THE NORMAL PRODUCT IN QUANTUM THEORY

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The aim of this paper is purely technical. We want to show the positive points and shortcomings in practical application of the two procedures in quantum field theory. The first — the canonical quantization approach which uses the normal product (NP) and the second procedure, the Feynman path integral approach without the normal product (WNP). To compare both procedures we have made detailed renormalization of the ϕ^4 theory and of the scalar electrodynamics.

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1. Introduction

There are two main approaches in the quantum field theory. The first, the so-called Canonical Quantization, where quantum fields are obtained by quantization of classical, canonical conjugate quantities of appropriate classical field theory (see e.g. [1]). The second is the Feynman Path Integral approach where generating functional for any Green's function is given by path integral of the appropriate classical action (see e.g. [2]). This way of the formulation of quantum theory have become popular after proving that, with unphysical particles called "ghosts", non-abelian gauge theories can be unitary [3] and renormalizable [4].

It is possible also to introduce "ghosts" to theory, without path integral formulation, by requirement, that total lagrangian (with gauge fixing term) is BRS invariant [5]. By introducing the BRS charge it is possible to state the conditions for Hilbert space where S-matrix is unitary, and give canonical quantization formulation of non-abelian gauge theories [6].

In practical applications, there are essential differences in both of the mentioned approaches to quantum field theory. In the canonical approach to remove vacuum infinite energy, the normal product is introduced for field operators. Then, in perturbative calcu-

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lation, using Wick's theorem, any time ordered connections between operators from the same normal product will vanish ($\langle 0|T(:\phi(x)\phi(y):)|0\rangle = 0$). This condition will eliminate many Feynman diagrams from perturbative series, which would be present without normal product. In the Feynman path integral approach, the generating functional is expressed by classical action and it is not usual to include normal product here. The aim of this paper is purely technical. We discuss some positive points and shorcomings of the two procedures — the quantum field theory with the normal product (NP procedure) and without normal product (WNP procedure). The main advantage of the NP procedure is that we have to calculate smaller number of infinite Feynman diagrams. On the other hand, in this procedure, dimensional regularization is not gauge symmetric, and there is no mass independent renormalization scheme.

In the next Chapter both procedures are discussed for a theory without gauge symmetry. To be more precise we have made detailed calculation for the ϕ^4 theory. Few comments for gauge theories, using the example of scalar electrodynamic, are given in Chapter 3. In Chapter 4 our conclusions are given.

2. *A theory without gauge symmetry*

The two procedures NP and WNP are being discussed in details taking as an example the theory ϕ^4. Most of our conclusions presented in this case are valid for the gauge theory and also for the theory with spontaneous symmetry breaking. To see the difference between two approaches we have done detailed calculation up to second order.

In the next subsection we give short descriptions of our calculation. All necessary Feynman diagrams in dimensional regularization [7] are given in Appendix A.

2.1. The renormalization procedure

In this section we briefly present our notations and the method of the calculation of the renormalization constants. In the paper we use the dimensional regularization of 't Hooft and Veltman [7]. As usual, $n = 4-\varepsilon$ is the new dimension. In the ϕ^4 theory the bare lagrangian is given by

$$\mathscr{L}(\phi_0, \partial_\mu\phi_0) = \tfrac{1}{2}\,\partial_\mu\phi_0\partial^\mu\phi_0 - \tfrac{1}{2}\,m_0^2\phi_0^2 - \frac{g_0}{4!}\,\phi_0^4. \tag{1}$$

Next we define the renormalization constants Z_3, Z_m (or δm^2) and Z_g by

$$\phi_0 = Z_3^{1/2}\phi, \quad Z_3 = 1-\Delta_3, \quad \Delta_3 = \sum_{n=1}^{\infty}\left(\frac{g}{4\pi}\right)^n a_{3n}, \tag{2}$$

$$m_0^2 = Z_m m^2 = m^2+\delta m^2, \quad \delta m^2 = m^2\sum_{n=1}^{\infty}\left(\frac{g}{4\pi}\right)^n d_n, \tag{3}$$

$$g_0 = \mu^{4-n} Z_g g, \quad Z_g = 1 - \Delta_g, \quad \Delta_g = \sum_{n=1}^{\infty} \left(\frac{g}{4\pi}\right)^n a_{gn}. \tag{4}$$

To find the constants Z_3, Z_m and Z_g to any order, it is enough to renormalize only two Green's functions:

$$\text{Graph in Fig. 1} = S_0 = \frac{i}{k^2 - m_0^2 - \Sigma_0} = Z_3 \frac{i}{k^2 - m^2 - \Sigma_R} \tag{5}$$

and

$$\text{Graph in Fig. 2} = \tau_0(k_1, k_2, k_3, k_4) = Z_3^2 \tau(k_1, k_2, k_3, k_4), \tag{6}$$

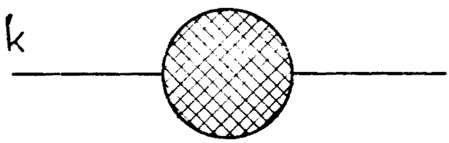

Fig. 1. The propagator in ϕ^4 theory

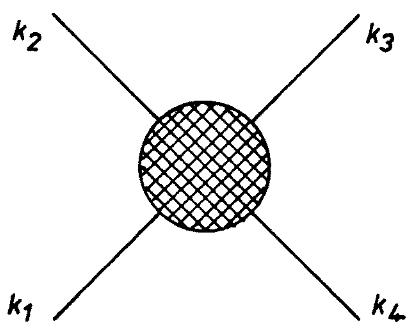

Fig. 2. The vertex Green function in ϕ^4 theory

where Σ_0 and Σ_R are the bare and the renormalized self energy for the ϕ particle respectively. As usual we take interaction Hamiltonian $\mathscr{H}_I$ in the form:

$$\mathscr{H}_I(\phi_0) = \tfrac{1}{2}\,\delta m^2 \phi_0^2 + \frac{g_0}{4!}\,\phi_0^4. \tag{7}$$

Now, using general perturbative prescriptions (see e.g. [1]) we expand Green's functions S_0 up to the second order, and τ_0 up to the third order, in the bare coupling constant g_0. Next both sides of the equations (8) and (9)

$$Z_3(k^2 - m_0^2 - \Sigma_0) = k^2 - m^2 - \Sigma_R, \tag{8}$$

$$Z_3^{-2} \tau_0(k_1, k_2, k_3, k_4) = \tau_R(k_1, k_2, k_3, k_4) \tag{9}$$

are expanded in the renormalized coupling constant g.

In each order, parameters a_{3n}, a_{gn} and d_n are chosen in such a way, that renormalized quantity Σ_R and τ_R are finite. Of course the values of these parameters and also Σ_R and τ_R depend on renormalization scheme.

2.2. Renormalization with the normal product (NP)

Only three graphs which are given in Fig. 3 will give contribution for the self-energy $\tilde{\Sigma}_0 = \Sigma_0 + \delta m^2$ in the second order.

$$\tilde{\Sigma}_0^{NP} = \delta m^2 + \Sigma_0^1 + \Sigma_0^2. \tag{10}$$

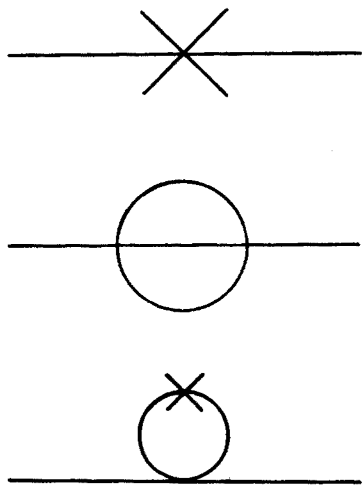

Fig. 3. Feynman graphs which give contributions to the self-energy $\tilde{\Sigma}_0$ in the NP procedure

Sign "NP" denotes that we calculate quantity using the normal product. Full formulae for Σ_0^1 and Σ_0^2 are given in Appendix A. All graphs given in Fig. 4 will contribute to the four particle Green's function $\tau_0^{NP}(k_1, k_2, k_3, k_4)$, in the third order. Adding contributions from all graphs (more complicated formulae are given in Appendix A) we obtain formula for the τ_0^{NP}. The full formula is of some length so is not given here. To get renormalized Green's function $\tau_R^{NP}(k_1, k_2, k_3, k_4)$ we follow the remarks from Chapter 2.1. In the minimal subtraction scheme MS [8] we obtain:
in the first order:

$$a_{31}^{NP} = 0, \quad d_1^{NP} = 0, \quad a_{g1}^{NP} = -\frac{3}{4\pi}\frac{1}{\varepsilon}, \tag{11}$$

in the second order:

$$a_{32}^{NP} = \frac{1}{12(4\pi)^2}, \quad a_{g2}^{NP} = \frac{1}{(4\pi)^2}\left(-\frac{9}{\varepsilon^2} + \tfrac{17}{6}\frac{1}{\varepsilon}\right),$$

$$d_2^{NP} = \frac{1}{(4\pi)^2}\left(-\frac{1}{\varepsilon^2} + \frac{1}{\varepsilon}(\gamma - \xi - \tfrac{17}{12})\right). \tag{12}$$

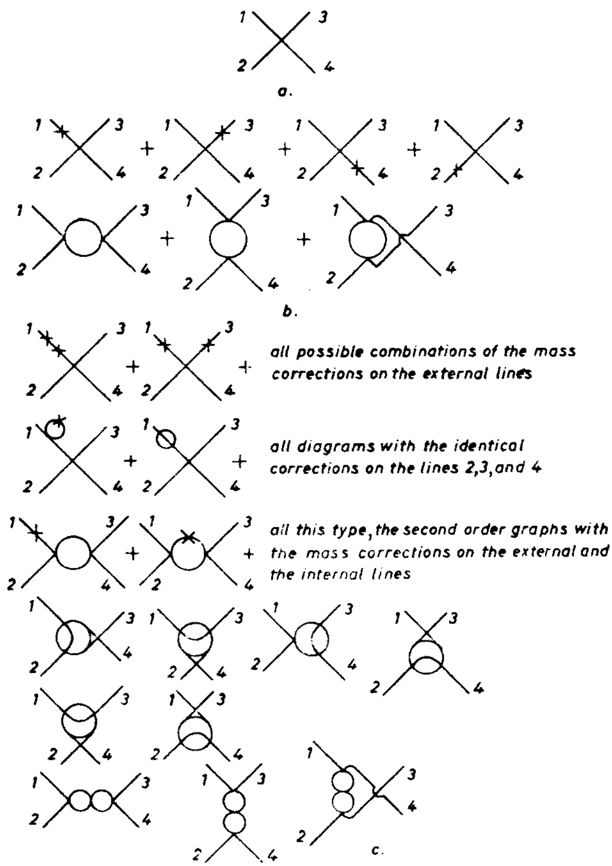

Fig. 4. Feynman graphs which contribute to the Green's function τ_0 in the NP procedure: (a) first order, (b) second order, (c) third order

As we see from these formulae, the mass renormalization constant $Z_m^{\mathrm{NP}} = 1 + \frac{g}{4\pi} d_1 + \left(\frac{g}{4\pi}\right)^2 d_2 + \ldots$ depends on the mass m^2 and dimensional parameter μ^2.

2.3. Renormalization without the normal product (WNP)

In comparison with the previous approach two additional graphs given in Fig. 5 will contribute to the self energy $\tilde{\Sigma}_0$ ane we have

$$\tilde{\Sigma}_0^{\mathrm{WNP}} = \tilde{\Sigma}_0^{\mathrm{NP}} + \Sigma_0^3 + \Sigma_0^4, \tag{13}$$

Σ_0^3 and Σ_0^4 are given in Appendix A. To find Green's function $\tau_0^{\mathrm{WNP}}(k_1, k_2, k_3, k_4)$, we have to add to $\tau_0^{\mathrm{NP}}(k_1, k_2, k_3, k_4)$ all contributions from graphs in Fig. 4:

$$\tau_0^{\mathrm{WNP}}(k_1, k_2, k_3, k_4) = \tau_0^{\mathrm{NP}}(k_1, k_2, k_3, k_4) + (\text{contributions from graphs in Fig. 6}). \tag{14}$$

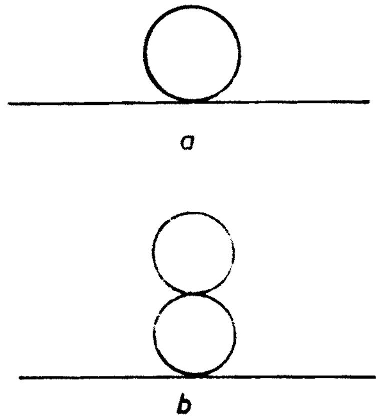

Fig. 5. Two additional Feynman graphs which appear for the particle self-energy in the WNP procedure

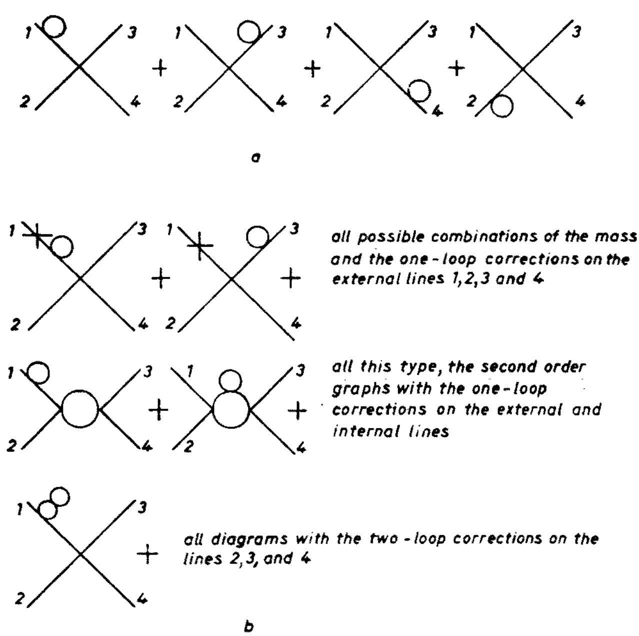

Fig. 6. Additional Feynman diagrams which appear in the scattering amplitude of the ϕ particles for the WNP procedures: (a) second order, (b) third order

Now we find the renormalization constant parameters in MS scheme:
in the first order:

$$a_{31}^{\mathrm{WNP}} = 0, \quad d_1^{\mathrm{WNP}} = \frac{1}{4\pi}\frac{1}{\varepsilon}, \quad a_{g1}^{\mathrm{WNP}} = -\frac{3}{4\pi}\frac{1}{\varepsilon}, \tag{15}$$

in the second order:

$$a_{32}^{\mathrm{WNP}} = \frac{1}{12(4\pi)^2}\frac{1}{\varepsilon}, \quad d_2^{\mathrm{WNP}} = \frac{1}{8\pi^2}\frac{1}{\varepsilon^2} - \frac{5}{12(4\pi)^2}\frac{1}{\varepsilon},$$

$$a_{g2}^{\mathrm{WNP}} = \frac{1}{(4\pi)^2}\left(-\frac{9}{\varepsilon^2} + \frac{17}{6}\frac{1}{\varepsilon}\right). \tag{16}$$

In contrary to the NP procedure, now mass renormalization constant Z_m^{WNP} does not depend on m^2 and μ^2.

2.4. Comparison of both procedures NP and WNP

To find the differences in both procedures we have made detailed calculation for the ϕ^4 theory but greater part of our conclusions are valid not only for this specific model.

I: using the same renormalization scheme (MS in our example) both procedures result in different renormalized quantity and different renormalization constants. But we can change renormalization scheme for the NP procedure to get the same renormalized quantity as in the WNP one. This means that we can make renormalization group transformation [9] for the first procedure (e.g. NP) to get physical quantity from the second one, thus to conclude — *both procedures are physically equivalent.*

II: comparing the formulae (11) and (12) with (15) and (16) we see the difference only for mass renormalization.

$$a_{3n}^{\mathrm{NP}} = a_{3n}^{\mathrm{WNP}}, \quad a_{gn}^{\mathrm{NP}} = a_{gn}^{\mathrm{WNP}} \quad \text{and} \quad d_n^{\mathrm{NP}} \neq d_n^{\mathrm{WNP}}. \tag{17}$$

The additional graphs in the WNP procedure (Fig. 5) change the propagator renormalization. These additional graphs appear also on the external and internal line in Fig. 6, but they do not change four point vertex renormalization ($a_{gn}^{\mathrm{NP}} = a_{gn}^{\mathrm{WNP}}$). Field renormalization constant Z_3 is introduced to cancel infinity which is multiplied by powers of momentum (in our case k^2, because kinetic energy counter term is the following $Z_3 \frac{1}{2} \partial_\mu\phi\partial^\mu\phi$). The graphs in Fig. 5 do not depend on particle momentum (formulae (A9) and (A10) from Appendix A), but only on mass m. So, they do not change renormalization constant Z_3 and $a_{3n}^{\mathrm{NP}} = a_{3n}^{\mathrm{WNP}}$. Formulae (A9) and (A10) from Appendix do not depend on momentum because four particle coupling is momentum independent (Graph in Fig. 7 = $-ig_0$ in our case). In any renormalizable field theory four particles cannot couple with derivatives, as a result of this Feynman four particle vertex does not depend on momentum. *For any renormalizable field theory in which three particle vertex is momentum independent the two procedures NP and WNP differ only in masses renormalization* ($\delta m^{2\mathrm{NP}} \neq \delta m^{2\mathrm{WNP}}$).

III: Weinberg [10] and 't Hooft [8] have found the practical way to investigate the renormalization group. The base of this approach is the existence of renormalization scheme independent of mass, where renormalization constants do not depend on mass. For the NP procedure d_2^{NP} does not fulfil this condition (see Eq. (12)). It means that the mass independent renormalization scheme does not exist for the ϕ^4 theory. On the other hand, if we add all graphs from Fig. 3 and Fig. 5, mass dependences disappear, in spite of that separate graphs depend on $\xi = \ln \frac{4\pi\mu^2}{m^2}$ (see Eqs. (A4), (A5), (A9) and (A10) from Appendix A). *Mass independent renormalization scheme exists only for the WNP procedure.*

IV: for on-mass shell renormalization scheme, to get renormalized particle self--energy we subtract appropriate bare graphs for the particle momentum $k^2 = m^2$ (see

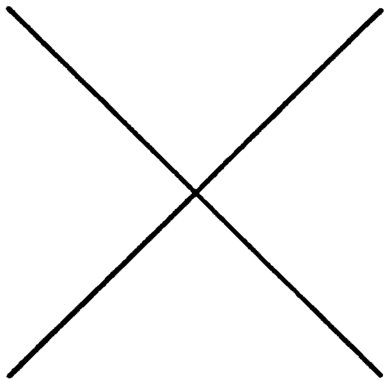

Fig. 7. The vertex Green function in tree approximation in ϕ^4 theory

e.g. [1]). In the case of the ϕ^4 theory, in agreement with the BPHZ prescription [11], for particle ϕ self-energy we have (index of the diagram $\omega = 2$):

$$\tilde{\Sigma}_R(k^2, m^2) = \tilde{\Sigma}_0(k^2, m^2) - \tilde{\Sigma}_0(k^2, m^2)|_{k^2=m^2}$$

$$- \left.\frac{d\tilde{\Sigma}_0}{dk^2}\right|_{k^2=m^2}(k^2-m^2) - 2\left.\frac{d^2\tilde{\Sigma}^0}{d(k^2)^2}\right|_{k^2=m^2}((\mathring{k}k)^2 - 2\mathring{k}km^2 + m^4), \tag{17}$$

where $\mathring{k}$ is any momentum which fulfils $\mathring{k}^2 = m^2$. But the WNP procedure differs from the NP procedure in the independent of momentum graphs (Fig. 5). So already after the first subtraction in Eq. (17) contributions from this additional graphs are cancelled and we get:

$$\tilde{\Sigma}_R^{NP}(k^2, m^2) = \tilde{\Sigma}_R^{WNP}(k^2, m^2). \tag{18}$$

The renormalized constant Z_m is obviously different in both cases. Other renormalized constants are the same as in the previous renormalization scheme MS. In our ϕ^4 theory we have:

$$\tilde{\Sigma}_R^{NP}(k^2, m^2) = \tilde{\Sigma}_R^{WNP}(k^2, m^2) = \left(\frac{g}{4\pi}\right)^2 \Big\{\Sigma_R^1(k^2, m^2)$$

$$- \Sigma_R^1(k^2, m^2)\Big|_{k^2=m^2} - \left.\frac{d\Sigma_R^1}{dk^2}\right|_{k^2=m^2}(k^2-m^2)\Big\}, \tag{19}$$

where Σ_R^1 is given by Eq. (A4) in Appendix A. *Both procedures, the NP and WNP, are of the same physical renormalized quantity if we use the on shell renormalization scheme.*

3. *Gauge theories*

For the gauge theories there is an additional problem connected with freedom of gauge transformation. Regularization and renormalization can disturb gauge symmetry but renormalized Green's functions must have such symmetry. Usually is more practical to use regularization which preserves gauge symmetry; then all the counter terms in lagrangian also have this symmetry.

Dimensional regularization is gauge symmetric [7], but only in the case, when we do not use the normal product. To see this let us consider vacuum polarization in the scalar electrodynamics. A charged, spinless particle can interact with one or with two photons (see Fig. 8). Then the one-loop approximation for the vacuum polarization $i\Pi_{\mu\nu}$ is:
the WNP procedure:

$$i\Pi_{\mu\nu}^{\mathrm{WNP}} = \text{Graph in Fig. 9} + \text{Graph in Fig. 10}, \tag{20}$$

the NP procedurc:

$$i\Pi_{\mu\nu}^{\mathrm{NP}} = \text{Graph in Fig. 9}. \tag{21}$$

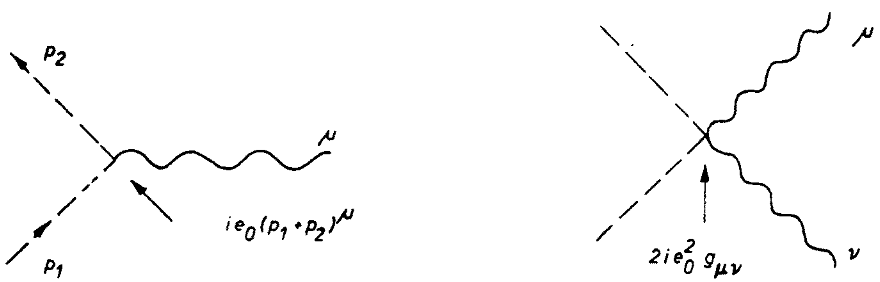

Fig. 8. The two way of interaction for the charged, spinless particle ϕ with photons A_μ

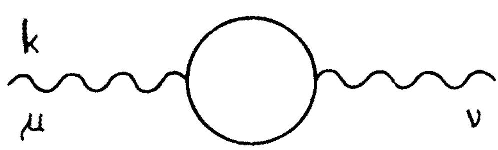

Fig. 9. The momentum dependent contribution to vacuum polarization in scalar electrodynamics

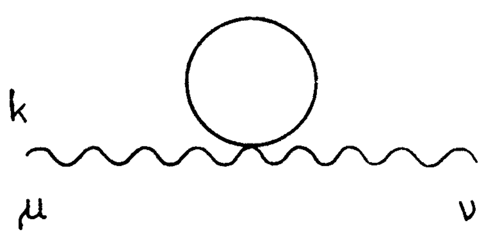

Fig. 10. The momentum independent contribution to vacuum polarization in scalar electrodynamics

From (20), in the case of the WNP procedure, we get:

$$i\Pi_{\mu\nu}^{\mathrm{WNP}}(k^2, m^2) = -i(k^2 g_{\mu\nu} - k_\mu k_\nu)\Pi(k^2, m^2), \tag{22}$$

where

$$\Pi(k^2, m^2) = e_0^2(\mu^2)^{-\varepsilon/2} \frac{1}{(4\pi)^2}\left\{\frac{2}{3}\cdot\frac{1}{\varepsilon} - \frac{1}{3}\gamma + \frac{1}{3}\ln\frac{4\pi\mu^2}{m^2}\right.$$

$$\left. - \int_0^1 dx(1-2x)^2 \ln\left(1 + \frac{k^2}{m^2}x(x-1)\right)\right\}.$$

Because of the factor $k^2 g_{\mu\nu} - k_\mu k_\nu$, the dimensional regularization is gauge independent, and $k^\mu \Pi_{\mu\nu}^{\mathrm{WNP}} = 0$. For the NP procedure from (21) we have:

$$i\Pi_{\mu\nu}^{\mathrm{NP}}(k^2, m^2) = -i(k^2 g_{\mu\nu} - k_\mu k_\nu)\Pi(k^2, m^2) + i g_{\mu\nu}\Omega(m^2), \tag{23}$$

where

$$\Omega(m^2) = e_0^2(\mu^2)^{-\varepsilon/2} \frac{2m^2}{(4\pi)^2}\left(\frac{2}{\varepsilon} - \gamma + 1 + \ln\frac{4\pi\mu^2}{m^2}\right),$$

and $\Pi(k^2, m^2)$ is the same as in Eq. (22). The term which disturbs gauge symmetry in $i\Pi_{\mu\nu}^{\mathrm{NP}}(ig_{\mu\nu}\Omega(m^2))$ depends only on mass m because the additional graphs in the WNP (Fig. 10) are momentum independent. For the on-mass shell renormalization scheme contribution from the additional graphs are cancelled and

$$i\Pi_{\mu\nu}^{\mathrm{WNP}}(\text{on mass shell}) = i\Pi_{\mu\nu}^{\mathrm{NP}}(\text{on mass shell}). \tag{24}$$

The mass renormalization constant Z_m is different in both cases. To remove infinity from the not gauge symmetric term in Eq. (23), we have to introduce photon mass counter-term $\left(\frac{\delta M^2}{2} A_\mu A^\mu\right)$ to the lagrangian. And we see that: *the dimensional regularization is not gauge symmetric for theory with the normal product.*

The question is now which procedure is more practical in application of the gauge theory. For the WNP procedure we have to calculate more Feynman graphs, but the dimensional regularization is gauge symmetric. So it is easier to make its renormalization because it is also gauge symmetric. If we know $i\Pi_{\mu\nu}^{\mathrm{WNP}}$ (Eq. (22)) then one easily calculate $i\Pi_{\mu\nu}^{\mathrm{NP}} = i\Pi_{\mu\nu}^{\mathrm{WNP}} -$ (Graph in Fig. 10). It is more difficult to get $i\Pi_{\mu\nu}^{\mathrm{NP}}$ (Eq. (23)) directly from the graph in Fig. 9. *For the gauge theories in spite of greater number of graphs, necessary to calculate, the WNP procedure is more practical because calculations are simpler, the dimensional regularization is gauge symmetric.* But the fastest way to get photon propagator $\Pi(k^2, m^2)$ in Eq. (22) is to calculate only the part which is proportional to $k_\mu k_\nu$ in the $i\Pi_{\mu\nu}^{\mathrm{NP}}$. Because additional graphs in the WNP procedure do not depend on momentum, the term proportional to $k_\mu k_\nu$ in the $i\Pi_{\mu\mu}^{\mathrm{NP}}$ is just $\Pi(k^2, m^2)$.

4. *Conclusions*

Using the very simple models we compare, from practical point of view, two approaches in quantum field theory. The first, the canonical quantization approach, which uses normal product (NP) and the second procedure — the Feynman patn integral approach without normal product (WNP).

We have found that:

I: both procedures are equivalent in the sense that there exist renormalization schemes in which the same physical results are obtained,

II: for any renormalizable field theory in which three particle vertex is momentum independent the two procedures NP and WNP differ only in masses renormalization,

III: the mass independent renormalization scheme, necessary for Weinberg and 't Hooft approach to renormalization group, exists only for the WNP procedure,

IV: using the on-shell renormalization scheme both procedures give the same physical renormalized quantity,

V: the dimensional regularization is (is not) gauge symmetric for the WNP procedure (the NP procedure),

VI: the WNP procedure is more practical for the gauge theories.

APPENDIX A

We give all the necessary one and two loop Feynman amplitudes using dimensional regularization. As usual we introduce the parameter $n = 4-\varepsilon$ and the mass dimensional parameter μ. To find renormalized Feynman amplitudes (not only renormalization constants) we need linear parts in the ε of same graphs. For the Euler Gamma function these linear parts are denoted [12]:

$$\Gamma(\varepsilon) = \frac{1}{\varepsilon} - \gamma + c_0\varepsilon, \tag{A1}$$

$$\Gamma(-1+\varepsilon) = -\frac{1}{\varepsilon} + \gamma - 1 + c_1\varepsilon, \tag{A2}$$

where

$$c_0 = \tfrac{1}{2}\left(\gamma^2 + \frac{\pi^2}{6}\right), \quad c_1 = \gamma - 1 - c_0,$$

and γ is the Euler constant $\gamma = 0.57721\ldots$. It is convenient to introduce the parameter

$$\xi = \ln\frac{4\pi\mu^2}{m^2}. \tag{A3}$$

Feynman graphs from Chapter 2.2

(A) for the self-energy Σ_0^{NP},

$$\text{Graph in Fig. 11a} \equiv -i\Sigma_0^1$$

$$= -ig_0^2(\mu^2)^{-\varepsilon}\frac{m^2}{(4\pi)^4}\left[\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\left(\tfrac{3}{2}-\gamma+\xi-\frac{k^2}{12m^2}\right)+\Sigma_{\mathrm{R}}^1\right] \qquad \text{(A4)}$$

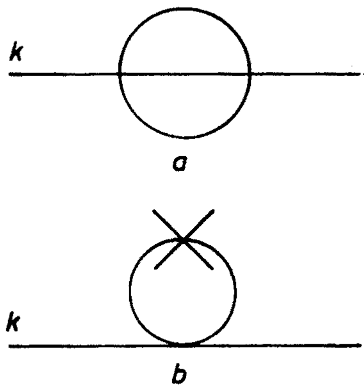

Fig. 11. Vacuum polarization in ϕ^4 theory second order contributions

where

$$\Sigma_{\mathrm{R}}^1 = \tfrac{1}{12}I(k^2, m^2)+\left(\frac{k^2}{12m^2}-\tfrac{1}{2}\right)(\gamma-1-\xi)-\xi(\gamma-1-\tfrac{1}{2}\xi),$$

and

$$I(k^2, m^2) = \int_0^1 dx\int_0^1 d\varrho\left[\left(\frac{1}{\varrho A^2}-\frac{k^2}{m^2}\frac{B}{A^3}\right)\left(-2\ln\left(1-\frac{k^2}{m^2}\frac{B}{A}\right)\right.\right.$$

$$\left.\left.+\ln(\varrho A)\right)-\frac{1}{\varrho}\ln\varrho-\frac{\varrho}{(1-\varrho(1-x))^2}\ln(1-\varrho(1-x))-\frac{\varrho}{(1-\varrho x)^2}\ln(1-\varrho x)\right],$$

where $A = \varrho x(1-x)+1-\varrho$, $B = \varrho x(1-x)(1-\varrho)$,

$$\text{Graph in Fig. 11b} \equiv -i\Sigma_0^2$$

$$= -ig_0\delta m^2(\mu^2)^{-\varepsilon/2}\frac{1}{2(4\pi)^2}\left[-\frac{2}{\varepsilon}+\gamma-\xi-\frac{\varepsilon}{2}(-\gamma\xi+\tfrac{1}{2}\xi^2+c_0)\right], \qquad \text{(A5)}$$

(B) for the Green's function τ_0^{NP},

$$\text{Graph in Fig. 12a} = i\frac{g_0^2}{2}(\mu^2)^{-\varepsilon/2}\frac{1}{(4\pi)^2}\left[\frac{2}{\varepsilon}+\xi-\gamma+A(s)+\varepsilon A_1(s)\right], \qquad \text{(A6)}$$

where

$$A(s) = 2 + \sqrt{1 - \frac{4m^2}{s}} \ln \left[\frac{1 - \sqrt{1 - \frac{4m^2}{s}}}{1 + \sqrt{1 - \frac{4m^2}{s}}} \right],$$

and

$$A_1(s) = \tfrac{1}{2}(c_0 - \gamma\xi + \tfrac{1}{2}\xi^2) + \tfrac{1}{2}(\xi - \gamma)A(s) + \tfrac{1}{4}R(s),$$

$$R(s) = \int_0^1 dx \ln^2 \left(1 - \frac{s}{m^2} x(1-x)\right), \qquad s = (k_1 + k_2)^2.$$

All other graphs in second order we get by transforming the Mandelstam variable s into t and u in Eq. (A6). In the third order for the τ_0^{NP}, we have kinds of graphs necessary to calculate.

$$\text{Graph in Fig. 12b} = \text{Graph in Fig. 12c} = -ig_0^2 \delta m^2 \frac{1}{(4\pi)^2} \frac{2 - A(s)}{s - 4m^2}. \tag{A7}$$

The other graphs of this type are obtained from (A7) by changing $s \to t$, and $s \to u$.

$$\text{Graph in Fig. 12d} = ig_0^3 (\mu^2)^{-\varepsilon} \frac{1}{(4\pi)^4}$$

$$\times \left[\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}(1 - 2\gamma + 2\xi + 2A(s)) + \tfrac{1}{2} + 2(c_0 - \gamma\xi + \tfrac{1}{2}\xi^2) \right.$$

$$\left. + (\xi - \gamma)(2A(s) + 1) + R(s) + A(s) + I_1(s, m^2) \right], \tag{A8}$$

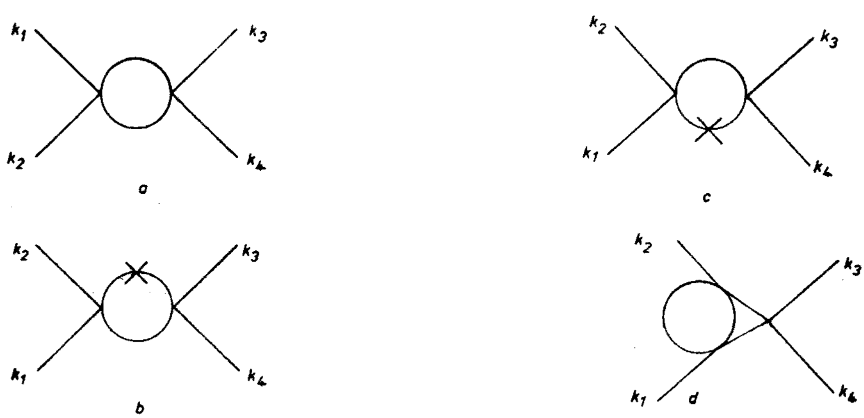

Fig. 12. Third order contributions to τ_0

where

$$I(s, m^2) = \int_0^1 dx \int_0^1 dy \int_0^1 d\varrho \left[\frac{1-x}{x(1-Px)^2} \ln \left(\frac{(1-Px)^{3/2}}{1-x-x^2(\varrho^2-\varrho)+(1-x)^2 (y^2-y) \frac{s}{m^2}} \right) \right.$$

$$\left. + \frac{x-1}{x} \ln \left(\frac{\sqrt{x}}{1+(y^2-y) \frac{s}{m^2}} \right) \right], \quad P = \varrho^2-\varrho+1.$$

As previously, to get remaining graphs we have to change $s \to t$ and $s \to u$.

The additional graphs appearing in Chapter 2.3

(C) for the self-energy Σ_0^{WNP}

$$\text{Graph in Fig. 5a} = -i\Sigma_0^3 = -ig_0(\mu^2)^{-\varepsilon/2} \frac{m^2}{2(4\pi)^2}$$

$$\times \left(-\frac{2}{\varepsilon} +\gamma-1-\xi+ \frac{\varepsilon}{2} (\xi(\gamma-1)-\tfrac{1}{2}\xi^2+c_1) \right), \tag{A9}$$

$$\text{Graph in Fig. 5b} = -i\Sigma_0^4 = ig_0^2(\mu^2)^{-\varepsilon} \frac{m^2}{4(4\pi)^2}$$

$$\times \left[-\frac{4}{\varepsilon^2} + \frac{4}{\varepsilon} (\gamma-\xi-\tfrac{1}{2})+2\xi\gamma-\xi-\xi^2+c_1-c_0+(\xi-\gamma)(\gamma-\xi-1) \right], \tag{A10}$$

(D) having the one-loop (A9) and the two-loop (A10) graphs it is easy to find all formulae for the graphs in Fig. 6. We do not give here the lengthy formulae.

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