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FERMION MASSES FROM GRAVITY INDUCED SUPERPOTENTIAL

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The Nanopoulos-Srednicki superpotential has been generalized to include Planck slot non-renormalizable terms, including one proportional to $\mathrm{Tr}(A^4)$, $A^a{}_b$ being the $\underline{24}$-adjoint representation. The symmetry breaking mechanism is shown to stay intact. We postulate a new generation of particles and estimate their masses from mass-generation number graphs. New mass ratios in terms of parameters is proposed. Those parameters can be adjusted to yield any mass ratio at the grand unification mass scale. The parameters are also compared under some very restrictive conditions, by equating the mass ratios to the experimental values.

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1. Introduction

As the attempt of unification of all interactions [1] in particle physics has continued, the introduction of fermion masses [2] has always been a difficult subject. The mass terms which have been proposed are invariably in conflict with the symmetries, which are required. Since the introduction of the toy model SU(5) by Georgi and Glashow [3], theorists have introduced fermion masses by postulating Yukawa potentials, where fermions interact with the Higgs particles, and by resorting to a conventional symmetry breaking Higgs mechanism. Unfortunately this mechanism yields mass ratios $m_b/m_\tau = m_s/m_\mu = m_d/m_e = 1$ [1]. However Buras, Ellis, Gaillard and Nanopoulos [4] suggested that such relations are only valid at the grand unification scale. They used the renormalization group to compute the masses at the present energy levels and obtained striking results. They showed under certain approximations that $m_b \approx 4.8$–5.6 GeV and $m_s \approx 0.4$–0.5 GeV. The value of m_b is reasonable but m_s is slightly low. Since then the prevailing tendency among particle physicists is to start to construct models at GUT mass level and then use renormalizable group procedures to compute the experimental mass at the present energy scale. But it is felt that some new ideas are needed to improve the situation. In an attempt

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to do exactly that, Ellis and Gaillard [5] suggested that we have to introduce the influence of gravitation in computing masses. They introduced a non-renormalizable interaction by coupling a $\underline{24}$-adjoint representation $A^i{}_j$ and $\bar{\underline{5}}$-Higgs field jointly with two fermion fields. This is also sometimes referred as an opening up of Planck's slot. They showed that such a term will have contributions to the order of $O(m_X m_W/m_P)$, where m_P in the Planck's mass ($\sim 10^{19}$ GeV), m_X is the SU(5)-breaking scale ($\sim 10^{14.5}$ GeV), and m_W ($\sim 10^2$ GeV) is the mass scale where SU(2)×U(1) breaks into $U_{em}(1)$. Effects of this order of magnitude have been estimated by Ellis and Gaillard [5]. The effect of these terms they found are of the same order of magnitude as the original mass terms. In their paper they used the experimental masses of the quarks and leptons to estimate the theoretical parameters.

In a recent paper, Nanopoulos and Srednicki [6] argued that when global supersymmetry is introduced into the GUT scale, this results in a larger unification scale about 10^{16} GeV in a minimal GUT. They estimated that such unrenormalizable terms would contribute effects of the order of magnitude proportional to 10^{-2}. They proposed a toy model in the context of supersymmetry. They took SU(5) as the gauge group and a definite form of superpotential. They then computed the mass ratios. They showed also that at the grand unification mass scale, we can improve on the ratio $|m_s/m_\mu| = 1$ to 2/3 by suitable choice of parameters.

It seems to us that it is not self-evident that the introduction of gravity induced effect would not give rise to a term of the form A^4 or even one with higher power of A in the superpotential. Whether such potential would yield symmetry breaking of the type SU(5) → SU(3)×SU(2)×U(1) → SU(3)×$U_{em}(1)$ has not been discussed anywhere. In this paper we have introduced a superpotential following the pattern of Nanopoulos and Srednicki [6] by adding higher non-renormalizable terms. We added an A^4-term in the superpotential to check whether symmetry breaking procedure works. We have demonstrated that there is a solution, for the SU(5) breaking condition enunciated by Witten [7] which corresponds to the process SU(5) → SU(3)×SU(2)×U(1) without breaking the supersymmetry.

We have postulated the existence of a fourth family which we called (ν_λ, λ, l, p). We have made a crude estimate for the masses of this generation by plotting masses against the family number. We have used these values and other approximate mass values to have some estimate of the parameters introduced in the model.

In Section 2 we define our new superpotential and indicate how it reduces to the Nanopoulos and Srednicki superpotential. We show in Section 3 that our superpotential yields a symmetry breaking pattern SU(5) → SU(3)×SU(2)×U(1), following Witten. In Section 4 we write down the mass ratios of different particles in terms of parameters introduced. We introduce the fourth generation in Section 5 and explain how we have estimated masses of different particles in the family. In Section 6 we propose a mass ratio model in the grand unification scale, which could be used to compute masses at present energy level using the renormalization group technique. In another approach we have attempted to compare the mass ratios of experimental and estimated masses and obtained under some assumptions some relations between different parameters.

2. *The extended Nanopoulos-Srednicki superpotential*

We extend the Nanopoulos and Srednicki [6] superpotential by introducing a term proportional to A^4 and two extra terms with order of magnitude proportional to M^{-3}, where M is the modified Planck scale introduced by them. The new superpotential runs as follows:

$$W = \mu^2 Z + \Delta + \tfrac{1}{2}\lambda_0 A^2 + \tfrac{1}{3}\lambda_1 A^3 + \tfrac{1}{4}\lambda_2 A^4 + a_1 \bar{H}H + a_2 \bar{H}AH + a_{3j}(\bar{H}A^2H)_j$$
$$+ \sum_i \alpha^i_{00}\bar{F}_i T_i \bar{H} + M^{-1}\sum_i \alpha^i_{1j}\{(\bar{F}_i A T_i \bar{H})_j + (\bar{F}_i T_i A\bar{H})_j\}$$
$$+ M^{-2}\sum_i \alpha^i_{2j}[\{(\bar{F}_i T_i \bar{H})(A^2)\}_j + \{\bar{F}_i T_i A^2 \bar{H}\}_j + \{\bar{F}_i A T_i A \bar{H}\}_j + \{\bar{F}_i A^2 T_i \bar{H}\}_j]$$
$$+ M^{-3}\sum_i \alpha^i_{3j}[\{(\bar{F}_i T_i \bar{H})(A^3)\}_j + \{\bar{F}_i T_i A^3 \bar{H}\}_j$$
$$+ \{(\bar{F}_i T_i A\bar{H})(A^2)\}_j + \{\bar{F}_i A T_i A^2 \bar{H}\}_j + \{\bar{F}_i A^2 T_i A \bar{H}\}_j + \{\bar{F}_i A^3 T_i \bar{H}\}_j]$$
$$+ \sum_i \beta^i_{00}(T_i^{\mathrm{T}} T_i H) + M^{-1}\sum_i \beta^i_{1j}(A T_i^{\mathrm{T}} T_i H)_j$$
$$+ M^{-2}\sum_i \beta^i_{2j}(A^2 T_i^{\mathrm{T}} T_i H)_j + M^{-3}\sum_i \beta^i_{3j}(A^3 T_i^{\mathrm{T}} T_i H)_j. \tag{2.1}$$

In the above superpotential A is a $\underline{24}$, H and $\bar{H}$ are the $\underline{5}$ and $\underline{\bar{5}}$ representations of the Higgs fields, T_i and $\bar{F}_i$ are the i-th generation $\underline{10}$ and $\underline{\bar{5}}$ fermion representations. In principle we should have taken the Cabbibo-Kobayashi-Maskawa (CKM) [8] rotated $\underline{10}$ and $\underline{\bar{5}}$ fermion representation as indicated by Nanopoulos and Srednicki, but since our superpotential is parametrized we will assume that CKM parameters are absorbed in the other constants. Z is a singlet used to break local supersymmetry, and Δ is a constant which must be adjusted to cancel cosmological constants.

In each term of the superpotential the fields have to be contracted in SU(5) indices in all possible ways. Thus for terms involving H and A we have

$$A^n = \mathrm{Tr}\, A^n \quad \text{for} \quad n = 2, 3 \text{ and } 4, \tag{2.2}$$

$$\bar{H}H = \bar{H}_a H^a, \tag{2.3}$$

$$\bar{H}AH = \bar{H}_a A^a{}_b H^b, \tag{2.4}$$

$$a_{3j}(\bar{H}A^2H)_j = a_{31}\bar{H}_a A^a{}_b A^b{}_c H^c + a_{32}\bar{H}_a A^b{}_c A^c{}_b H^a. \tag{2.5}$$

In the terms of the form $\alpha^i_{mj}\{(\ )_j + \ldots\}$, the quantities in the parenthesis have to be contracted in all possible ways in SU(5) indices and then multiplied by a parameter α^i_{mj} belonging to i-th generation. We have thus:

$$\alpha^i_{00}\bar{F}_i T_i \bar{H} = \alpha^i_{00} F^{\mathrm{T}}_{ai\mathrm{L}} C T^{ab}_{i\mathrm{L}} H_b, \tag{2.6}$$

$$\alpha^i_{1j}\{(\bar{F}_i A T_i \bar{H})_j + \ldots\} = \alpha^i_{11} F^{\mathrm{T}}_{ai\mathrm{L}} C T^{ab}_{i\mathrm{L}} A^c{}_b H_c + \alpha^i_{12} F^{\mathrm{T}}_{ai} C T^{cd}_{i\mathrm{L}} A^a{}_c H_d, \tag{2.7}$$

$$\alpha^i_{2j}[\{(\bar{F}_i T_i \bar{H})(A)^2\}_j + \ldots] = \alpha^i_{21} F^{\mathrm{T}}_{ai\mathrm{L}} C T^{ab}_{i\mathrm{L}} H_b A^d{}_e A^e{}_d + \alpha^i_{22} F^{\mathrm{T}}_{ai\mathrm{L}} C T^{ab}_{i\mathrm{L}} A^d{}_b A^e{}_d H_e$$

$$+\alpha^i_{23} F^{\mathrm{T}}_{ai\mathrm{L}} C A^a{}_b T^{bc}_{i\mathrm{L}} A^d{}_c H_d + \alpha^i_{24} F^{\mathrm{T}}_{ai\mathrm{L}} C A^a{}_b A^b{}_c T^{cd}_{i\mathrm{L}} H_d, \tag{2.8}$$

$$\alpha^i_{3j}[\{(\bar{F}_i T_i \bar{H})(A^3)\}_j + \ldots]$$

$$= \alpha^i_{31} F^{\mathrm{T}}_{ai\mathrm{L}} C T^{ab}_{i\mathrm{L}} H_b A^c{}_d A^d{}_e A^e{}_c + \alpha^i_{32} F^{\mathrm{T}}_{ai\mathrm{L}} C T^{ab}_{i\mathrm{L}} A^c{}_b H_c A^d{}_e A^e{}_d$$

$$+\alpha^i_{33} F^{\mathrm{T}}_{ai\mathrm{L}} C T^{ab}_{i\mathrm{L}} A^c{}_b A^d{}_c A^e{}_d H_e + \alpha^i_{34} F^{\mathrm{T}}_{ai\mathrm{L}} C A^a{}_b T^{bc}_{i\mathrm{L}} H_c \operatorname{Tr} A^2$$

$$+\alpha^i_{35} F^{\mathrm{T}}_{ai\mathrm{L}} C A^a{}_b T^{bc}_{i\mathrm{L}} A^d{}_c A^e{}_d H_e + \alpha^i_{36} F^{\mathrm{T}}_{ai\mathrm{L}} C A^a{}_b A^b{}_c T^{cd}_{i\mathrm{L}} A^e{}_d H_e$$

$$+\alpha^i_{37} F^{\mathrm{T}}_{ai\mathrm{L}} C A^a{}_b A^b{}_c A^c{}_d T^{de}_{i\mathrm{L}} H_e. \tag{2.9}$$

Similarly

$$\beta^i_{00} T^{\mathrm{T}}_i T_i H = \beta^i_{00} \varepsilon_{abcde} T^{\mathrm{T}ab}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^e, \tag{2.10}$$

$$\beta^i_{1j}(A T^{\mathrm{T}}_i T_i H)_j = \beta^i_{11} \varepsilon_{abcde} A^a{}_f T^{\mathrm{T}fb}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^e + \beta^i_{12} \varepsilon_{abcde} A^a{}_f T^{\mathrm{T}bc}_{i\mathrm{L}} C T^{de}_{i\mathrm{L}} H^f, \tag{2.11}$$

$$\beta^i_{2j}(ATTH)_j = \varepsilon_{abcde}\{\beta^i_{21} T^{\mathrm{T}ab}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^e \operatorname{Tr}(A^2)$$

$$+\beta^i_{22} A^a{}_f A^b{}_g T^{\mathrm{T}cd}_{i\mathrm{L}} C T^{ef}_{i\mathrm{L}} H^g + \beta^i_{23} A^a{}_f A^b{}_g T^{\mathrm{T}fg}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^e$$

$$+\beta^i_{24} A^a{}_f A^b{}_g T^{\mathrm{T}cf}_{i\mathrm{L}} C T^{de}_{i\mathrm{L}} H^g + \beta^i_{25} A^a{}_f A^b{}_g T^{\mathrm{T}fc}_{i\mathrm{L}} C T^{gd}_{i\mathrm{L}} H^e$$

$$+\beta^i_{26} A^a{}_f A^f{}_g T^{\mathrm{T}bg}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^e + \beta^i_{27} A^a{}_f A^f{}_g T^{\mathrm{T}bc}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^g\}, \tag{2.12}$$

and finally

$$\beta^i_{3j}(A^3 T^{\mathrm{T}}_i T_i H)_j = \varepsilon_{abcde}\{\beta^i_{31} \operatorname{Tr}(A^3) T^{\mathrm{T}ab}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^e + \beta^i_{32} A^a{}_f \operatorname{Tr}(A^2) T^{\mathrm{T}bf}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^e$$

$$+\beta^i_{33} A^a{}_h \operatorname{Tr}(A^2) T^{\mathrm{T}bc}_{i\mathrm{L}} C T^{de}_{i\mathrm{L}} H^h + \beta^i_{34} A^a{}_f A^b{}_g A^c{}_h T^{\mathrm{T}de}_{i\mathrm{L}} C T^{fg}_{i\mathrm{L}} H^h$$

$$+\beta^i_{35} A^a{}_f A^b{}_g A^c{}_h T^{\mathrm{T}df}_{i\mathrm{L}} C T^{eg}_{i\mathrm{L}} H^h + \beta^i_{36} A^a{}_f A^b{}_g A^c{}_h T^{\mathrm{T}df}_{i\mathrm{L}} C T^{gh}_{i\mathrm{L}} H^e$$

$$+\beta^i_{37} A^f{}_h A^g{}_f A^a{}_g T^{\mathrm{T}bh}_{i\mathrm{L}} C T^{cd}_{i\mathrm{L}} H^e + \beta^i_{38} A^f{}_h A^g{}_f A^a{}_g T^{\mathrm{T}bc}_{i\mathrm{L}} C T^{de}_{i\mathrm{L}} H^h\}. \tag{2.13}$$

In the above expressions we have followed the terminology defined by Langacker [1]. All Latin suffixes run from 1 through 5. The constant $M = m_{\mathrm{P}}/(8\pi)^{1/2} = 2.4 \times 10^{18}$ GeV has been introduced following the prescription of Nanopoulos and Srednicki [6].

The superpotential W reduces to Nanopoulos and Srednicki potential if we set

$$\lambda_2 = 0, \quad a_{3j} = 0, \tag{2.14}$$

$$\alpha^1_{00} = \alpha^2_{00} = 0, \tag{2.15}$$

$$\alpha^1_{1j} = \alpha^3_{1j} = 0, \tag{2.16}$$

$$\alpha^2_{2j} = \alpha^2_{2j} = 0, \tag{2.17}$$

$$\alpha^{i}_{3j} = 0, \tag{2.18}$$

$$\beta^{1}_{00} = \beta^{2}_{00} = 0, \tag{2.19}$$

$$\beta^{1}_{1j} = \beta^{3}_{1j} = 0, \tag{2.20}$$

$$\beta^{2}_{2j} = \beta^{3}_{2j} = 0, \tag{2.21}$$

$$\beta^{i}_{3j} = 0. \tag{2.22}$$

3. SU(5)-breaking

We wish to generate models, including ours, to follow the supersymmetric theory building concept. Although our scheme is phenomenological, we would follow the pattern of symmetry breaking developed by Witten [7] and used extensively by Dimopoulos and Georgi [9]. The objective is to keep the supersymmetry unbroken but break the gauge invariance at the begining. The terms which play the dominant role of gauge symmetry breaking are

$$\begin{aligned} v &= W - \text{terms involving non-adjoint fields} \\ &= \tfrac{1}{2}\lambda_0 A^2 + \tfrac{1}{3}\lambda_1 A^3 + \tfrac{1}{4}\lambda_2 A^4. \end{aligned} \tag{3.1}$$

We want to stress the fact that terms with higher powers of fields are the contracted expressions in SU(5) indices. The conditions, which have to be satisfied to get the vacuum expectation values of the fields by minimizing v with respect to them, are

$$v_a(\phi) = \frac{\partial v(\phi)}{\partial \phi_a} = 0 \tag{3.2}$$

and

$$K_\alpha = \bar{\Phi}_a T^{\alpha}_{ab} \phi^b = 0, \tag{3.3}$$

where T^{n}_{ab} are the generators of the gauge group.

If we minimize the potential for

$$\langle H \rangle = \langle \bar{H} \rangle = \langle Z \rangle = \langle T_i \rangle = \langle F_i \rangle = 0 \tag{3.4}$$

then the adjoint 24-fields, $A^a{}_b$, satisfy under the restriction of tracelessness and the Eq. (3.2):

$$\lambda_0 A^b{}_a + \lambda_l (A^b{}_z A^z{}_a - \tfrac{1}{5}\delta^b{}_a \operatorname{Tr} A^2) + \lambda_2 (A^b{}_z A^z{}_s A^s{}_a - \tfrac{1}{5}\delta^b{}_a \operatorname{Tr} A^3) = 0. \tag{3.5}$$

Since we are looking for diagonal solutions, we set

$$A^x{}_y = c_x \delta^x{}_y. \tag{3.6}$$

For $x = y$, we must have

$$\lambda_0 c_x + \lambda_1 c_x^2 - N + \lambda_2 c_x^3 - K = 0, \tag{3.7}$$

where

$$N = \tfrac{1}{5}\lambda_1 \sum_{y=1}^{5} c_y^2 \quad \text{and} \quad K = \tfrac{1}{5}\lambda_2 \sum_{y=1}^{5} c_y^3. \tag{3.8}$$

Eq. (3.7) has a solution

$$c_x = 0 \tag{3.9}$$

which corresponds to unbroken SU(5) symmetry.

If we look for a solution of the form $c_1 = c_2 = c_3 = c_4 = c$ and $c_5 = -4c$, the c must satisfy the equation

$$\lambda_0 c - 3\lambda_1 c^2 + 13\lambda_2 c^3 = 0. \tag{3.10}$$

Therefore if $c \neq 0$, we must have

$$c = \frac{3\lambda_1 \pm \sqrt{9\lambda_1^2 - 52\lambda_0\lambda_2}}{26\lambda_2}. \tag{3.11}$$

We can remove the ambiguity of c by setting

$$9\lambda_1^2 = 52\lambda_0\lambda_2 \tag{3.12}$$

and obtain

$$c = (3\lambda_1)/(26\lambda_2). \tag{3.11a}$$

This solution corresponds to the symmetry breaking

$$\mathrm{SU}(5) \to \mathrm{SU}(4) \times \mathrm{U}(1) \tag{3.13}$$

To obtain a solution of the type $c_1 = c_2 = c_3 = c$ and $c_4 = c_5 = -(3/2)c$, the c must satisfy the equation

$$4\lambda_0 c - 2\lambda_1 c^2 + 7\lambda_2 c^3 = 0. \tag{3.14}$$

If $c \neq 0$, we have

$$c = \frac{\lambda_1 \pm \sqrt{\lambda_1^2 - 28\lambda_0\lambda_2}}{7\lambda_2}. \tag{3.15}$$

To remove the ambiguity we set

$$\lambda_1^2 = 28\lambda_0\lambda_2. \tag{3.15a}$$

We get thus

$$c = \lambda_1/(7\lambda_2). \tag{3.15b}$$

We have therefore demonstrated that the following options exist for the type of superpotential we have introduced:

$$[1] \qquad A^a{}_b = 0, \tag{3.16}$$

$$[2] \qquad A^a{}_b = c[\delta^a{}_b - 5\delta^a{}_5\delta^5{}_b], \tag{3.17}$$

with the restriction Eq. (3.12) and c given by the relation (3.11a), and

$$[3] \qquad A^a{}_b = c[\delta^a{}_b - (\tfrac{5}{2})(\delta^a{}_4\delta^4{}_b + \delta^a{}_5\delta^5{}_y)] \tag{3.18}$$

with c given by the Eq. (3.15b) and the restriction Eq. (3.15a).

As we know the Eq. (3.16) corresponds to unbroken symmetry of SU(5). The Eq. (3.17) corresponds to SU(5) → SU(4) × U(1) breaking and the Eq. (3.18) corresponds to the symmetry breaking SU(5) → SU(3) × SU(2) × U(1).

We have thus shown that even after introduction of a non-renormalizable term proportional to Tr A^4, the symmetry breaking procedure of Witten [7] stays intact but we have to subject the λ's to constraints stated in the relations of Eqs. (3.12) and (3.15a).

We now introduce the next step of symmetry breaking $\langle H_5\rangle \neq 0$ and $\langle \bar{H}\rangle \neq 0$, but $\langle F_i\rangle = \langle T_i\rangle = 0$, which leads finally to SU(2) × U(1) → $U_{em}(1)$. Our choice will be

$$\langle 0|H_5|0\rangle = \delta^5{}_a v_0'/2, \quad v_0' = 2m\mathrm{W}/g' \tag{3.19}$$

and

$$\langle 0|\bar{H}_5|0\rangle = \delta^a{}_5 v_0'/2, \quad v_0 = 2m\mathrm{W}/g, \tag{3.20}$$

where m_W, g, and g' are well-known quantities.

4. *Mass coefficients*

We now recall (see for example [1]) that $\underline{\bar{5}}$-fermion fields are given by

$$\bar{F}_{1a\mathrm{L}} = \begin{bmatrix} d_1^c \\ d_2^c \\ d_3^c \\ \bar{e} \\ -\nu_e \end{bmatrix}_\mathrm{L}, \quad \bar{F}_{2a\mathrm{L}} = \begin{bmatrix} s_1^c \\ s_2^c \\ s_3^c \\ \mu^- \\ -\nu_\mu \end{bmatrix}_\mathrm{L}, \quad \text{and} \quad \bar{F}_{3a\mathrm{L}} = \begin{bmatrix} b_1^c \\ b_2^c \\ b_3^c \\ \tau^- \\ -\nu_\tau \end{bmatrix}_\mathrm{L} \tag{4.1}$$

and $\underline{10}$-fermions by

$$T_{1\mathrm{L}}^{ab} = (1/\sqrt{2}) \begin{bmatrix} 0 & -u_3^c & -u_2^c & -u^1 & -d^1 \\ -u_3^c & 0 & u_1^c & -u^2 & -d^2 \\ u_2^c & -u_1^c & 0 & -u^3 & -d^3 \\ u^1 & u^2 & u^3 & 0 & -e^+ \\ d^1 & d^2 & d^3 & e^+ & 0 \end{bmatrix}_\mathrm{L}, \tag{4.2}$$

$$T_{2\mathrm{L}}^{ab} = \begin{bmatrix} \text{same as Eq. (4.2), only replace } u \text{ by c,} \\ d \text{ by } s, \text{ and } e \text{ by } \mu \end{bmatrix}_\mathrm{L}, \tag{4.3}$$

$$T_{3\mathrm{L}}^{ab} = \begin{bmatrix} \text{same as Eq. (4.2), only replace } u \text{ by } t, \\ d \text{ by } b, \text{ and } e \text{ by } \tau \end{bmatrix}_\mathrm{L}. \tag{4.4}$$

In the above expressions ψ^c stands for the charge conjugate field of ψ and ψ_L for the left chiral field of ψ.

To obtain the mass matrix, we now compute from the superpotential W, terms which contribute to fermion mass terms by substituting sequential breaking: SU(5) $\xrightarrow{A}$ SU(3)×SU(2)×U(1) $\xrightarrow{H}$ SU(3)×U_{em}(1) given by Eqs. (3.16), (3.17), and (3.18). We get

$$M = v_0[m^1_{01}+m^1_{11}(c/M)+m^1_{21}(c/M)^2+m^1_{31}(c/M)^3]\,(\bar{d}_1 d^1+\bar{d}_2 d^2+\bar{d}_3 d^3)$$

$$+v_0[m^1_{02}+m^1_{12}(c/M)+m^1_{22}(c/M)^2+m^1_{32}(c/M)^3]\,(\bar{e}^+ e^+)$$

$$+(v_0'/\sqrt{2})\,[m^1_{03}+m^1_{13}(c/M)+m^1_{23}(c/M)^2+m^1_{33}(c/M)^3]\,(\bar{u}_1 u^1+\bar{u}_2 u^2+\bar{u}_3 u^3)$$

+ similar terms involving second and third generation leptons and quarks, (4.5)

where

$$m^i_{01} = \alpha^i_{00}/2, \tag{4.6}$$

$$m^i_{11} = -(\tfrac{3}{4})\alpha^i_{11}+(\tfrac{1}{2})\alpha^i_{12}, \tag{4.7}$$

$$m^i_{21} = (\tfrac{15}{4})\alpha^i_{21}+(\tfrac{9}{8})\alpha^i_{22}-(\tfrac{3}{4})\alpha^i_{23}+(\tfrac{1}{2})\alpha^i_{24}, \tag{4.8}$$

$$m^i_{31} = -(\tfrac{15}{4})\alpha^i_{31}-(\tfrac{45}{4})\alpha^i_{32}-(\tfrac{27}{16})\alpha^i_{33}+(\tfrac{15}{4})\alpha^i_{34}+(\tfrac{9}{8})\alpha^i_{35}-(\tfrac{3}{4})\alpha^i_{36}+(\tfrac{1}{2})\alpha^i_{37}, \tag{4.9}$$

$$m^i_{02} = \alpha^i_{00}/2 = m^i_{01}, \tag{4.10}$$

$$m^i_{12} = -(\tfrac{3}{4})\alpha^i_{11}-(\tfrac{3}{4})\alpha^i_{12}, \tag{4.11}$$

$$m^i_{22} = (\tfrac{15}{4})\alpha^i_{21}+(\tfrac{9}{8})\alpha^i_{22}+(\tfrac{9}{8})\alpha^i_{23}+(\tfrac{9}{8})\alpha^i_{24}, \tag{4.12}$$

$$m^i_{32} = -(\tfrac{15}{4})\alpha^i_{31}-(\tfrac{45}{4})\alpha^i_{32}-(\tfrac{27}{16})\alpha^i_{33}-(\tfrac{45}{8})\alpha^i_{34}-(\tfrac{27}{16})\alpha^i_{35}$$
$$+(\tfrac{27}{16})\alpha^i_{36}-(\tfrac{27}{16})\alpha^i_{37}, \tag{4.13}$$

$$m^i_{03} = 4\beta^i_{00}, \tag{4.14}$$

$$m^i_{13} = (\tfrac{3}{2})\beta^i_{11}-6\beta^i_{12}, \tag{4.15}$$

$$m^i_{23} = 30\beta^i_{21}-(\tfrac{9}{4})\beta^i_{22}-\beta^i_{23}-(\tfrac{9}{4})\beta^i_{24}+\beta^i_{25}-(\tfrac{21}{4})\beta^i_{26}+9\beta^i_{27}, \tag{4.16}$$

$$m^i_{33} = -15\beta^i_{31}-(\tfrac{45}{4})\beta^i_{32}-45\beta^i_{33}+(\tfrac{3}{2})\beta^i_{34}-(\tfrac{3}{4})\beta^i_{35}$$
$$+(\tfrac{7}{2})\beta^i_{36}-(\tfrac{3}{8})\beta^i_{37}-(\tfrac{27}{2})\beta^i_{38}. \tag{4.17}$$

Therefore in our calculation for the d-quark and electron the most general mass ratio turns out to be

$$\frac{m_d}{m_e} = \frac{m^1_{01}+m^1_{11}(c/M)+m^1_{21}(c/M)^2+m^1_{31}(c/M)^3}{m^1_{02}+m^1_{12}(c/M)+m^1_{22}(c/M)^2+m^1_{32}(c/M)^3}\,. \tag{4.18}$$

We can also express the mass ratio between the d- and u-quarks as

$$\frac{m_d}{m_u} = \frac{m_{01}^1 + m_{11}^1(c/M) + m_{21}^1(c/M)^2 + m_{31}^1(c/M)^3}{m_{03}^1 + m_{13}^1(c/M) + m_{23}^1(c/M)^2 + m_{33}^1(c/M)^3} \cdot \left[\frac{\sqrt{2}\, v_0}{v_0'}\right]. \tag{4.18a}$$

Similar results can be derived for other families. We can also compare the masses of different families. For example, the ratio of the masses of d- and s-quark can be written down as

$$\frac{m_d}{m_s} = \frac{m_{01}^1 + m_{11}^1(c/M) + m_{21}^1(c/M)^2 + m_{31}^1(c/M)^3}{m_{01}^2 + m_{11}^2(c/M) + m_{21}^2(c/M)^2 + m_{31}^2(c/M)^3}. \tag{4.19}$$

As we have mentioned earlier, we can go over to the results of Nanopoulos and Srednicki if we use Eqs. (2.14) through (2.22). We note that those conditions lead to constraints on m's as follows:

$$m_{01}^1 = m_{01}^2 = 0, \tag{4.20}$$

$$m_{1j}^1 = m_{1j}^3 = 0, \tag{4.21}$$

$$m_{2j}^2 = m_{2j}^3 = 0, \tag{4.22}$$

$$m_{3j}^i = 0, \tag{4.23}$$

$$m_{03}^1 = m_{03}^3 = 0, \tag{4.24}$$

$$m_{2j}^1 = m_{2j}^3 = 0, \tag{4.25}$$

$$m_{2j}^2 = m_{2j}^3 = 0, \tag{4.26}$$

$$m_{3j}^i = 0. \tag{4.27}$$

When the above conditions satisfy, the mass ratio for d and e becomes

$$\frac{m_d}{m_e} = \frac{m_{21}^1}{m_{22}^1} = \frac{(\frac{15}{4})\alpha_{21}^1 + (\frac{9}{8})\alpha_{22}^1 - (\frac{3}{4})\alpha_{23}^1 + (\frac{1}{2})\alpha_{24}^1}{(\frac{15}{4})\alpha_{21}^1 + (\frac{9}{8})\alpha_{22}^1 + (\frac{9}{8})\alpha_{23}^1 + (\frac{9}{8})\alpha_{24}^1} \tag{4.28}$$

which is the same as their result.

We can proceed now in two different directions. Firstly the parameters of all these equations could be adjusted in a new way to yield suitable mass ratios at the grand unification mass scale. These ratios could then be used in the renormalization group theory technique to evaluate the masses of the quarks at the present energy level.

In the second procedure, we note that it is reasonable to adjust the parameters to compare the masses of quarks and leptons at our present energy level. This process has been carried out by Ellis and Gaillard [5] to estimate the magnitude of parameters. We shall also proceed along this alternative line and try to determine the ratios of these constants under certain restrictions at a later stage. We are well aware of the fact that the number of parameters in our technique is more than necessary for our purpose. However it is not totally unreasonable to carry out the comparison of parameters. We intend to do exactly that in Section 6.

Before we go into these two studies, we would like to introduce a new generation of leptons and quarks, which we speculate to exist, in Section 5.

5. *Fourth generation*

We conjecture in this section that a fourth generation of particles exists and we designate it as (ν_λ, λ, l, p). We estimate their masses tentatively by extrapolating graphs of masses of the particles having the same quantum numbers versus the family number. We have taken the mass values of fitted experimental results from a table produced by Marciano [10]. Table I shows the data of first three families. The fourth generation is a speculation and the masses of the new particles are extrapolated by a prescription, which we explain now. In Fig. 1, we have plotted the generation number along the x-axis and the neutrino mass bounds along the y-axis. A smooth extrapolation gives us a mass bound of $\nu_\lambda < 1.75$ GeV. From Fig. 2, we have determined the estimate for the λ-mass to be 5.6 GeV.

Since we do not know the mass of top quark, we have estimated the l-quark mass for two different values of top quark mass, $m_t = 20$ GeV and $m_t = 30$ GeV in Fig. 3. We find corresponding values to be $\sim$ 100 Gev and 125 Gev respectively.

TABLE I

Fermions in standard model

Family No.	Name	Charge	Color	Mass
1	ν_e	0	0	<60 eV
	e	−1	0	0.51 MeV
	u	2/3	3	4.00 MeV
	d	−1/3	3	10.00 MeV
2	ν_μ	0	0	<0.52 MeV
	μ	−1	0	106 MeV
	c	2/3	3	1.25 GeV
	s	−1/3	3	200 MeV
3	ν_τ	0	0	<250 MeV
	τ	−1	0	1.78 GeV
	t	2/3	3	>20 GeV
	b	−1/3	3	4.5 GeV
4	ν_λ	0	0	<1.08 GeV
	λ	−1	0	~5.6 GeV
	l	2/3	3	~100 GeV (if $m_t = 20$ GeV)
				~125 GeV (if $m_t = 30$ GeV)
	p	−1/3	3	~14 GeV

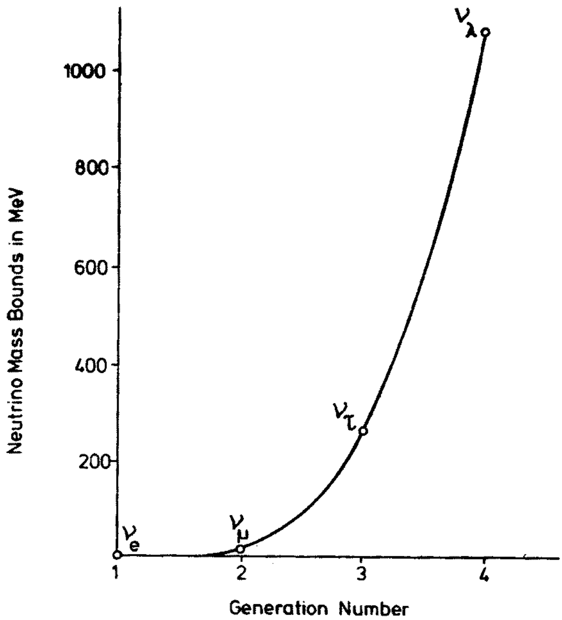

Fig. 1. Neutrino mass bounds in MeV

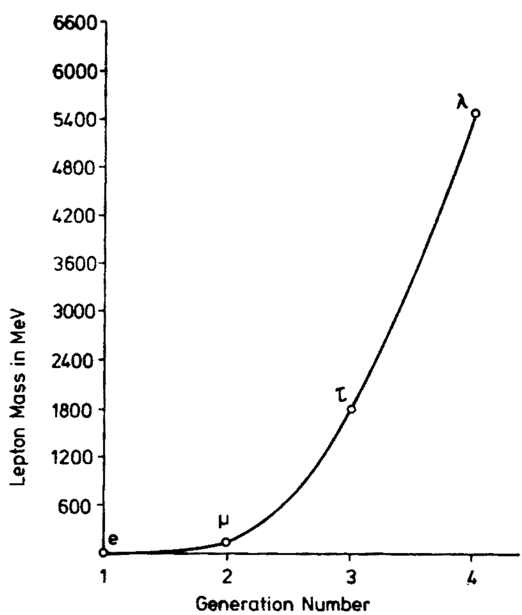

Fig. 2. Lepton mass against generation number

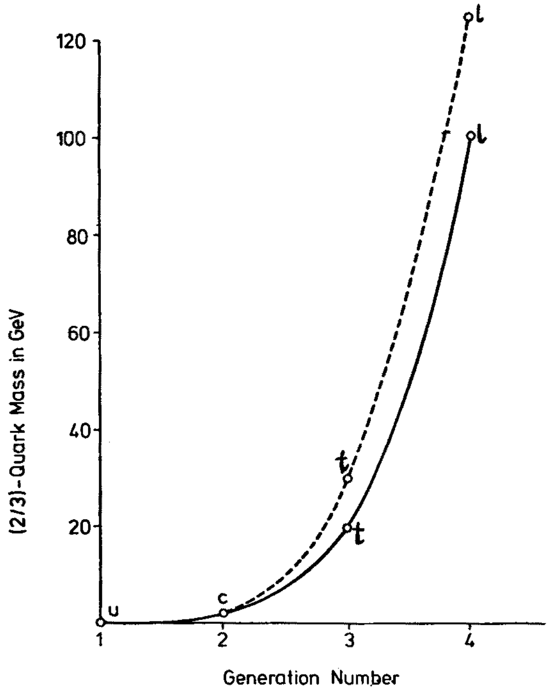

Fig. 3. (2/3)-quark mass against generation number. Solid curve for top quark mass around 20 GeV. Broken curve for top quark mass 30 GeV

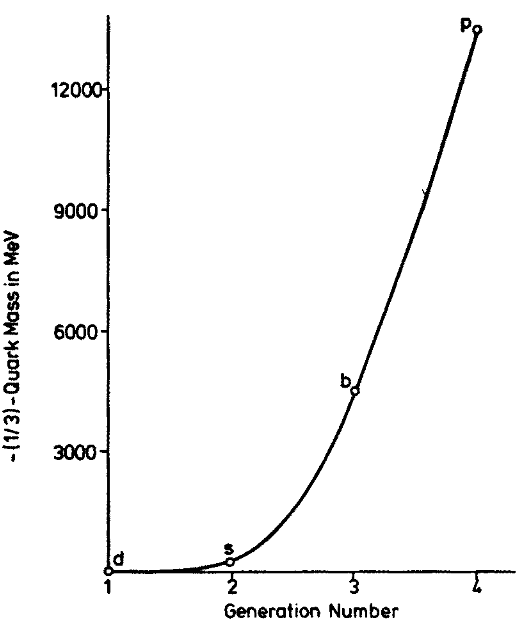

Fig. 4. $-(1/3)$-quark mass against generation number

From Fig. 4 we obtain the mass of the p quark to be around 14 GeV. All these values are put in fourth family data of Table I.

We can easily compute the contribution of the superpotential for the fourth generation. The $\underline{\bar{5}}$-fermion has to be written as

$$\bar{F}_{4aL} = \begin{bmatrix} p_1^c \\ p_2^c \\ p_3^c \\ \lambda^- \\ -\nu_\lambda \end{bmatrix}_L \tag{5.1}$$

and $\underline{10}$-representation as

$$T_{4L}^{ab} = (1/\sqrt{2}) \begin{bmatrix} 0 & -l_3^c & -l_2^c & -l^1 & -p^1 \\ -l_3^c & 0 & l_1^c & -l^2 & -p^2 \\ l_2^c & -l_1^c & 0 & -l^3 & -p^3 \\ l^1 & l^2 & l^3 & 0 & -\lambda^+ \\ p^1 & p^2 & p^3 & \lambda^+ & 0 \end{bmatrix}_L . \tag{5.2}$$

Using these values we can easily extend Eq. (4.5) to incorporate the fourth generation.

6. Parameter adjustment

A. In this section, we shall proceed in two different ways to adjust the theoretical parameters. We shall first approach the problem from the GUT mass-scale level. At this level we propose the following relations for different generations which have wider adjustment possibilities then those used by Nanopoulos and Srednicki. For the fourth generation we set

$$\frac{m_p}{m_\lambda} = \frac{m_{01}^4}{m_{02}^4} = 1 \tag{6.1a}$$

with

$$m_{01}^i = m_{02}^i = 0 \quad \text{for} \quad i = 1, 2 \text{ and } 3. \tag{6.1b}$$

For the third generation, we set

$$\frac{m_b}{m_\tau} = \frac{m_{11}^3}{m_{12}^3} = \frac{\alpha_{11}^3 - (\frac{2}{3})\alpha_{12}^3}{\alpha_{11}^3 + \alpha_{12}^3} \tag{6.2a}$$

with

$$m_{11}^i = m_{12}^i = 0 \quad \text{for} \quad i = 1,\ 2 \text{ and } 4. \tag{6.2b}$$

For the second generation we propose

$$\frac{m_s}{m_\mu} = \frac{m_{21}^2}{m_{22}^2} = \frac{(\frac{10}{3})\,\alpha_{21}^2 + \alpha_{22}^2 - (\frac{2}{3})\,\alpha_{23}^2 + (\frac{4}{9})\,\alpha_{24}^2}{(\frac{10}{3})\,\alpha_{21}^2 + \alpha_{22}^2 + \alpha_{23}^2 + \alpha_{24}^2} \tag{6.3a}$$

with

$$m_{21}^i = m_{22}^i = 0 \quad \text{for} \quad i = 1, 3 \text{ and } 4. \tag{6.3b}$$

Finally for the first generation we suggest

$$\frac{m_d}{m_e} = \frac{m_{31}^1}{m_{32}^1} = \frac{4\alpha_{31}^1 + 12\alpha_{32}^1 + (\frac{9}{5})\alpha_{33}^1 - 4\alpha_{34}^1 - (\frac{6}{5})\alpha_{35}^1 + (\frac{4}{5})\alpha_{36}^1 - (\frac{8}{5})\alpha_{37}^1}{4\alpha_{31}^1 + 12\alpha_{32}^1 + (\frac{9}{5})\alpha_{33}^1 + 6\alpha_{34}^1 + (\frac{9}{5})\alpha_{35}^1 - (\frac{9}{5})\alpha_{36}^1 + (\frac{9}{5})\alpha_{37}^1} \tag{6.4a}$$

with

$$m_{31}^i = m_{32}^i = 0 \quad \text{for} \quad i = 2, 3 \text{ and } 4. \tag{6.4b}$$

In Eq. (6.2a), we can retain the ratio $m_b/m_\tau = 1$ if we just choose $\alpha_{12}^3 = 0$. On the other hand from Eq. (6.3a), we can generate a GUT mass scale ratio $m_s/m_\mu = 2/3$ if we set $\alpha_{21}^2 = (6/5)\alpha_{23}^2$ and $\alpha_{22}^2 = \alpha_{24}^2 = 0$. As suggested by Nanopoulos and Srednicki, this ratio may improve the renormalization group computation of the mass of the s-quark at present energy levels. The ratio of d and e masses can be adjusted to 1, that is $m_d/m_e = 1$, if we choose $\alpha_{34}^1 = \alpha_{35}^1 = \alpha_{36}^1 = \alpha_{37}^1 = 0$. It can be brought to any other value at that scale if we choose appropriate α_{ij}^1's.

B. On the other hand we can ignore the renormalization group technique of computation of mass at present energy level from the GUT mass level, as has been done by Ellis and Gaillard [5], and concentrate on the parameter fitting of experimental masses to form an idea about the magnitude of non-renormalizable components.

Although there is a large number of possibilities, by which we could adjust the parameters to get the experimental mass ratios, we are tempted to assume a kind of "partial universality of parametrization", implying that

$$\alpha_{jk}^i = \alpha_{jl}^i = \alpha_j^i, \tag{6.5}$$

$$\beta_{jk}^i = \beta_{jl}^i = \beta_j^i. \tag{6.6}$$

Both of the above relations should hold for a fixed pair, i, j but k and l could be arbitrary. The motivation for introducing such universality is to design a way to see how (c/M) influences the parametrization at least at lowest power terms. Using Eqs. (4.18), (4.6) through (4.13), and (6.5) we get

$$\frac{m_d}{m_e} = \frac{\alpha_0^1 - (\frac{1}{2})\alpha_1^1(c/M) + (\frac{37}{4})\alpha_2^1(c/M)^2 - (\frac{193}{8})\alpha_3^1(c/M)^3}{\alpha_0^1 - 3\alpha_1^1(c/M) + (\frac{57}{4})\alpha_2^1(c/M)^2 - 4\alpha_3^1(c/M)^3}. \tag{6.7}$$

We can equate this value to experimental ratio $m_d/m_e = (1/200)$. Assuming $\alpha_2^1 = \alpha_3^1 = 0$, we find

$$\alpha_1^1(c/M) = 2.05\, \alpha_0^1. \tag{6.8}$$

Thus the zeroeth and the first order terms are comparable. For $m_s/m_\mu = 2$, under the similar assumption we find

$$\alpha_1^2(c/M) = 0.18\, \alpha_0^2. \tag{6.9}$$

For $m_b/m_\tau = 2.5$, and $\alpha_2^3 = \alpha_3^3 = 0$, we obtain

$$\alpha_1^3(c/M) = 0.21\,\alpha_0^3. \tag{6.10}$$

Finally from the extrapolated mass values of the new generation of particles with $m_p/m_\lambda = 2.5$, we get again

$$\alpha_1^4(c/M) = 0.20\,\alpha_0^4. \tag{6.11}$$

Making the same approximation by setting the coefficients of $(c/M)^2$ and $(c/M)^3$ equal to zero and assuming Eqs. (6.5) and (6.6), and requiring $m_d/m_u = 10/4$, $m_s/m_c = 1/6.3$, $m_b/m_t = 4.5/30$, and $m_p/m_l = 14/125$, (setting $v_0 = v_0'$), we obtain relations

$$\beta_1^1(c/M) = 0.89\,\beta_0^1 + 0.0016\,\alpha_0^1, \tag{6.12}$$

$$\beta_1^2(c/M) = 0.89\,\beta_0^2 - 0.96\,\alpha_0^2, \tag{6.13}$$

$$\beta_1^3(c/M) = 0.89\,\beta_0^3 - 0.935\,\alpha_0^3, \tag{6.14}$$

$$\beta_1^4(c/M) = 0.89\,\beta_0^4 - 1.25\,\alpha_0^4, \tag{6.15}$$

It is perhaps worth mentioning that utilizing the masses of d- and s-quarks for example, we can obtain relations between α_0^2 and α_0^1, such as

$$\alpha_0^2 = -0.027\,\alpha_0^1. \tag{6.16}$$

The derivation of the above relations justifies the conjectures originally put forward by Ellis and Gaillard [5] that the terms proportional to c/M are comparable with renormalizable components.

7. *Concluding remarks*

In the above treatment a generalization of Nanopoulos-Srednicki superpotential has been carried out. We have shown that even if we add a non-renormalizable term ($\sim A^4$) in the superpotential, we can break the symmetry in the pattern SU(5) → SU(3) × SU(2) × U(1) → SU(3) × U_{em}(1). This superpotential has been used to compute the fermion mass term. Our result yields those derived earlier by Ellis and Gaillard and Nanopoulos and Srednicki formulae of the mass ratios. We have postulated the existence of a new generation of fundamental particles whose masses we have conjectured following a graphical extrapolation. We have then proposed a new mass ratio scheme between different particles in the GUT mass scale. By adjusting the parameters suitable mass ratios can be obtained, which finally could be used in a renormalization group computation of mass at present energy scales.

We have also related these parameters under certain special conditions comparing the masses with experimental or conjectured values. With our present assumption the constants cannot be completely determined. But we have seen that the non-renormalizable terms yield comparable quantities as conjectured by Ellis and Gaillard.

We are looking for other physical assumptions which would reduce the number of parameters used in this computation. This type of non-renormalizable potential terms can also be used in other processes. One can estimate the effect of such potential on magnetic moments of particles. The topic is at present under investigation.

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