STATIC AND DYNAMICAL PROPERTIES OF LIGHT HADRONS IN QCD*

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The review of QCD determination of static and dynamical properties of hadrons is given. Hadron masses, their transition constants into quark currents, meson formfactors at intermediate momentum transfers, mesonic partial widths and structure functions at small x are considered. A special attention is paid to calculation of static parameters of hadrons in external fields (nucleon and hyperon magnetic moments, interaction constants with axial currents).

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1. *Introduction*

In the last few years we have a considerable progress in QCD in finding various static and dynamical properties of hadrons using the nonperturbative method based on the assumption of existence of nontrivial (nonperturbative) vacuum expectation values of quark and gluon fields in QCD. A great number of results has been obtained this way: masses of almost all the lower hadronic states with spin $s \leqslant 2$ were determined, a number of widths and formfactors at small and intermediate momentum transfers were found, static characteristics of hadrons in external constant fields were calculated etc.

This approach possesses great predictive power since it proceeds from the basic principles of QCD, does not resort to any model considerations and exploits a very small number of phenomenological constants for obtaining the results. In my lectures I will try to bring the light to the basic ideas of this method, to the main results concerning the properties of light (consisting of u,d,s-quarks) hadrons, obtained with this method, and possible outlooks for its development. I will dwell also on comparing these results with those following from some models which enable one to make certain conclusions on advantages and disadvantages of these models.

First, some general remarks. QCD is the strong interaction theory: even at the highest accessible in the foreseen future momentum transfers the coupling constant $\alpha_s = g^2/4\pi$

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is not very small, $\alpha_s \geqslant 0.1$. At not very large momentum transfers, besides perturbative effects, there appear nonperturbative ones which exceed them. Calculation of high orders of perturbative theory is unreal and because of divergence of the series it may be even unreasonable. Exact taking into account of nonperturbative effects is even more unreal. What can be expected is the development of some approximate methods. That is why neither at the present time nor in the future one cannot expect a high accuracy of QCD predictions, such as, say, in QED. As a rule, (with rare though very important exceptions) the accuracy of such predictions is at one or few tens of per cent. But in my opinion, this is not the reason to be grieved. As not only that experiment is brilliant which has high accuracy but also that one in which a new result is obtained with a sufficient confidence, so the reliability of theoretical results in QCD controllable inside the theory itself is important first of all. And this is an essential difference between the QCD and the model approaches where usually one has to leave the framework of a model to control reliability of its results.

2. *Light hadron masses*

The method in view which is often called the sum rule QCD method, has been originally suggested by Shifman, Vainshtein and Zakharov [1] and has been applied by them for determining masses and leptonic widths of light mesons (ρ, π, A_1, K*) and for studying some parameters of charmonium.

The method is based on the following considerations:

(*i*) in the virtuality region of order $Q^2 \sim 1\ \mathrm{GeV}^2$ the strong interaction constant α_s is already rather small, $\alpha_s \sim 0.3$–0.4, so that perturbative terms are small, $\alpha_s/\pi \sim 0.1$ and the leading logarithmic corrections $\sim [\alpha_s(Q^2) \ln Q^2/\Lambda^2]^n$ can be easily taken into account,

(*ii*) the nonperturbative effects which reduce to appearance of vacuum condensates play a fundamental role. Of the vacuum condensates the most essential are the quark condensate density $\langle 0|\bar{q}q|0\rangle$, $q = u, d, s$, and the gluon condensate density $\langle 0|G^n_{\mu\nu}\, G^n_{\mu\nu}|0\rangle$. Vacuum condensates in this approach are considered as phenomenological parameters determined either from experiment or from self-consistency of the sum rules obtained.

It is convenient to present characteristic features of the method on an example of proton mass calculation [2, 3] to which I now turn. Consider the polarization operator

$$\Pi(p) = i \int d^4x e^{ipx} \langle 0|T\{\eta(x), \bar{\eta}(0)\}|0\rangle, \tag{1}$$

where $\eta(x)$ is the quark current with proton quantum numbers and p^2 is chosen to be space-like: $p^2 < 0$, $|p^2| \sim 1\ \mathrm{GeV}^2$. The current η is the colourless product of three quark fields $\eta = \varepsilon^{abc} q^a q^b q^c$, $q = u, d$, the form of the current will be specialized below. The general tensor structure of $\Pi(p)$ is

$$\Pi(p) = \hat{p} f_1(p^2) + f_2(p^2). \tag{2}$$

For each of the functions $f_i(p^2)$, $i = 1, 2$ the following operator expansion can be written:

$$f_i(p^2) = \sum_n C^{(i)}_n(p^2)\, \langle 0|O^{(i)}_n|0\rangle, \tag{3}$$

where $\langle 0|O_n^{(i)}|0\rangle$ are vacuum expectation values (v.e.v.) of different operators (vacuum condensates), $C_n^{(i)}(p^2)$ are functions calculated in QCD.

As is known, the current u- and d-quark masses entering the lagrangian of QCD are very small — of order of several MeV, so they can be neglected with a very good accuracy, i.e. u- and d-quarks can be taken massless. Then accounting for only u- and d-quarks, the QCD lagrangian is chirally-invariant and if chiral invariance would not be spontaneously broken, the function $f_2(p^2)$ would be identically zero. In reality, however, the chiral invariance is spontaneously broken in QCD. The first evidence of this is, of course, the existence of baryon masses. Another signal of the chiral invariance spontaneous breaking in QCD is the fact that the chirality violating density of quark condensate $\langle 0|\bar{q}q|0\rangle$ is non-zero. As is well known, in the low-energy limit

$$\langle 0|\bar{q}q|0\rangle = -\tfrac{1}{2}\,\frac{f_\pi^2 m_\pi^2}{m_u+m_d} = -(240\ \mathrm{MeV})^3, \tag{4}$$

where m_π is the pion mass, $f_\pi = 133$ MeV is the $\pi \to \mu\nu$ decay constant. The question arises, how these two phenomena are connected and, particularly, if the proton mass can be expressed through $\langle 0|\bar{q}q|0\rangle$. This question can be answered affirmatively and a bit later I shall demonstrate the corresponding formula. Since $\langle 0|\bar{q}q|0\rangle$ is the lowest in dimension chirality violating operator, the operator expansion for $f_2(p^2)$ starts from the term proportional to $\langle 0|\bar{q}q|0\rangle$.

To demonstrate how the operator expansion (3) is built I present several of its first terms, classifying the terms according to the operator dimension. The zero term of the operator expansion with $d = 0$ corresponding to the unit operator is described by the graph of Fig. 1 and has the form

$$C_0 \hat{p} p^4 \ln(\Lambda_u^2/(-p^2)) + \text{polynomial}, \tag{5}$$

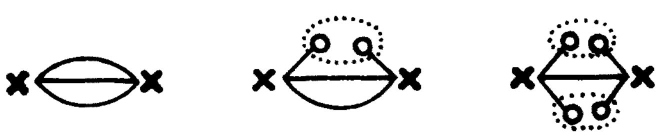


Fig.1 Fig.2 Fig.3

Fig. 1. The simplest quark loop contributing to polarization operator $\Pi(p)$: solid lines — quark propagators

Fig. 2. The graph corresponding to v.e.v. $\langle 0|\bar{q}q|0\rangle$ contribution into polarization operator: circles surrounded by dots mean v.e.v.

Fig. 3. The graph corresponding to v.e.v. $\langle 0|\bar{q}q|0\rangle^2$ contribution to the polarization operator

where C_0 is a constant, Λ_u is the ultraviolet cut-off. This term preserves chiral invariance and contributes to the function $f_1(p^2)$. The chirality violating operator $\bar{q}q$ with $d = 3$ is the next-in-dimension. The diagram of Fig. 2 corresponding to operator $\bar{q}q$ gives to $f_2(p^2)$ a contribution which is equal to

$$C_1 p^2 \ln\frac{\Lambda_u^2}{-p^2}\langle 0|\bar{q}q|0\rangle + \text{polynomial}. \tag{6}$$

The most essential correction in the operator expansion for $f_2(p^2)$ arises from the four--quark operators $\bar{q}\Gamma^{(k)}q \cdot \bar{q}\Gamma^{(k)}q$ with $d = 6$ whose contribution is described by the graph of Fig. 3 and has the form

$$\sum_k C_2^{(k)}\langle 0|\bar{q}\Gamma^{(k)}q \cdot \bar{q}\Gamma^{(k)}q|0\rangle\,(\hat{p}/p^2). \tag{7}$$

The contribution (7) is very important numerically since unlike the graph of Fig. 1 containing two-loop integration which is suppressed by the factor $(2\pi)^{-4}$, the diagram of Fig. 3 has no such integration, so that $C_2/C_0 \sim (2\pi)^4$.

When calculating v.e.v. (7) there is often used the factorization hypothesis according to which in expansion of the four-quark operator product over intermediate states the main contribution is given by the vacuum state. The arguments based on the $1/N_c$ expansion are in favour of this hypothesis (N_c is the colour number, in QCD $N_c = 3$). As can be shown for any colourless operators O_1 and O_2 at large N_c

$$\langle 0|O_1O_2|0\rangle = \langle 0|O_1|0\rangle\,\langle 0|O_2|0\rangle\left(1+O\left(\frac{1}{N_c}\right)\right), \tag{8}$$

i.e. in the limit $N_c \to \infty$ factorization becomes exact. The same considerations based on the $1/N_c$ expansion indicate suppression of v.e.v. of nonfactorized operators comparing with those of factorized operators of the same dimension

$$\frac{\text{v.e.v. of nonfactor operators}}{\text{v.e.v. of factor operators}} \sim O\left(\frac{1}{N_c}\right). \tag{9}$$

Relation (9) will be needed in what follows when considering magnetic moments. By virtue of factorization and taking into account the relation

$$\langle 0|q_\alpha^a(0)\bar{q}_\beta^b(0)\,|0\rangle = -\tfrac{1}{12}\,\delta^{ab}\delta_{\alpha\beta}\langle 0|\bar{q}q|0\rangle \tag{10}$$

(a, $b = 1, 2, 3$ are colour, α, β are Lorentz indices) all four-quark v.e.v. reduce to the quark condensate square $\langle 0|\bar{q}q|0\rangle^2$.

The above three terms are basic in the operator expansion $\Pi(p)$. To improve and control the accuracy in the mass calculation other v.e.v. will be also taken into account: gluonic condensate $\langle 0|(\alpha_s/\pi)G_{\mu\nu}^nG_{\mu\nu}^n|0\rangle$, v.e.v. $\langle 0|\bar{q}\sigma_{\mu\nu}(\lambda^n/2)G_{\mu\nu}^nq|0\rangle$ and (assuming factorization) higher dimension v.e.v.'s $\langle 0|\bar{q}q|0\rangle\,\langle O|\bar{q}\sigma_{\mu\nu}(\lambda^n/2)G_{\mu\nu}^nq|0\rangle$, $\alpha_s\langle 0|\bar{q}q|0\rangle^3$, $\langle 0|\bar{q}q|0\rangle$ $\langle 0|(\alpha_s/\pi)G_{\mu\nu}^nG_{\mu\nu}^n|0\rangle$. The gluonic condensate gives a contribution into the chirality preserving structure $f_1(p^2)$. Though its dimension $d = 4$ is smaller than the dimension $d = 6$ of the four-quark operator (7) its role in the sum rules for baryons is inessential since its contribution is determined by the two-loop diagram and is numerically suppressed.

The polarization operator $\Pi(p)$ in QCD is calculated in this way, i.e. the left-hand side of the desired sum rules is found. On the other hand, functions $f_i(s)$, $s = -p^2$ may be expressed via the characteristics of physical states using the dispersion relations

$$f_i(s) = \frac{1}{\pi}\int_0^\infty \frac{\operatorname{Im} f_i(p^2)}{p^2+s}\,dp^2 + \text{polynomial}. \tag{11}$$

It is useless to directly equate (3) and (11) because left-hand side and right-hand side of the resultant equality contain unknown polynomials. In order for this equality to acquire a meaning one has to apply to both its sides the Borel (Laplace) transformation [1] defined as

$$\mathcal{B}_{M^2} f(s) = \lim_{\substack{n\to\infty, s\to\infty \\ s/n = M^2 = \text{const}}} \frac{s^{n+1}}{n!}\left(-\frac{d}{ds}\right)^n f(s)$$

$$= \frac{1}{\pi}\int_0^\infty \exp\left(-\frac{p^2}{M^2}\right) \operatorname{Im} f(p^2) dp^2, \tag{12}$$

if $f(s)$ is given by dispersion relation (11). Notice that

$$\mathcal{B}_{M^2} \frac{1}{s^n} = \frac{1}{(M^2)^{n-1}(n-1)!}. \tag{13}$$

The Borel transformation permits to attain three goals at once:

(1) to nullify subtraction terms;

(2) to suppress the contribution of the highest excited states compared to the desired lowest state (proton);

(3) to suppress the contributions of high order terms in the operator expansion (owing to factor $1/(n-1)!$ in (13)).

The lowest state (proton) contribution to the imaginary part of $\Pi(p)$ has the form

$$\operatorname{Im} \Pi(p)_p = \pi\langle 0|\eta|\mathrm{p}\rangle\langle \mathrm{p}|\bar{\eta}|0\rangle\delta(p^2-m^2) = \pi\lambda_\mathrm{N}^2(\hat{p}+m)\delta(p^2-m^2), \tag{14}$$

where

$$\langle 0|\eta|\mathrm{p}\rangle = \lambda_\mathrm{N} v(p), \tag{15}$$

λ_N is a constant and v is the proton spinor. It is clear from (14) that the proton contribution will dominate in some region of the Borel parameter M^2 only in the case when both QCD calculated functions f_1 and f_2 are of the same order, and the spontaneous violation of chiral invariance characterized by the value of quark condensate has to explain the numerical value of the proton mass.

To improve and control the accuracy in the dispersion representation (14) one should also take into account the highest states contribution. It is usually done by replacing $\operatorname{Im} f(p^2)$ by contributions of the simplest quark loops (Figs. 1, 2) starting from some "continuum threshold" W.

It should be emphasized that in the sum rule method the presence of structure in hadronic spectra in the small mass region, i.e. the appearance of resonances separated by a dip from the region of smooth continuum which corresponds to parton model, is not introduced into the theory from outside but follows from existence of power corrections.

A few words on the choice of the quark current $\eta(x)$. In case of baryons (unlike mesons) even if we restrict ourselves by currents without derivatives, there exist, as a rule, several currents with quantum numbers of a given baryon. The choice between them should be done from physical reasons in order to provide: (1) renormcovariance, (2) existence of nonrelativistic limit, (3) the above formulated requirement (for proton) for the functions f_1 and f_2 to be of the same order, (4) covergence of the operator expansion series within accounted terms. In case of proton all these requirements are satisfied by the current [2, 4]

$$\eta = u^a C\gamma_\mu u^b \gamma_\mu \gamma_5 d^c \cdot \varepsilon^{abc}. \tag{16}$$

I will present now the explicit form of the sum rules for calculation of the proton mass [2, 5]

$$M^6 E_2\left(\frac{W^2}{M^2}\right) L^{-4/9} + \tfrac{4}{3}a^2 L^{4/9} + \tfrac{1}{4} bM^2 E_0\left(\frac{W^2}{M^2}\right) - \tfrac{1}{3}a^2\frac{m_0^2}{M^2} = \tilde{\lambda}_N^2 \exp\left(-\frac{m^2}{M^2}\right) \tag{17}$$

$$2aM^4 E_1\left(\frac{W^2}{M^2}\right) + \tfrac{272}{81}\frac{\alpha_s}{\pi}\frac{a^3}{M} - \tfrac{1}{12}ab = m\tilde{\lambda}_N^2 \exp(-m^2/M^2). \tag{18}$$

Here

$$a^2 = -(2\pi)\langle 0|\bar{q}q|0\rangle = 0.55\ \mathrm{GeV}^3, \tag{19}$$

$$b = (2\pi)^2\left\langle 0\left|\frac{\alpha_s}{\pi} G^n_{\mu\nu}G^n_{\mu\nu}\right|0\right\rangle \approx 0.5\ \mathrm{GeV}^4, \tag{20}$$

$$-g\left\langle 0\left|\bar{q}\sigma_{\alpha\beta}\frac{\lambda^n}{2}G^n_{\alpha\beta}q\right|0\right\rangle \equiv m_0^2\langle 0|\bar{q}q|0\rangle, \tag{21}$$

$$m_0^2 \approx 0.8\ \mathrm{GeV}^2.$$

The factors

$$E_0(x) = 1-e^{-x}, \quad E_1(x) = 1-e^{-x}(1+x), \quad E_2(x) = 1-e^{-x}\left(1+x+\frac{x^2}{2}\right)$$

take into account (transferred into the left-hand part) continuum contribution,

$$\tilde{\lambda}_N^2 = 32\pi^4\lambda_N^2, \tag{22}$$

the factors $L = \ln(M/\Lambda)/\ln(\mu/\Lambda)$ take into account the anomalous dimensions of the operators (Λ is the QCD parameter, μ is the normalization point, numerical values hereafter correspond to $\mu = 0.5$ GeV).

The proton mass m and the proton transition constant into the quark current λ_N (W is also a variable parameter) may be found from the sum rules by the best fit. Such a fit should be made within a restricted interval of M^2 where, on one hand, the continuum contribution is rather small (say, less than 50%) which restricts M^2 from above, and, on the other hand, highest power corrections are small (say, < 10%) which restricts M^2

from below. Outside of this interval the accuracy of the theory is noncontrollable, for instance, at very large M^2 the lowest state (proton) contribution is hidden in the background and the result for m and λ_N depends cardinally on continuum model. I wish to emphasize that without estimating the contribution of highest states and highest power corrections the results obtained by the QCD sum rule method cannot be considered as reliable. The best fit in the permissible interval $0.7 < M^2 < 1.2\ \mathrm{GeV}^2$ at the chosen values of v.e.v.'s (for discussion of their numerical values see below) and $W = 1.5$ GeV gives

$$m = 1.0 \pm 0.1\ \mathrm{GeV}. \tag{23}$$

Note that the value of m can be determined dividing (18) by (17). In doing so, a simple approximated formula arises for m

$$m \approx [-2(2\pi)^2 \langle 0|\bar{q}q|0\rangle]^{1/3} \tag{24}$$

which reflects the fact that appearance of proton mass is connected with spontaneous violation of chiral invariance — i.e., with the presence of quark condensate.

The accuracy of sum rules (17), (18) can also be checked by another method — giving experimental value of m and plotting the graphs of M^2 dependence of $\tilde{\lambda}_N^2$ from (17), (18). As is seen from Fig. 4, the difference between $\tilde{\lambda}_N^2$ determined from (17) (solid curve) and (18)

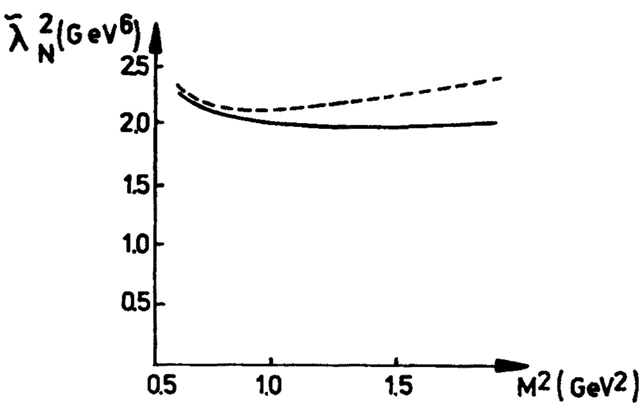


Fig. 4

(dashed curve) and deviation of $\tilde{\lambda}_N^2$ from a constant within the interval $0.7 < M^2 < 1.2\ \mathrm{GeV}^2$ does not exceed 10%. Such an accuracy is natural because highest power corrections are in this case of the same order. As a result of such procedure we find

$$\tilde{\lambda}_N^2 = 2.1 \pm 0.2\ \mathrm{GeV}^6. \tag{25}$$

This value will be used in the following, particularly, in calculating nucleon magnetic moments.

Masses and leptonic widths of meson decays and masses of other baryons are calculated analogously. Without dwelling on details I give the results of the calculations (Tables I and II).

Table I does not contain predictions for η, η′ and for scalar mesons: QCD sum rules do not work in the case of scalar and pseudoscalar channels as well as in the case of longitudinal axial channel with isospin zero. In particular, the sum rule approach cannot explain the 100% violation of the Okubo-Zweig-Iizuki rule in the pseudoscalar (or in the longitudinal axial) channel with $T = 0$, i.e. the fact that η-meson is an octet component and η′ is a unitary singlet. A possible (though not proved) explanation of this fact is in an important effect of direct instantons [9, 10]. When calculating meson and baryon masses with no strange quarks, only the above mentioned v.e.v.'s (19)–(21) enter. In considering hyperons and mesons with strange quarks there appear two new parameters — the strange quark

TABLE I

Meson (Ref.)	J^{PC}	Mass (GeV)		Leptonic decay constant	
		theor.	exp.	theor.	exp.
ρ [1]	1−−	0.78±0.04	0.770	$g_\varrho^2/4\pi = 2.4\pm0.2$	2.54±0.23
ω [1]	1−−	0.78±0.04	0.783	22±2	18.4±1.8
φ [1]	1−−	1.07±0.05	1.02	14	11.7±0.9
K* [1]	1−−	0.93±0.05	0.892	1.4	—
π [1]	0−+	—	0.140	$f_\pi = 125$ MeV	133 MeV
A_1 [6]	1++	1.25±0.15*	1.27	$g^2/4\pi \simeq 6$	≈ 6
D [6]	1++	1.25±0.15*	1.28	$g^2/4\pi \approx 6$	
f [7]	2++	1.25±0.05	1.27	$g_f = 0.040$	—
				$\Gamma_{f\pi\pi} = 200\pm30$ MeV	180 MeV
A_2 [7]	2++	1.25±0.05	1.32	—	—
A_3 [7]	2−+	1.63±0.1	1.68	—	—

* Corrected numbers (B.I.)

TABLE II

Baryons

Baryon	J^P	T	Calculated value (GeV)	Theor. [2, 5]	Exp.
N	1/2+	1/2	m_N	1.0 ±0.1	0.94
Δ	3/2+	3/2	m_Δ	1.37±0.15	1.23
N*	3/2−	1/2	m_{N^*}	1.75±0.25	1.52
Λ	1/2+	0	$m_\Lambda - m_N$	0.19	0.175
Σ	1/2+	1	$m_\Sigma - m_N$	0.23	0.25
Ξ	1/2+	1/2	$m_\Xi - m_N$	0.40	0.38
Σ*	3/2+	1	$m_{\Sigma^*} - m_\Delta$	0.14	0.15
Λ**	3/2−	0	$m_{\Lambda^{**}} - m_{N^*}$	0.14	0.17
Σ**	3/2−	1	$m_{\Sigma^{**}} - m_{N^*}$	0.18	0.15
Ξ**	3/2−	1/2	$m_{\Xi^{**}} - m_{N^*}$	0.30	0.30

mass m_s and the parameter

$$f = \frac{\langle 0|\bar{s}s|0\rangle}{\langle 0|\bar{u}u|0\rangle} - 1 \tag{26}$$

which characterizes the difference between quark condensates of strange and usual quarks. In Ref. [8] the values of m_s and f were determined from the best fit conditions of the sum rules for all the given in the Table mass differences in the linear approximation in m_s, f and $m_Y - m_N$, $m_{Y^*} - m_{N^*}$, $m_{\Sigma^*} - m_\Delta$ and it was obtained

$$m_s = 0.105 \pm 0.030 \text{ GeV}, \quad f = -0.11 \pm 0.05 \tag{27}$$

(the mass differences given in Table II correspond to these values of m_s and f). In fact, the linear approximation in the hyperon and nucleon mass differences e.g., in kinematic factors $\exp[-(m_\Xi^2 - m_N^2)/M^2]$ gives poor accuracy and to find m_s and f with a higher accuracy one should substitute into sum rules the experimental values of hyperon masses and make the best fit not employing the expansion in the mass differences $m_Y - m$. This gives

$$m_s = 150 \pm 30 \text{ MeV}, \quad f = -0.2^{+0.1}_{-0.05}. \tag{28}$$

A few words on the quantity λ_N which determines the transition amplitude into quark current η. As was seen from (25) and from Fig. 4 λ_N can be found from sum rules with a good accuracy. Exact knowledge of the constant λ_N and of other constants of such type is very important since they: (1) are used in calculating baryon magnetic moments (see below); (2) determine the proton life time in the grand unification SU(5) theory; (3) determine the constant in the asymptotics of the neutron electromagnetic formfactor. The latter point needs a more detailed explanation. The neutron electromagnetic formfactor in the asymptotics at $Q^2 \to \infty$ has the form [11, 12]

$$F_{1n}(Q^2)_{\text{asymp}} = [4\pi\alpha_s(Q^2)]^2 \left[\frac{\alpha_s(Q)}{\alpha_s(\mu^2)}\right]^{4/27} \tfrac{100}{3} f_0^2 \frac{1}{Q^4} \tag{29}$$

where f_0 is determined by proportional to p_μ part of the matrix element

$$\varepsilon^{abc}\langle 0|\,(u^a C\gamma_\mu u^b) d^c_\cdot|p\rangle = (p_\mu f_0 + \gamma_\mu \beta) v(p). \tag{30}$$

The matrix element (30) was determined from the sum rules [5] and it was found that $f_0 = 0.8 \cdot 10^{-2}$ GeV^2. Substituting it into (29) at the largest experimentally accessible $Q^2 = 25$ GeV^2 gives

$$F_{1n}(Q^2)_{Q^2=25\text{GeV}^2} = \frac{1.2 \cdot 10^{-2}}{Q^4} \tag{31}$$

i.e. the value by two orders of magnitude smaller than the experimental one,

$$F_{1n}^{\exp}(Q^2) \approx -1/Q^4, \tag{32}$$

(and of the opposite sign). Hence it follows that asymptotic formulae for the nucleon electromagnetic formfactor have nothing to do with what is being observed (or will be observed) at accessible Q^2. The analogous conclusion has been obtained by Isgur and Llewellyn-Smith [13] on the basis of the nucleon quark wave function model. Constants λ_N can also be used for checking and correcting the bag model. Since in the bag model $\lambda_N^2 \sim |\psi(0)|^2 \sim R^{-6}$ where R is the bag radius, the knowledge of λ_N enables one to find R with a high accuracy [14].

3. *Formfactors and hadronic widths*

The sum rule method was generalized to consideration of hadronic formfactors and widths [15, 16]. The initial point of such generalization is the studying of the vertex function

$$\Gamma_{AB}(p, p'; q) = -\int d^4x d^4y \exp[i(p'x - qy)]$$
$$\times \langle 0|T\{j_B^+(x), j(y), j_A(0)\}|0\rangle, \quad q = p' - p, \tag{33}$$

where j_A, j_B are quark currents with quantum numbers of hadrons A and B and $j = j^{el}$ if we are interested in the electromagnetic formfactor $\Gamma_{A\to B}(q^2)$. $\Gamma_{AB}(p, p'; q)$ (33) is studied in the kinematic region $p^2 < 0, p'^2 < 0$, $q^2 = -Q^2 < 0$ at $|p^2| \sim p'^2 \sim 1$ GeV^2, $Q^2 \gtrsim 1$ GeV^2. After this, on one hand, Γ_{AB} is calculated in QCD using the operator expansion and on the other hand, is represented via physical state contributions with the help of the double dispersion relation in p^2 and p'^2. In QCD calculations two condensates $\langle 0|G^n_{\mu\nu}G^n_{\mu\nu}|0\rangle$ and $\alpha_s\langle 0|\bar{q}q|0\rangle^2$ are taken into account. The effect of power corrections and of excited state transition increases with Q^2 rising. The latter is evident since at large Q^2 the main role belongs to transitions with production of a great number of particles. This circumstance restricts from above the Q^2 region where the approach in view is applicable. At small Q^2 this approach is also inapplicable because at $q^2 \to 0$ in t-channel large distances are impor-

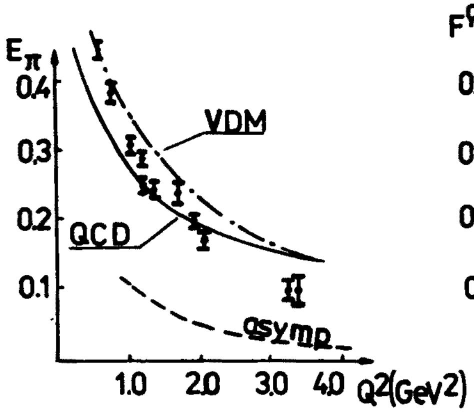


Fig.5

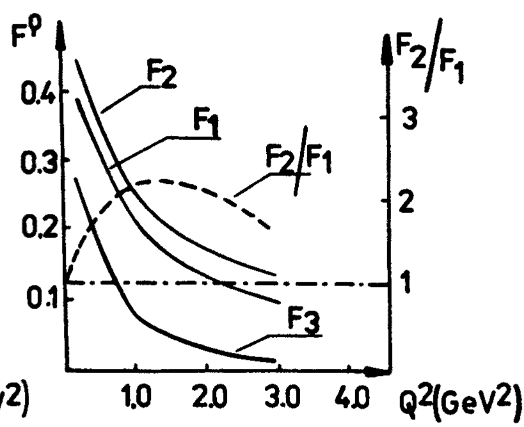


Fig.6

tant which is signaled by appearance of nonphysical singularities $\sim 1/Q^2$, ln Q^2 etc. As a result, the whole consideration appears to be valid in the region of intermediate Q^2, $0.5 \lesssim Q^2 \lesssim 3\ \text{GeV}^2$. Several hadronic formfactors were calculated on this way and I give here the results. Fig. 5 represents the QCD found values of the pion formfactor [15] (solid curve) in comparison with the experimental data. Fig. 5 shows also the vector dominance model predictions (dot-dash curve) and the asymptotic QCD formula [11, 17–20]

$$F_\pi(Q^2)_{\text{asymp}} = \frac{8\pi\alpha_s(Q^2)}{Q^2} f_\pi^2, \quad f_\pi = 133\ \text{MeV}. \tag{34}$$

The QCD calculated curve agrees with experiment in the region where QCD formulae are valid. The VDM curve differs a little from QCD and also sufficiently describes the experiment. However, the asymptotic formula (34) in the region $Q^2 \sim 3\ \text{GeV}^2$ results in the values of $F_\pi(Q^2)$ few times smaller than QCD formulae (and experiment). Fig. 6 shows the QCD calculated ϱ-meson formfactors [15]: electric $F_1(Q^2)$, anomalous magnetic $F_2(Q^2)$ and quadrupole $F_3(Q^2)$ as well as the ratio $F_2(Q^2)/F_1(Q^2)$ (dot-dash curve, the right-hand scale). In Fig. 6 of importance is that at $Q^2 \to 0$ $F_2(Q^2)/F_1(Q^2) \to 1$. The property $F_2(0)/F_1(Q) = 1$, i.e. that the gyromagnetic ratio for ϱ-meson is 2, is a characteristic feature of VDM in which ϱ-meson is considered as the Yang-Mills boson [21–23]. Thereby, at small Q^2 QCD surprisingly confirms VDM. The coincidence of the QCD and VDM results disappears, however, with the rise of Q^2.

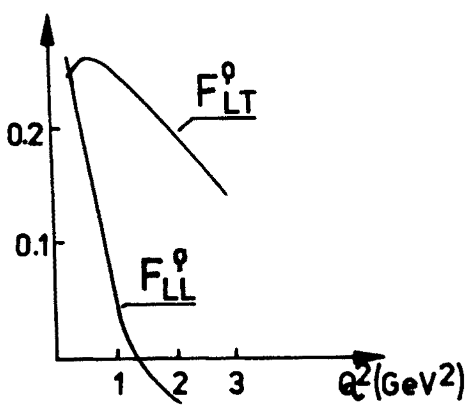


Fig. 7

One can also compare the ϱ-meson formfactor behaviour with expectations of asymptotic QCD. To this end, it is convenient to plot graphs for longitudinal-longitudinal $F_{LL}(Q^2)$ and longitudinal-transverse $F_{LT}(Q^2)$ formfactors (Fig. 7). It is expected that in the asymptotics [17, 24] $F_{LL}(Q^2) \sim 1/Q^2$, $F_{LT}(Q^2) \sim 1/Q^3$, i.e. $F_{LL} \gg F_{LT}$. We see that at intermediate Q^2 there is an inverse relation.

In Ref. [25] the electromagnetic $\varrho-\pi$ transition formfactor in the intermediate region $0.5 \lesssim Q^2 \lesssim 3$ GeV^2 was calculated. Now and again, there is a good agreement between QCD and VDM resul's $F_{\varrho\pi}(Q^2)_{VDM} = (g_{\omega\varrho\pi}/g_\omega)(m_\omega^2/m_\omega^2+Q^2)$ and strong disagreement with the asymptotic QCD where $F_{\varrho\pi}(Q^2) \sim 1/Q^4$ is expected.

Knowledge of formfactors in the intermediate Q^2 region enables one to determine some interaction constants and, respectively, hadronic widths using dispersion relation for $F(Q^2)$ in Q^2 and saturating it by the lowest state contributions. Thus it was found

$$g^2_{\varrho\pi\pi}/4\pi = 3.4 \pm 0.3 \text{ [15]}, \quad \Gamma_{\varrho\pi\pi} = 175 \text{ MeV} \pm 10\% (\Gamma^{\exp}_{\varrho\pi\pi} = 155 \pm 5 \text{ MeV}), \tag{35}$$

$$g_{\omega\varrho\pi} = 17 \pm 3 \text{ GeV}^{-1} \text{ [26]}, \quad \Gamma_{\omega\to 3\pi} = 11.4 \text{ MeV} \pm 30\% (\Gamma^{\exp}_{\omega\to 3\pi} = 8.9 \text{ MeV}). \tag{36}$$

Finally, I will demonstrate the proton and neutron magnetic formfactor curves in the region of small $Q^2 < 1$ GeV^2 region obtained in the recent paper [27] (Figs. 8, 9). Form-

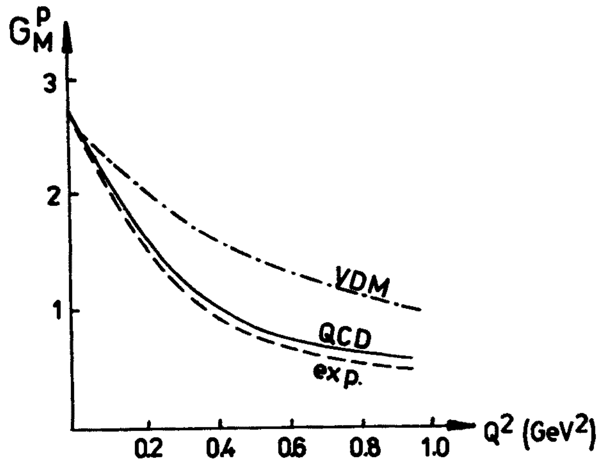


Fig. 8

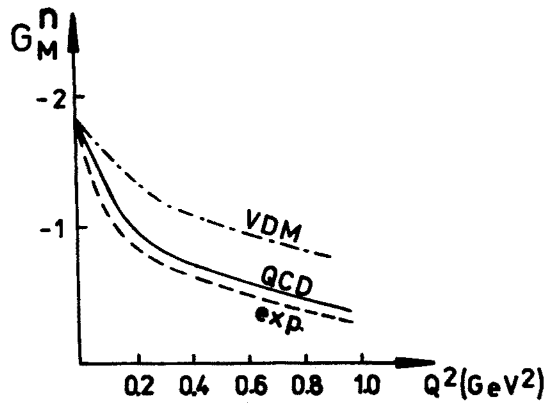


Fig. 9

factors G_M^p and G_M^n are calculated employing a rather different technique than one discussed above: to calculate them one should know field induced v.e.v's, particularly, the determined below quantity $\chi(q^2)$. The advantage of the method used in Ref. [27] is that it makes it possible to pass into the small Q^2 region. As is seen from Figs. 8, 9, the agreement with experiment is good enough. In this case the VDM curves significantly differ from QCD (and from experiment).

4. *Hadrons in external fields*

I will turn now to the problem of calculating hadron characteristics connected with their behaviour in static (or slowly varying) external fields.

Among such quantities of primary interest is, of course, the calculation of proton and neutron magnetic moments. Hopeless attempts to calculate them were made starting from 40-ties (for a review of earlier papers see Pauli [28]); The necessity of calculation of proton magnetic moment in quantum chromodynamics was especially emphasized by Feynman [29].

Recently the problem of determining hadronic properties in static external fields has been solved and, in particular, nucleon and hyperon magnetic moments have been calculated. The method developed is general enough and can be applied not only to calculating baryon and meson magnetic moments but also to find a number of other static characteristics of hadrons, such as axial weak interaction constant of nucleon g_A, interaction constants of baryons with soft pions etc. It should be noticed that together with calculation of various physical characteristics of hadrons one managed to obtain (using the same method) some information on the properties of vacuum in QCD which, as it should be expected, will prompt a better understanding of the structure of QCD and may be useful in developing various models.

In this lecture I will describe the grounds of the method in view on an example of calculation of nucleon magnetic moments. I shall follow here papers [30, 31].

The standard QCD sum rule method with which the masses and formfactors were calculated, is inapplicable for calculating magnetic moments since it is based on the calculation of the polarization operator $\Pi(p)$ or of the vertex function $\Gamma(p^2, p'^2, q^2)$ in euclidean region $p^2, p'^2, q^2 < 0$ at large enough virtualities in all the channels when nontrivial QCD effects reduce to calculable corrections in perturbation theory. Thus, in this case one cannot exploit the technique presented in Sections 2, 3. It we try to analytically continue the formulae for electromagnetic formfactors valid at intermediate $Q^2 = -q^2$ to the point $Q^2 = 0$ we encounter with singularities of the type $1/Q^2$, $1/Q^4$, $\ln Q^2$ etc. or if they are absent in some particular cases with unknown contributions not accessible to a standard operator expansion. The appearance of such terms makes the whole procedure invalid.

To solve the problem we consider quarks to be in a constant electromagnetic field $F_{\mu\nu}$. The initial point of our approach is a statement that even if one of the external momenta is zero, formulae of the operator expansion including, however, new phenomenological parameters, can be written. The meaning of these parameters is evident: they describe a response of v.e.v. to the presence of external colourless field. For instance, v.e.v.

$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle$ where q is the quark field $q = u, d$, by virtue of Lorenz invariance is zero when external fields are absent (the mean value of quark spin in the condensate is zero). If quarks are in a constant electromagnetic field, then there exists an external tensor $F_{\mu\nu}$ and in general case one can write

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F = \sqrt{4\pi\alpha}\,\chi_q F_{\mu\nu}\langle 0|\bar{q}q|0\rangle, \tag{37}$$

where χ_q is a new parameter. The physical meaning of Eq. (37) is clear: in the presence of external magnetic field the quark spin in condensate is oriented along the field and in the case of small fields the value of the mean spin is proportional to the field. The proportionality coefficient χ_q can be called magnetic susceptibility of quark condensate and hereafter I shall use this term. If in QCD with massless u- and d-quarks the chiral invariance would not be violated, the l.h.s. of Eq. (37) would be zero ($\bar{q}\sigma_{\mu\nu}q$ is non-invariant at chiral transformations). But, as we know, in QCD chiral invariance is spontaneously broken. The simplest and the most important v.e.v. which features chiral invariance violation is the quark condensate density $\langle 0|\bar{q}q|0\rangle$. For this reason it appears to be convenient to explicitly insert it into the r.h.s. of Eq. (37). As will be seen in the following, the role of magnetic susceptibility is of great importance in the problem of calculating baryon magnetic moments.

4.1. The method

Consider the polarization operator of quark currents with nucleon quantum numbers assuming quarks to be in a constant weak external electromagnetic field $F_{\mu\nu}$ and restrict ourselves to the linear in $F_{\mu\nu}$ terms. Then

$$\begin{aligned}\Pi(p) &= i\int d^4x e^{ipx}\langle 0|T\{\eta(x), \bar{\eta}(0)\}\,|0\rangle_F \\ &= \Pi^{(0)}(p)+\sqrt{4\pi\alpha}\,\Pi_{\mu\nu}(p)F_{\mu\nu},\end{aligned} \tag{38}$$

where $\Pi^{(0)}(p)$ is the polarization operator in the absence of field, $\eta = \eta_p, \eta_n$ are the currents with proton and neutron quantum numbers

$$\eta_p(x) = u^a(x)C\gamma_\mu u^b(x)\gamma_\mu\gamma_5 d^c(x)\varepsilon^{abc}, \tag{39}$$

$$\eta_n(x) = d^a(x)C\gamma_\mu d^b(x)\gamma_\mu\gamma_5 u^c(x)\varepsilon^{abc}, \tag{40}$$

$u^a(x)$, $d^a(x)$ are the fields of u- and d-quarks.

Following the idea of the QCD sum rule method we calculate the polarization operator in euclidean region $p^2 < 0$, $|p^2| \sim 1\ \mathrm{GeV}^2$ as an operator expansion whose coefficients are expressed via v.e.v.'s of various operators. On the other hand, let us write for $\Pi_{\mu\nu}(p)$ dispersion relations and saturate them by the contributions of the lowest states. As usual, in the QCD sum rule method, in order to suppress excited state contributions into sum rules, apply the Borel transformation to structure functions of the polarization operator $f_i(p^2)$.

Let us now dwell on the operator expansion and classify v.e.v.'s of the operators according to their dimensions d. Suppose that u- and d-quarks are massless. Since in (38) we are interested in the linear in $F_{\mu\nu}$ terms, $F_{\mu\nu}$ is itself the lowest dimension operator ($d = 2$).

The next-in-dimension, with $d = 3$, is the electromagnetic field induced v.e.v. $\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F$, $\sigma_{\mu\nu} = i/2(\gamma_\mu\gamma_\nu - \gamma_\nu\gamma_\mu)$. The minimal value of dimension of this v.e.v. is the origin of its significant role in the problem under discussion. V.e.v.'s with $d = 4$ are absent, since a possible in principle operator $\bar{q}(\nabla_\mu\gamma_\nu - \nabla_\mu\gamma_\nu)q$ has positive C-parity and its v.e.v. cannot be induced by C-even electromagnetic field. Three v.e.v.'s, multiplied by $F_{\mu\nu}$ quark condensate density $\langle 0|\bar{q}q|0\rangle F_{\mu\nu}$ and field induced v.e.v.'s

$$g\langle 0|\bar{q}(\lambda^n/2)G^n_{\mu\nu}q|0\rangle_F = \sqrt{4\pi\alpha}\,\kappa_q F_{\mu\nu}\langle 0|\bar{q}q|0\rangle, \tag{41}$$

$$g\langle 0|\bar{q}\gamma_5(\lambda^n/2)\varepsilon_{\mu\nu\lambda\sigma}G^n_{\lambda\sigma}q|0\rangle_F = i\sqrt{4\pi\alpha}\,\zeta_q F_{\mu\nu}\langle 0|\bar{q}q|0\rangle, \tag{42}$$

have $d = 5$. Here κ_q and ξ_q are new unknown parameters. Of the v.e.v.'s of $d = 6$ operators the most essential is

$$\langle 0|\bar{q}q|0\rangle\,\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F. \tag{43}$$

For this v.e.v. the factorization hypothesis is adopted.

Except for (43) there are still four v.e.v.'s with $d = 6$, these are $\langle 0|G^n_{\alpha\beta}G^n_{\alpha\beta}|0\rangle F_{\mu\nu}$, and three field induced nonfactorizable v.e.v.'s. The contribution from all of them is small comparing to (43) and will be neglected in the following since they enter (38) with small numerical factors: the first with $(2\pi)^{-4}$ and the rest three with $(2\pi)^{-2}N_c^{-1}$ (according to (9)). A great number of operator v.e.v.'s corresponds to $d = 7$. Some of them are known but a considerable number of $d = 7$ v.e.v.'s is nonfactorizable and unknown. In order not to introduce a great number of unknown parameters into theory it is thus reasonable to avoid the sum rules where v.e.v.'s with $d = 7$ can be essential. Restricting ourselves with the operator v.e.v.'s with $d = 6$ (and in the case of chirality conserving structure with factorizable v.e.v.'s with $d = 8$) we are left with three v.e.v.'s induced by electromagnetic field and defined in (37), (41), (42).

Let us now make the basic assumption on which, as will be seen from what follows, the calculation of baryon magnetic moments is grounded, namely, assume that χ_q, κ_q and ζ_q are proportional to the quark q charge

$$\chi_q = e_q\chi, \quad \kappa_q = e_q\kappa, \quad \zeta_q = e_q\zeta. \tag{44}$$

This assumption corresponds to taking into account only the graphs of the type Fig. 10a where the same quark whose field enters v.e.v. interacts with electromagnetic field. So the diagrams of the type Fig. 10b are neglected. It is clear that for massless quarks the diagram Fig. 10b with gluon exchange are zero in any order of perturbation theory by virtue of chirality conservation. Chirality breaking could appear due to instantons but, as was

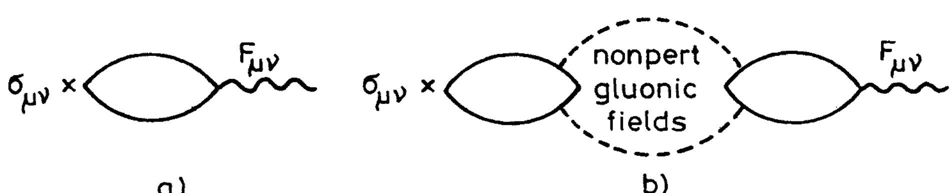


Fig.10

shown in [9], instantons (in the diluted gas approximation) do not contribute to diagrams of Fig. 10b. The Fig. 10 amplitudes rather resemble the $\varphi-\omega$ mixing amplitude. The only difference between these two amplitudes is that in the first chirality is violated and in the second conserved. That is why the experimental smallness of $\varphi-\omega$ mixing is also an argument in favour of the assumption made.

Now we can start to construct sum rules for calculating proton and neutron magnetic moments. The l.h.s. of the desired sum rules, the term proportional to $F_{\mu\nu}$ in the polarization operator (38) is calculated at $p^2 < 0$ in QCD from the operator expansion, the r.h.s. is represented using dispersion relation via physical state contributions. $\Pi_{\mu\nu}(p)$ is composed of three different tensor structures: $\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p}$, $i(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}$ and $\sigma_{\mu\nu}$. The first structure contains three γ-matrices and conserves chirality. The second and the third structures contain even number of γ-matrices and violate chirality.

Let us first consider the l.h.s. of the sum rules with odd structure $\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p}$. The lowest dimension operator contributing to this structure is $F_{\mu\nu}$ ($d = 2$), the corresponding Feynman graph is shown in Fig. 11. Because of chiral invariance the $d = 3$ and $d = 5$

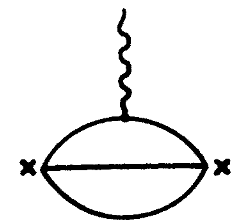
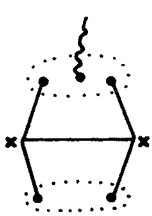
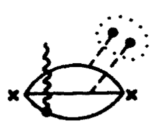


Fig.11

Fig. 11. The simplest quark loop contributing to the polarization operator in external field. Here (and in the following figures) solid lines — quark propagators, dashed lines — gluon propagators, wavy lines correspond to electromagnetic field





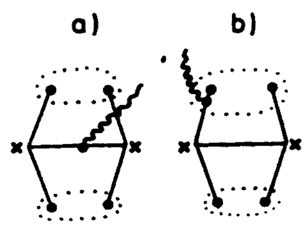


Fig.12 **Fig.13** **Fig.14**

operators do not contribute to the odd structure so that the next in dimension are the $d = 6$ operators. Taking into account of the fied induced v.e.v. $\langle 0|\bar{q}q|0\rangle \langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F$ ($d = 6$) corresponds to the graph of Fig. 12. The contribution of the $F_{\mu\nu}G^n_{\alpha\beta}G^n_{\alpha\beta}$ ($d = 6$) operator is determined by the two-loop graph of Fig. 13 which contains a small factor

$(2\pi)^{-4}$. The direct calculation of the operator $G^n_{\alpha\beta}G^n_{\alpha\beta}$ contribution in $\Pi^{(0)}(p)$ showed that its presence did not practically affect the nucleon mass [5]. Thereby the term in the operator expansion proportional to v.e.v. $F_{\mu\nu}\langle 0|G^n_{\alpha\beta}G^n_{\alpha\beta}|0\rangle$ can be omitted.

In the sum rules with the odd structure $\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p}$ we take into account also (assuming factorization) the $d = 8$ operators contribution. There exist four such v.e.v.'s: $F_{\mu\nu}\langle 0|\bar{q}q|0\rangle^2$, $-g\langle 0|\bar{q}\sigma_{\alpha\beta}(\lambda^n/2)G^n_{\alpha\beta}q|0\rangle\,\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F = \sqrt{4\pi\alpha}e_q\chi m_0^2F_{\mu\nu}\langle 0|\bar{q}q|0\rangle^2$ where m_0^2 was determined in Ref. [9], $m_0^2 = 0.8$ GeV^2, $\sqrt{4\pi\alpha}e_q\kappa F_{\mu\nu}\langle 0|\bar{q}q|0\rangle^2$ and $\sqrt{4\pi\alpha}e_q\zeta F_{\mu\nu}\langle 0|\bar{q}q|0\rangle^2$. The corresponding diagrams are presented in Figs. 14–16. The estimate of the $d = 8$ terms is very

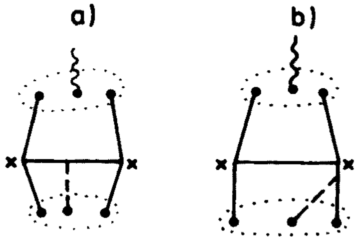


Fig.15

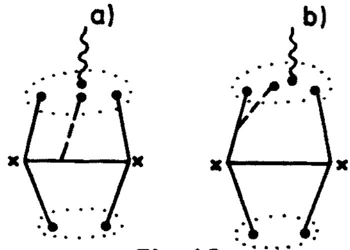


Fig.16

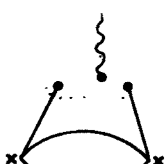


Fig.17

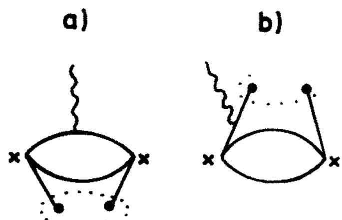


Fig.18

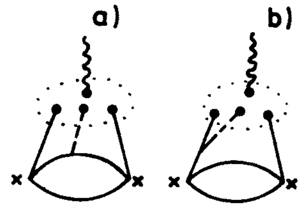


Fig.19

mportant. Indeed, as will be seen in the following, the $d = 6$ term contribution to the sum rule is very large, even larger than the contribution of the diagram of Fig. 11 since the magnitude of magnetic susceptibility of quark condensate is large. The $d = 8$ terms (especially the term $\chi m_0^2\langle 0|\bar{q}q|0\rangle^2$) can be considered as the first corrections to the $d = 6$ terms. Thereby the size of their contribution is necessary to know in order to check the convergence of the operator expansion series. Of course, the results will be convincing enough if it will appear (and really this is the case) that the $d = 8$ term contribution is much smaller than that with $d = 6$.

The lowest dimension operator contributing to chirality violating structures in $\Pi_{\mu\nu}(p)$ is $\bar{q}\sigma_{\mu\nu}q$, $d = 3$, see Fig. 17. The next in dimension ($d = 5$) operator v.e.v.'s are $\langle 0|\bar{q}q|0\rangle F_{\mu\nu}$ and (41), (42) corresponding to the diagrams of Figs. 18, 19. Assuming factorization again we have at even structures two operator v.e.v.'s with $d = 7$:

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle_F\,\langle 0|G^n_{\alpha\beta}G^n_{\alpha\beta}|0\rangle$$

and

$$-g\langle 0|\bar{q}\sigma_{\alpha\beta}\frac{\lambda^n}{2}G^n_{\alpha\beta}q|0\rangle F_{\mu\nu} = m_0^2 F_{\mu\nu}\langle 0|\bar{q}q|0\rangle.$$

The corresponding diagrams are shown in Figs. 20, 21. (Note that the higher the dimension the less reliable is the factorization hypothesis.)

As is seen, the accounted for dimension interval $d = 2$–8 for odd structures is larger than that of dimensions $d = 3$–7 for even structures. Thereby, one can expect a better

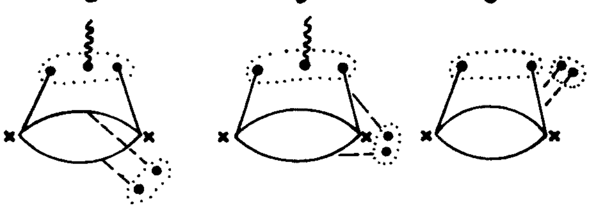


Fig. 20

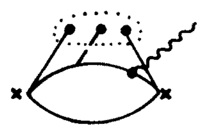


Fig. 21

accuracy for the results obtained from sum rules at odd structures. As for even structures it is reasonable to use only sum rule with the structure $i(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p}$. The reasons for this are the following:

(1) The structure $i(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p}$ comparing with $\sigma_{\mu\nu}$ contains two extra momenta in the numerator. This leads to appearance of an extra factor $1/p^2$ in nonperturbative corrections. As a result, the Borel transformation (see Eq. (13)) brings about an extra factor $1/n$ in the higher-dimensional terms to the structure $i(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p}$ as compared to those in $\sigma_{\mu\nu}$, which improves the power series convergence. At the same time, the role of excited states (continuum) in the r.h.s. of sum rules with the structure $i(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p}$ is also less than that with $\sigma_{\mu\nu}$.

(2) The sum rule with the structure $\sigma_{\mu\nu}$ contains infrared divergence which evidences the presence of unknown nonfactorizable v.e.v.'s. Under the procedure used in the following the unknown higher-dimensional contributions change the results obtained from sum rules with this structure rather significantly.

Let us turn to calculating the r.h.s. of the sum rules expressed versus physical state contributions. The part of the polarization operator $\Pi(p)$ linear in the field $F_{\mu\nu}$ can be phenomenologically presented as a sum of the contributions shown in Figs. 22–24. Our interest is concentrated on the graph of Fig. 22 where current $\bar{\eta}$ produces a nucleon which interacts

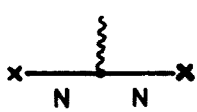


Fig.22

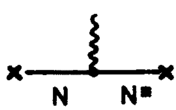


Fig.23

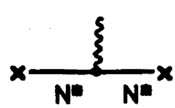


Fig.24

with electromagnetic field and afterwards is annihilated by the current η. The contribution of this graph contains the double pole and is proportional to $\lambda_N^2\mu(p^2-m^2)^{-2}$ where λ_N is the transition amplitude of current η into nucleon state $\langle 0|\eta|N\rangle = \lambda_N v$, v is nucleon spinor, μ is the nucleon magnetic moment (the total magnetic moment for the structures $\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p}$ and $\sigma_{\mu\nu}$ and anomalous magnetic moment for the structure $i(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}$). After the Borel transformation it gives the term $\sim\lambda_N^2\mu\exp(-m^2/M^2)/M^2$. The diagram of Fig. 24 corresponds to transitions between excited states which appear to be exponentially suppressed by the Borel transformation and as usual, can be approximated by continuum. The graph of Fig. 23 which contains transitions from nucleon to excited states is a nondesirable background. The contribution of such transitions to $\Pi_{\mu\nu}$ is proportional to $1/(p^2-m^2)(p^2-m^{*2})$ where m^* is the excited state mass. The Borel transform of this expression contains the term $(m^{*2}-m^2)^{-1}\exp(-m^2/M^2)$ not suppressed exponentially as compared with Fig. 22. At reasonable values of M^2 it is of the same order and must be taken into account. The transitions N $\to$ excited states can be taken into account by introducing two new phenomenological constants A_N and B_N

$$\langle 0|\eta|N\rangle_F = \int d^4x\langle 0|T\{j_\mu^{el}(x)A_\mu(x),\eta(0)\}|N\rangle - \text{pole term}$$

$$= \sqrt{4\pi\alpha}\,\tfrac{1}{4}\,\lambda_N[A_N\sigma_{\mu\nu}+iB_N(p_\mu\gamma_\nu-p_\nu\gamma_\mu)/m]F_{\mu\nu}v. \quad (45)$$

As a result, the one-proton contribution into $\Pi_{\mu\nu}(p)$ is given by

$$\Pi_{\mu\nu}(p) = -\tfrac{1}{4}\frac{\lambda_N^2}{(p^2-m^2)^2}\{\mu_p(\sigma_{\mu\nu}\hat{p}+\hat{p}\sigma_{\mu\nu})+2\sigma_{\mu\nu}m\mu_p$$

$$+\sigma_{\mu\nu}\mu_p^a(p^2-m^2)/m+2i\mu_p^a(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}/m\}, \quad (46)$$

where μ_p and μ_p^a are the proton total and anomalous magnetic moments in nuclear magnetons, λ_N is the transition amplitude of proton into current η defined in (15). The term resulting from proton-excited state transitions can be safely obtained using Eq. (45). It is equal to

$$\Pi_{\mu\nu}(p)_{p\to N^*} = \tfrac{1}{4}\,\lambda_N^2\frac{1}{p^2-m^2}\{A_p(\sigma_{\mu\nu}\hat{p}+\hat{p}\sigma_{\mu\nu})$$

$$+2A_p\sigma_{\mu\nu}m+2iB_p(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}/m\}. \quad (47)$$

It should be emphasized that nucleon-excited state transitions give a significant contribution to sum rules and they can be by no means neglected. This statement refers not only to the problem of magnetic moment calculation but to all processes of hadron interaction with external fields. (For this reason the calculation of the constants $g_{\pi NN}$, $g_{\pi N\Delta}$ made in [32] where such transitions were neglected are not convincing.) Such transitions must also be taken into account in calculating magnetic moments on lattices. Their contribution at large times t is here suppressed as compared with the ground term but only in the power-like way, as m/t but not exponentially (for details see [31]).

As usual, in QCD sum rules the continuum contribution into imaginary part of the polarization operator is represented by imaginary parts of quark loops and starts from some threshold value W^2. It is convenient to transfer it into the l.h.s. of the sum rule.

Omitting intermediate calculations I will present now the sum rules for invariant functions with the structures $\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p}$ and $i(p_\mu\gamma_\nu-p_\nu\gamma_\mu)\hat{p}$

$$e_u M^4 E_2\left(\frac{W^2}{M^2}\right)L^{-4/9}+\frac{a^2}{3M^2}L^{4/9}[-(e_d+\tfrac{2}{3}e_u)+\tfrac{1}{3}e_u(\kappa-2\zeta)$$

$$-2e_u\chi(M^2L^{-16/27}-\tfrac{1}{8}m_0^2L^{-28/27})]=\frac{\tilde{\lambda}_N^2}{4}e^{-m^2/M^2}\left(\frac{\mu_p}{M^2}+A_p\right), \quad (48)$$

$$ma\left\{(e_u+\tfrac{1}{2}e_d)+\tfrac{1}{3}e_d\chi M^2\left[E_1\left(\frac{W^2}{M^2}\right)+\frac{b}{24M^4}\right]L^{-16/27}\right\}$$

$$=\tfrac{1}{4}\tilde{\lambda}_N^2 e^{-m^2/M^2}\left(\frac{\mu_p^a}{M^2}+B_p\right), \quad (49)$$

where the same notations as in (19)–(22) are used. In the sum rule (49) we neglect the contribution of $d=7$ operator $g\bar{q}\sigma_{\alpha\beta}\lambda^n G^n_{\alpha\beta}qF_{\mu\nu}$ (for its account see below).

When calculating the continuum contribution into polarization operator it was assumed that it is described by the double dispersion integral

$$\int_{W^2}^{\infty}\frac{\varrho(s')}{(s'-s)^2}\,ds'.$$

In numerical calculations the continuum threshold W was taken to be $W=1.5$ GeV (continuum does not affect determination of μ_p and μ_n but is essential in finding χ and constants A, B — see below).

4.2. Determination of proton and neutron magnetic moments and of quark condensate magnetic susceptibility

The sum rules (48), (49) contain, except for μ_p, many unknown parameters: χ, κ and ζ in the left-hand sides, A_p and B_p in the right-hand sides (for neutron there appear additional parameters A_n and B_n). At first sight, this is a serious nuisance so that it seems impossible to determine magnetic moments from these sum rules with sufficient accuracy. But in fact the situation is not so bad.

Note that parameters χ, κ and ζ enter the sum rule (48) being multiplied by the u-quark charge e_u while in the sum rule (49) they are multiplied by e_d. This is a direct consequence of neglecting closed loops and of assuming the quark condensate magnetic susceptibility with the given flavour to be proportional to the quark charge. Using this fact one can get rid of parameters χ, κ and ζ as well as of chirality violating v.e.v.'s of higher-dimensional biquark operators induced by electromagnetic field.

Let us multiply the sum rule (48) for proton by e_d, for neutron by e_u and subtract one from another. Similarly, let us multiply the sum rule (49) for proton by e_u, for neutron by e_d and subtract one from another. The resultant expressions can be written as

$$\mu_p e_d - \mu_n e_u + M^2(A_p e_d - A_n e_u) = \frac{4a^2}{3\tilde{\lambda}_N^2} e^{m^2/M^2}(e_u^2 - e_d^2)L^{4/9}, \tag{50a}$$

$$\mu_p^a e_u - \mu_n e_d + M^2(B_p e_u - B_n e_d) = \frac{4am}{\tilde{\lambda}_N^2} M^2 e^{m^2/M^2}(e_u^2 - e_d^2).$$

In order to eliminate the unknown one-pole contributions remaining in the l.h.s. of Eq. (50) we apply to Eq. (50) the differential operator $1 - M^2\partial/\partial M^2$. We get

$$\mu_p e_d - \mu_n e_u = \frac{4a^2}{3\tilde{\lambda}_N^2}(e_u^2 - e_d^2)\left[1 - M^2 \frac{\partial}{\partial M^2}\right] e^{m^2/M^2} L^{4/9}, \tag{51a}$$

$$\mu_p e_u - \mu_n e_d = e_u + \frac{4am}{\tilde{\lambda}_N^2}(e_u^2 - e_d^2)\left(1 - M^2 \frac{\partial}{\partial M^2}\right) M^2 e^{m^2/M^2}. \tag{51b}$$

Magnetic moments μ_p and μ_n can approximately be determined if we put in (51) $M = m$, neglect anomalous dimensions, and substitute instead of the residue $\tilde{\lambda}_N^2$ the value

$$\tilde{\lambda}_N^2 = \frac{2aM^4}{m} e^{m^2/M}|_{M=m} \tag{52}$$

which follows from the mass sum rules (17), (18) neglecting anomalous dimensions and continuum contribution. Solving equations (51) in this approximation we arrive at simple formulae

$$\mu_p = \tfrac{8}{3}\left(1 + \tfrac{1}{6}\frac{a}{m^3}\right) = 2.96, \tag{53a}$$

$$\mu_n = -\tfrac{4}{3}\left(1 + \tfrac{2}{3}\frac{a}{m^3}\right) = -1.93. \tag{53b}$$

Theoretical values (53) can be compared with the experimental $\mu_p = 2.79$, $\mu_n = -1.91$.

Let us now turn to a more rigorous treatment. To do this we need to know the exact value of the residue $\tilde{\lambda}_N^2$. If the nucleon mass is not a free parameter but is fixed at the experimental value $m = 0.94$ GeV, the best fit of the sum rule for the nucleon mass can be achieved at $\tilde{\lambda}_N^2 = 2.1\ \text{GeV}^6$, $W^2 = 2.3\ \text{GeV}^2$ (see (25) and Fig. 4). The solutions of Eqs. (51a, b) are plotted in Fig. 25 versus M^2. Variations of μ_p and μ_n with the change of M^2 reflect the uncertainty of our predictions. A permissible interval for M^2 variation is 0.8 GeV $< M^2 <$ 1.4 GeV² where continuum contribution and power corrections are still controllable. Though continuum does not give a direct contribution to the r.h.s. of Eqs. (51) its concellation is a consequence of our model of continuum which may not work if this contribution is too large. Besides, at large M^2 in the r.h.s. of (48), (49) single-pole terms play the main role and extracting of μ_p, μ_n in the presence of a large background becomes inadequate. Note finally that changes in the $\tilde{\lambda}_N^2$ do not significantly affect the values of magnetic moments because of the presence of the e_u term in the r.h.s. of Eq. (51b) stemming from the difference between the total and anomalous magnetic moments. The final results for proton and neutron magnetic moments do not practically differ from (53): $\mu_p = 3.0$, $\mu_n = -2.0$ ($\pm$10–15%).

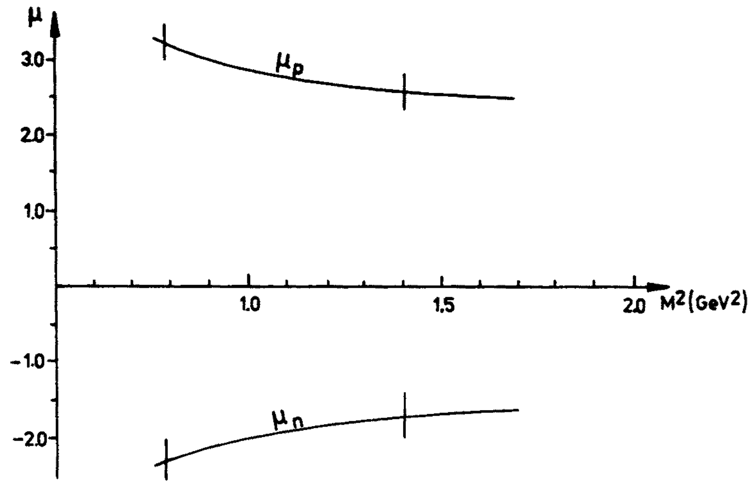


Fig. 25. The Borel parameter M^2 dependence of proton and neutron magnetic moments defined from Eq. (51). Vertical lines separate the interval inside which in Eqs. (48), (49) and in the sum rules for the nucleon mass the continuum contribution <50% and the higher power correction contribution <15% of the total theoretical result

To control the accuracy of calculations of magnetic moments one may take into account the contribution of the operator $g\bar{q}\sigma_{\alpha\beta}\lambda^n G^n_{\alpha\beta} g F_{\mu\nu}$ to structure $i(p_\mu\gamma_\nu - p_\nu\gamma_\mu)\hat{p}$ omitting all the remaining nonfactorizable $d = 7$ operators. The account of this operator results in appearance of factor $1-(m_0^2/4M^2)L^{-4/9}$ at the second term in the r.h.s. of Eq. (51b). As a result, the absolute values of μ_p and μ_n decrease, correspondingly, by 15 and 10% and the agreement with experiment is approximately the same as before. It can be thought that taking account of this operator leads to overestimating the accuracy since effects of comparable scale may arise due to nonfactorizable operators, from corrections to relation (44) and from uncertainty in the values of $\langle 0|\bar{q}q|0\rangle$ and $\tilde{\lambda}_N^2$.

Substituting the obtained values of μ_p and μ_n into (48), (49) and excluding from them the unknown constants A, B by applying operator $(1-M^2\partial/\partial M^2)M^2 \exp(m^2/M^2)$, one can find the value of magnetic susceptibility of quark condensate χ. Putting $\kappa = \zeta = 0$ we find from (48)

$$\chi = -6\ \mathrm{GeV}^{-2}(\pm 25\%). \tag{54}$$

The influence of κ and ζ on the estimate of χ is not large: $d\chi/d\kappa = -\frac{1}{2}d\chi/d\zeta \approx \frac{1}{3}\ \mathrm{GeV}^{-2}$. (Determination of χ from (49) is less reliable.)

The sign and the value of χ are "natural" in the following sense. Let us consider a simple model in which the current quark acquires a mass in the field of vacuum fluctuation and the resulting constituent quark interacts with electromagnetic field. In such a model v.e.v.

$$\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle = \sqrt{4\pi\alpha}\, e_q \chi F_{\mu\nu}\langle 0|\bar{q}q|0\rangle = -\int d^4p \,\mathrm{Tr}\,[S(p)\sigma_{\mu\nu}], \tag{55}$$

where $S(p)$ is the Green function of constituent quark in electromagnetic field

$$S(p) = \frac{i}{(2\pi)^4}\left\{\frac{1}{\hat{p}-m_q} - \sqrt{4\pi\alpha}\,\frac{e_q}{2}F_{\alpha\beta}\left[\frac{i}{\hat{p}-m_q}\gamma_\alpha\frac{1}{\hat{p}-m_q}\gamma_\beta\frac{1}{\hat{p}-m_q} - \frac{\mu_q^a}{2m_q}\frac{1}{\hat{p}-m_q}\sigma_{\alpha\beta}\frac{1}{\hat{p}-m_q}\right]\right\}, \tag{56}$$

m_q is the quark mass, μ_q^a its anomalous magnetic moment (in the units $e_q/2m_q$). Substituting of (56) into (55) gives

$$\chi\langle 0|\bar{q}q|0\rangle = \frac{3}{4\pi^2}m_q \ln\frac{\Lambda_u^2}{m_q^2}(1+\tfrac{1}{2}\mu_q^a), \tag{57}$$

where Λ_u is the ultraviolet cut-off. Since the r.h.s. of (57) is positive and $\langle 0|\bar{q}q|0\rangle < 0$, χ must be negative. The numerical value of χ (54) must be reproduced using (57) at $m_q = 350$ MeV, $\ln \Lambda_u^2/m_q^2 \simeq 3$, $\mu_q^a = 0$. Note that the same model leads to a correct sign of the quark condensate density.

The phenomenological constants A and B can be found in the same way. They appeared to be $A_p = 1.3\ \mathrm{GeV}^{-2}$, $A_n = -0.6\ \mathrm{GeV}^{-2}$, $B_p = 1.0\ \mathrm{GeV}^{-2}$, $B_n = -1.5\ \mathrm{GeV}^{-2}$. In the region $M^2 \sim 1\ \mathrm{GeV}^2$ the contribution of the single-pole terms into sum rules (48), (49) are of the same order as the contributions of the terms proportional to magnetic moments.

The value of magnetic susceptibility of quark condensate (54) was found from the sum rules for magnetic moments. It can be, however, determined independently using to this end a special sum rule [33]. The idea of such an approach is in consideration of the quantity $\chi(q)$ defined by the equality

$$\int d^4x e^{iqx}\langle 0|T\{\bar{u}(x)\gamma_\lambda u(x), \bar{u}(0)\sigma_{\mu\nu}u(0)\}|0\rangle = (\delta_{\mu\lambda}q_\nu - \delta_{\nu\lambda}q_\mu)\langle 0|\bar{u}u|0\rangle\chi(q^2). \tag{58}$$

It can be easily seen that neglecting the diagrams Fig. 10b $\chi(0) = \chi$. Let us assume $|q^2|$ to be large and write for $\chi(q^2)$ the operator expansion. Since in (58) there is a chirality violation, then the operator expansion will be contributed only by v.e.v.'s of chirality violating operators: $\langle 0|\bar{q}q|0\rangle$, $-g\langle 0|\bar{q}\sigma_{\alpha\beta}(\lambda^n/2)G^n_{\alpha\beta}q|0\rangle = m_0^2\langle 0|\bar{q}q|0\rangle$ etc. Taking into account the contribution of the two mentioned operators only and neglecting anomalous dimensions, we find [33]

$$-\chi(Q^2) = \frac{2}{Q^2} - \frac{2}{3}\frac{m_0^2}{Q^4}, \quad Q^2 = -q^2. \tag{59}$$

For $\chi(Q^2)$ there is a subtractionless dispersion relation

$$-\chi(Q^2) = \int_0^\infty \frac{\varrho(s)}{s+Q^2}\, ds. \tag{60}$$

Let us equate (59) and (60) and make the Borel transformation. Because of fast convergence of integral in (60) it is enough to restrict oneself with the lowest state contributions of ϱ and ϱ' mesons. (Strictly speaking, Eqs. (58) for isovector and isoscalar currents should be considered separately, taking into account, respectively, ϱ and ω mesons in them, but by virtue of $\varrho-\omega$ degeneracy this would not change the result.) Then

$$\varrho(s) = f_\varrho \delta(s-m_\varrho^2) - f_{\varrho'}\delta(s-m_{\varrho'}^2). \tag{61}$$

The equality obtained after the Borel transformation makes it possible to determine the constants $f_\varrho = 4$ and $f_{\varrho'} = 2$ and using Eq. (60) find

$$-\chi(Q^2) = \frac{4}{Q^2+m_\varrho^2} - \frac{2}{Q^2+m_{\varrho'}^2}. \tag{62}$$

One can extrapolate Eq. (62) to the point $Q^2 = 0$ and determine

$$\chi = \chi(0) = -(5.7 \pm 0.6)\ \mathrm{GeV}^{-2} \tag{63}$$

(the numerical value of (63) is determined from [33] with the account of anomalous dimensions).

The value of $\chi(0)$ (63) depends very weakly on $m_{\varrho'}^2$: Variation of $m_{\varrho'}^2$ from 2 to 10 GeV^2 changes $\chi(0)$ less than by 10%. The value of χ (63) completely agrees with (54).

In Ref. [34] which was dedicated, just as Refs. [30, 31], to calculation of proton and neutron magnetic moments, an attempt was made to determine μ_p and μ_n from only one sum rule with the structure $\hat{p}\sigma_{\mu\nu}+\sigma_{\mu\nu}\hat{p}$ by substituting into it the value of χ determined with the VDM (and neglecting other unknown parameters κ and ζ). In VDM (see (59)–(61) at $f_{\varrho'} = 0$) $\chi = -2/m_\varrho^2 = -3\ \mathrm{GeV}^{-2}$, i.e., by a factor of two smaller than the values of (54), (63). Since in the sum rule (48) the contribution of the term proportional to χ comprises about 70–80% of the value of magnetic moments, then according to the arguments given above, such method is incorrect and must result in the values of μ_p and $|\mu_n|$ by a factor of 1.5–2 smaller than the experimental ones. (This was not the case in Ref. [34] since the authors used the values of $\langle 0|\bar{q}q|0\rangle$ larger by factor 1.5 comparing with the adopted above.)

4.3. Hyperon magnetic moments

The approach discussed above can be used also for calculation of hyperon magnetic moments [35]. The method of consideration is the same and only two new moments arise: one must take into account the strange quark mass m_s and the difference of v.e.v. $\langle 0|\bar{s}s|0\rangle$ from $\langle 0|\bar{q}q|0\rangle$, $q = u, d$, i.e. the value of f (28).

Just as in calculating nucleon magnetic moments, the sum rules for hyperon magnetic moments include field induced v.e.v.'s. The most unpleasant here is the magnetic susceptibility of strange quark condensate $\langle 0|\bar{s}\sigma_{\mu\nu}s|0\rangle$ whose value may differ from $\langle 0|\bar{q}\sigma_{\mu\nu}q|0\rangle$ and is unknown. For the case of Σ and Ξ hyperons these field induced v.e.v.'s can be eliminated using a trick analogous to that exploited in calculation of nucleon magnetic moments. In such a way the sum rules without new unknown parameters are obtained. The results of the calculation of Σ and Ξ hyperon magnetic moments are given in Table III. For Λ-hyperon such a procedure is impossible and Table III gives the value found from

TABLE III

Baryon octet magnetic moments

	p	n	Σ^+	Σ^0	Σ^-	Ξ^0	Ξ^-	Λ	$\Lambda\to\Sigma^0$
Sum rule	3.0	−2.0	2.4	0.7	−1.0	−1.40	−0.90	−0.7[a]	1.55[a]
Quark model	2.79[b]	−1.91[b]	2.67	0.78	−1.09	−1.44	−0.49	−0.61[b]	1.63
Experiment	2.79	−1.91	2.37 ±0.02		−1.18 ±0.03	−1.25 ±0.015	−0.69 ±0.04	−0.61	$1.82^{+0.25}_{-0.18}$

[a] Approximated formulae   [b] Input data.

relation of SU(3) symmetry $\mu_\Lambda = \mu_n/2$ and from formula (53b) into which we substitute m_Λ instead of m and take into account the kinematic factor m/m_Λ for transforming Λ magnetic moment into nuclear magnetons. (Σ and Ξ magnetic moments can be determined in a similar way. Such an approximated procedure leads to values not strongly differing from those shown in Table III.) To determine Λ hyperon magnetic moment one could employ a sum rule of the type (48) in which v.e.v. $\langle 0|\bar{s}\sigma_{\mu\nu}s|0\rangle$ is not excluded, determining the latter from QCD sum rules, analogously to that which was done in the preceding Section (Eqs. (58)–(63)), but taking into account the strange quark mass. It is, however, difficult to expect that such a method will permit one to significantly improve the calculation accuracy of Λ magnetic moment. The magnetic moment of $\Sigma^0 \to \Lambda$ transition presented in Table III was obtained using relation [36]

$$\mu_{\Sigma\Lambda} = \frac{1}{2\sqrt{3}}(3\mu_\Lambda + \mu_{\Sigma^0} - 2\mu_n - 2\mu_{\Xi^0}) \tag{64}$$

valid in the linear approximation in SU(3)-symmetry violation when this violation is a 3,3 component of an octet (i.e. strange quark mass).

For comparison, Table III presents also the latest experimental data [37] and the results of the calculations in nonrelativistic quark model in which μ_p, μ_n, and μ_Λ are input data determining magnetic moments of constituent quarks. As is seen from the Table, QCD calculations agree with experiment up to expected accuracy — 10–15%.

4.4. Calculation of the nucleon coupling constants with axial current

The method in view can be also applied to calculation of the weak interaction constant g_A [38]. To this end one should introduce a constant isovector axial field A_μ as an external field by adding to Lagrangian the term

$$\Delta L = (\bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d)A_\mu. \tag{65}$$

Similarly as before, let us consider the polarization operator

$$\Pi(p) = i\int d^4x e^{ipx}\langle 0|T\{\eta(x), \bar{\eta}(0)\}\,|0\rangle, \tag{66}$$

where quark current $\eta(x)$ is defined according to (39) and quarks are moving in external axial field A_μ. The presence of external axial field results in appearance of field induced v.e.v.'s. Classifying these v.e.v.'s according to their dimension one can see that v.e.v.'s of the lowest dimension operators are

$$d = 3: \quad \langle 0|\bar{u}\gamma_\mu\gamma_5 u|0\rangle_A = -\langle 0|\bar{d}\gamma_\mu\gamma_5 d|0\rangle_A, \tag{67}$$

$$d = 5: \quad \tfrac{1}{2}\varepsilon_{\alpha\beta\mu\nu}g\left\langle 0\left|\bar{u}\gamma_\alpha\frac{\lambda^n}{2}G^n_{\mu\nu}u\right|0\right\rangle_A$$

$$= -\tfrac{1}{2}\varepsilon_{\alpha\beta\mu\nu}g\left\langle 0\left|\bar{d}\gamma_\alpha\frac{\lambda^n}{2}G^n_{\mu\nu}d\right|0\right\rangle_A \equiv f_\pi^2 m_1^2 A_\beta. \tag{68}$$

V.e.v. of the first of these operators can be easily calculated using the axial current conservation condition. According to (65) we have

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u|0\rangle_A = \frac{i}{2}\int d^4x e^{iqx}A_\nu\langle 0|T\{j^A_\nu(x), j^A_\mu(0)\}\,|0\rangle_{q\to 0} \equiv \Pi_{\mu\nu}(q)A_\nu|_{q\to 0}, \tag{69}$$

where

$$j^A_\mu = \bar{u}\gamma_\mu\gamma_5 u - \bar{d}\gamma_\mu\gamma_5 d. \tag{70}$$

In the r.h.s. of (69) $\Pi_{\mu\nu}(q)$ is the polarization operator of the axial isovector current. Neglecting quark masses the axial current is conserved, so that

$$\Pi_{\mu\nu}(q) = -(\delta_{\mu\nu}q^2 - q_\mu q_\nu)\Pi(q^2). \tag{71}$$

At $q \to 0$ the non-zero contribution into $\Pi_{\mu\nu}(q)$ may arise only due to pole at $q^2 \to 0$ in $\Pi(q^2)$, i.e. due to massless state. The single-pion state is such a state. (In the massless quark approximation the pion mass is zero.) The constant of the pion transition into axial current is known — this is the $\pi \to \mu\nu$ decay constant $f_\pi = 133$ MeV. Extracting in (71) the single-pion contribution and substituting it into (69) we find

$$\langle 0|\bar{u}\gamma_\mu\gamma_5 u|0\rangle_A = f_\pi^2 A_\mu. \tag{72}$$

Determination of v.e.v. (69) is a more complicated problem. It is expressed via the matrix element

$$\tfrac{1}{2}\,\varepsilon_{\alpha\beta\mu\nu} g \left\langle 0 \left| \bar{d}\gamma_\alpha \frac{\lambda^n}{2}\, G^n_{\mu\nu} d \right| \pi \right\rangle = -if_\pi m_1^2 q_\beta \tag{73}$$

and was found in Ref. [39] from selfconsistency of some sum rules. It was obtained that $m_1^2 \approx 0.2\ \mathrm{GeV}^2$. (The m_1^2 determination accuracy is rather low and is given by the factor $2^{\pm 1}$, but luckily, v.e.v. (73) enters the expression for g_A with a small coefficient). An essential moment in g_A calculation is that the difference of g_A from 1 results only from nonperturbative effects connected with quark condensate or field induced v.e.v.'s (67), (68). This can be easily understood. In perturbation theory with massless quarks chirality is conserved and the axial interaction constant must be equal to the vector one, i.e. in the adopted normalization, to unity. Therefore, an appropriate method of g_A calculation consists in writing the Borel sum rules for g_A and in subtracting from them the sum rule (equal to 1) for vector constant, i.e. by virtue of the Ward theorem, the sum rule for the nucleon mass. As a result, we have a sum rule for $g_A - 1$ with no uncertainties due to continuum contribution. Among the coefficient functions at different tensor structures of the polarization operator (66) the most appropriate for the writing sum rule is the function at the structure $2(Ap)\hat{p}\gamma_5$ where the contribution of continuum and higher power corrections is suppressed. Omitting the technical details of the calculations I will give the sum rules for this structure function [38]:

$$g_A - 1 + CM^2 = \tfrac{8}{9}\,\frac{1}{\tilde{\lambda}_N^2}\, e^{m^2/M^2}\left[a^2 L^{4/9} + 2\pi^2 m_1^2 f_\pi^2 M^2 L^{-8/9}\right]. \tag{74}$$

Here C is an unknown constant corresponding to nondiagonal transitions from nucleon to excited states due to the axial current, analogous to constants A, B in the problem of magnetic moment determination. Of importance is the fact that v.e.v. (67) did not enter (74) and here only higher dimension v.e.v.'s are left. To exclude C let us apply the differential operator $1 - M^2\partial/\partial M^2$ to (74). After substituting numbers we have

$$g_A - 1 = 0.40 \pm 0.20. \tag{75}$$

The error in (75) comes from a certain dependence of $g_A - 1$ determined according to (74) from the Borel parameter. (The value of $g_A - 1$ and the error in (75) differ from the values given in Ref. [38] where $g_A - 1 = 0.30 \pm 0.05$. The origin of this difference is explained in Ref. [40].)

The nucleon interaction constant g_A^s with the axial isoscalar current

$$g_A^s = 0.5 \pm 0.2 \tag{76}$$

was determined also [40] from analoguous calculations. The value of this constant is of interest from the viewpoint of checking the SU(6) symmetric quark model where $(g_A^s)_{SU(6)} = 1$ (I recall that $(g_A)_{SU(6)} = 5/3$). It is interesting to note that the value g_A^s (76)

is rather close to its value in SU(6) symmetry due to large contribution of power corrections — the taking into account of only bare quark loop would give a value $g_A^s = -1$ quite different from the one of SU(6) symmetry. Constant g_A^s enters the Bjorken sum rule for the deep-inelastic scattering of polarized electrons on polarized nucleons (see, e.g. [43])

$$\int_0^1 g_1^{p,n}(x)dx = \tfrac{1}{12}(\pm g_A + \tfrac{5}{3} g_A^s), \tag{77}$$

where the upper and lower signs refer, respectively, to proton and neutron. At g_A^s (76) the r.h.s. of (77) for neutron is negative. As can be shown at large x $g_1^n(x) \geqslant 0$. From (77) it thereby follows that at small x $g_1^n(x) < 0$. It would be interesting to experimentally check the fact that $g_1(x)$ changes its sign.

Knowing g_A and using the Goldberger-Treiman relation one can determine the pion-nucleon interaction constant

$$\frac{g_{\pi NN}^2}{4\pi} = \frac{1}{4\pi}\frac{2m^2}{f_\pi^2} g_A = 16 \pm 3$$

comparing with $(g_{\pi NN}^2/4\pi)_{exp} = 14.5$.

4.5. Discussion

The method of consideration of hadron properties in external field presented above is very general. It is applicable not only to determining static electromagnetic properties and constants of semileptonic processes of the low-lying hadronic states which were considered above but to calculating any emission and absorption amplitudes of the fields with the wavelengths much larger than hadronic dimensions.

In particular, one can in principle determine interaction constants of soft pions with hadrons. In this case, an extra term $\Delta L = j_{\mu 5}^a A_\mu^a$ is introduced into Lagrangian, where $j_{\mu 5}^a$ is the axial isovector current of quarks, A_μ^a is external field considered in the limit when its momentum q tends to zero. The transversality of the emission amplitude of the axial field $q_\mu T_\mu = 0$ (or $q_\mu T_\mu$ = terms with the current commutators) enables one to express soft pion emission amplitudes via the interaction amplitudes with the field A_μ in the limit $q = 0$. An essential moment facilitating solution of the problem of soft pion-hadron interaction is that in this case the field induced v.e.v.'s can be determined (or be reduced to the simplest ones) using the current algebra commutation relations. Various relations of such type, besides their own significance, would be of great interest also from the viewpoint of their comparison with the chiral models developed nowadays, in particular, with the bag chiral model (see, e.g. [41]).

A possible comparison of the results obtained with this method with another quark model — the constituent quark model based on SU(6) symmetry, is also interesting. As is known in such a model quarks are massive and their interaction is assumed spin independent. In the approach under consideration quarks are massless and, at least, in the zero approximation without taking into account power corrections one should, generally speaking, expect a strong spin dependence. It would be instructive to trace how (it seems; due to power corrections) SU(6) symmetry arises in this approach and in which cases one could expect its noticeable violation.

The method discussed can be extended to the case of particles containing heavy quarks. For example, charmed baryon magnetic moments, $D^* \to D\gamma$ and $D^* \to D\pi$ decay widths etc. can be calculated with it. It should be, however, borne in mind that when applying this method to nondiagonal transitions of the type $A \to B+\gamma$, $A \to B+\pi$ it works rather well only if A and B state masses are rather close, $m_A - m_B \ll m_{ch} \sim m_\rho$ and A and B are the lowest states in their channels. Otherwise, it is difficult to separate the contribution of transition of interest from nondiagonal transitions into excited states which comprise an essential background.

5. *Structure functions*

The problem of a modelless calculation of structure functions is one of the most important in QCD. Recently some progress has been achieved in this field: the valence quark contribution into structure functions at small x and intermediate $Q^2 \sim$ 5–20 GeV^2 has been calculated [42]. It was assumed in the calculation that at small x for quark distributions there is the Regge behaviour

$$u_v(x) - d_v(x) = \beta^{(\rho)}(0)x^{-\alpha_\rho(0)},$$

$$u_v(x) + d_v(x) = \beta^{(\omega)}(0)x^{-\alpha_\omega(0)}, \qquad (78)$$

the intercepts $\alpha_\rho(0) \approx \alpha_\omega(0) \approx 0.5$ were taken from experiment and coefficients $\beta^{(\rho)}(0)$ and $\beta^{(\omega)}(0)$ were determined. The idea of the calculation was QCD based consideration of the upper vertex in the diagram Fig. 26 in the vicinity of $t = (q-q')^2 = m_\rho^2$ and afterwards extrapolation of the amplitude as a function of t from $t = m_\rho^2$ to $t = 0$.

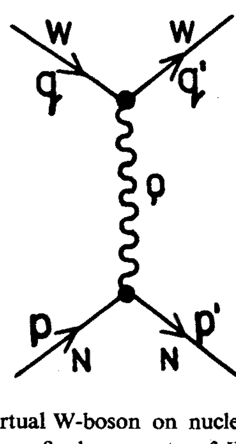


Fig. 26. The scattering amplitude of virtual W-boson on nucleon due to ρ reggeon exchange, q, p are initial, q', p' are final momenta of W and nucleon

As a result, it was obtained for small x-distributions of valence quarks in the proton (at $Q^2 \approx 10\ \text{GeV}^2$)

$$u_v(x) = 1.65x^{-1/2}, \quad d_v(x) = 0.83x^{-1/2}, \tag{79}$$

and in pion

$$u_v^{\pi^+}(x) = 0.68x^{-1/2}. \tag{80}$$

The accuracy of the coefficient in front of $x^{-1/2}$ is about 30%. Within the errors the valence quark distributions (79), (80) agree with experiment.

6. *Numerical values of vacuum condensates*

All the physical results discussed above are expressed through various vacuum condensates. Therefore their numerical values are very important. When considering baryons the most essential is the value of quark condensate $\langle 0|\bar{q}q|0\rangle$, $q = u, d$. Its low-energy limit is determined by Eq. (4). The thorough analysis made by Gasser and Leutwyler [44] taking into account the first terms of expansion in m_s showed that

$$2\,\frac{m_s}{m_u+m_d} = 25.7 \pm 2.6.$$

I take for m_s the value (28) (at normalization point $\mu = 0.5$ GeV). Then

$$\langle 0|\bar{q}q|0\rangle = -(240\ \text{MeV})^3 \pm 25\% \quad \text{at} \quad \mu = 0.5\ \text{GeV}.$$

If Λ_{QCD} is taken to be 100 MeV then the renorminvariant quantity

$$\alpha_s \langle 0|\bar{q}q|0\rangle^2 = (0.8^{+0.6}_{-0.4})10^{-4}\ \text{GeV}^6.$$

The numerical value of 5-dimensional quark-gluon condensate

$$-g\left\langle 0\left|\bar{q}\sigma_{\mu\nu}\frac{\lambda^n}{2}G^n_{\mu\nu}q\right|0\right\rangle \equiv m_0^2\langle 0|\bar{q}q|0\rangle$$

was found from selfconsistency conditions of baryon sum rules [5]. The result is $m_0^2 = (0.8 \pm 0.3)\ \text{GeV}^2$.

The value of gluon condensate $\langle 0|(\alpha_s/\pi)G^n_{\mu\nu}G^n_{\mu\nu}|0\rangle = 0.012\ \text{GeV}^4$ (see [1]) was taken in the most of ITEP calculations. It is possible that its real value is 30–40% higher (for a detailed discussion see Ref. [39]).

7. *Conclusion*

As has been shown, the above described nonperturbative method in QCD based on the operator expansion explains and predicts a great number of phenomena in the low energy hadron physics. The facts concerning light hadrons enumerated above do not exhaust all the results obtained with this method: there are many of them concerning hadrons with open and hidden charm (or beauty) which were not included in these lectures.

The possibilities of the method are far from being exhausted. There is a hope, more, a certainty that a lot of new unknown parameters both of heavy and light hadrons would be possible to calculate with it.

At the same time, the method is not universal. In some cases two of its basic requirements — the smallness of higher power corrections and the smallness of higher excited states contributions are in contradiction with each other and the method stops working. Therefore, as a rule, application of this method demands calculations of higher terms of operator expansion and a proof of the fact that the choice of a continuum model does not affect the answer. Without fulfilling these requirements the results obtained with this method cannot be believed reliable.

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