mathscr{L}_{Sk}$ in interpolating two phases. The non-triviality of the two-phase communication can be most transparently illustrated by a baryon-number calculation. We shall perform such a calculation in more detail and represent a one-dimensional toy model which is exactly solvable. We shall also give and discuss pertinent results for a three-dimensional model.

4. *Two-phase chiral bag models*

4.1. Chiral solitonic field in one dimension [58]

To introduce a 1+1-dimensional scenario which will mimic the two-phase, bag--Skyrme description (3.12), one has to confine a quark field to a finite (inner) segment of a line, whereas a solitonic field θ of a hedgehog type permeates the outer region (Fig. 3). At the surface of the bag the quark and solitonic fields are coupled in a U(1) × U(1) chirally symmetric way, so that a Lagrangian density is given by

$$\mathscr{L}(1+1) = (\tfrac{1}{2}\bar{q}\overleftrightarrow{\partial}q - B)\theta_V - \frac{g}{2}\bar{q}(\phi_1 + i\gamma_5\phi_2)q\Delta_S + \mathscr{L}_S(\phi_1, \phi_2)\theta_{\bar{V}}. \tag{4.1}$$

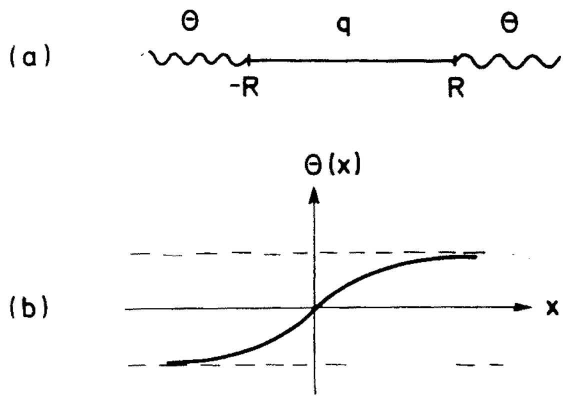

Fig. 3. A one-dimensional bag (a) with a modified sine-Gordon field (b) in the outher region

It is convenient to parametrise $\phi_1(x)$ and $\phi_2(x)$ by a chiral angle $\theta(x)$:

$$\phi_1 = \frac{1}{g} \cos \theta, \quad \phi_2 = - \frac{1}{g} \sin \theta, \tag{4.2a}$$

which enables us to introduce the analogs of the 3+1-dimensional fields:

$$v(x) = e^{-i\theta(x)} = \frac{1}{g} (\phi_1 + i\phi_2), \tag{4.2b}$$

$$v_5(x) = e^{-i\theta(x)\gamma_5} = \frac{1}{g} (\phi_1 + i\gamma_5\phi_2).$$

The solitonic background field may be represented by a modified sine-Gordon field [59]

$$\mathscr{L}_S = \tfrac{1}{2} (\partial_\mu\theta)(\partial^\mu\theta) - \kappa^2(1+\cos\theta) = -\tfrac{1}{2} (v^\dagger\partial_\mu v)(v^\dagger\partial^\mu v) - 2\kappa^2(v+v^\dagger)^2, \tag{4.3}$$

the modification being contained in the sign of the κ^2 term. The corresponding equation of motion

$$(\partial_t^2 - \partial_x^2)\theta - \kappa^2 \sin\theta = 0 \tag{4.4}$$

gives the configuration of a solitonic field (Fig. 3b) symmetric at the origin:

$$\theta(x) = \varepsilon(x) \left[-\pi + 4 \tan^{-1}(e^{\kappa\varepsilon(x)x}) \right],$$

$$\theta(-x) = -\theta(x), \tag{4.5}$$

which resembles a three-dimensional hedgehog solution. The existence of solitons in Eq. (4.3) is ensured by the non-trivial homotopy group $\Pi_1(S^1) = Z$ given by the mapping $v(x)$ from the conpactified line $R\cup\{\pm\infty\}$ onto S^1. The corresponding topological current and charge are, respectively,

$$j^\mu = \frac{i}{2\pi} \varepsilon^{\mu\nu} v^\dagger \partial_\nu v = \frac{1}{2\pi} \varepsilon^{\mu\nu} \partial_\nu \theta \tag{4.6}$$

and

$$N = \int_{-\infty}^{\infty} j^0(x)dx = \frac{1}{2\pi}[\theta(+\infty)-\theta(-\infty)] = 1. \tag{4.7a}$$

Inserting a bag in such a solitonic background field represents a defect in the configuration and causes a "fractional" topological charge

$$N_S = \int_{-\infty}^{-R} j^0(x)dx + \int_{R}^{\infty} j^0(x)dx. \tag{4.7b}$$

Thus, all ingredients seem to be present in order to simulate the physics of the two-phase bag-Skyrme picture. The $1+1$-dimensional counterpart of Eq. (3.12) is given by

$$\mathscr{L}(1+1) = \left(\frac{i}{2}\bar{q}\overleftrightarrow{\partial}q - B\right)\theta_V - \tfrac{1}{2}\bar{q}e^{-i\theta\gamma_5}q$$

$$+[\tfrac{1}{2}(\partial_\mu\theta)(\partial^\mu\theta)-\kappa^2(1+\cos\theta)]\theta_{\bar{V}}. \tag{4.8}$$

Still, the analogy is not complete. By Coleman's theorem [60], there is no spontaneous breaking of a continuous symmetry in $1+1$ dimensions and thus no Goldstone boson. On the other hand, the "$\cos\theta$" term of the outer phase in Eq. (4.8) breaks the continuous chiral transformation $e^{-i\beta\gamma_5}$ explicitly:

$$\phi_1 \to \phi_1\cos\beta-\phi_2\sin\beta \simeq \phi_1-\beta\phi_2,$$

$$\phi_2 \to \phi_1\sin\beta+\phi_2\cos\beta \simeq \phi_2+\beta\phi_1,$$

$$v = e^{-i\theta} \to v+\delta v = e^{-i(\theta+\delta\theta)},$$

where $\delta\theta = -\beta$.

4.2. Baryon anomaly in one dimension

Now we focus our attention on the chirally symmetric quark phase represented by a massless, flavour singlet Dirac field governed by the equations

$$i\partial\!\!\!/ q(x,t) = 0, \quad |x| < R,$$

$$[v_5(x)+i\hat{n}(x)\cdot\gamma]q(x,t) = 0, \quad |x| = R. \tag{4.9}$$

Here $\hat{n}(\pm R) = \varepsilon(\pm R)$ represents an outer normal, and the choice of γ matrices is the one of Jackiw and Rebbi [61]:

$$\gamma^0 = \sigma_1 = \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}, \quad \gamma^1 = i\sigma_3 = \begin{pmatrix}i & 0\\ 0 & -i\end{pmatrix}, \quad \gamma_5 = \gamma^0\gamma^1 = \sigma_2 = \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix}$$

This leads to

$$[i\sigma_1(\partial_t+\sigma_2\partial_x)-m]q(x,t) = 0, \quad |x| < R,$$

$$[v_5(\pm R)-\sigma_3\varepsilon(\pm R)]q(\pm R,t) = 0. \tag{4.10}$$

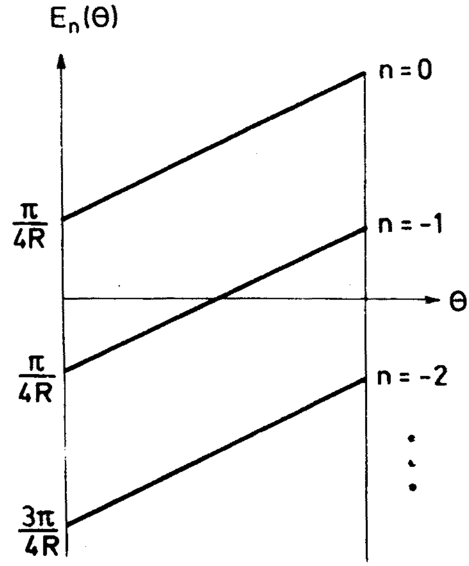

Fig. 4. The quark spectrum symmetric about zero energy for the values $\theta = 0$, $\pi/2$ and π

Imposing the solution of the form

$$q(x,t) = q(x)e^{-iEt} = N\begin{pmatrix} f(x) \\ ig(x) \end{pmatrix} e^{-iEt}, \tag{4.11}$$

one obtains the explicit expression

$$q_n(x) = \frac{1}{\sqrt{2R}}\, e^{i\gamma_5 \frac{\theta}{2}} \begin{pmatrix} \cos E_n(x-R) \\ -\sin E_n(x-R) \end{pmatrix}, \tag{4.12}$$

where $\theta \equiv \frac{1}{2}[\theta(+R)-\theta(-R)]$ amounts to a global chiral rotation and therefore to the parity mixing in Eq. (4.12).

The chiral boundary conditions determine the energy spectrum (Fig. 4)

$$E_n(\theta) = \left(2n+1+\frac{2\theta}{\pi}\right)\frac{\pi}{4R} \tag{4.13}$$

which has the following symmetry properties:

$$E_n(\theta+\pi) = E_{n+1}(\theta),$$
$$E_n(-\theta) = -E_{-n-1}(\theta),$$
$$E_n\left(\frac{\pi}{2}\right) = (n+1)\frac{\pi}{2R}, \tag{4.14}$$

and the zero-mode (self-conjugate) state $E_{-1}(\pi/2) = 0$. The quark system is symmetric about zero energy for the values $\theta = 0$, $\pi/2$ and π.

Unwinding the chiral angle from π to zero causes that the $n = -1$ valence state sinks

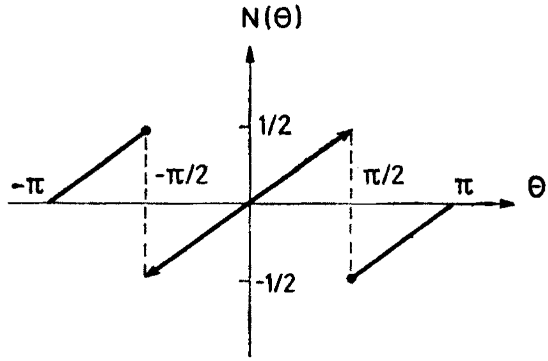

Fig. 5. The regularised baryon number of the vacuum

in the Dirac sea. This changes the baryon (fermion) number of the vacuum by an amount which can be easily calculated in the one-dimensional model at hand.

The baryon density is given by the formal expression in terms of field operators

$$\varrho = q^\dagger(x)q(x) = -\tfrac{1}{2}[q(x), q^\dagger(x)] + \tfrac{1}{2}\{q(x), q^\dagger(x)\}, \tag{4.15}$$

where the commutator part conventionally defines the spectral asymmetry function

$$N_{\text{non-reg}} = -\tfrac{1}{2}\int_{-R}^{R} dx \langle 0|\,[q(x), q^\dagger(x)]\,|0\rangle = -\tfrac{1}{2}\Big(\sum_{\text{particle}} + \sum_{\text{hole}}\Big). \tag{4.16}$$

This undetermined sum may be regularised by a symmetric suppression of the high particle and hole energies:

$$N = -\tfrac{1}{2}\lim_{s\to 0^+}\int_{-R}^{R} dx\langle 0|\,[q(x;s), q^\dagger(x;0)]\,|0\rangle$$
$$= -\tfrac{1}{2}\langle c_0^\dagger c_0\rangle - \tfrac{1}{2}\lim_{s\to 0^+}\Big[\sum_{E_n>0} e^{-sE_n} - \sum_{E_n<0} e^{sE_n}\Big], \tag{4.17}$$

where $q(x;s)$ is the Wick-rotated fermionic field operator. Evaluation of a simple geometrical series appearing in Eq. (4.17) gives the regularised baryon number of the vacuum:

$$N(\theta) = \begin{cases} \theta/\pi & 0 \leqslant \theta < \pi/2, \\ \dfrac{\theta}{\pi} - 1 & \dfrac{\pi}{2} \leqslant \theta \leqslant \pi, \end{cases} \tag{4.18}$$

displayed in Fig. 5.

The discontinuity in $N(\theta)$ results from the redefinition of the vacuum state at the points where the valence state sinks in the Dirac sea. Counting the baryon number of the valence quark

$$n_{\text{val}}(\theta) = \begin{cases} 0 & 0 \leqslant \theta < \pi/2 \\ 1 & \dfrac{\pi}{2} \leqslant \theta \leqslant \pi \end{cases} \tag{4.19}$$

gives the continuous function in the inside region

$$N_B(\theta) = n_{val}(\theta)+N(\theta) = \frac{\theta}{\pi}. \tag{4.20}$$

An explicit evaluation of the topological charge in the outer region as given by Eq. (4.7b) yields

$$N_S(\theta) = \frac{1}{2\pi}\int_{-\infty}^{R} \frac{d\theta}{dx}\,dx+\frac{1}{2\pi}\int_{R}^{\infty} \frac{d\theta}{dx}\,dx = 1-\frac{\theta}{\pi}. \tag{4.21}$$

The fraction of the topological charge carried by the chiral field is complementary to the baryon number sitting in the bag,

$$N_S(\theta)+N_B(\theta) = 1, \tag{4.22}$$

providing the consistency check on the description of baryons in the exposed two-phase model.

4.3. Results in three dimensions and some open questions

A three-dimensional chiral bag model based on Eq. (3.12) restores chiral invariance by a surface coupling with an external pion field. Analogously to the one-dimensional case, the baryon number leaks out into the pion cloud by an amount [62]

$$N_S(\theta) = 1-\frac{1}{\pi}[\theta(R)-\sin\theta(R)\cos\theta(R)]. \tag{4.23}$$

The value (4.23) is ascribed to the background pion field which should be distinguished [63] from the fluctuating pion field of the traditional low-energy physics.

A more difficult calculation for the inner region performed by Goldstone and Jaffe [64] yields

$$N_B(\theta) = \frac{1}{\pi}[\theta(R)-\sin\theta(R)\cos\theta(R)], \quad \theta\in\left[-\frac{\pi}{2},\frac{\pi}{2}\right], \tag{4.24}$$

$$N_B(\theta+\pi) = N_B(\theta),$$

and completes the proof that the baryon number in the 3+1-dimensional two-phase model equals 1. Note that in both one- and three-dimensional spaces there is a "magic angle" $\theta(R) = \pi/2$ at which one half of the baryon number is carried by the quark vacuum inside the bag, and the other half by the chiral solitonic field. The fact that these two add up to the winding number independently of the bag radius indicates the artificial nature of the bag radius in a two-phase model. The bag appears as the Cheshire Cat [65] from Lewis Carroll's "Alice's Adventures in Wonderland". Accordingly, one expects to witness the insensitivity [63] of low-energy quantities to the bag radius.

The result for the baryon number, Eqs. (4.23) and (4.24), being also confirmed by the authors of Refs. [66–69], provides an important consistency check on the two-phase model. Similar leakage from the bag can be expected for other low-energy observables, such as axial-vector current, charge and magnetisation [63, 68, 69]. In this way, the axial-current flow and the Casimir energy seem to be plagued with infinities (note the slight difference in conclusions between Refs. [66, 69] and Ref. [68]), and the last word has not been said yet.

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