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ON THE PROTON SPIN DEPENDENT STRUCTURE FUNCTION G_2

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The purpose of this paper is to present possible experimental test for the yet unobservable proton structure function G_2. We discuss the Drell-Yan process on transverse polarized proton and antiproton and find the way to measure this function.

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1. Introduction

In this paper we discuss the problem of the one of proton spin dependent structure functions — the function G_2 [1]. For testing QCD predictions it is important to have complete set of the proton structure functions — up to now its spin dependent part is less known.

The problem of the proton structure in the longitudinally polarized state has been studied both theoretically [2, 3] and experimentally [4] but there are very few papers on the structure of the proton in the transversely polarized state. The usual way for testing spin-dependent part of structure functions, i.e. examining the spin-spin asymmetry, is not useful in this problem. The reason is that the predictions for this asymmetry for transversely polarized protons/antiprotons are by factor of 100 smaller than the corresponding asymmetry for longitudinally polarized hadrons and therefore are hardly measurable [5]. So our task is to find and calculate such a process where transverse polarization can give measurable effects. We proposed the Drell-Yan muon pair production [6] on transverse polarized proton-antiproton as an interesting process to examine. In this case the spin configuration determines the azimutal distribution of the muon pair production. The asymmetry in this distribution tests the structure of the transverse polarization state. We have constructed the model of the structure of the transversely polarized proton based on the Wandzura-Wilczek sum rule [7] and models for longitudinally polarized proton [2, 3]. It turned out that the magnitude of the asymmetry obtained from this calculation makes the measurment possible.

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Our paper is arranged as follows. In the next section we present some definitions concerning structure functions and its interpretation in the partcn model. Section 3 describes the Drell-Yan process in the transverse spin configuration. Section 4 gives the results of the calculations. Summary (Section 5) closes the paper. More detailed information about the used models is given in the Appendix.

2. *The spin-dependent proton structure functions*

Let us present some notation for proton structure functions. They are usually defined in DIS process in ep collisicns. We can introduce two structure functions G_1 and G_2 by the Fourier transform of the comutator of the two electromagnetic currents sandwiched between one-nucleon state with momentum p and covariant spin s [8]

$$W_{\mu\nu}(p, q, s) = \frac{1}{2\pi}\int d^4x e^{iqx}\langle p, s|\,[J_\mu(x), J_\nu(0)]\,|p, s\rangle = W^{(S)}_{\mu\nu}+iW^{(A)}_{\mu\nu}, \tag{1}$$

the spin dependent part $W^{(A)}_{\mu\nu}$ is equal to

$$W^{(A)}_{\mu\nu} = \frac{M}{2p\cdot q}\,\varepsilon_{\mu\nu\lambda\sigma}q^\lambda\left\{s^\sigma G_1(\nu, q^2)+\left(s^\sigma-\frac{s\cdot qp^\sigma}{p\cdot q}\right)G_2(\nu, q^2)\right\}, \tag{2}$$

here s is a spin 4-vector and structure functions G_1, G_2 are defined to be dimensionless (q is the photon 4-momentum, $\nu = p\cdot q$ and M is proton mass).

The parton model [1] gives a simple interpretation of the function G_1 and G_2:

— For polarized state of proton we deal with densities, helicity densities L and transversity densities T for quarks:

$$\begin{aligned} q(x) &= q^+(x)+q^-(x),\\ \Delta q^{\rm L}(x) &= q^{+\rm L}(x)-q^{-\rm L}(x),\\ \Delta q^{\rm T}(x) &= q^{+\rm T}(x)-q^{-\rm T}(x), \end{aligned} \tag{3}$$

x denotes the parton momentum fraction, + − denote the polarization state parallel (antiparallel) to the polarization of the parent proton.

— In the case of longitudinally polarized proton the interpretation of the function G_1 is as follows [1]

$$G_1(x) = \sum_f e_f^2\Delta q_f^{\rm L}(x), \tag{4}$$

where e_f denote charge of quark q_f, f means quarks flavour. The factor which multiplies the function G_2 in Eq. (2) is equal to zero so in this case this quantity is unmeasurable.

— In the case of transversely polarized proton we can interpret the sum of functions G_1 and G_2 as [1]

$$(G_1(x)+G_2(x)) = \sum_f e_f^2\Delta q_f^{\rm T}(x). \tag{5}$$

From Eqs. (4), (5) one can see that we cannot interpret function G_2 in the parton model separately from function G_1. The interesting problem is thus to find the connection between them. Such analysis was performed [7] using operator product expansion for the spin--dependent part of the electroproduction cross section and the approximate sum-rule was found in the following form

$$\int_0^1 x^{J-1}dx \left\{\frac{J-1}{J} G_1(x)+G_2(x)\right\} \approx 0 \quad J \geqslant 1. \tag{6}$$

Eq. (6) can be inverted to yield [5]

$$G_1(x)+G_2(x) = \int_x^1 \frac{dy}{y} G_1(y)+\varepsilon(x)G_1(x) \tag{7}$$

where $\varepsilon(x)$ denotes the possible error factor which is connected with the approximate nature of the sum rule (6). This sum rule (Eq. (7)) gives the possibility to construct the model for transversely polarized proton based on the model for the longitudinally polarized one. Some more detailed information about this construction is given in the Appendix.

To summarize this section we want to stress that we cannot interpret function G_2 separately from function G_1. We can study either function G_1 (at longitudinally polarized state of the particle, Eq. (4)) or functions G_2 and G_1 together (at transversely polarized state of the particle (Eq. (5)). Our aim is to propose a reasonable way to measure the function (G_1+G_2).

3. *The Drell-Yan process* $p_\uparrow \bar{p}_\downarrow \to \mu^+\mu^- X$

Let us examine the Drell-Yan process [6] in the case of collision of the transversely polarized proton and antiproton (see Fig. 1). On the Born level we have only one graph to calculate (Fig. 2). The angle ϕ and the initial polarization vectors enter formula for the hard cross-section only in the form of the simple multiplicative [9]

$$\frac{d\hat{\sigma}}{d\Omega} = (1+\cos^2\phi\hat{s}_1(x_1)\hat{s}_2(x_2))\frac{d\hat{\sigma}_{\mathrm{NP}}}{d\Omega}. \tag{8}$$

Here x_1, x_2 denote as usual momentum fractions. Angle ϕ and the spin configuration is defined in Fig. 1. $\frac{d\hat{\sigma}_{\mathrm{NP}}}{d\Omega}$ denotes the spin and ϕ angle independent part of the hard cross-section. $\hat{s}_{1,2}(x)$ are the spin 4-vectors defined in the corresponding rest frames

$$\hat{s}_1(x) = (0,1,0,0)\frac{\Delta q^{\mathrm{T}}(x)}{q(x)} \quad \text{for proton quarks,}$$

$$\hat{s}_2(x) = (0,-1,0,0)\frac{\Delta \bar{q}^{\mathrm{T}}(x)}{\bar{q}(x)} \quad \text{for antiproton antiquarks.} \tag{9}$$

We define the asymmetry in the azimutal angle distribution for the muon μ^- production as follows

$$A|_{Q^2 \text{fixed}} = \frac{\int_I \frac{d\sigma^{DY}}{dQ^2} d\phi - \int_{II} \frac{d\sigma^{DY}}{dQ^2} d\phi}{\int_{I+II} \frac{d\sigma^{DY}}{dQ^2} d\phi}, \tag{10}$$

where the integration regions in the ϕ angle are as follows:

$$I = \left(-\frac{\pi}{4}, \frac{\pi}{4}\right) \cup \left(\frac{3\pi}{4}, \frac{5\pi}{4}\right),$$

$$II = \left(\frac{\pi}{4}, \frac{3\pi}{4}\right) \cup \left(\frac{5\pi}{4}, \frac{7\pi}{4}\right),$$

Q^2 is the muon pair energy.

Due to the simple way in which spin enters the hard-scattering formula (8) one can invert order of integration in formula (10) and perform integration over the angle before convoluting hard cross-section with the structure functions.

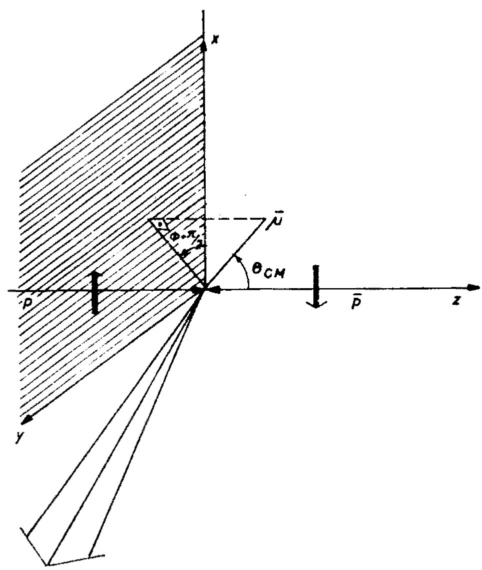

Fig. 1. Definition of the θ_{CM} and ϕ angles in the Drell-Yan process. θ_{CM} is defined here as the angle between proton and μ^- momenta. $\left(\phi + \frac{\pi}{2}\right)$ is an angle between proton spin polarization vector and μ^- momentum vector projection on the plane perpendicular to the beam axis

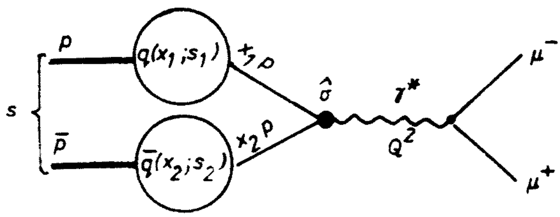

Fig. 2. Kinematical structure of the Drell-Yan process in a proton-antiproton collision and leading-log order

Then asymmetry takes the form

$$A\Big|_{\tau=Q^2/s\text{ fixed}} = \frac{[\sum e_f^2 q_f(x_1)\bar{q}_f(x_2)]\otimes[2\hat{s}_1(x_1)\cdot\hat{s}_2(x_2)]\,\frac{d\hat{\sigma}_{NP}}{d\Omega}}{[\sum e_f^2 q_f(x_1)\bar{q}_f(x_2)]\otimes[2\pi+\pi\hat{s}_1(x_1)\hat{s}_2(x_2)]\,\frac{d\hat{\sigma}_{NP}}{d\Omega}}\Bigg|_{\tau\text{ fixed}} \tag{11}$$

where the symbol $\otimes$ stands for appropriate integrations (see e.g. [10]). The maximum value of this asymmetry is 21% (for $s_\perp(x) = -s_2(x) = (0, 1, 0, 0)$). One can notice (Eq. 11)) that we obtain not negligible value of asymmetry A from such regions of $\tau = x_\perp x_2$ where partons which mainly take part in the process are significantly transversely polarized (for the models analyzed here this means $x_1 \approx x_2 \approx \sqrt{\tau}$).

4. *Numerical results and conclusions*

We have calculated spin dependent structure functions for the proton. Calculations were based on the Wandzura-Wilczek sum rule (7) and models of Refs [2, 3] (see also Appendix and Table I). Results of this calculation are shown in Fig. 3–6, some interesting normalization integrals are summarized in Table II.

The transverse polarization vector $\hat{s}(x)$ is shown in Fig. 3 for two flavours of valence quarks. One can see that the Carlitz-Kaur model predicts slightly bigger polarization for this quarks than the SU(6) one. The density of the polarization was calculated for two limiting values of the error factor $\varepsilon(x)$ [5, 7] i.e. $\varepsilon(x) = 0.0$ and $\varepsilon(x) = 0.2$.

The next two figures present the ratio between spin-dependent and spin-independent structure functions:

$$\frac{G_1(x)}{\mathscr{F}_2(x)} = \frac{\sum e_f^2 \Delta q^L(x)}{\sum e_f^2 q(x)}$$

for longitudinally polarized state (Fig. 4) and

$$\frac{G_1(x)+G_2(x)}{\mathscr{F}_2(x)} = \frac{\sum e_f^2 \Delta q^T(x)}{\sum e_f^2 q(x)}$$

for transversely polarized state (Fig. 5).
For $x < 0.08$ ($\varepsilon = 0.0$) and for $x < 0.2$ ($\varepsilon = 0.2$) this ratio is bigger for transversely polarized state than the corresponding ratio for longitudinally polarized one but drops off

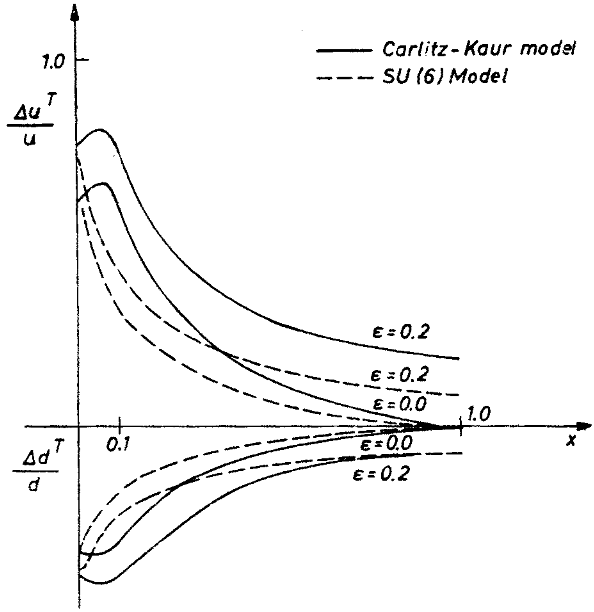

Fig. 3. The density of the polarization four-vector $\hat{s}$ for valence quarks. Details on the distribution functions are given in Table I. Some integrals are given in Table II

TABLE I

The valence quark densities, helicity densities and transversity densities used in the calculations. The valence quarks helicity densities are predicted by Carlitz-Kaur model [2] and SU(6) model [3]. The transversity densities are obtained using Wandzura-Wilczek sum rule [7]

	Carlitz-Kaur model	SU(6) model
	$A_0 = 1.64\sqrt{x}\,(1-x)^3$ $A_1 = 1.216(1-x)A_0$ $\cos 2\theta = \frac{1}{1+0.052(1-x)^2/\sqrt{x}}$	
U_v	$(A_0 + \frac{1}{3}A_1)/x$	
d_v	$\frac{2}{3}A_1/x$	
Δu^L	$\cos 2\theta(u_v - \frac{2}{3}d_v)$	$0.44u_v$
Δd^L	$\cos 2\theta(-\frac{1}{3}d_v)$	$-0.35d_v$
Δu^T	$\int_x^1 \frac{\Delta u^L}{x}dx + \varepsilon(x)\Delta u^L$	
Δd^T	$\int_x^1 \frac{\Delta d^L}{x}dx + \varepsilon(x)\Delta d^L$	

rapidly as x increases. It means that at large x the quarks do not "remember" well the transverse polarization of the proton (Fig. 5), in contrast with the case of longitudinally polarized proton for which quarks carry the helicity information almost exclusively at large x (Fig. 4). So the transverse spin of the proton is carried mainly by the valence quarks with the moderately small x.

We draw also the ratio $G_2(x)/\mathcal{F}_2(x)$ (Fig. 6). The predictions obtained from two models [2, 3] are different in the important region i.e. for small x (in this region the funcion $G_2(x)$ is significantly different from the function $G_1(x)$).

TABLE II

Normalization integrals describing distributions of the spin among proton components

	Carlitz-Kaur model		SU(6) model	
	$\varepsilon(x) = 0.0$	$\varepsilon(x) = 0.2$	$\varepsilon(x) = 0.0$	$\varepsilon(x) = 0.2$
$\frac{1}{2}\int_0^1 [\Delta u^{L}(x)+\Delta d^{L}(x)]dx$	0.38		0.27	
$\frac{1}{2}\int_0^1 [\Delta u^{T}(x)+\Delta d^{T}(x)]dx$	0.38	0.45	0.34	0.39
$\int_0^1 \frac{\Delta u^{T}(x)}{u(x)} dx$	0.20	0.32	0.13	0.22
$\int_0^1 \frac{\Delta d^{T}(x)}{d(x)} dx$	−0.08	−0.13	−0.06	−0.12
$\int_0^1 \frac{(G_1(x)+G_2(x))}{\mathcal{F}_2(x)}$	0.17	0.28	0.12	0.19

TABLE III

Asymmetry A in the ϕ angle (Eq. (11)) for the muon pair produced via the Drell-Yan mechanism in the $p\bar{p}$ collisions. $\sqrt{s} = 540$ GeV, for distribution functions see Table I

$\tau = x_1x_2$	Q^2 [GeV]^2	Asymmetry A			
		Carlitz-Kaur model		SU(6) model	
		$\varepsilon = 0.0$	$\varepsilon = 0.2$	$\varepsilon = 0.0$	$\varepsilon = 0.2$
$0.05 < \tau < 0.1$	$(120)^2 < Q^2 < (170)^2$	<0.5%	3%	<0.5%	<0.5%
$0.01 < \tau < 0.05$	$(54)^2 < Q^2 < (120)^2$	4.5%	8%	2.8%	4.5%
$0.005 < \tau < 0.01$	$(38)^2 < Q^2 < (54)^2$	6%	9%	3.7%	6%

In the second part we calculated the asymmetry A (Eq. (11)) for transversely polarized pp collisions. The calculations were made using Monte Carlo simulations, some part of the program Coral-B [9] was applied. We calculated it for fixed energy $\sqrt{s} = 540$ GeV and for few intervals of $\tau = Q^2/s$. Its results are shown in Table III.

We can expect asymmetry about 6–9% for the Carlitz-Kaur model [2] and 4–6% for the SU(6) model [3] if we choose $\tau = x_1x_2$ small enough (e.g. $0.005 \leqslant \tau < 0.01$). The asymmetry decreases fast for larger value of τ.

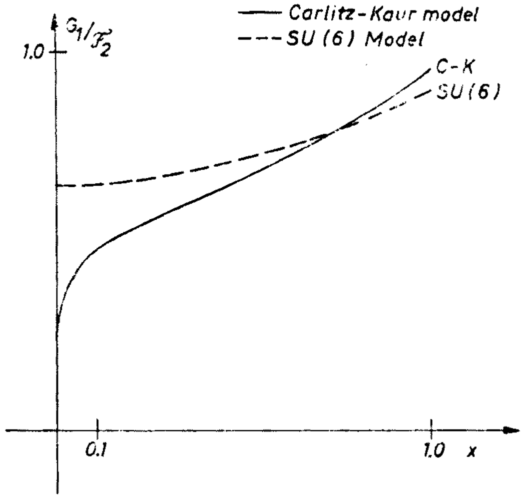

Fig. 4. The ratio of the structure functions G_1 and $\mathcal{F}_2$ for Carlitz-Kaur [2] and SU(6) [3] models. Details of the distribution functions are given in Table I

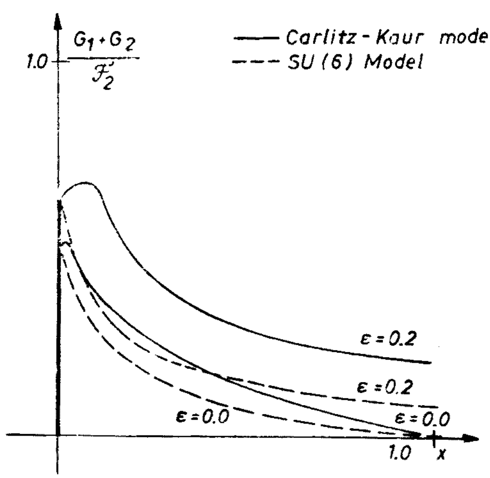

Fig. 5. The ratio of structure functions (G_1+G_2) and $\mathcal{F}_2$ for Carlitz-Kaur [2] and SU(6) models. Details of the distribution functions are given in Table I. $\varepsilon(x)$ labels the possible error factor in the Wandzura--Wilczek sum rule (Eq. (7))

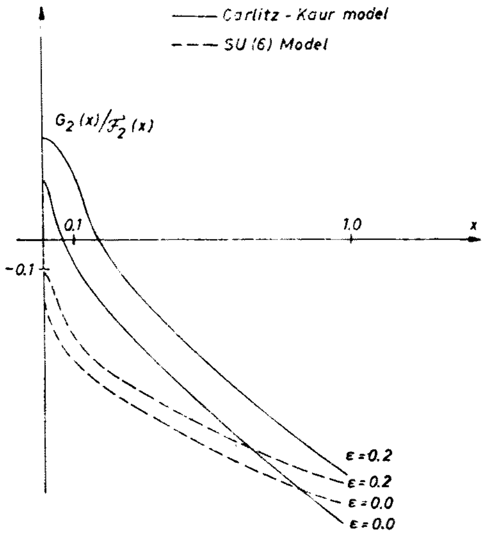

Fig. 6. The ratio of structure functions G_2 and $\mathcal{F}_2$ for Carlitz-Kaur [2] and SU(6) [3] models. Details of the distribution functions are given in Table I. $\varepsilon(x)$ labels the possible error factor in the Wandzura-Wilczek sum rule (Eq. (7))

If we can measure asymmetry A as a function of τ we can investigate regions of $x_1 \approx x_2 \approx \tau$ for which partons taking part in the reaction are significantly transverse polarized. This result gives us possible way to measure function G_2.

5. *Summary*

In this paper we propose the experimental test for proton spin dependent structure functions G_2.

The principle of this test is to measure the azimuthal angle asymmetry in the Drell-Yan process with the transversely polarized protons and antiprotons.

Our calculation was based on the Wandzura-Wilczek sum rule [7] and the Carlitz-Kaur or the SU(6) model for longitudinally polarized proton. We found that at $\tau = 0.007$ the asymmetry can be as big as 6% for the SU(6) model and 9% for the Carlitz-Kaur model. On the other hand it is possible to reconstruct approximately the function G_2 from the asymmetry A.

I would like to thank Jerzy Szwed for suggesting the subject and for constant interest in this work. I thank Zbigniew Wąs for some technical remarks.

APPENDIX

The Wandzura-Wilczek sum rule (Eq. (7)) gives the relation between proton spin-dependent structure functions in the transversely and longitudinally polarized state.

This relation is flavour and charge independent

$$\Delta q^{\mathrm{T}} = \varepsilon(x)\Delta q^{\mathrm{L}}(x) + \int_x^1 \frac{\Delta q^{\mathrm{L}}(y)}{y}\, dy. \tag{A.1}$$

We have calculated transverse distributions functions for two models of longitudinally polarized proton.

— The SU(6) "conservative" model [3]

This model assumed that ratio $\Delta q^{\mathrm{L}}/q$ takes an average value independent of x, i.e.

$$\Delta u_{\mathrm{v}}^{\mathrm{L}}(x) = 0.44 u_{\mathrm{v}}(x), \quad \Delta d_{\mathrm{v}}^{\mathrm{L}}(x) = -0.35 d_{\mathrm{v}}(x), \tag{A.2}$$

which satisfies the Bjorken sum rule. The sea quark distributions are assumed to be flavour independent and based on gluon bremsstrahlung and quark pair creation calculations.

$$s(x) = 0.071(1-x)^8\,(1+(1-x)^2)/x,$$
$$\Delta s^{\mathrm{L}}(x) = 0.024(1-x)^8\,(1-(1-x)^2)/x. \tag{A.3}$$

— The Carlitz-Kaur model [2]

This model is based on the idea that the valence quarks carry most of the helicity only as $x \to 1$. One introduces a spin dilution factor

$$\cos 2\theta = \frac{1}{1+0.052(1-x)^2/\sqrt{x}}, \tag{A.4}$$

which becomes important for small x. In this model the spin distributions for quarks are

$$\Delta u_{\mathrm{v}}^{\mathrm{L}}(x) = [u_{\mathrm{v}}(x) - \tfrac{2}{3}\, d_{\mathrm{v}}(x)] \cos 2\theta,$$
$$\Delta d_{\mathrm{v}}^{\mathrm{L}}(x) = -\tfrac{1}{3}\, d_{\mathrm{v}}(x) \cos 2\theta. \tag{A.5}$$

The sea quarks are assumed to be unpolarized. These distributions seem to be in better agreement with the SLAC data [11].

This way we have constructed two models of transversely polarized proton. The detailed formulae for both models are summarized in Table I, some interesting curves are shown in Figs 3–5. In Table II the normalization integrals are collected.

Let us notice that the Wandzura-Wilczek sum rule [7] gives negligible predictions for transverse polarization of the sea (for SU(6) model $\frac{\Delta s^{\mathrm{T}}(x)}{s(x)} \leqslant 0.025$, for comparison $\frac{\Delta s^{\mathrm{L}}(x)}{s(x)} \leqslant 0.3$). The structure functions evolution is irrelevant for the results of our calculation so we omit it here.

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