VELOCITY DISTRIBUTION OF RESIDUAL NUCLEI AFTER SEQUENTIAL NUCLEON EVAPORATION

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A simple formula for the velocity distribution of nuclei after sequential nucleon evaporation from a compound nucleus is derived. The Gaussian distribution for a single act of nucleon emission was assumed. Results are compared with experimental data.

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1. *Introduction*

A direct observation of evaporation residua is a useful method of investigation of different aspects of heavy-ion reactions. In particular, it is easy to confirm, in this way, that an observed fragment has originated from a compound nucleus and, subsequently, to determine fusion cross-section. Furthermore, this analysis can be done independently of statistical model calculations [1]. In recent years many measurements of evaporation residua distributions have been performed [1–4].

It has been suggested [1, 2] that velocity distributions of evaporation residua are determined by kinematics and time-consuming statistical model computations can be substituted by simple models. Gomez del Campo et al. [1] have argued that a velocity distribution after emission of an evaporation cascade is Maxwellian. In that model the whole evaporation process has been treated as a single act of emission and a recoil of a nucleus at subsequent steps has not been taken into account. Such approach is unable to determine the standard deviation of the distribution.

The aim of this paper is to give a simple expression for a velocity-angle distribution of nuclei after sequential emission of certain number of nucleons from a compound nucleus. Our considerations are based on the statistical model prediction that a distribution of nucleons emitted by a compound nucleus is Gaussian.

In order to compare the results of our approach with experiments we have made some additional assumptions (concerning nuclear temperature, velocity distributions

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of protons and neutrons and relative multiplicities of particles of each kind), but they can be easily modified. Agreement with experimental data is good.

This paper is organized as follows. Section 2 is devoted to obtaining a statistical distribution of residual nuclei from a one-nucleon evaporation distribution. In Section 3 theoretical predictions are compared with experimental data. In the last section a summarizing discussion is given.

2. *Derivation of velocity distribution of residual nuclei*

We start with an assumption that a velocity distribution of nucleons emitted from an excited nucleus in the rest frame of the nucleus (RFN) is of the form

$$\varrho^{\mathrm{RFN}}(\vec{w}) = Ke^{-\vec{w}^2/2}\sigma_1^2, \tag{1}$$

where the normalization constant K is given by

$$K = (2\pi\sigma_1^2)^{-3/2} \tag{1'}$$

and $\vec{w}$ is an emitted nucleon velocity in RFN.

Let N be a number of nucleons in the nucleus and let i enumerates the evaporation sequence: $i = 1, 2, \ldots, n$. The change of momentum of an emitted nucleon is $m\vec{w}$, where m is the nucleon mass. Hence, if $\vec{p}_i$, $\vec{v}_i$ denote momentum and velocity of the nucleus in the LAB frame after i-th emission then the recoil momentum is

$$\vec{p}_i-\vec{p}_{i-1} \equiv m(N-i)\vec{v}_i-m(N-i+1)\vec{v}_{i-1} = -\vec{w}_i m. \tag{2}$$

Here, $\vec{w}_i$ is in RFN and we have to transform it to the LAB frame. As the nucleus is moving in LAB with a velocity $\vec{v}_{i-1}$, we have this transformation in the form: $\vec{w}_i \to \vec{w}_i-\vec{v}_{i-1}$. Hence, the distribution (1) is

$$\varrho_i^{\mathrm{LAB}}(\vec{w}_i) = K\exp\left(-(\vec{w}_i-\vec{v}_{i-1})^2/2\sigma_i^2\right). \tag{3}$$

On the other hand, from (2), we have

$$\vec{w}_i-\vec{v}_{i-1} = (\vec{v}_{i-1}-\vec{v}_i)(N-i) \tag{4}$$

and

$$\varrho_i(\vec{v}_i) = K\exp\left(-(\vec{v}_i-\vec{v}_{i-1})^2(N-i)^2/2\sigma_i^2\right), \tag{5}$$

where the superscript LAB, from now on, will be omitted.

In this simple way we have obtained a distribution of residual nuclei in LAB after i-th emission, where an initial velocity of the nucleus was $\vec{v}_{i-1}$. As a result, we have the following formula for the final distribution $\varrho(\vec{v}_n)$ after emission of the whole evaporation cascade:

$$\varrho(\vec{v}_n) = \int d^3\vec{v}_1 \ldots d^3\vec{v}_{n-1}\varrho_1\varrho_2 \ldots \varrho_n. \tag{6}$$

Now, our task is to calculate the above $3(n-1)$-ple integral. First we change variables: $\vec{v}_i \to \vec{u}_i$, $i = 1, 2, \ldots, n-1$, where

$$\vec{u}_i = (\vec{v}_i - \vec{v}_{i-1})(N-i)/\sigma_i$$

and hence

$$\vec{v}_i = \sum_{k=0}^{i} \frac{\sigma_k}{N-k} \vec{u}_k.$$

$\vec{v}_0$ is an initial velocity (before the first emission) of the nucleus and, by definition, $\sigma_0 \equiv 1$ Now, (6) is of the form[1]

$$\varrho(\vec{v}_n) = \int d^3\vec{u}_1 \ldots d^3\vec{u}_{n-1} K \exp\left(-\tfrac{1}{2}\vec{u}_i A_{ij} \vec{u}_j + \vec{y}_i \vec{u}_i\right) \exp\left(-\tfrac{1}{2}(\vec{v}_n - \vec{v}_0)^2/\alpha_n^2\right), \tag{7}$$

where we have introduced the following notation

$$\alpha_i \equiv \sigma_i/(N-i), \tag{8a}$$

$$A_{ij} \equiv \delta_{ij} + \alpha_n^2 \alpha_i \alpha_j, \tag{8b}$$

$$\vec{y}_i \equiv \alpha_n^{-2}(\vec{v}_n - \vec{v}_0)\alpha_i. \tag{8c}$$

Finally, using the well-known formula for multiple Gaussian integrals we have (see e.g. [5] and Appendix)

$$\varrho(\vec{v}_n) = K e^{-(\vec{v}_n - \vec{v}_0)^2/2\sigma^2}, \tag{9}$$

where K is a normalization factor: $K \equiv (2\pi\sigma^2)^{-3/2}$ and

$$\sigma^2 = \sum_{i=1}^{n} \frac{\sigma_i^2}{(N-i)^2}. \tag{10}$$

Formula (9) is in agreement with a suggestion that this distribution should be Gaussian and peaked at an initial compound nucleus velocity $\vec{v}_0$ [1]; but here also the variance σ^2 is given explicitly by (10). It appears to be a "weighed sum" of one-nucleon variances.

3. *Comparison with experimental data*

The statistical model implies that

$$\sigma_i^2 = T_i/m, \tag{11}$$

where T_i is a temperature of a residual nucleus after an i-th act of emission. For not very large excitation energies E_i^* a compound nucleus behaves like the degenerate Fermi-gas

[1] Here, summation convention over $i, j = 1, 2, \ldots, n-1$ is assumed throughout this paper; n-th term is not included in the sum.

and the following formula is valid [6]:

$$E_i^* = \frac{N-i}{8} T_i^2. \tag{12}$$

In general, nucleon velocities can be arbitrary but in the first approximation we can calculate a sequence of excitation energies as if each nucleon were emitted with a velocity corresponding to an average value of the distribution. This assumption enables us to calculate E_i^* independently of $\vec{w}^2$. We obtain:

$$E_i^* = E_{i-1}^* - E_s - \frac{4}{\pi} T_{i-1}, \tag{13}$$

where E_s is a separation energy.

Expressions (10)–(13) allow us to calculate a distribution of residual nuclei if only neutrons are evaporated. In order to compare results with experimental data we must

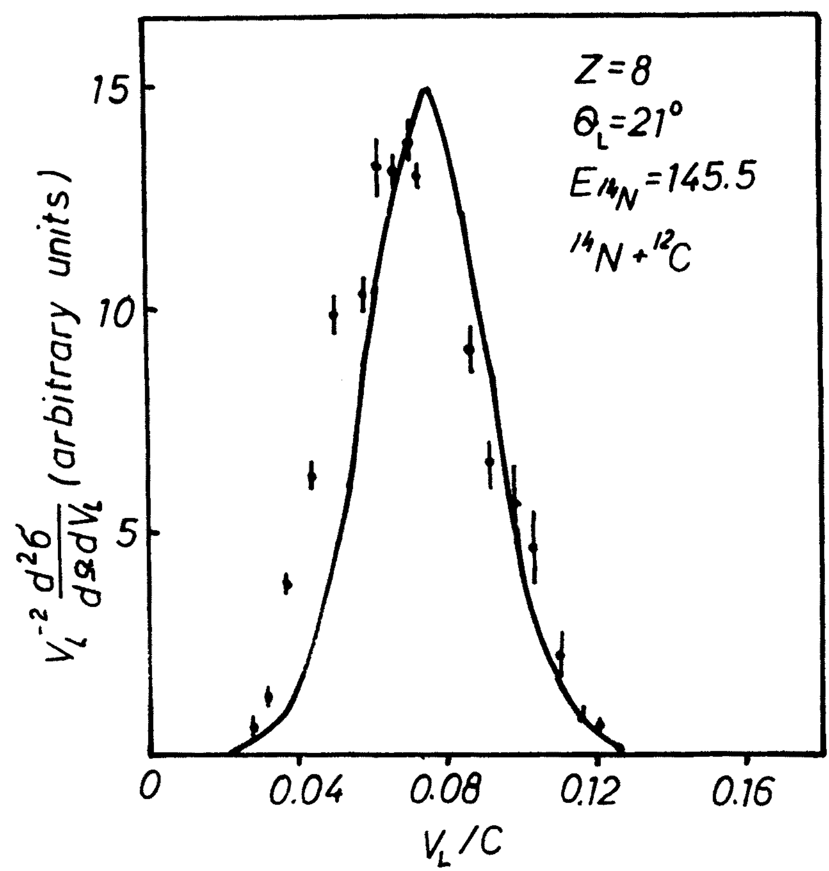

Fig. 1. Experimental (from Ref. [1]) and theoretical (solid line) velocity distributions of evaporation residua

estimate a modification of the distribution due to proton emission. A spectrum of charged particles is shifted toward higher values of $|\vec{w}|$, compared to (1) and sharply cut at small $|\vec{w}|$. The latest effect is due to the Coulomb barrier and is especially important for heavy nuclei. We neglect it in this paper. On the other hand, an emitted proton interacts with the nucleus still after emission and imposes an additional recoil on the nucleus. The final proton velocity is then

$$\vec{w}' = \left(1+\frac{k}{w}\right)\vec{w} \approx \left(1+\frac{k}{w_{av}}\right)\vec{w}, \tag{14}$$

where w_{av} stands for the average velocity and k is an additional velocity due to the Coulomb interaction. Finally, we obtain a standard deviation of residua distribution for emission of protons only

$$(\sigma_i^p)^2 \approx \frac{T_i}{m}\left(1+\frac{0.72\,[\mathrm{MeV}^{1/2}]}{\sqrt{T_i}}(N-i)^{1/3}\right)^2. \tag{15}$$

Calculations have been performed for evaporation from compound nuclei produced in reactions: $^{14}N+^{12}C$ and $^{20}Ne+^{27}Al$. Experimental data are taken from Refs. [1] and [3]. Fig. 1 presents experimental and theoretical double-differential cross-sections

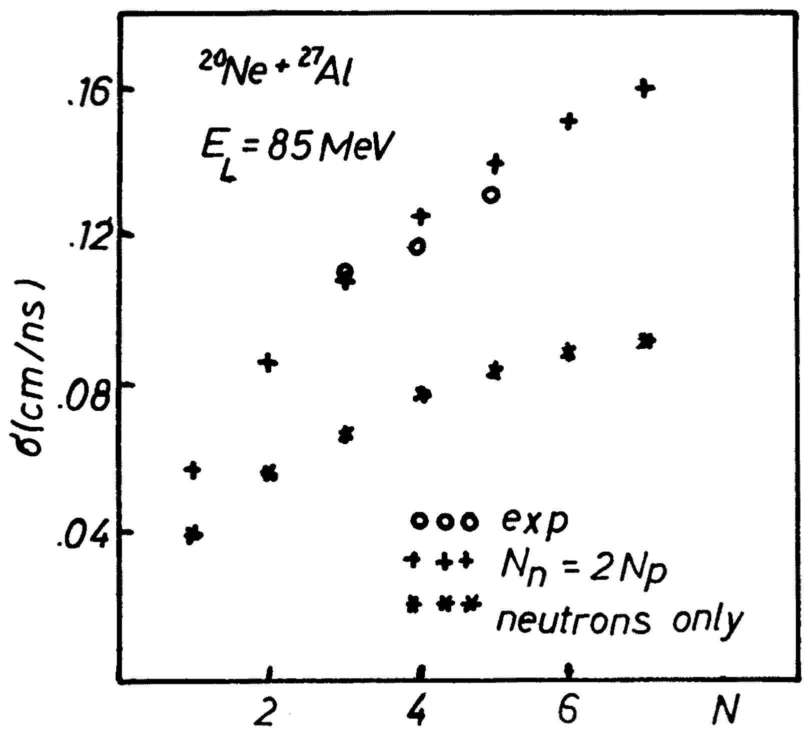

Fig. 2. Standard deviation of the velocity distribution of residual nuclei as a function of the number of evaporated nucleons (data are from [3])

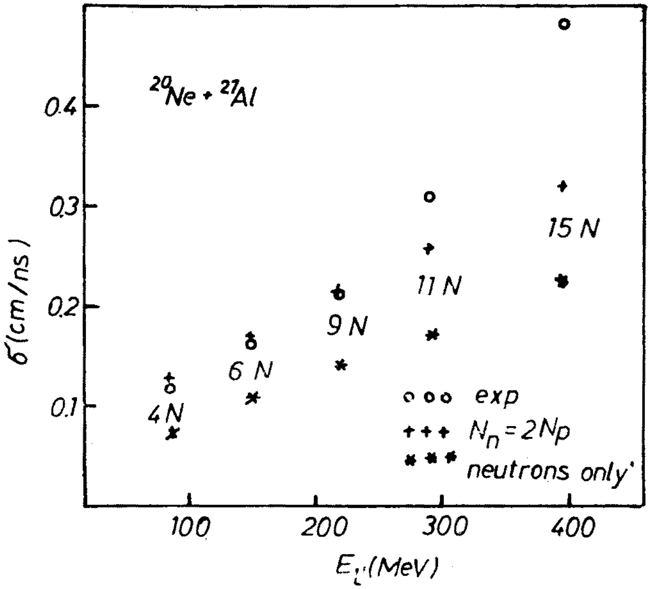

Fig. 3. Standard deviation of the velocity distribution of residual nuclei as a function of the projectile incident energy. A number of emitted nucleons is depicted for each energy

in LAB. In calculations equal numbers of emitted neutrons and protons have been assumed. Evaporation of other particles has been neglected.

Evaporation residua produced after collision $^{20}Ne + ^{27}Al$ were measured for different rates of emitted alpha-particles. Data presented here refer to the case of emission of nucleons alone. In Fig. 2 and 3 a standard deviation is presented as a function of the number of emitted particles and the incident projectile energy, respectively. Two sets of theoretical points have been obtained assuming: (*i*) only neutrons are evaporated and (*ii*) a number of neutrons is twice as large as a number of protons. The last assumption leads to the good agreement with experimental data.

4. *Final remarks*

Obtained results are in good agreement with experimental data, as illustrated in Figs. 1, 2, 3. However, at high energies substantial discrepancies appear. Such discrepancies are quite comprehensible because in this energy range non-equilibrium phenomena play an important role. An immediate indication that pre-equilibrium emission takes place is a substantial shift of the experimental spectra [3] (for evaporation from a compound nucleus they should be centered at the zero velocity in CM — see (9) and Refs. [1, 4]). Moreover, temperatures exceed noticeably the degeneration temperature in this case and the formula (12) may be no longer valid. However, the classical Boltzmann formula gives standard deviations even smaller and the lack of degeneracy can not be responsible for discrepancies with experimental data.

Finally, we want to mention about yet another approximation we have done. Calculating integral (6) we have assumed infinite integration limits (Gaussian distribution). In fact, an excitation energy of the compound nucleus is finite and distributions should be cut-off at a maximum velocity allowed by kinematics. Fortunately, a value of the Gaussian distribution at the point of the cut is very small and an error committed in this way is negligible.

APPENDIX

Here, we calculate the $3(n-1)$-ple integral (7) to obtain the formulae (9) and (10). To this end we use the following well-known formula for multiple Gaussian integrals[2]

$$\int dx_1 \dots dx_L e^{-\frac{1}{2}x_i M_{ij} x_j + \varphi_i x_i} = C \det |M_{ij}|^{-1/2} \exp\left(-\tfrac{1}{2}\varphi_i M_{ij}^{-1} \varphi_j\right), \tag{A1}$$

where C is a normalization constant, M_{ij} is a $L \times L$ symmetric matrix. In (7) the matrix M_{ij} is of the form

$$M_{ij} = A_{ij} \otimes \delta^{AB} \equiv (\delta_{ij} + \alpha_n^{-2}\alpha_i\alpha_j) \otimes \delta^{AB}, \tag{A2}$$

where $A, B = 1, 2, 3$ are Cartesian coordinates.

It is easy to show that the matrix A_{ij} has eigenvalues: $(1, 1, \dots, 1, (1+\alpha_n^2\alpha_i\alpha_i))$; the last eigenvalue corresponds to the eigenvector parallel to the vector α_i. Hence, we have

$$\alpha_i A_{ij}^{-1} \alpha_j = \alpha_i (1 + \alpha_n^2 \alpha_i \alpha_i)^{-1} \alpha_i$$

and finally

$$\tfrac{1}{2}\vec{y}_i A_{ij}^{-1} \vec{y}_j - \tfrac{1}{2}\alpha_n^{-2}\vec{v}_n^2 = -\tfrac{1}{2}\vec{v}_n^2[\alpha_n^{-2} - \alpha_n^{-4}\alpha_i A_{ij}^{-1}\alpha_j] = -\tfrac{1}{2}\vec{v}_n^2 \frac{1}{\alpha_i\alpha_i + \alpha_n^2}. \tag{A3}$$

Using (8a) this yields (10) immediately.

Note added in proof: The problem of neutrons evaporation from a compound nucleus has been also investigated in: R. W. Lide, ORNL-Report-3358 (1967). We are indebted to Prof. A. Budzanowski for pointing at this reference.

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[2] Continuous generalization of this formula is widely used to evaluate functional integrals in field theory and statistical physics, see e.g. [5].