Vol. B16 (1985) ACTA PHYSICA POLONICA No 6

THE PIONIC DECAYS OF THE $J^{PC} = 1^{++}$ MULTIPLET IN THE HARMONIC OSCILLATOR QUARK MODEL

BY M. AYUB

Department of Physics, Gomal University*

(*Received May 21, 1984; final version received November 20, 1984*)

The mesons $A_1(1270)$, $Q_1(1280)$, D(1285), E(1400) are tentatively classified according to the $J^{PC} = 1^{++}$ nonet of SU(3) and the quark antiquark contents of the two isoscalars are estimated by the use of the usual SU(3) symmetry-breaking mechanism. Then the harmonic oscillator quark model for mesons is used to calculate the widths of the decays $A_1(1270) \to \rho\pi$, $Q_1(1280) \to K^*\pi$ and $D(1285) \to \delta(980)\pi$. We get reasonable widths for these decays with the harmonic oscillator parameter $\alpha^2 \simeq 0.055$ GeV^2 and the quark-pion coupling constant $f_q^2 \simeq 0.371$. The main result is that a much smaller α^2-value (than the one used in the description of baryon decays) is needed to describe the mesonic processes. This conclusion is consistent with the model results obtained for the radiative widths of the "old" $L = 0$ mesons, the decay $\psi \to \gamma\pi^0$, the radiative widths of the low-lying positive parity $L = 1$ "old" mesons and the decays $A_2(1310) \to \rho\pi$, $A_2(1310) \to \eta\pi$; $\psi \to \rho\pi$, $D^* \to D\pi$; $A_3(1660) \to f\pi$ and $A_3(1660) \to \rho\pi$.

PACS numbers: 12.35.Ht, 13.25.+m, 14.80.Dq

1. Introduction

The quark model of elementary particles has proved very useful in classifying the internal quantum numbers of hadrons and also very useful in describing the hadron dynamics with simple additional dynamical assumptions [1]. Assuming an appropriate interaction between quark (antiquark) and field quantum, this model also gives quite reasonable decay widths of the hadrons [2]. In previous papers [3] we performed calculations based on the harmonic oscillator quark model for the radiative widths of the low-lying positive parity $L = 1$ "old"[1] mesons and the widths of the decays $A_2(1310) \to \rho\pi$, $A_2(1310) \to \eta\pi$; $\psi \to \rho\pi$, $D^* \to D\pi$; $A_3(1660) \to f\pi$ and $A_3(1660) \to \rho\pi$. Our results for all these decay widths are in reasonable agreement with the available experimental information for these processes. In all these calculations, we have two parameters, that is, the quark pion coupling constant (with the p- and n-type quarks) f_q where $f_q = \frac{3}{5} f_{\pi N}$ and the harmonic oscillator

* Address: Department of Physics, Gomal University, Dera Ismail Khan, Pakistan.

[1] Old mesons refer to those which can be interpreted as the bound states of the ordinary quarks (p, n, λ) and the antiquarks ($\bar{p}$, $\bar{n}$, $\bar{\lambda}$).

parameter α^2 (for the ordinary[1] quark antiquark system). As all the processes dealt with in the previous papers could be reasonably described with the values $\alpha^2 \simeq 0.055$ GeV^2 and $f_q^2 \simeq 0.371$, we continue to use these values in the present calculations. Using the well known SU(3) symmetry-breaking mechanism [4] the mass-mixing of the two isoscalars in the $J^{PC} = 1^{++}$ multiplet containing $A_1(1270)$, $Q_1(1280)$, D(1285), E(1400) is considered and the unitary spin state obtained for D(1285) is as follows:

$$D(1285) = 0.631(p\bar{p}+n\bar{n})-0.449\lambda\bar{\lambda}. \tag{1}$$

We take these states as members of $J^{PC} = 1^{++}$ multiplet in the $N = 2$, $L = 1$ energy band of the armonic oscillator potential and the total quark antiquark spin $S = 1$. $\delta(980)$ is assumed to belong to the 0^{++} multiplet in the $N = 2$, $L = 1$ band of the harmonic oscillator potential and the total quark antiquark spin $S = 1$.

2. *The model*

The quark antiquark system (meson) is assumed to be described by the Hamiltonian

$$H = \frac{1}{2M}(p_1^2+p_2^2)+\frac{\kappa}{2}(r_1-r_2)^2 \tag{2}$$

in obvious notation [5]. The Hamiltonian can be reduced to one which describes the internal motion of the quark antiquark system which in turn can be solved easily [6]. The physical spin of the meson is obtained by coupling the spin state with the orbital angular momentum of the spatial wave functions. Total wave functions for the mesons are obtained by combining the spin, unitary spin and the spatial wave functions.

The mechanism for a pion emission is taken as the de-excitation of a single quark (antiquark). The interaction which we use is the one which is the non-relativistic limit of the pion–nucleon interaction given as

$$\mathscr{H} = \frac{f_q}{m_\pi}(\boldsymbol{\sigma}^q\cdot\nabla)(\boldsymbol{\tau}^q\cdot\boldsymbol{\phi})-\frac{f_q}{m_\pi}(\boldsymbol{\sigma}^{\bar{q}}\cdot\nabla)(\boldsymbol{\tau}^{\bar{q}}\cdot\boldsymbol{\phi}) \tag{3}$$

in obvious notation [2]. The superscript q ($\bar{q}$) stands for quark (antiquark). Calculations become considerably easier if we take the pion emission along the z-axis.

The decay width of a particle with spin J is given as

$$\Gamma = \frac{1}{(2\pi)^2}\frac{1}{(2J+1)}\frac{E_1E_2}{E_c}k\int\sum_{m'}|A_{m'}d^J_{m'm}(\theta)|^2 d\Omega^\pi(\theta,\phi) \tag{4}$$

where k is the c.m. momentum of the pion and $d\Omega^\pi(\theta,\phi)$ is the solid angle for pion. $|A_{m'}\, d^J_{m'm}(\theta)|^2$ is the intensity of emission (of pion) at some angle θ to the z-axis. $A_{m'}$ denotes the amplitude where m' refers to the decaying particle. In formula (4) E_c is the energy of the decaying particle while E_1 and E_2 are the c.m. energy of the outgoing particles. The matrix elements which contribute are: $A_{\pm1} = \langle \rho,\ J_z = \pm1|\mathscr{H}|\,A_1,\ J_z = \pm1\rangle$, $A_{\pm1} = \langle K^*, J_z = \pm1|\mathscr{H}|Q_1,\ J_z = \pm1\rangle$, $A_0 = \langle \delta, J = 0|\mathscr{H}|D^0,\ J_z = 0\rangle$. With the values of the parameters mentioned in the introduction, we get $\Gamma(A_1 \to \rho\pi) \simeq 60$ MeV, $\Gamma(Q_1 \to K^*\pi)$

$\simeq$ 6 MeV and $\Gamma(D \to \delta\pi) \simeq 18$ MeV. These values are to be compared with the experimental values [7] $\Gamma(Q_1 \to K^*\pi) = 8$–23 MeV and $\Gamma(D \to \delta\pi) = 6$—13 MeV. For the decay width of the process $A_1 \to \rho\pi$ there is no experimental measurement up to date. The calculated decay widths of the decays $Q_1 \to K^*\pi$ and $D \to \delta\pi$ are as good as the harmonic oscillator quark model results for most of the decays of other mesons [3], which vary by a factor of two to three with the experimental values.

3. *Conclusion*

From the reasonable agreement of the harmonic oscillator quark model results with the experimental measurements for the radiative and pionic widths of a large number of mesons [3, 8] including the excited states with the same α^2-value ($\alpha^2 \simeq 0.055$ GeV^2), we infer that the harmonic oscillator parameter for the ordinary quark antiquark system α^2 has got a smaller value of about 0.055 GeV^2 as compared to the one ($\alpha^2 \simeq 0.1$ GeV^2) used in fitting the pionic and radiative widths [9] of baryons. The present calculations also favour the identification of the mesons $A_1(1270)$, $Q_1(1280)$, D(1285), E(1400) with the $J^{PC} = 1^{++}$ nonet of SU(3) (with the usual SU(3) symmetry-breaking mechanism) in the $N = 2$, $L = 1$ energy band of the harmonic oscillator potential with the total quark antiquark spin $S = 1$ and that of the $\delta(980)$ with the $J^{PC} = 0^{++}$ multiplet in the $N = 2$, $L = 1$ energy band of the harmonic oscillator potential with the total quark antiquark spin $S = 1$. That a smaller α^2-value is needed to describe the mesonic processes is also supported by the observations of Ono [10]. It is clear that one cannot attach much significance to parameters like the quark masses and charges (involved in the radiative widths of mesons), the harmonic oscillator constant for quark-quark or quark-antiquark system or any parameter related to quarks but the important fact which comes out of this model calculations is that a large number of radiative and strong (especially pionic) decays of mesons can be described by the same values of the parameters. Calculations for the partial decay widths of other orbitally or radially excited mesons (for which there is recent experimental information) are in progress and the results will be reported.

REFERENCES

[1] For a review see: (a) A. J. G. Hey, *Quarks and Symmetries*, Physics Department, Southampton University, 1975; (b) A. J. G. Hey, D. Morgon, *Meson Spectroscopy*, RL — 77/060/A (April, 1977).
[2] R. Van Royen, V. F. Weisskopf, *Nuovo Cimento* **40**, 617 (1967).
[3] (a) M. Ayub, M. Yar Khan, Saadullah Jan, *Islamabad J. Sci.* 7, 78 (1980); (b) M. Ayub, *The Gomal University Journal of Research* **2**, 101 (1982).
[4] W. M. Gibson, B. R. Pollard, *Symmetry Principles in Elementary Particle Physics*, Cambridge University Press, London 1976, p. 292.
[5] M. Hirano et al., *Prog. Theor. Phys.* **45**, 545 (1970).
[6] de ter Haar (ed.), *Problems in Quantum Mechanics*, Pion Limited, London 1975, p. 234.
[7] *Particle Properties Data Booklet*, April 1982.
[8] M. Ayub, Ph. D. Thesis, University of Edinburgh, December 1978.
[9] D. Faiman, A. W. Hendry, *Phys. Rev.* **173**, 1720 (1968).
[10] S. Ono, *Nucl. Phys.* **B107**, 522 (1976).