SIMPLE COSMOLOGICAL MODELS WITH TORSION IN THE GRAVITATIONAL THEORY WITH LAGRANGIAN $L_g = \frac{c^4}{16\pi G}(\Omega^i_{.k}\Lambda\eta_i^{\ k}+\Theta^i\Lambda * \Theta_i)+\frac{\hbar c}{16\pi}\Omega^i_{.k}\Lambda * \Omega^k_{.i}$ *

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In the paper we consider the simple spatially homogeneous and isotropic cosmological models with torsion in the framework of the gauge gravitational theory with quadratic Lagrangian $L_g = \frac{c^4}{16\pi G}(\Omega^i_{.k}\Lambda\eta_i^{\ k}+\Theta^i\Lambda * \Theta_i)+\frac{\hbar c}{16\pi}\Omega^i_{.k}\Lambda * \Omega^k_{.i}$. These models were obtained by using of the ansatz $\frac{\dot{a}}{ac}+h = (\pm)\frac{1}{a}$.

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1. Introduction

The gauge gravitational theory with quadratic Lagrangian

$$L_g = \alpha(\Omega^i_{.k}\Lambda\eta_i^{\ k}+\Theta^i\Lambda * \Theta_i)+\beta\Omega^i_{.k}\Lambda * \Omega^k_{.i}, \tag{1}$$

where $i, j, b, k, l, m, n, p, r, s, t = 0, 1, 2, 3$, and $\alpha = \frac{c^4}{16\pi G}$, $\beta = \alpha\mathscr{A} = \frac{\alpha\hbar G}{c^3} = \frac{\hbar c}{16\pi}$ was presented in the papers [1].

In the Lagrangian (1) $\hbar$ is the Planck's constant, c is the value of the velocity of light in vacuum and G denotes the Newtonian gravitational constant. $\Omega^i_{.k}$ is the curvature two-form, Θ^i is the torsion two-form and η_{ik} means the pseudotensorial two-form introduced by Trautman [2]. $*$ denotes the Hodge-star-operator [3].

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In tensor notation the equations of the theory take the following form

$$\beta(\nabla_m R_{li}{}^{pm} + R_{li}{}^{pt} Q^k_{.tk} + \tfrac{1}{2} R_{li}{}^{tn} Q^p_{.tn})$$

$$+ \frac{\alpha}{2}(Q_{i.l}^{\,p} - Q_{l.i}^{\,p} + Q^k_{.lk}\delta^p_i - Q^k_{.ik}\delta^p_l + Q^p_{.il}) = (-)\tfrac{1}{4} S^p_{.il}, \tag{2}$$

$$\alpha\left(\nabla_k Q_{bp}{}^k + Q_{bp}{}^k Q^l_{.kl} + \tfrac{1}{2} Q_b{}^{lk} Q_{plk} - G_{pb} + \frac{g_{bp}}{4} Q^{itr} Q_{itr} - Q^i_{.br} Q_{ip.}{}^r\right)$$

$$+\beta\left(R^{ij}{}_{bt} R_{ijp}{}^t - \frac{g_{bp}}{4} R^{ijrt} R_{ijrt}\right) = (-)\tfrac{1}{2} t_{pb}, \tag{3}$$

where

$$G_{pb} := R_{pb} - \tfrac{1}{2} g_{pb} R \tag{4}$$

are the components of the Einstein tensor.

In the above formulae $R^i_{.klm} = (-)R^i_{.kml}$ are the curvature components and $Q^i_{.kl} = (-)Q^i_{.lk}$ mean the torsion components; $S^p_{.il} = (-)S^p_{.li}$ denote the components of the canonical spintensor of matter and t_{pb} are the canonical energy-momentum tensor of matter components. ∇ means the covariant derivative with respect to the Riemann--Cartan connection $\omega^i_{.k}$.

In the paper we restrict ourselves to the so-called "classical spin" [4] with properties

$$S^p_{.ik} = u^p S_{ik}, \quad S_{ik} = (-)S_{ki}, \quad u^l S_{lk} = 0. \tag{5}$$

u^k are here the four-velocity components ($u^k u_k = 1$).

The gravitational theory based on the Lagrangian (1) is satisfactory from the different points of view [5]. It gives a classical, microscopic gravitational theory (MicGT) satisfying the Birkhoff theorem [1].

The limiting process $\hbar \to 0$ performed in the Lagrangian (1) and in the field equations (2)–(3) of the theory leads to the new macroscopic gravitational theory (MGT) which is based on the following equations

$$Q_{i.l}^{\,p} - Q_{l.i}^{\,p} + Q^k_{.lk}\delta^p_i - Q^k_{.ik}\delta^p_l + Q^p_{.il} = (-)\frac{S^p_{.il}}{2\alpha}, \tag{6}$$

$$\nabla_k Q_{bp}{}^k + Q_{bp.}{}^k Q^l_{.kl} + \tfrac{1}{2} Q_b{}^{lk} Q_{plk} - G_{pb}$$

$$+ \frac{g_{bp}}{4} Q^{itr} Q_{itr} - Q^i_{.br} Q_{ip.}{}^r = (-)\frac{t_{pb}}{2\alpha}. \tag{7}$$

The MGT corresponds as well to the Newtonian gravitational theory (NGT) as General Relativity (GRT) does [1]. It is the theory which is identical with the GRT in vacuum and inside of spinless matter and gives the same results inside of the Solar System as GRT gives.

The solutions with torsion are submitted, in both theories, MGT and MicGT, to the same constraints. We obtain these constraints by comparison of the antisymmetric part $R_{[pb]}$ of the Ricci tensor R_{pb} calculated from the field equations of the theory with the antisymmetric part of the tensor obtained from the torsion Bianchi identities [1]. We get

$$\nabla_k Q_{[bp]}{}^k + Q_{[bp]}{}^k Q_k + \frac{1}{2\alpha} t_{[pb]} = \tfrac{1}{2} \nabla_k Q^k_{.bp} + \nabla_{[b} Q_{p]} + \tfrac{1}{2} Q_n Q^n{}_{bp}, \tag{8}$$

where

$$Q_n := Q^k_{.nk}.$$

The constraints (8) essentially restrict the set of the solutions with torsion in the theory, in comparison with a theory of gravitation having quadratic Lagrangian without Einsteinian term. Namely, in a given problem in vacuum or inside of matter with a symmetric energy-momentum tensor, the solutions with torsion exist if the following Criterion "C" is satisfied[1].

Criterion "C". In vacuum or inside of matter with a symmetric energy-momentum tensor, the solutions with torsion exist when:

1. The constraints (8) are compatible with the field equations and the compatibility conditions neither lead to an overdetermined resulting system of equations which we must solve nor imply vanishing torsion or
2. The constraints (8) are identically satisfied in the conditions which neither lead to an overdetermined resulting system of equations which we must solve nor imply vanishing torsion or
3. The constraints (8) immediately follow from the field equations (the trivial consistency of the constraints with the field equations) and the system of the field equations is not overdetermined.

If the Criterion "C" is not fulfilled, then there exist torsionless solutions only.

2. *Equations of the theory in spatially homogeneous and isotropic cosmology*

We take the Robertson-Walker linear element in the form [6]

$$ds^2 = c^2 dt^2 - a^2(t)\left[d\chi^2 + R^2(\chi)(d\Theta^2 + \sin^2\Theta d\varphi^2)\right], \tag{9}$$

where $a = a(t)$ is here the so-called "scale parameter" and

$$R = R(\chi) = \begin{cases} \sin\chi \text{ if } k = 1, \\ \chi \text{ if } k = 0, \\ \text{sh}\,\chi \text{ if } k = (-)1. \end{cases} \tag{10}$$

The parameter t means here and in the following the cosmic time.

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[1] Inside of matter having asymmetric energy-momentum tensor may exist only the solutions with torsion.

In the case of the SO(3) isotropy group the linear element (9) is consistent with the following form of torsion in the orthonormal tetrad (OT) determined by (9) [7, 8]

$$Q^{1}_{.01} = (-)Q^{1}_{.10} = Q^{2}_{.02} = (-)Q^{2}_{.20} = Q^{3}_{.03} = (-)Q^{3}_{.30} = :h(t),$$

$$Q^{1}_{.23} = (-)Q^{1}_{.32} = Q^{2}_{.31} = (-)Q^{2}_{.13} = Q^{3}_{.12} = (-)Q^{3}_{.21} = :f(t) \qquad (11)$$

or, equivalently

$$Q^{1}_{.01} = Q^{2}_{.02} = Q^{3}_{.03} = :h(t),$$

$$Q_2 = Q_4 = :Q(t), \qquad (12)$$

where

$$Q_4(t) := (-)\tfrac{1}{2}Q^{1}_{.23},$$

$$Q_2(t) := Q^{2}_{.13} + \tfrac{1}{2}Q^{1}_{.23}, \qquad (13)$$

and

$$Q^{2}_{.13} = (-)Q^{3}_{.12}. \qquad (14)$$

The remaining torsion components vanish. In the following we shall use (12) as the form of the admissible torsion.

In the case of the O(3) isotropy group (rotational and space reflections symmetry at every point) there remain only the components

$$Q^{1}_{.01} = (-)Q^{1}_{.10} = Q^{2}_{.02} = (-)Q^{2}_{.20} = Q^{3}_{.03} = (-)Q^{3}_{.30} = :h(t) \qquad (15)$$

of torsion [7, 8].

Using the above cited information we may write out the field equations (2)–(3) in the orthonormal tetrad determined by the Robertson-Walker linear element (9). We get the following systems of the four equations on the four unknown functions: $a(t)$, $h(t)$, $Q(t)$ and $\varepsilon(t)$ or $p(t)$:

(*i*) $k = 1$

$$(-)9h\frac{\dot a}{ac} - \tfrac{15}{2}h^2 - \frac{3\dot a^2}{a^2c^2} - \frac{3}{a^2} - 3Q^2 + \frac{3\beta}{\alpha}\left\{\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{ac}\right]^2 - \left[\frac{(aQ)^{\cdot}}{ac}\right]^2 + 4Q^2\left(\frac{\dot a}{ac}+h\right)^2 - \left[\left(\frac{\dot a}{ac}+h\right)^2 + \frac{1}{a^2} - Q^2\right]^2\right\} = (-)\frac{\varepsilon}{2\alpha},$$

$$(-)\frac{3\dot h}{c} - 6h\frac{\dot a}{ac} - \tfrac{9}{2}h^2 - Q^2 - \frac{1}{a^2} - \frac{\dot a^2}{a^2c^2} - 2\frac{\ddot a}{ac^2} - \frac{\beta}{\alpha}\left\{\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{ac}\right]^2\right.$$

$$+4Q^2\left(\frac{\dot a}{ac}+h\right)^2-\left[\frac{(aQ)^{\cdot}}{ac}\right]^2-\left[\left(\frac{\dot a}{ac}+h\right)^2+\frac{1}{a^2}-Q^2\right]^2\Bigg\}=\frac{p}{2\alpha},$$

$$\frac{1}{c^2}\left[\frac{(aQ)^{\cdot}}{a}\right]^{\cdot}-2Q^3+\frac{2Q}{a^2}+2\left(\frac{\dot a}{ac}+h\right)\frac{(aQ)^{\cdot}}{ac}-2Q\left(\frac{\dot a}{ac}+h\right)^2$$

$$+\frac{2h}{ac}(aQ)^{\cdot}-\frac{\alpha}{\beta}Q=0, \tag{16}$$

$$\frac{1}{c^2}\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{a}\right]^{\cdot}-2Q^2\left(\frac{\dot a}{ac}+h\right)+\frac{2}{ac}\left(\frac{\dot a}{ac}+h\right)\left(\frac{\dot a}{c}+ah\right)^{\cdot}$$

$$-2\left(\frac{\dot a}{ac}+h\right)^3+\frac{2h}{ac}\left(\frac{\dot a}{c}+ah\right)^{\cdot}-\frac{2}{a^2}\left(\frac{\dot a}{ac}+h\right)+\frac{\alpha}{2\beta}h=0.$$

(ii) $k=0$

$$(-)9h\frac{\dot a}{ac}-\tfrac{15}{2}h^2-\frac{3\dot a^2}{a^2c^2}-3Q^2+\frac{3\beta}{\alpha}\left\{\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{ac}\right]^2-\left[\frac{(aQ)^{\cdot}}{ac}\right]^2\right.$$

$$\left.+4Q^2\left(\frac{\dot a}{ac}+h\right)^2-\left[\left(\frac{\dot a}{ac}+h\right)^2-Q^2\right]^2\right\}=(-)\frac{\varepsilon}{2\alpha},$$

$$(-)\frac{3\dot h}{c}-\frac{6h\dot a}{ac}-\tfrac{9}{2}h^2-Q^2-\frac{\dot a^2}{a^2c^2}-2\frac{\ddot a}{ac^2}-\frac{\beta}{\alpha}\left\{\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{ac}\right]^2\right.$$

$$\left.+4Q^2\left(\frac{\dot a}{ac}+h\right)^2-\left[\frac{(aQ)^{\cdot}}{ac}\right]^2-\left[\left(\frac{\dot a}{ac}+h\right)^2-Q^2\right]^2\right\}=\frac{p}{2\alpha},$$

$$\frac{1}{c^2}\left[\frac{(aQ)^{\cdot}}{a}\right]^{\cdot}-2Q^3+\frac{2}{ac}\left(\frac{\dot a}{ac}+h\right)(aQ)^{\cdot}-2Q\left(\frac{\dot a}{ac}+h\right)^2$$

$$+\frac{2h}{ac}(aQ)^{\cdot}-\frac{\alpha}{\beta}Q=0, \tag{17}$$

$$\frac{1}{c^2}\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{a}\right]^{\cdot}-2Q^2\left(\frac{\dot a}{ac}+h\right)+\frac{2}{ac}\left(\frac{\dot a}{ac}+h\right)\left(\frac{\dot a}{c}+ah\right)^{\cdot}$$

$$-2\left(\frac{\dot a}{ac}+h\right)^3+\frac{2h}{ac}\left(\frac{\dot a}{c}+ah\right)^{\cdot}+\frac{\alpha}{2\beta}h=0.$$

(iii) $= k = (-)1$

$$(-)9h\frac{\dot a}{ac}-\tfrac{15}{2}h^2-\frac{3\dot a^2}{a^2c^2}+\frac{3}{a^2}-3Q^2+\frac{3\beta}{\alpha}\left\{\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{ac}\right]^2\right.$$

$$\left.-\left[\frac{(aQ)^{\cdot}}{ac}\right]^2+4Q^2\left(\frac{\dot a}{ac}+h\right)^2-\left[\left(\frac{\dot a}{ac}+h\right)^2-\frac{1}{a^2}-Q^2\right]^2\right\}=(-)\frac{\varepsilon}{2\alpha},$$

$$(-)3\frac{\dot h}{c}-\frac{6h\dot a}{ac}-\tfrac{9}{2}h^2-Q^2-\frac{\dot a^2}{a^2c^2}-2\frac{\ddot a}{ac^2}+\frac{1}{a^2}-\frac{\beta}{\alpha}\left\{\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{ac}\right]^2\right.$$

$$\left.+4Q^2\left(\frac{\dot a}{ac}+h\right)^2-\left[\frac{(aQ)^{\cdot}}{ac}\right]^2-\left[\left(\frac{\dot a}{ac}+h\right)^2-\frac{1}{a^2}-Q^2\right]^2\right\}=\frac{p}{2\alpha},$$

$$\frac{1}{c^2}\left[\frac{(aQ)^{\cdot}}{a}\right]^{\cdot}-2Q^3-\frac{2Q}{a^2}+\frac{2}{ac}\left(\frac{\dot a}{ac}+h\right)(aQ)^{\cdot}-2Q\left(\frac{\dot a}{ac}+h\right)^2$$

$$+\frac{2h}{ac}(aQ)^{\cdot}-\frac{\alpha}{\beta}Q=0,$$

$$\frac{1}{c^2}\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{a}\right]^{\cdot}-2Q^2\left(\frac{\dot a}{ac}+h\right)+\frac{2}{ac}\left(\frac{\dot a}{ac}+h\right)\left(\frac{\dot a}{c}+ah\right)^{\cdot}$$

$$-2\left(\frac{\dot a}{ac}+h\right)^3+\frac{2h}{ac}\left(\frac{\dot a}{c}+ah\right)^{\cdot}+\frac{2}{a^2}\left(\frac{\dot a}{ac}+h\right)+\frac{\alpha}{2\beta}h=0. \tag{18}$$

We have denoted by dot the differentiation with respect to the cosmic time t.

The classical spin is eliminated from the above equations. Therefore, there is admissible only symmetric energy-momentum tensor of matter. We have taken this tensor in the form of the energy-momentum tensor of perfect fluid.

The constraints (8) are identically fulfilled in the case with $h \neq 0$ and $Q \neq 0$ and the Criterion "C" is satisfied. Therefore, the systems (16)–(18) have the solutions with torsion.

3. *Spatially homogeneous and isotropic cosmology in the framework of MGT*

We give here information about spatially homogeneous and isotropic cosmology in the framework of MGT. This cosmology is the macroscopic limit given by $\hbar \to 0$ of the cosmology based on the equations (16)–(18).

In the case of the O(3) isotropy group the field equations and imposed symmetry exclude the classical, macroscopic spin[2]. Thus, in the case, the cosmological equations of the theory reduce to the Friedmann equations and we have the same spatially homogeneous and isotropic cosmology as in GRT.

In the case of the SO(3) isotropy group the field equations and imposed symmetry admit the only one intrinsic, nonzero component $S^{0}_{.23} = u^0 S_{23} = S_{23}$ of the classical macroscopic spin. In consequence, only the two intrinsic torsion components $Q^{2}_{.03} = (-)Q^{3}_{.02} = :Q_1(t)$ may be different from zero. But this kind of torsion is not consistent with the postulate of spatial homogeneity and isotropy [7, 8].

The field equations take the following form in the orthonormal tetrad determined by the Robertson-Walker linear element (9)

$$(-)\frac{3\dot{a}^2}{a^2c^2} + \frac{2R''}{a^2R} - \frac{1}{a^2R^2} + \frac{R'^2}{a^2R^2} - Q_1^2 = (-)\frac{\varepsilon}{2\alpha},$$

$$(-)\frac{1}{a^2R^2} - \frac{\dot{a}^2}{a^2c^2} + \frac{R'^2}{a^2R^2} - 2\frac{\ddot{a}}{ac^2} + Q_1^2 = \frac{p}{2\alpha},$$

$$Q_1^2 - \frac{\dot{a}^2}{a^2c^2} + \frac{R''}{a^2R} - 2\frac{\ddot{a}}{ac^2} = \frac{p}{2\alpha},$$

$$\dot{Q}_1 + Q_1\frac{\dot{a}}{a} = 0, \qquad Q_1 = (-)\frac{1}{4\alpha}S_{32}. \tag{19}$$

$a(t)$ is here the scale parameter and $R = R(\chi)$ is given by (10). The dot, as usual, means the differentiation with respect to the cosmic time t; $R' := \dfrac{dR}{d\chi}$ etc.

The constraints on torsion reduce to the following, single equation

$$\dot{Q}_1 + \frac{3\dot{a}}{a}Q_1 = 0. \tag{20}$$

This equation can also be obtained from the differential conservation laws existing in the theory [1].

Comparing the constraints equation (20) and the field equations (19) one can see that the constraints are compatible with the field equations if and only if $\dfrac{\dot{a}}{a}Q_1 = 0$, i.e., if either $Q_1 = 0$ or $\dot{a} = 0$ which implies $\dot{Q}_1 = 0$. In both cases the constraints equation (20) is identically fulfilled and the resulting system of the equations is not overdetermined, i.e., the Criterion "C" is satisfied.

In the first case $Q_1 = 0$ which implies $S_{23} = 0$, we get the same equations (Friedmann equations) and the same torsionless, spatially homogeneous and isotropic cosmology as in the GRT.

[2] By macroscopic spins we mean the spins of planets, stars and galaxies (see [1]).

In the second case $\dot{a} = 0$, we have the static cosmological solutions with torsion of the form:

(*i*) $k = 1$

$$a^2 = \frac{8\alpha}{\varepsilon - p}, \quad Q_1^2 = \frac{\varepsilon + 3p}{8\alpha}, \quad S_{23} = 4\alpha Q_1, \tag{21}$$

$$\varepsilon = \text{const}; \quad p = \text{const}; \quad Q_1 = \text{const},$$

(*ii*) $k = 0$

$$Q_1^2 = \frac{\varepsilon}{2\alpha} = \frac{p}{2\alpha}, \quad S_{23} = 4\alpha Q_1, \tag{22}$$

$$Q_1 = \text{const}, \quad \varepsilon = p = \text{const}.$$

The scale parameter a is not determined in the case by the field equations alone.

(*iii*) $k = -1$

$$a^2 = \frac{8\alpha}{p - \varepsilon}, \quad Q_1^2 = \frac{3p - \varepsilon}{8\alpha}, \quad S_{23} = 4\alpha Q_1, \tag{23}$$

$$\varepsilon = \text{const}, \quad p = \text{const}, \quad Q_1 = \text{const}.$$

The non-Einsteinian solutions (21)–(23) are not consistent with the postulate of the spatial homogeneity and isotropy. Therefore, we must reject these static models. Finally, we can say that the spatially homogeneous and isotropic cosmology is here spinless and torsionless and the same as in GRT. This is the cosmology given by the first case $Q_1 = 0$.

4. Simple spatially homogeneous and isotropic cosmological models with O(3) isotropy group

In the paper we consider, for simplicity, only cosmological models having O(3) isotropy group. The more general cosmological models described by the equations (16)–(18) will be studied in the forthcoming paper.

In the case of the O(3) isotropy group the equations (16)–(18) take the following, simpler form:

(*i*) $k = 1$

$$(-)\frac{9h\dot{a}}{ac} - \tfrac{15}{2}h^2 - \frac{3\dot{a}^2}{a^2c^2} - \frac{3}{a^2} + \frac{3\beta}{\alpha}\left\{\frac{\left[\left(\frac{\dot{a}}{c} + ah\right)^{\cdot}\right]^2}{a^2c^2} - \left[\left(\frac{\dot{a}}{ac} + \quad\right)^2 + \frac{1}{a^2}\right]^2\right\} = (-)\frac{\varepsilon}{2\alpha},$$

$$(-)\frac{3\dot{h}}{c} - \frac{6h\dot{a}}{ac} - \tfrac{9}{2}h^2 - \frac{\dot{a}^2}{a^2c^2} - 2\frac{\ddot{a}}{ac^2} - \frac{1}{a^2} - \frac{\beta}{\alpha}\left\{\frac{\left[\left(\frac{\dot{a}}{c} + ah\right)^{\cdot}\right]^2}{a^2c^2}\right.$$

$$-\left[\left(\frac{\dot a}{ac}+h\right)^2+\frac{1}{a^2}\right]^2\Bigg\}=\frac{p}{2\alpha},$$

$$\frac{1}{c^2}\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{a}\right]^{\cdot}-2\left(\frac{\dot a}{ac}+h\right)^3-\frac{2}{a^2}\left(\frac{\dot a}{ac}+k\right)+\frac{2\dot a}{a^2c^2}\left(\frac{\dot a}{c}+ah\right)^{\cdot}$$

$$+\frac{4h}{ac}\left(\frac{\dot a}{c}+ah\right)^{\cdot}+\frac{\alpha}{2\beta}h=0. \tag{24}$$

(*ii*) $k=0$

$$(-)9h\frac{\dot a}{ac}-\tfrac{15}{2}h^2-\frac{3\dot a^2}{a^2c^2}+\frac{3\beta}{\alpha}\left\{\frac{\left[\left(\frac{\dot a}{c}+ah\right)^{\cdot}\right]^2}{a^2c^2}-\left(\frac{\dot a}{ac}+h\right)^4\right\}=(-)\frac{\varepsilon}{2\alpha},$$

$$3\frac{\dot h}{c}-\frac{6h\dot a}{ac}-\tfrac{9}{2}h^2-\frac{\dot a^2}{a^2c^2}-2\frac{\ddot a}{ac^2}-\frac{\beta}{\alpha}\left\{\frac{\left[\left(\frac{\dot a}{c}+ah\right)^{\cdot}\right]^2}{a^2c^2}-\left(\frac{\dot a}{ac}+h\right)^4\right\}=\frac{p}{2\alpha},$$

$$\frac{1}{c^2}\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{a}\right]^{\cdot}-2\left(\frac{\dot a}{ac}+h\right)^3+\frac{4h}{ac}\left(\frac{\dot a}{c}+ah\right)^{\cdot}+\frac{2\dot a}{a^2c^2}\left(\frac{\dot a}{c}+ah\right)+\frac{\alpha}{2\beta}h=0. \tag{25}$$

(*iii*) $k=-1$

$$(-)9h\frac{\dot a}{ac}-\tfrac{15}{2}h^2-\frac{3\dot a^2}{a^2c^2}+\frac{3}{a^2}+\frac{3\beta}{\alpha}\Bigg\{\frac{\left[\left(\frac{\dot a}{c}+ah\right)^{\cdot}\right]^2}{a^2c^2}$$

$$-\left[\left(\frac{\dot a}{ac}+h\right)^2-\frac{1}{a^2}\right]^2\Bigg\}=(-)\frac{\varepsilon}{2\alpha},$$

$$(-)\frac{3\dot h}{c}-\frac{6h\dot a}{ac}-\tfrac{9}{2}h^2-\frac{\dot a^2}{a^2c^2}-\frac{2\ddot a}{ac^2}+\frac{1}{a^2}-\frac{\beta}{\alpha}\Bigg\{\frac{\left[\left(\frac{\dot a}{c}+ah\right)^{\cdot}\right]^2}{a^2c^2}$$

$$-\left[\left(\frac{\dot a}{ac}+h\right)^2-\frac{1}{a^2}\right]^2\Bigg\}=\frac{p}{2\alpha},$$

$$\frac{1}{c^2}\left[\frac{\left(\frac{\dot a}{c}+ah\right)^{\cdot}}{a}\right]-2\left(\frac{\dot a}{ac}+h\right)^3+\frac{2}{a^2}\left(\frac{\dot a}{ac}+h\right)+\frac{4h}{ac}\left(\frac{\dot a}{c}+ah\right)^{\cdot}$$

$$+\frac{2\dot a}{a^2c^2}\left(\frac{\dot a}{c}+ah\right)^{\cdot}+\frac{\alpha}{2\beta}h=0. \tag{26}$$

The systems (24)–(26) are very well determined if the state equation $p = p(\varepsilon)$ is given. As in the case of the SO(3) isotropy group the Criterion "C" is satisfied in the case and, therefore, the systems (24)–(26) have the solutions with torsion. We will obtain the solutions of that kind with the help of a suitable ansatz.

(a) The solutions to the system (24).

Using ansatz

$$\frac{\dot a}{ac}+h=0 \overset{a\neq 0}{\equiv} \frac{\dot a}{c}+ah=0 \tag{27}$$

we get the static and torsionless solution representing planckeon [1]. On the other hand, using of the ansatz

$$\frac{\dot a}{ac}+h=\frac{1}{a} \overset{a\neq 0}{\equiv} \frac{\dot a}{c}+ah=1 \tag{28}$$

leads to the following dynamical solutions with torsion:

(*i*) $a > \sqrt{8\mathscr{A}}$, where $\mathscr{A} = \frac{\hbar G}{c^3}$

$$h=\frac{8\mathscr{A}}{a^3},\quad \varepsilon=\frac{12\alpha}{a^2}\left(1+\frac{6\mathscr{A}}{a^2}+\frac{16\mathscr{A}^2}{a^4}\right),$$

$$p=\frac{4\alpha}{a^2}\left(\frac{6\mathscr{A}}{a^2}-1-\frac{144\mathscr{A}^2}{a^4}\right),$$

$$a-\frac{\sqrt{8\mathscr{A}}}{2}\ln\left(\frac{a+\sqrt{8\mathscr{A}}}{a-\sqrt{8\mathscr{A}}}\right)=ct+\text{const}. \tag{29}$$

It is conveniently to put the constant equal to zero because, then we have for $a \gg \sqrt{8\mathscr{A}}$

$$a = ct. \tag{30}$$

(*ii*) $a < \sqrt{8\mathscr{A}}$

$$h=\frac{8\mathscr{A}}{a^3},\quad \varepsilon=\frac{12\alpha}{a^2}\left(1+\frac{6\mathscr{A}}{a^2}+\frac{16\mathscr{A}^2}{a^4}\right),$$

$$p = \frac{4\alpha}{a^2}\left(\frac{6\mathscr{A}}{a^2} - 1 - \frac{144\mathscr{A}^2}{a^4}\right),$$

$$a - \frac{\sqrt{8\mathscr{A}}}{2}\ln\left(\frac{a+\sqrt{8\mathscr{A}}}{\sqrt{8\mathscr{A}}-a}\right) = ct + \text{const.} \tag{31}$$

Here, it is also convenient to put the constant equal to zero.

The cosmological model given by (29) expands from the initial value $a = \sqrt{8\mathscr{A}}$ at the moment $t = (-)\infty$ to the final value $a = \infty$ at the moment $t = \infty$. The velocity of this expansion is monotonically increasing from zero value at $a = \sqrt{8\mathscr{A}}$ to the value of the light velocity as $a(t)$ goes to infinity.

The model given by (31) contracts from the initial value $a = \sqrt{8\mathscr{A}}$ at the moment $t = (-)\infty$ to the final value $a = 0$ at the moment $t = 0$ with the velocity increasing from zero at $t = (-)\infty$ to infinity at $t = 0$. In our opinion it is unphysical cosmological model.

The ansatz

$$\frac{\dot a}{ac} + h = (-)\frac{1}{a} \overset{a\neq 0}{\equiv} \frac{\dot a}{c} + ah = (-)1 \tag{32}$$

leads to the following models being the time reflection of the models given by (29) and (31):

(*i*) $a > \sqrt{8\mathscr{A}}$

$$h = (-)\frac{8\mathscr{A}}{a^3}, \quad \varepsilon = \frac{12\alpha}{a^2}\left(1 + \frac{6\mathscr{A}}{a^2} + \frac{16\mathscr{A}^2}{a^4}\right),$$

$$p = \frac{4\alpha}{a^2}\left(\frac{6\mathscr{A}}{a^2} - 1 - \frac{144\mathscr{A}^2}{a^4}\right),$$

$$a - \frac{\sqrt{8\mathscr{A}}}{2}\ln\left(\frac{a+\sqrt{8\mathscr{A}}}{a-\sqrt{8\mathscr{A}}}\right) = \text{const} - ct. \tag{33}$$

This cosmological model contracts from $a = \infty$ at $t = (-)\infty$ (const $= 0$) to $a = \sqrt{8\mathscr{A}}$ as t goes to infinity. The velocity of the contraction is monotonically decreasing from the velocity of light at $t = (-)\infty$ to zero at $t = \infty$. The model is the time reflection of the expanding model given by (29).

(*ii*) $a < \sqrt{8\mathscr{A}}$

$$h = (-)\frac{8\mathscr{A}}{a^3}, \quad \varepsilon = \frac{12\alpha}{a^2}\left(1 + \frac{8\mathscr{A}}{a^2} + \frac{16\mathscr{A}^2}{a^4}\right),$$

$$p = \frac{4\alpha}{a^2}\left(\frac{6\mathscr{A}}{a^2} - 1 - \frac{144\mathscr{A}^2}{a^4}\right),$$

$$(-)a + \frac{\sqrt{8\mathscr{A}}}{2}\ln\left(\frac{\sqrt{8\mathscr{A}}+a}{\sqrt{8\mathscr{A}}-a}\right) = ct + \text{const.} \tag{34}$$

For convenience we will put the constant equal to zero.

The model given by (34) expands from the initial value $a = 0$ at $t = 0$ to the final value $a = \sqrt{8\mathscr{A}}$ when t goes to infinity. The velocity of this expansion is decreasing from infinity at $t = 0$ to zero at $t = \infty$. This is also unphysical model being the time reflection of the cosmological model given by (31).

(b) The solutions to the system (25) obtained by means of the ansatz method Ansatz

$$\frac{\dot{a}}{ac} + h = \frac{1}{a} \overset{a\neq 0}{\equiv} \frac{\dot{a}}{c} + ah = 1 \tag{35}$$

leads to the following solutions:

(*i*) $a > \sqrt{4\mathscr{A}}$

$$h = \frac{4\mathscr{A}}{a^3}, \quad \varepsilon = \frac{6\alpha}{a^2}\left(1 + \frac{5\mathscr{A}}{a^2} + \frac{8\mathscr{A}^2}{a^4}\right),$$

$$p = \frac{2\alpha}{a^2}\left(\frac{5\mathscr{A}}{a^2} - 1 - \frac{72\mathscr{A}^2}{a^4}\right),$$

$$a - \frac{\sqrt{4\mathscr{A}}}{2} \ln\left(\frac{a+\sqrt{4\mathscr{A}}}{a-\sqrt{4\mathscr{A}}}\right) = ct + \text{const.} \tag{36}$$

For convenience we put the constant equal to zero. Then we have for $a \gg \sqrt{4\mathscr{A}}$

$$a = ct. \tag{37}$$

The (36) represents the expanding cosmological model. The model expands from $a = \sqrt{4\mathscr{A}}$ at the moment $t = (-)\infty$ to infinity as t goes to infinity with velocity which monotonically increases from zero value at $t = (-)\infty$ to the light velocity c when t goes to infinity.

(*ii*) $a < \sqrt{4\mathscr{A}}$

$$h = \frac{4\mathscr{A}}{a^3}, \quad \varepsilon = \frac{6\alpha}{a^2}\left(1 + \frac{5\mathscr{A}}{a^2} + \frac{8\mathscr{A}^2}{a^4}\right),$$

$$p = \frac{2\alpha}{a^2}\left(\frac{5\mathscr{A}}{a^2} - 1 - \frac{72\mathscr{A}^2}{a^4}\right),$$

$$a - \frac{\sqrt{4\mathscr{A}}}{2} \ln\left(\frac{\sqrt{4\mathscr{A}}+a}{\sqrt{4\mathscr{A}}-a}\right) = ct + \text{const.} \tag{38}$$

The constant we also take equal to zero.

This model contracts from the initial value $a = \sqrt{4\mathscr{A}}$ at $t = (-)\infty$ to the final value $a = 0$ as t goes to zero. The velocity of this contraction is monotonically increasing from zero at $t = (-)\infty$ to infinity when $t \to 0$. Thus, it is an unphysical cosmological model.

Ansatz

$$\frac{\dot{a}}{ac}+h=(-)\frac{1}{a}\overset{a\neq 0}{\equiv}\frac{\dot{a}}{c}+ah=(-)1 \tag{39}$$

leads to the solutions which are the time reflection of the solutions (36) and (38). Namely, we have:

(*i*) $a>\sqrt{4\mathscr{A}}$

$$h=(-)\frac{4\mathscr{A}}{a^3},\quad \varepsilon=\frac{6\alpha}{a^2}\left(1+\frac{5\mathscr{A}}{a^2}+\frac{8\mathscr{A}^2}{a^4}\right),$$

$$p=\frac{2\alpha}{a^2}\left(\frac{5\mathscr{A}}{a^2}-1-\frac{72\mathscr{A}^2}{a^4}\right),$$

$$a-\frac{\sqrt{4\mathscr{A}}}{2}\ln\left(\frac{a+\sqrt{4\mathscr{A}}}{a-\sqrt{4\mathscr{A}}}\right)=\text{const}-ct. \tag{40}$$

As in the previous cases, we put here the constant equal to zero.

This model contracts from the initial value $a=\infty$ at the moment $t=(-)\infty$ to the final value $a=\sqrt{4\mathscr{A}}$ at the moment $t=\infty$. The contraction velocity monotonically decreases from the velocity of light at $t=(-)\infty$ to zero at the moment $t=\infty$. We see that the model is the time reflection of the model (36).

(*ii*) $a<\sqrt{4\mathscr{A}}$

$$h=(-)\frac{4\mathscr{A}}{a^3},\quad \varepsilon=\frac{6\alpha}{a^2}\left(1+\frac{5\mathscr{A}}{a^2}+\frac{8\mathscr{A}^2}{a^4}\right),$$

$$p=\frac{2\alpha}{a^2}\left(\frac{5\mathscr{A}}{a^2}-1-\frac{72\mathscr{A}^2}{a^4}\right),$$

$$(-)a+\frac{\sqrt{4\mathscr{A}}}{2}\ln\left(\frac{\sqrt{4\mathscr{A}}+a}{\sqrt{4\mathscr{A}}-a}\right)=ct+\text{const}. \tag{41}$$

The constant we put, for convenience, equal to zero.

The cosmological model given by (41) expands from the initial value $a=0$ at the moment $t=0$ to the final value $a=\sqrt{4\mathscr{A}}$ when t goes to infinity. The expansion velocity is decreasing from infinity at the moment $t=0$ to zero as t goes to infinity. Therefore, the model is unphysical and it is the time reflection of the model given by (38).

(c) The solutions to the system (26) obtained by using of the ansatz $\frac{\dot{a}}{ac}+h=(\pm)\frac{1}{a}$

The ansatz $\frac{\dot{a}}{ac}+h=(\pm)\frac{1}{a}$ leads only to the trivial solutions to the system (26) representing Minkowskian space-time. These solutions are given by $\varepsilon=h=p=0$, $a=\pm ct$.

5. *Discussion and conclusions*

The astronomical observation shows that the two following solutions from the solutions presented in the paper may have physical meaning:

(*i*) the solution (29) for $k = 1$,

(*ii*) the solution (36) for $k = 0$. (42)

These solutions describe expanding universes without singularity, defined for all values of the cosmic time t. On the other hand, for macroscopic values of $a(t) \gg \sqrt{8\mathscr{A}} > \sqrt{4\mathscr{A}}$ the evolution of the Universe should be described by cosmological solutions to the MGT equations, i.e., by the solutions to the equations (19) followed by the torsion constraints (20). Therefore, in macroscopic domain, our solutions should correspond to the suitable cosmological solutions to the equations of the MGT.

It is very interesting that for macroscopic values of $a: a \gg \sqrt{8\mathscr{A}} > \sqrt{4\mathscr{A}}$ the solutions cited in (42) really, with a very good accuracy, turn into spatially homogeneous and isotropic cosmological solutions to the macroscopic equations (19). Namely, we have with a very good accuracy that for macroscopic $a \gg \sqrt{8\mathscr{A}} > \sqrt{4\mathscr{A}}$:

(*i*) The solution (29) takes the form

$$h = 0, \quad \varepsilon = \frac{12\alpha}{a^2}, \quad p = (-)\frac{\varepsilon}{3}, \quad a = ct \tag{43}$$

and represents an exact expanding, spatially homogeneous and isotropic cosmological solution to the macroscopic equations (19) for $k = 1$.

(*ii*) The solution (36) takes the form

$$h = 0, \quad \varepsilon = \frac{6\alpha}{a^2}, \quad p = (-)\frac{\varepsilon}{3}, \quad a = ct \tag{44}$$

and represents an exact, expanding and spatially homogeneous and isotropic cosmological solution to the macroscopic equations (19) in the case $k = 0$. Summing up, one can say that the above mentioned, desired correspondence does exist.

The macroscopic cosmological models described by (43) and (44) predict sensible mean matter density for the present state of the Universe (at present $a = 10^{28}$ cm):

$$\frac{\varepsilon}{c^2} = \frac{12\alpha}{a^2c^2} \approx 10^{-29}\mathrm{g} \cdot \mathrm{cm}^{-3} \tag{45}$$

in the case $k = 1$ and

$$\frac{\varepsilon}{c^2} = \frac{6\alpha}{a^2c^2} \approx 0{,}5 \cdot 10^{-29}\mathrm{g} \cdot \mathrm{cm}^{-3} \tag{46}$$

in the case $k = 0$.

The negative pressure existing in our cosmological models leads to the gravitational repulsion between the particles of the cosmological substratum.

At the cost of the work performed by the negative pressure and torsion during expansion the continual "creation" of matter takes place in the presented models, e.g. in the case $k = 1$ the total mass of matter contained in the model increases proportionally to the $a(t)$ from the initial value of order 10^{-4} g. However, these models are evolutionary models, not steady-state models, because the expansion dominates over the creation and, in consequence, the mean matter density in the models decreases as a^{-2} from the initial density of the order 10^{93} g · cm^{-3}. At present the matter is probably "created" in the active nuclei of galaxies.

The models of expanding Universe presented in the paper are very interesting; especially the model with $k = 0$. They are without singularities and, in macroscopic domain, they are simpler than the standard Friedmann models. Moreover, in the framework of these models, it is possible to explain the observed isotropy of Universe [9]. The standard Friedmann models are powerless in this field [9].

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