SOME ASPECTS OF SUPERSYMMETRIC COMPOSITE MODELS OF QUARKS AND LEPTONS*

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(*Received November 5, 1984*)

Supersymmetric confining gauge theories are interesting candidates for a theory of quark-lepton substructure. We review some general features of supersymmetric composite models and illustrate various technical developments by means of a toy model based on the group U(6). We also discuss some constraints on models with composite W-vector bosons.

PACS numbers: 12.35.–i

1. Are quarks and leptons composite?

The topic of supersymmetric preon models is very speculative. There is neither experimental evidence for supersymmetry or quark-lepton substructure [1] nor does a satisfactory theoretical model exist. Yet the considerable amount of recent work [2] on this subject is not without motivation: it is based on the belief that the Higgs sector of the standard model is only a low energy effective Lagrangian and the experience that the dynamical understanding of a mass spectrum generally involves more fundamental constituents. Indeed, focusing on the family replication and the quark-lepton mass spectrum, it seems difficult to escape the problem of quark-lepton substructure. If quarks and leptons are composite, however, their structure must be very different from the bound states we know. Contrary to atoms, nuclei and hadrons, quarks and leptons are very pointlike, i.e., their size $r_{q,l}$ is much smaller than their Compton wavelength:

$$\xi_{q,l} = \left(\frac{1}{m_{q,l}}\right)\frac{1}{r_{q,l}} \gg 1. \tag{1}$$

This inequality represents the main dynamical problem of composite quarks and leptons. 't Hooft [3] has shown that unbroken chiral symmetries imply the existence of massless

* Presented at the XXIV Cracow School of Theoretical Physics, Zakopane, June 6–19, 1984 and the International School of Physics "Enrico Fermi", XCII Course, "Elementary Particles", Varenna, June 26–July 6, 1984.

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composite fermions. Furthermore, it appears that the most interesting models which satisfy 't Hooft's consistency conditions require spin-0 preons in addition to spin-$\frac{1}{2}$ preons. Fundamental scalars, however, are "unnaturally" light unless they are part of a supersymmetric theory. Thus one is led to supersymmetric theories as the most promising candidates for a theory of quark-lepton substructure. Indeed, as we will see in the next section, supersymmetric confining gauge theories lead almost unavoidably to light composite fermions.

An important issue in the context of composite models is the nature of the weak interactions. It is conceivable that the W-vector bosons are also composite and that the substructure scale is related to the Fermi scale, i.e., $1/r_{q,l} \sim G_F^{-\frac{1}{2}} \sim 300$ GeV. It is believed [1, 2] that composite W-bosons are consistent with the neutral current phenomenology as well as the successful mass predictions of the standard model if the heavy bound states predicted by the preon theory have masses of the order of 1 TeV.

In these lectures we will discuss a supersymmetric toy model in which the left-handed particles of one family and the W-bosons are bound states, and we will use it to illustrate some techniques which are important in the context of supersymmetric composite models. In Section 2 we discuss the idea of quasi-Goldstone fermions and the structure of their residual interactions. Section 3 deals with the coset space U(6)/SU(2) × U(4), and in Section 4 a particular preon model is described which realizes this symmetry breaking. Section 5 contains some remarks on how the U(6) toy model may be extended to a more realistic theory and some constraints on models with composite W-bosons are listed.

2. *Quasi-Goldstone fermions*

The only known bound states, which are light compared to their inverse size, are the pions for which one has $\xi_\pi \sim 2.0$ [4] (cf. Eq. (1)). They arise as pseudo-Goldstone bosons from the spontaneous breaking of chiral invariance. As the Goldstone mechanism plays a crucial role in supersymmetric preon models, let us briefly recall some features of dynamical symmetry breaking in QCD. In the case of two flavours $u_{L,R}$ and $d_{L,R}$ the QCD Lagrangian possesses the (approximate) global symmetry

$$G = SU(2)_L \times SU(2)_R \times U(1)_v, \tag{2}$$

where the two SU(2) subgroups are generated by the charges

$$Q^A_{L,R} = \tfrac{1}{2}(T^A \mp X^A),$$

with

$$T^A = \int d^3x \bar{q}(x)\gamma_0 \tfrac{1}{2}\tau^A q(x),$$

$$X^A = \int d^3x \bar{q}(x)\gamma_0\gamma_5 \tfrac{1}{2}\tau^A q(x), \quad q = \begin{pmatrix} u \\ d \end{pmatrix}. \tag{3}$$

The formation of the vacuum expectation values

$$\langle 0|\bar{u}u|0\rangle = \langle 0|\bar{d}d|0\rangle \sim \Lambda^3_{QCD} \tag{4}$$

breaks the chiral symmetry G dynamically to the diagonal subgroup $H = SU(2)_v \times U(1)_v$. The spontaneous breaking of the axial generators X^A leads to an isotriplet of (almost) massless (pseudo) Goldstone bosons, the pions,

$$\pi^A(x) \sim \bar{q}(x)\gamma_5 \tfrac{1}{2}\tau^A q(x). \tag{5}$$

It is a crucial feature of the Goldstone phenomenon that the interactions of the Goldstone bosons at energies small compared to Λ_{QCD} are determined entirely by the coset space G/H [5] and do not depend on details of the dynamics of the underlying theory. In order to obtain the effective low energy Lagrangian one constructs a representation of the full group G acting on the pion fields where the broken generators X^A are realized non-linearly,

$$\frac{1}{i}[T^A, \pi^B] = \varepsilon_{ABC}\pi^C,$$

$$\frac{1}{i}[X^A, \pi^B] = 2f_\pi\delta^{AB} + \ldots, \tag{6}$$

and $f_\pi \sim \Lambda_{QCD}$ is the pion decay constant:

$$\langle 0|\bar{q}(0)\gamma_\mu\gamma_5 \tfrac{1}{2}\tau^A q(0)|\pi^B(p)\rangle = ip_\mu\delta^{AB}f_\pi. \tag{7}$$

The effective Lagrangian describing the $\pi\pi$-interaction is now obtained by demanding that its variation with respect to T^A and X^A is a total derivative. This yields the result [5]

$$\mathscr{L}_{eff} = \tfrac{1}{2}\partial^\mu\pi^A\partial_\mu\pi^A - \frac{1}{f_\pi^2}\pi^A\pi^A\partial^\mu\pi^B\partial_\mu\pi^B + O(\pi^6), \tag{8}$$

which incorporates the low energy theorems of current algebra.

It is expected that in general also in supersymmetric (SUSY) confining gauge theories dynamical symmetry breaking will take place. SUSY gauge theories are built from chiral superfields $\phi_i = (\chi_i, \eta_{Li})$, containing complex scalars χ_i and left-handed Weyl fermions η_{Li}, and vector superfields $V^I = (\lambda_L^I, V_\mu^I)$, containing Weyl fermions λ_L^I and vector bosons V_μ^I. Vacuum expectation values of scalar fields

$$\langle 0|\chi_i\chi_j|0\rangle \sim \Lambda_{HC}^2, \tag{9}$$

which are of the order of the hypercolour scale Λ_{HC}, can break the symmetry G of the Lagrangian to a subgroup H. Due to supersymmetry, the resulting Goldstone bosons have to be part of chiral superfields $\phi_i = (\phi_i, \psi_{Li})$. Applying Weinberg's method to the supersymmetric case an effective low energy Lagrangian for the Goldstone superfields can be constructed [6] which has the generic form

$$\mathscr{L}_{eff} = -\frac{1}{f^2}\bar{\phi}_i\bar{\phi}_j\phi_i\phi_j|_{\theta\theta\bar{\theta}\bar{\theta}} + \ldots$$

$$= -\frac{1}{f^2}\phi_i^*\phi_i\partial^\mu\phi_j^*\partial_\mu\phi_j - \frac{1}{2f^2}\bar{\psi}_{Li}\gamma^\mu\psi_{Li}\bar{\psi}_{Lj}\gamma_\mu\psi_{Lj} + \ldots, \qquad f \sim \Lambda_{HC}. \tag{10}$$

The almost unavoidable appearance of massless composite fermions in supersymmetric theories leads naturally to the conjecture [7, 6], that quarks and leptons may be identified as "quasi-Goldstone fermions", i.e., as superpartners of Goldstone bosons. In such a scenario (cf. Fig. 1), light composite Goldstone supermultiplets arise from a spontaneous symmetry breaking at a large mass scale (e.g. 1–100 TeV). Mass splittings within the Goldstone multiplets which render the quasi-Goldstone fermions lighter than their scalar superpartners can occur as a consequence of soft SUSY breaking, explicit breaking of the original global symmetry — for instance through gauge interactions — and unbroken chiral symmetries [8]. Some interesting explicit examples have been discussed by Lerche and Lüst [9].

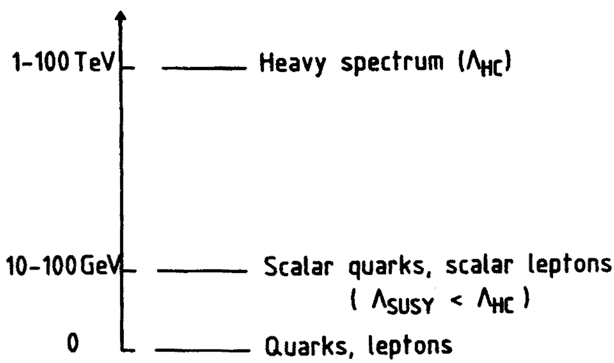

Fig. 1. Possible mass spectrum of scalar and fermionic components of Goldstone supermultiplets in supersymmetric preon models

An important feature of the effective Lagrangian Eq. (10) is the current-current self--interaction of the fermions. This suggests that the weak interactions of quarks and leptons may be residual interactions which are mediated by the exchange of composite W-bosons, a possibility which we will pursue in the following sections. The current-current form of the residual fermion interactions in Eq. (10) is a special property of quasi-Goldstone fermions. If quarks and leptons are (pseudo) Goldstone fermions arising from the spontaneous breakdown of an extended supersymmetry, as suggested by Bardeen and Višnjić [10], their residual interactions will involve derivatives, in accord with the low energy theorems for Goldstone particles,

$$\mathscr{L}_{\text{eff}}^{\text{GF}} = -\frac{1}{4f^4}\bar{\psi}_i\gamma^\mu\partial^\nu\psi_i\bar{\psi}_j\gamma_\nu\partial_\mu\psi_j + \cdots, \tag{11}$$

and can therefore not be identified as weak interactions.

3. *The coset space $U(6)/SU(2)\times U(4)$*

In models with composite W-bosons weak isospin has to be introduced as a global symmetry. Furthermore one has to ensure that quarks and leptons couple universally to the composite W-bosons. This can be achieved by imposing a global Pati-Salam SU(4) invariance [11] with lepton number as fourth colour. In the case of residual weak interactions parity violation must be due to a different bound state structure of left- and right-handed fermions. It is therefore natural to consider in first approximation only the left-handed fields as composite and to treat, as in the Abbott-Farhi model [12], right-handed fields as elementary spectators.

The left-handed fermions u_L^α, d_L^α, ν_L and e_L^- of one family transform with respect to SU(2) × SU(4) as

$$\begin{pmatrix} u_L^\alpha \nu_L \\ d_L^\alpha e_L^- \end{pmatrix} \equiv (\psi_L^{ia}) \sim (2,4). \tag{12}$$

It is easy to see that one can obtain the ψ_L^{ia} as quasi-Goldstone fermions from the spontaneous symmetry breaking U(6) → U(2) × U(4) [8, 13–16]:

$$U(6): \left(\begin{array}{c|c} U(2) & (2,4^*) \\ \hline (2^*,4) & U(4) \end{array}\right). \tag{13}$$

The 16 Goldstone bosons transform as (2, 4*)+(2*, 4) and can be embedded into 8 chiral multiplets $\phi_i^a \equiv (\phi_i^a, \psi_{iL}^a)$. The assignment of Goldstone bosons to chiral superfields is not unique. The only restriction is that the coset space G/H is embedded in a Kähler manifold [17]. The above choice of 8 chiral multiplets is clearly the minimal one and it is possible because U(6)/U(2) × U(4) is a Grassmann manifold, i.e., a special Kähler manifold. One can also associate, however, the 16 Goldstone bosons with 16 chiral superfields [13]. It is a dynamical question which case is realized. Another important point concerns the two U(1) factors in H = U(2) × U(4). One or both of them may also be spontaneously broken. This leads to one or two additional neutral chiral multiplets and as we will see, changes the low energy effective Lagrangian of the Goldstone multiplets ϕ_i^a.

It turns out that the relevant coset space is U(6)/SU(2) × U(4). The broken U(1) factor yields one neutral Goldstone superfield ϕ, the "novino". The effective Lagrangian for the superfields ϕ_i^a and ϕ can be constructed in a straightforward manner following Weinberg's method [5]. The first step is the construction of a non-linear realization of the U(6) algebra. The generators T_β^α ($\alpha, \beta = 1, ..., 6$) which satisfy the commutation relation

$$[T_\beta^\alpha, T_\delta^\gamma] = \delta_\delta^\alpha T_\beta^\gamma - \delta_\beta^\gamma T_\delta^\alpha \tag{14}$$

are split into the unbroken generators ($\alpha = (i, a)$; $i = 1, 2$; $a = 3, ..., 6$)

$$L_j^i = T_j^i - \tfrac{1}{2}\,\delta_j^i T_k^k, \quad L_b^a = T_b^a, \tag{15}$$

which belong to the unbroken subgroup SU(2) × U(4), and the broken generators

$$X_i^a = T_i^a, \quad X_a^i = T_a^i, \quad X = \frac{1}{\sqrt{2}}\,T_i^i. \tag{16}$$

The Goldstone superfields ϕ_i^a and ϕ transform linearly with respect to H = SU(2) × U(4):

$$[L_j^i, \phi_k^a] = \delta_k^i \phi_j^a - \tfrac{1}{2}\,\delta_j^i \phi_k^a,$$
$$[L_b^a, \phi_i^c] = -\delta_b^c \phi_i^a. \tag{17}$$

For the broken generators in G/H a non-linear realization has to be constructed $\left[A, B, \ldots = \binom{a}{i}, \binom{i}{a}, \binom{i}{i}\right]$:

$$[X_{A1}\phi_B] = F_{AB}(\phi), \tag{18}$$

where $F_{AB}(\phi)$ satisfies the "Jacobi identities" [5] $\left(P, Q = \binom{i}{j}, \binom{a}{b}\right)$:

$$\begin{aligned} [X_A, [X_B, \phi_C]] - [X_B, [X_A, \phi_C]] &= [[X_A, X_B], \phi_C], \\ [X_A, [L_P, \phi_B]] - [L_P, [X_A, \phi_B]] &= [[X_A, L_P], \phi_B], \\ [L_P, [L_Q, \phi_A]] - [L_Q, [L_P, \phi_A]] &= [[L_P, L_Q], \phi_A]. \end{aligned} \tag{19}$$

Eqs. (19) ensure that the functions $F_{AB}(\phi)$ define a representation of the U(6) Lie algebra. One can check by explicit calculation that an exact solution of Eqs. (19), which is unique up to redefinition of the Goldstone superfields, is given by [16]

$$\begin{aligned} &\frac{1}{i}[X^a_i, \phi^b_j] = \frac{1}{f_2}\phi^a_j\phi^b_i, \quad \frac{1}{i}[X^i_a, \phi^b_j] = f_2\delta^i_j\delta^b_a, \\ &\frac{1}{i}[X^a_i, \phi] = \frac{i}{\sqrt{2}}\phi^a_i, \quad \frac{1}{i}[X^i_a, \phi] = 0, \\ &\frac{1}{i}[X, \phi^a_i] = -\frac{i}{\sqrt{2}}\phi^a_i, \quad \frac{1}{i}[X, \phi] = f_1. \end{aligned} \tag{20}$$

Zumino has shown [17] that in general the Lagrangian of a supersymmetric σ-model takes the form

$$\mathscr{L} = K(\bar{\phi}_A, \phi_A)|_{\theta\theta\bar{\theta}\bar{\theta}}, \tag{21}$$

where K is the Kähler potential of the associated Kähler manifold. Therefore the effective Lagrangian for Goldstone superfields ϕ^a_i and ϕ can be constructed by making an SU(2) $\times$ U(4) invariant Ansatz for K and by demanding that a variation of K yields a sum of chiral and antichiral superfields:

$$\begin{aligned} [X_A, K(\bar{\phi}^i_a, \phi; \phi^a_i, \phi)] = {} & h^{(0)}_A + h^{(1)}_{AB}\phi_B + h^{(2)}_{ABC}\phi_B\phi_C + \ldots \\ & + \bar{h}^{(1)}_{AB}\bar{\phi}_B + \bar{h}^{(2)}_{ABC}\bar{\phi}_B\bar{\phi}_C + \ldots . \end{aligned} \tag{22}$$

Eqs. (21) and (22) imply that the variation of the Lagrangian is a total derivative and that the action is consequently invariant under U(6) transformations. Using the described procedure one finds [16]:

$$K = \bar{\phi}^i_a\phi^a_i + \bar{\phi}\phi - \frac{1}{2f_2^2}\bar{\phi}^i_a\bar{\phi}^j_b\phi^a_j\phi^b_i + \frac{1}{4f_1^2}\bar{\phi}^i_a\bar{\phi}^j_b\phi^a_i\phi^b_j + F\bar{\phi}^i_a\bar{\phi}\phi^a_i\phi + G\bar{\phi}\bar{\phi}\phi\phi + \ldots \tag{23}$$

where F and G are unconstrained parameters. This arbitrariness of the Kähler potential is related to the presence of one "quasi-Goldstone boson" in ϕ and reflects the different ways in which the odd-dimensional coset space $U(6)/SU(2)\times U(4)$ can be embedded in a Kähler manifold.

Of particular interest are the quark-lepton residual interactions which are contained in Eq. (23). Using the identity

$$\vec{\tau}^i_j\vec{\tau}^k_l = 2(\delta^i_l\delta^k_j - \tfrac{1}{2}\delta^i_j\delta^k_l) \tag{24}$$

one obtains [16]:

$$\mathscr{L}^{(q,l)}_{\text{eff}} = -\frac{1}{2f_2^2}[\bar{\psi}^i_{\text{L}a}\gamma_\mu(\tfrac{1}{2}\vec{\tau})^j_i\psi^a_{\text{L}j}]^2 - \frac{f_2^2-f_1^2}{2f_2^2}[\tfrac{1}{2}\bar{\psi}^i_{\text{L}a}\gamma_\mu\psi^a_{\text{L}i}]^2 + \dots . \tag{25}$$

Eq. (25) contains the phenomenologically wanted isovector exchange term as well as an isoscalar exchange term whose presence is a familiar problem of models with composite W-bosons [1, 2]. Indeed, in the limit $v_1 \to 0$, i.e., in the absence of the novino, the result is unacceptable because isovector and isoscalar contributions are of equal magnitude. In the case $v_1 \approx v_2$, however, which one may expect in a preon theory with a single scale, the isoscalar exchange term is suppressed. We are thus led to the unexpected result that in supersymmetric preon models with composite W-bosons there is a direct relation between the weak interactions of quarks and leptons and the existence of a new neutral Goldstone superfield, the novino.

In this section we have shown how one can construct the effective Lagrangian for Goldstone superfields in a direct pedestrian way. More elegant methods which are crucial if one wants to construct the σ-models beyond the quartic terms can be found in the literature [18]. The very interesting subject of gauged supersymmetric σ-models [19] is beyond the scope of these lectures.

4. *A U(6) model*

The simplest supersymmetric "preon model" with global U(6) invariance is a SU(2) gauge theory with 6 doublets of chiral superfields $\chi^p_\alpha = (\tilde{\chi}^p_\alpha, \eta^p_{\text{L}\alpha})$, where $\alpha = 1, \dots, 6$ denotes the U(6) flavour index and $p = 1, 2$ is the SU(2) hypercolour index. The interaction Lagrangian for the chiral multiplets χ^p_α and the gauge vector multiplets $V = \frac{1}{2}\tau^{\text{I}}V^{\text{I}}$, $V^{\text{I}} = (\lambda^{\text{I}}_{\text{L}}, V^{\text{I}}_\mu)$, is given by [20]

$$\mathscr{L} = \int d^4\theta \bar{\chi}^\alpha_p(e^{2gV})^p_q\chi^q_\alpha. \tag{26}$$

The simplest gauge invariant composite operators are the bilinear chiral superfields

$$\Phi_{\alpha\beta} = \varepsilon_{pq}\chi^p_\alpha\chi^q_\beta \tag{27}$$

and the vector superfields

$$J^\alpha_\beta = \bar{\chi}^\alpha_p(e^{2gV})^p_q\chi^q_\beta, \tag{28}$$

which, except for the anomalous U(1) factor J^α_α, represent the conseved currents of the theory:

$$D^2 J^\alpha_\beta = \bar{D}^2 J^\alpha_\beta = 0. \tag{29}$$

This U(6) "preon model" corresponds to SUSY QCD with two colours and three flavours. As the fundamental representation of SU(2) is pseudoreal, the global symmetry is U(6) rather than U(3)×U(3). Vacuum expectation values of the operators $\Phi_{\alpha\beta}$ and J^α_β can break the U(6) symmetry down to $(SU(2))^3$ ($i = 1, 2$; $a_1 = 3, 4$; $a_2 = 5, 6$):

$$\langle\Phi_{ij}\rangle = v_1^2\varepsilon_{ij}, \quad \langle\Phi_{a_1b_1}\rangle = v_2^2\varepsilon_{a_1b_1},$$
$$\langle\Phi_{a_2b_2}\rangle = v_3^2\varepsilon_{a_2b_2}, \quad \langle J^i_j\rangle = \tilde{v}_1^2\delta^i_j,$$
$$\langle J^{a_1}_{b_1}\rangle = \tilde{v}_2^2\delta^{a_1}_{b_1}, \quad \langle J^{a_2}_{b_2}\rangle = \tilde{v}_3^2\delta^{a_2}_{b_2}. \tag{30}$$

Constraints [21] on the possible values of $v_1^2, \ldots, \tilde{v}_3^2$ can be obtained by breaking the U(6) invariance explicitly to $(SU(2))^3$ through the superpotential

$$g_m = -\tfrac{1}{2}\, m_1\varepsilon^{ij}\Phi_{ij} - \tfrac{1}{2}\, m_2\varepsilon^{a_1b_1}\Phi_{a_1b_1} - \tfrac{1}{2}\, m_3\varepsilon^{a_2b_2}\Phi_{a_2b_2} \tag{31}$$

and considering the chiral limit $m_i \to 0$. The effective Lagrangian approach [22], analyticity arguments [23] and instanton calculations [24] lead to the following mass dependence of the chiral condensates:

$$v_i^2 \sim \frac{1}{m_i}(m_1m_2m_3)^{1/2}. \tag{32}$$

If one demands that in the chiral limit all vacuum expectation values are finite, only two cases are possible: either the full U(6) symmetry remains unbroken ($v_1^2 = v_2^2 = v_3^2 = 0$) or the unbroken subgroup is SU(2)×U(4) ($v_1^2 \neq 0$, $v_2^2 = v_3^2 = 0$). Constraints on the vector condensates follow from the SUSY Dashen formulae [25, 26]

$$f_A(M^2)_{AB}f_B = \tfrac{1}{2}\langle 0|\,[X_A, [X_B, \mathscr{L}_F]]\,|0\rangle,$$
$$f_AM_{AB}f_B = \langle 0|\,[X_A, [X_B, \mathscr{L}_S]]\,|0\rangle, \tag{33}$$

where X_A are the broken charges, f_A the related decay constants and M_{AB} the pseudo-Goldstone boson mass matrix. $\mathscr{L}_F$ and $\mathscr{L}_S$ are the fermionic and scalar terms in the symmetry breaking part of the Lagrangian:

$$\mathscr{L}_F = \tfrac{1}{2}\frac{\partial^2 g_m}{\partial\chi^p_\alpha\partial\chi^q_\beta}\eta^p_\alpha\eta^q_\beta + \text{c.c.},$$

$$\mathscr{L}_S = \tfrac{1}{2}\frac{\partial^2 g_m}{\partial\chi^p_\alpha\partial\chi^q_\beta}\tilde{\chi}^p_\alpha\tilde{\chi}^q_\beta. \tag{34}$$

Demanding that the decay constants f_1 and f_2 of the Goldstone superfields ϕ and ϕ_i^a remain finite in the chiral limit, one finds [21] that the vector condensates follow the pattern of the chiral condensates: either the U(6) symmetry remains unbroken ($\tilde{v}_1^2 = \tilde{v}_2^2 = \tilde{v}_3^2 = 0$) or the unbroken subgroup is SU(2) × U(4) ($\tilde{v}_1^2 \neq 0$, $\tilde{v}_2^2 = \tilde{v}_3^2 = 0$). The size of the condensates is expected to be large, i.e., $\alpha(v_1^2) \sim \alpha(\tilde{v}_1^2) \sim 1$ ($\alpha = (g^2/4\pi)$).

The analysis of the bilinear condensates suggests that the model may have a phase in which the full U(6) symmetry is unbroken. It is a remarkable feature of the model that this possibility is indeed compatible [27] with 't Hooft's anomaly conditions [3]. The symmetry of the classical Lagrangian is $G_{cl} = U(6) \times U(1)_R$ where the R symmetry acts differently on the scalar and the fermionic components of the superfield χ_α^p:

$$\tilde{\chi}_\alpha^p \to \tilde{\chi}_\alpha^p, \qquad \eta_{L\alpha}^{\;p} \to e^{-i\gamma}\eta_{L\alpha}^{\;p}. \tag{35}$$

Instanton effects reduce the classical invariance G_{cl} to $G_{qu} = SU(6) \times U(1)_X$ with [16]

$$X = T_\alpha^\alpha + 3R. \tag{36}$$

The preons $\eta_{L\beta}^{\;\alpha}$, the gauginos $\lambda_{Lq}^{\;p}$ and the composite fermions $\psi_{L\alpha\beta}$, which are contained in the chiral superfields $\Phi_{\alpha\beta}$, transform with respect to $SU(6) \times U(1)_X$ as follows:

$$(\eta_{L\alpha}^{\;p}) \sim 2(\bar{6})_{-2}, \qquad (\lambda_{Lq}^{\;p}) \sim 3(1)_3, \qquad (\psi_{L\alpha\beta}) \sim (\overline{15})_{-1}. \tag{37}$$

At the preon level one obtains for the triangle anomalies (in units of $(6)_1$):

$$\begin{aligned} [SU(6)]^3 &: -2, \\ X[SU(6)]^2 &: 2 \cdot (-2) = -4, \\ X^3 &: 12 \cdot (-2)^3 + 3 \cdot 3^3 = -15. \end{aligned} \tag{38}$$

In terms of the composite fermions, the anomalies read:

$$\begin{aligned} [SU(6)]^3 &: K(\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}) = -(6-4) = -2, \\ X[SU(6)]^2 &: (-1)C(\overline{\begin{smallmatrix}\square\\\square\end{smallmatrix}}) = (-1)(6-2) = -4, \\ X^3 &: 15(-1)^3 = -15, \end{aligned} \tag{39}$$

where the quantities K and C in Eq. (39) are defined [3] through

$$\begin{aligned} \mathrm{tr}\,[\lambda^a(R)\lambda^b(R)] &= C(R)\,\mathrm{tr}\,[\lambda^a(\square)\lambda^b(\square)], \\ \mathrm{tr}\,[\{\lambda^a(R), \lambda^b(R)\}\lambda^c(R)] &= K(R)\,\mathrm{tr}\,[\{\lambda^a(\square), \lambda^b(\square)\}\lambda^c(\square)]. \end{aligned} \tag{40}$$

The matching of the anomalies for all $SU(6) \times U(1)_X$ currents does not necessarily imply that a phase with unbroken SU(6) symmetry exists. In the "Higgs phase", where the SU(2) gauge invariance is spontaneously broken through large vacuum expectation values of the fundamental scalar fields $\tilde{\chi}_\alpha^p$, i.e., $\alpha(\langle\tilde{\chi}\rangle^2) \ll 1$, the unbroken global symmetry is $SU(2) \times U(4)$ [16]. Up to U(6) transformations, the only solution to the D-term constraint

$$\langle \tilde{\chi}_p^{*\alpha} \rangle (\tau^I)^p_q \langle \tilde{\chi}^q_\alpha \rangle = 0, \quad I = 1, \ldots, 3, \tag{41}$$

reads [28]:

$$\langle \tilde{\chi}^p_i \rangle = v\delta^p_i, \quad \langle \tilde{\chi}^p_a \rangle = 0. \tag{42}$$

This corresponds to the result obtained in the "confining phase" (cf. Eq. (30)) with $v_1^2 = \tilde{v}_1^2 = v^2$. According to the idea of complementarity [29], there is no phase transition between the "Higgs phase" and the "confining phase". This suggests that also in the strong coupling regime the unbroken global symmetry is $SU(2) \times U(4)$ rather than U(6).

An important question concerns the existence of heavy vector bosons in the theory. In analogy with QCD one expects naively that the currents J^α_β create 36 massive vector supermultiplets from the vacuum. The suppression of the isoscalar exchange term in the effective Lagrangian Eq. (25) suggests [16] that among these 36 vector bosons the isotriplet of W-bosons plays a special role. The additional 33 vector bosons should be either heavier or weaker coupled than the W-bosons. Such a qualitative difference can only be caused by the scalar condensates. A means to study the effect of scalar condensates on the physical spectrum of a theory are the SVZ-sum rules [30]. In the standard manner, the 2-point function of the currents J^α_β in the Euclidean region can be evaluated using dispersion relations and an Ansatz for the physical spectrum on one side, and in perturbation theory (including power corrections) on the other side. A straightforward supergraph calculation (cf. Fig. 2) yields the result [21]:

$$\begin{aligned} &i \int d^4x e^{iqx} \langle T(J^\alpha_\beta(x, \theta, \bar{\theta}) J^\gamma_\delta(0, 0, 0)) \rangle \\ &= \tfrac{1}{2} P_T \delta^4(\theta) q^2 \pi^{\alpha\gamma}_{\beta\delta}(q^2) - \tfrac{1}{2}(P_1 - P_2)\delta^4(\theta)\delta^\alpha_\delta \delta^\gamma_\beta (\langle \bar{\chi}^\alpha \chi_\alpha \rangle - \langle \bar{\chi}^\beta \chi_\beta \rangle) \\ &\quad - \tfrac{1}{2} \delta^4(\theta) \delta^\alpha_\delta \delta^\gamma_\beta (\langle \bar{\chi}^\alpha \chi_\alpha \rangle + \langle \bar{\chi}^\beta \chi_\beta \rangle), \end{aligned} \tag{43a}$$

with

$$\begin{aligned} \pi^{\alpha\gamma}_{\beta\delta}(q^2) &= \delta^\alpha_\delta \delta^\gamma_\beta \left[-\frac{1}{8\pi^2} \ln \frac{q^2}{\Lambda^2} + \frac{1}{q^2} (\langle \bar{\chi}^\alpha \chi_\alpha \rangle + \langle \chi^\beta \chi_\beta \rangle) \right] \\ &-2 \frac{2\pi\alpha}{(q^2)^2} \langle (\bar{\chi}^\alpha \tau^A \chi_\beta)(\bar{\chi}^\gamma \tau^A \chi_\delta) \rangle + 2 \frac{(2\pi\alpha)^2}{(q^2)^3} \langle (\bar{\chi}^\alpha \tau^A \chi_\beta)(\bar{\chi}^\gamma \tau^A \chi_\delta)(\bar{\chi}\chi) \rangle + \ldots, \\ \alpha &= \frac{g^2}{4\pi}, \quad \bar{\chi}\chi \equiv \sum_\varepsilon \bar{\chi}^\varepsilon \chi_\varepsilon. \end{aligned} \tag{43b}$$

The polarization tensor $\Pi^{\alpha\gamma}_{\beta\delta}(q^2)$ has 5 irreducible components with respect to the unbroken $SU(2) \times U(4)$ symmetry: (3, 1), (1, 1), (2, 4), (1, 15) and (1, 1)′. In the channels (1, 1) and (2, 4), the Goldstone superfields ϕ and ϕ_{ia} contribute in addition to the vector superfields. Using the results for the bilinear condensates (30) and factorization for higher condensates, one finds large power corrections in the (3, 1), (1, 1) and (2, 4) channels which are directly related to the mass of the composite W-bosons and the decay constants

f_1 and f_2 [21]:

$$M_W^2 \approx 2\pi\alpha_W \langle \bar{\chi}\chi \rangle, \tag{44a}$$

$$f_1^2 \approx f_2^2 \approx \langle \bar{\chi}\chi \rangle, \quad \left| \frac{f_1^2 - f_2^2}{f_2^2} \right| \ll 1. \tag{44b}$$

The sum rule in the (3, 1) channel, which yields the standard model mass formula (44a), is identical with a sum rule [31] which has previously been derived in the context of the Abbott-Farhi model.

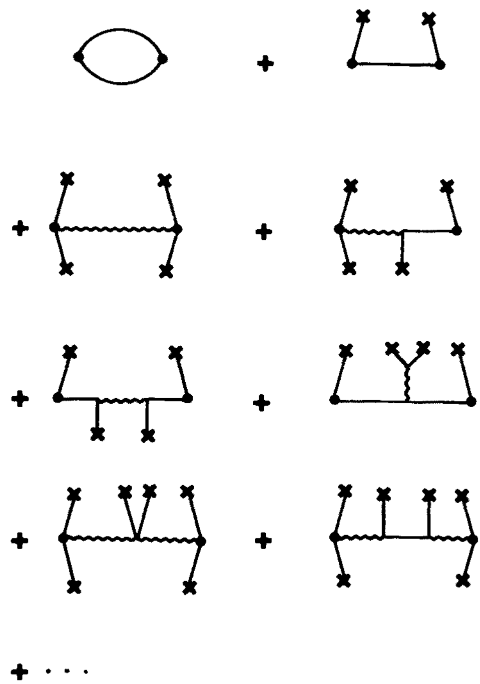

Fig. 2. Supergraphs contributing to the two-point function of the U(6)-currents J^{α}_{β} (cf. Eq. 43)). Full lines denote chiral superfield propagators, wavy lines vector superfield propagators. Crosses indicate scalar condensates

The result Eq. (44) shows clearly the importance of scalar condensates for the vector boson mass spectrum. The W-mass is directly related to the size of the condensate and, as a consequence of Eqs. (25) and (44b), the additional 33 vector bosons must be substantially heavier than the W-bosons, if they exist at all. Given a spectrum of resonances and the condensates of the underlying theory sum rules constrain the resonance parameters. In general, it is not possible to prove the existence or absence of certain states by means of sum rules. The qualitative features of the sum rules [21] obtained from Eq. (43) indicate, however, that the physical states in the strong coupling regime may be completely identical with the ones in the Higgs phase and that an additional strongly interacting spectrum of heavy states may not exist. This would mean that the intuition derived from ordinary

QCD is totally misleading and that the spectrum of physical states can essentially be read off from the classical Lagrangian, a question which clearly deserves further studies. Even if such radical conclusions are unjustified, there remains at least a striking similarity in the vacuum structure between the Higgs phase and the confining phase.

5. *Towards a realistic preon model*

In the previous sections we have discussed the idea of quasi-Goldstone fermions and we have illustrated some techniques, which are used in the context of supersymmetric composite models, by means of a U(6) toy model. This "preon model" yields the left--handed sector of one family and the W-bosons as bound states. The right-handed part of one family can be incorporated by extending the group U(6) to U(6) × U(6) [8, 13, 15, 32]. The preons now consist of two sextets of chiral superfields which are usually chosen to transform as $N+N^*$ with respect to the hypercolour group SU(N). A multiplicity of families, which are labelled by means of a discrete or a broken U(1) symmetry, could emerge in such models as a consequence of the dimension of the hypercolour group [33], but no fully satisfactory example has been found so far. In U(6) × U(6) models with composite W-bosons one has to understand the origin of parity violation. It is conceivable that, like in ordinary left-right symmetric models, parity is broken spontaneously, yet a detailed mechanism has not yet been proposed. It is also possible, although not particularly attractive, to break parity explicitly by choosing SU(2) × SU(2)′ as hypercolour group with different coupling constants for the two SU(2) subgroups [32].

In general, one expects in models with composite W-bosons and global SU(2) × SU(4) invariance additional composite vector bosons which transform as (1, 15) [34, 35]. The effect of these V-bosons on low energy weak interactions within one family can be estimated and one finds a lower mass bound of ~ 500 GeV [34]. Much more stringent bounds are obtained if families are included, especially from the process $K_L^0 \to \mu^{\pm}e^{\mp}$ [11, 36]. The exchange of the leptoquark [11] among the V-bosons yields the effective four fermion Lagrangian (cf. Fig. 3)

$$L_{V_3} = -\frac{g_V^2}{2M_{V_3}^2}(\bar{\mu}_L\gamma^\mu s_L^\alpha \bar{d}_{L\alpha}\gamma_\mu e_L + \text{c.c.}), \tag{45}$$

where the coupling constant g_V satisfies the bound [34] $g_V^2/4\pi > \alpha_s M_V^2 \sim 0.1$. From the experimental bound [37]

$$\frac{\Gamma(K_L^0 \to \mu^{\pm}e^{\mp})}{\Gamma(K^- \to \mu^-\bar{\nu}_\mu)} < 2\cdot 10^{-6}$$

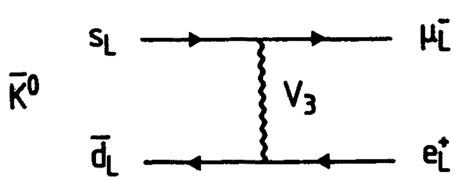

Fig. 3. Contribution of the leptoquark V_3 to the decay $\bar{K}^0 \to \mu^- e^+$

one obtains

$$\frac{M_{V_3}}{M_W} > \left[\frac{\alpha_s^2}{\alpha_W^2 \sin^{-2}\theta_c} \frac{\Gamma(K^+ \to \mu^+ \nu_\mu)}{\Gamma(K_L^0 \to \mu^\pm e^\mp)}\right]^{1/4} > 0.8 \cdot 10^2. \tag{46}$$

In models with a single scale one is thus faced with the problem of explaining dynamically vector boson masses which differ by a factor of about 100! The bound Eq. (46) is based on the assumption that the V-bosons couple universally to different families. This is indeed very likely to be a consequence of the same mechanism which enforces a universal coupling of the W-bosons. On the other side, without a "standard composite model" providing a convincing explanation for families, one can always hope for some mechanism which circumvents the bound Eq. (46). In supersymmetric theories, for instance, the V-bosons may simply not exist, as we saw in the previous section.

Within SUSY composite models, the weak interactions may also be treated as fundamental gauge interactions. Such models [8, 13, 32] are based on the coset space $U(6) \times U(6)/U(4) \times U(4) \times SU(2)_D$, and the W-bosons acquire mass from the spontaneous symmetry breaking $SU(2) \times SU(2) \to SU(2)_D$. Thus the confining hypercolour group acts like technicolour [39] and quarks and leptons are superpartners of the pseudo-Goldstone bosons familiar from extended technicolour theories. As in models with composite W--bosons, the substructure scale is related to the Fermi scale. A potential problem for these technicolour type models are the residual interactions [32] due to compositeness which now compete with the weak gauge interactions. An interesting possibility is that the two scales are separated as a result of soft SUSY breaking terms in the preon Lagrangian [38].

Such soft SUSY breaking terms are needed in any case in order to obtain a realistic boson and fermion mass spectrum [9, 13, 38]. In addition explicit breaking of the original global symmetry is required in order to generate a mass for the Goldstone bosons. Part of this explicit breaking is provided by the colour and electromagnetic gauge interactions, but presumably this will not be sufficient. In general, some neutral fermions and bosons, such as the novino, are likely to remain massless.

Supersymmetric gauge theories are interesting candidates for a theory of quark--lepton substructure because they provide naturally light composite fermions. During the last two years, we have become more familiar with various technical aspects of such theories, but the main challenge of composite models, with or without supersymmetry, remains to find the solution of the family problem.

It is a pleasure to thank R. D. Peccei, M. G. Schmidt and T. Yanagida for collaboration on the novino model.

REFERENCES

[1] For a general review, see R. Petronzio, lectures given at the 1984 Varenna school on "Elementary Particles", to be published in the proceedings.

[2] For a recent review, see R. D. Peccei, Talk given at the Fourth Topical Workshop on $p\bar{p}$ Collider Physics, Bern, 1984, preprint MPI-PAE/PTh 35/84 (1984).

[3] G. 't Hooft, in *Recent developments in gauge theories*, Cargèse 1979, eds. G. 't Hooft et al., Plenum, New York and London.

[4] O. Dumbrajs et al., *Nucl. Phys.* **B216**, 304 (1983).
[5] S. Weinberg, *Phys. Rev.* **166**, 1568 (1968); For a recent review, see M. E. Peskin, preprint SLAC--PUB-3021 (1982)
[6] W. Buchmüller, R. D. Peccei, T. Yanagida, *Phys. Lett.* **124B**, 67 (1983).
[7] W. Buchmüller, S. T. Love, R. D. Peccei, T. Yanagida, *Phys. Lett.* **115B**, 233 (1982).
[8] R. Barbieri, A. Masiero, G. Veneziano, *Phys. Lett.* **128B**, 493 (1983).
[9] W. Lerche, D. Lüst, *Nucl. Phys.* **B244**, 157 (1984).
[10] W. A. Bardeen, V. Višnjić, *Nucl. Phys.* **B194**, 422 (1982).
[11] J. C. Pati, A. Salam, *Phys. Rev.* **D10**, 275 (1974).
[12] L. F. Abbott, E. Farhi, *Phys. Lett.* **101B**, 69 (1981).
[13] W. Buchmüller, R. D. Peccei, T. Yanagida, *Nucl. Phys.* **B227**, 503 (1983).
[14] F. Bordi, R. Casalbuoni, D. Dominici, R. Gatto, *Phys. Lett.* **127B**, 419 (1983).
[15] O. W. Greenberg, R. N. Mohapatra, M. Yasue, *Phys. Lett.* **128B**, 65 (1983).
[16] W. Buchmüller, R. D. Peccei, T. Yanagida, *Nucl. Phys.* **B231**, 53 (1984).
[17] B. Zumino, *Phys. Lett.* **87B**, 203 (1979).
[18] C. K. Lee, H. S. Sharatchandra, preprint MPI-PAE/PTh 54/83 (1983); W. Lerche, *Nucl. Phys.* **B238**, 582 (1984); T. Kugo, I. Ojima, T. Yanagida, *Phys. Lett.* **135B**, 402 (1984); M. Bando, T. Kuramoto, T. Maskawa, S. Uehara, *Phys. Lett.* **138B**, 94 (1984).
[19] C. L. Ong, *Phys. Rev.* **D27**, 911 (1983); J. Bagger, E. Witten, *Phys. Lett.* **118B**, 103 (1982); A. Buras, W. Słominski, *Nucl. Phys.* **B223**, 157 (1983).
[20] We use the conventions of J. Wess and J. Bagger, *Supersymmetry and supergravity*, Princeton University Press, Princeton 1983.
[21] W. Buchmüller, M. G. Schmidt, preprint CERN TH-4005 (1984).
[22] T. R. Taylor, G. Veneziano, S. Yankielowicz, *Nucl. Phys.* **B231**, 493 (1983).
[23] G. Veneziano, *Phys. Lett.* **124B**, 357 (1983).
[24] G. C. Rossi, G. Veneziano, *Phys. Lett.* **138B**, 195 (1984); M. G. Schmidt, *Phys. Lett.* **141B**, 236 (1984).
[25] G. Veneziano, *Phys. Lett.* **128B**, 199 (1983).
[26] G. M. Shore, *Nucl. Phys.* **B231**, 139 (1984); T. E. Clark, S. T. Love, *Nucl. Phys.* **B232**, 306 (1984); W. Lerche, preprint MPI-PAE/PTh 23/84 (1984).
[27] J. M. Gerard, private communication.
[28] F. Buccella, J. P. Derendinger, S. Ferrara, C. A. Savoy, *Phys. Lett.* **115B**, 375 (1982).
[29] E. Fradkin, S. Shenker, *Phys. Rev.* **D19**, 3682 (1979); S. Dimopoulos, S. Raby, L. Susskind, *Nucl. Phys.* **B173**, 208 (1980).
[30] M. A. Shifman, A. I. Vainshtein, V. T. Zakharov, *Nucl. Phys.* **B147**, 385, 448 (1979).
[31] H. G. Dosch, M. Kremer, M. G. Schmidt, *Phys. Lett.* **137B**, 88 (1984); Heidelberg preprint HD--THEP 12 (1984).
[32] W. Buchmüller, R. D. Peccei, T. Yanagida, *Nucl. Phys.* **B244**, 186 (1984).
[33] O. W. Greenberg, R. N. Mohapatra, M. Yasue, *Phys. Rev. Lett.* **51**, 1737 (1983).
[34] W. Buchmüller, *Phys. Lett.* **145B**, 151 (1984).
[35] B. Schrempp, F. Schrempp, preprint DESY 84-055 (1984).
[36] I thank J. L. Rosner for pointing out the importance of this process which has recently also been emphasized by J. C. Pati, Maryland University preprint 84–146 (1984); see also R. N. Cahn, H. Harari, *Nucl. Phys.* **B176**, 135 (1980).
[37] Review of Particle Properties, *Rev. Mod. Phys.* **56**, S12 (1984).
[38] A. Masiero, G. Veneziano, CERN preprint TH-3950 (1984).
[39] L. Susskind, *Phys. Rev.* **D20**, 2619 (1979); S. Weinberg, *Phys. Rev.* **D19**, 1277 (1979).