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PROSPECTS FOR MEASURING THE MASS OF GALACTIC NEUTRINOS WITH COHERENT DETECTORS

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Detection of a cosmic neutrino background may become possible in the future by observation of the second order coherent weak interaction with suitably isolated and tuned physical systems. By considering the example of reflection forces on a material slab it is shown that, by variations in detector geometry and orientation, it would be possible in principle to separately determine the number density, mean velocity, and rest mass of the interacting particles, and to establish their identity as neutrinos through consistency with both the expected interaction potential and astrophysical bounds on the measured parameters. The possibility of separate estimation of neutrino and antineutrino fluxes, by using material lens arrays to produce changes in particle density dependent on the sign of the interaction potential, is also discussed.

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In previous papers [1–4] we have discussed a number of topics relating to the possibility of detecting the cosmic neutrino background by utilizing the coherent weak interaction with bulk matter. Neutrinos with mass of order 20 eV may have clustered in our galaxy to a density of $10^7/\text{cm}^3$, and the macroscopic scale of the corresponding neutrino wavelength (60 microns for a typical momentum of about 0.02 eV) would result in significant reflection and refraction effects at material surfaces. The anisotropic flux arising from motion of the solar system through the galaxy could then give, for example, a very small — but in principle detectable — force on an isolated target. Non-clustered neutrinos would have a lower average density ($10^2/\text{cm}$) but would, if greater than about 1 eV in mass, be accelerated by the gravitational potential of the galaxy to form a nearly mono-energetic stream through the solar system, and this may also be detectable by means of the coherent forces on a suitably 'tuned' target [4]. Measurement of such forces could be based on the ideas and techniques already developed for gravitational wave detection, although new levels of noise reduction would be required — possibly utilizing an orbital (zero-g) environ-

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ment to provide the necessary degree of mechanical isolation. Alternatives to purely mechanical forces might also be considered, including the possibility of interaction with superconductors or other bulk quantum fluids.

The purpose of the present paper is to show that, if the highly refined techniques needed to measure these small effects could be developed, it would be possible not only to verify the presence of an ambient flux of weakly interacting particles but also, by variations in detector configuration, to identify these as neutrinos and separately determine

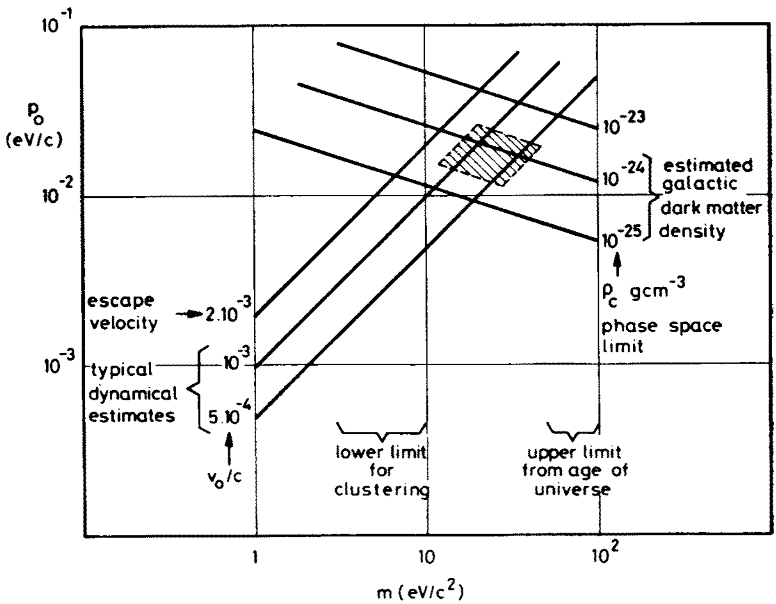

Fig. 1. Theoretical constraints on rest mass m and momentum parameter p_0 for galactically clustered neutrinos. Shaded region indicates most probable range of m and p_0 compatible with observations on visible matter

their number density, momentum, and rest mass. It would also be possible in principle to distinguish between neutrinos and antineutrinos.

To provide a framework for discussion of the different types of measurement procedure, we first summarize in Fig. 1 the range of numerical values of characteristic neutrino momentum p_0 and rest mass m (for the case of galactically clustered neutrinos) expected from basic astrophysical considerations. With the least restrictive assumption, that one neutrino type dominates the mass density, an upper limit on the rest mass in the region of 50–100 eV follows from considerations of the critical density and age of the universe [5, 6]. For these background neutrinos to have subsequently clustered gravitationally with visible matter various lower limits to the rest mass have also been estimated [5–13], ranging typically from 3 eV for neutrino clustering on the scale of galactic clusters to 10 eV or more for clustering on the scale of single galaxies. A more specific restriction results from the requirement that the maximum phase space density of the clustered distribution should not exceed that of the original Fermi distribution. Thus, assuming a clustered number distribution of the form:

$$d\varrho_\nu(p) = \varrho_\nu(\pi p_0^2)^{-3/2} \exp(-p^2/p_0^2) d^3p \tag{1}$$

and an original number distribution:

$$d\varrho_\nu(p) = h^{-3}[\exp(p/p_1)+1]^{-1} d^3p \tag{2}$$

the phase space restriction, applied at $p = 0$, gives the inequality (assuming both neutrinos and antineutrinos of one dominant type):

$$p_0^3 m > 2 \cdot 10^{20} \varrho_c, \tag{3}$$

where p_0 and m are in eV, and ϱ_c is the clustered dark matter density in g cm^{-3}.

Taking as an example the three-component galactic model of Caldwell and Ostriker [14] a typical fit to the visible matter rotation curve for our galaxy gives a dark matter density ϱ_c, averaged over a sphere at the radial position of the solar system, of about $6(\pm 2)10^{-25}$ g cm^{-3}. This could be equivalent locally to $1\text{–}2 \cdot 10^{-24}$ g cm^{-3} or more if the dark matter also has some degree of flattening towards the galactic plane. The corresponding limits imposed on p_0 and m by the inequality (3) are shown in Fig. 1 for various values of ϱ_c. The same galactic mass distribution also provides some constraints on the modal neutrino velocity $v_0/c = p_0/m$. The galactic escape velocity at the position of the solar system is about $2 \cdot 10^{-3}c$, and the rms velocity of galactic material (computed from the virial theorem) about $8 \cdot 10^{-4}c$, which is also the galactic orbital velocity of the solar system. Independently of the exact form of the clustered distribution, therefore, we can expect the characteristic momentum/mass ratio to be of order 10^{-3} and most probably in the range:

$$4 \cdot 10^{-4} < p_0/m < 12 \cdot 10^{-4}. \tag{4}$$

It can be seen that constraints (3) and (4) intersect in Fig. 1 to give a rather narrow range of possible neutrino parameters centred on $m = 25(\pm 10)$ eV and $p_0 = 0.02(\pm 0.005)$ eV.

We now show that experiments based on coherent interaction with macroscopic targets could, in principle, be designed to provide a number of intersecting lines on the same diagram, as shown in Fig. 2. For a momentum distribution of incident flux ϕ_z (in a direction z normal to a detecting surface) given by:

$$d\phi_z(\mathrm{cm}^{-2}\mathrm{s}^{-1}) = D(p_z)dp_z \tag{5}$$

the force per unit area is given by the general expression:

$$F(\mathrm{dyn\ cm}^{-2}) = \int D(p_z) \cdot C(p_z) \cdot 2p_z \cdot dp_z \tag{6}$$

where $C(p_z)$ is the reflection coefficient of the target for particles with momentum component p_z.

Eq. (6) indicates the possibility of a number of different types of experiment, based on the directionality of D, the momentum and geometric dependence of C, and the absolute value of F. We begin by noting a basic property of the reflection coefficient discussed in detail in a previous paper [4], that $C(p_z)$ will in general be "tuned", either naturally or by design, to specific values or bands of p_z (or corresponding wavelengths λ_z) related to the Fourier transform of the target density, i.e. to the overall target thickness in the case of a single uniform slab and to the additional Bragg wavelengths in the case of a periodic (laminated) structure. Since the integral (6) was shown in [4] to be essentially independent of such geometric variations, this would allow the option of selecting a region of the

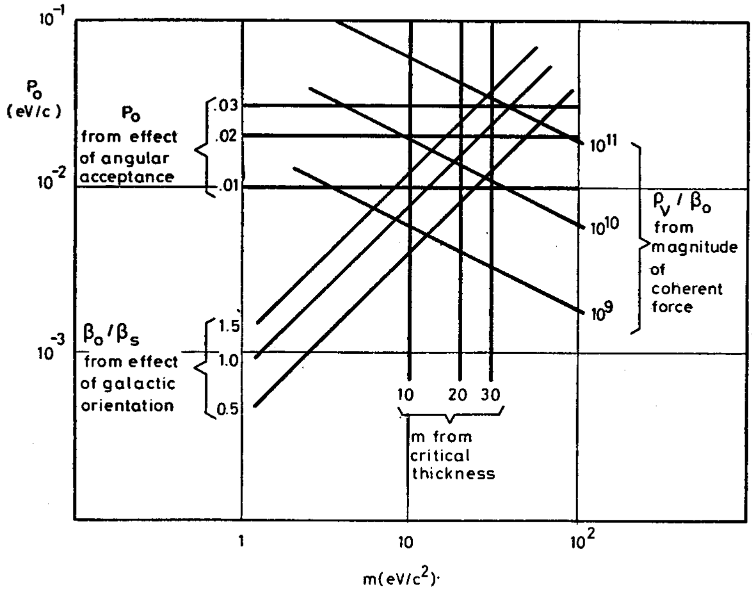

Fig. 2. Parameters measurable through coherent interactions, using different experimental procedures

spectrum with $p_z \ll p_0$, for which the neutrino refractive index is much larger than for $p_z = p_0$. It would then be feasible to utilize arrangements of Bragg mirrors to screen or modulate portions of the incident flux, and to design 'single-sided' experiments in which the target receives flux (of the selected normal component p_z) predominantly from one hemisphere.

We next consider the angular and directional dependence of the flux incident on a plane surface. If the solar system moves with velocity v_s, at an angle θ to the z-direction (Fig. 3) through an isothermal neutrino distribution given by (1), the total z-component of flux incident on a surface from one hemisphere is given by:

$$D(p_z) = D_0(cp_z/m)\exp\left[-(p_s\cos\theta - p_z)^2/p_0^2\right], \tag{7}$$

where

$$p_s = mv_s/c, \quad D_0 = K_1\varrho_c/mp_0, \quad K_1 = \pi^{-1/2}(300c^2/e) = 3.2\cdot 10^{32}.$$

In practice this integration will effectively extend to a maximum incident angle somewhat less than $\pi/2$, making some small angle $\alpha > 0$ with the surface (Fig. 4), either as a result of varying the screening arrangements or as a natural consequence of the finite thickness/width ratio (a/w) of the target — since the infinite slab approximation will overestimate C for those particle waves incident at an angle to the surface less than about $2a/w$. The momentum distribution (7) is then modified to:

$$D(p_z, \alpha) = D(p_z, 0)\,[1-\exp(-f^2)], \tag{8}$$

where

$$f = p_z/p_0 \tan\alpha \approx p_z/p_0\alpha. \tag{9}$$

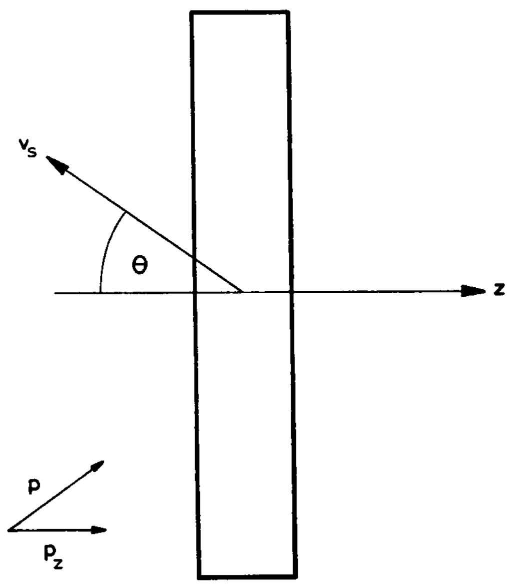

Fig. 3. Notation for modified momentum distribution arising from velocity v_s of detector through galaxy Note that the neutrino momentum p, and its z-component p_z, are now defined relative to the detector

As indicated above, the dominant contribution to the force integral (6) will in general come from a small interval Δp_z about some selected value $p_z = p_d$ say. The variation of F with α will then follow that of D which, from (8), is a maximum at $f = 1$. Thus, by varying the experimental geometry to limit α to different values, the sharp decrease in F in the vicinity of $f = 1$ can be used to determine p_0 as the ratio p_d/α, both the latter quantities being calculable from the geometry of the apparatus.

Other significant design criteria which follow from (8) and (9) are:

(a) experiments based on the use of a low value of p_d (i.e. $\ll p_0$) would require targets with a correspondingly large aspect ratio $(w/2a > p_0/p_d)$.

(b) targets with an aspect ratio ≈ 1 (and of size $a \gg \lambda\!\!\!^{-}_0 = 2 \cdot 10^{-5}/p_0 \approx 10^{-3}$ cm) would have negligible response unless 'tuned' (by lamination) to a momentum close to p_0, relying on the galactic motion to produce an asymmetric flux via (7). This provides an alternative geometric method of estimating or confirming the value of p_0, together with a method of separating out and identifying the coherent neutrino force by measuring the differential motion between adjacent (or concentric) tuned and untuned targets of the same mean density.

The variation of incident flux with galactic direction depends, from (7), on the parameter $r = p_s \cos\theta/p_0 = v_s \cos\theta/v_0$, the variation being of the form $\exp(-r^2)$ for a single hemisphere with $p_z = p_d \ll p_0$, or of the form $\exp[-(r-1)^2]-\exp[-(r+1)^2]$ for the

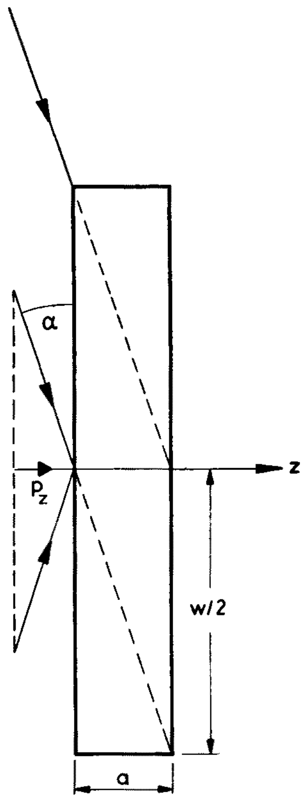

Fig. 4. Restriction of cone angle of incident flux (and hence attenuation of low values of p_z) by finite aspect ratio of detecting slab

net flux with $p_z = p_0$. Since v_s/v_0 is expected to be of order unity, $|r|$ varies between 0 and ≈ 1, and the resulting variation of F with the galactic orientation of the detector would enable v_0 to be estimated from the known value of v_s. Combining this with the measured p_0 would then provide a value for m.

The variation of F with thickness in the infinite slab approximation appears to offer a more direct method of determining m. Initially F increases linearly with a (for $a \gg h/p_0$), but subsequently approaches a saturation value for a critical thickness $a \approx a_c$ given by

$$a_c^2 = |(2\cdot 10^{-5})^2/2mU| = |[4\pi N_a f(0)]^{-1}|, \tag{10}$$

where U(eV) is the effective interaction potential for N_a atoms/cm^3 with forward scattering amplitude $f(0)$. This can be shown by numerical evaluation of (6) with the exact reflection coefficient for a plane slab of thickness a:

$$C = 1-\{4n_z^2/[4n_z^2+(1-n_z^2)^2S^2]\} \tag{11}$$

where

$$n_z^2 = 1-2mU/p_z^2 \tag{12}$$

and $S = \sin(n_z p_z a/\hbar)$ for $n_z^2 > 0$, or[1] $S = i\sinh(i\, n_z p_z a/\hbar)$ for $n_z^2 < 0$.

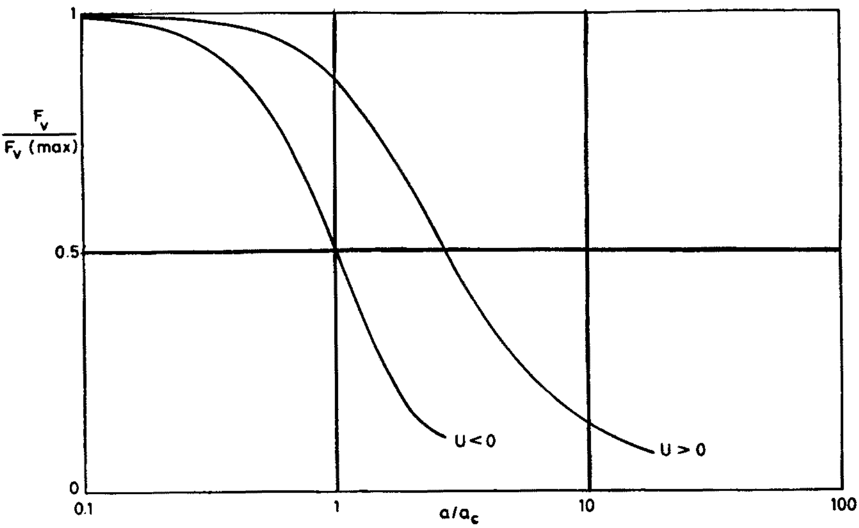

Fig. 5. Variation of force per unit volume with thickness, for slab of infinite transverse dimensions

The resulting normalized force per unit volume F_V is plotted as a function of a/a_c in Fig. 5, showing the difference between the U positive ($\bar{\nu}_e$, ν_μ, ν_τ) and U negative (ν_e, $\bar{\nu}_\mu$, $\bar{\nu}_\tau$) cases. On the assumption of an equal mixture of neutrinos and antineutrinos, the half--value of F_V occurs at $a \approx 2a_c$ which, together with the value of U obtained from the Weinberg-Salam model (summarized in [1] as a function of the atomic Z/A and neutrino type) can be substituted in (10) to provide a value for m. However, for typical values of m and U the critical thickness will be in the range 5–10 cm, corresponding to $p_z = 2-4.10^{-6}$ eV and hence requiring (from the above arguments) an aspect ratio $w/a \gtrsim p_0/p_z \sim 10^4$. Thus this experimental principle appears very difficult to realize in practice, the physical reason being that it is based on the detection of incident momenta $< 10^{-4}p_0$, which must be collected at correspondingly small angles $\lesssim 10^{-4}$ radians to the plate surface.

Each of the previous experiments requires only relative measurements of the coherent force, i.e. each is based simply on the value of a directional or geometric parameter for

[1] Note that the expression for $C(n_z^2 < 0)$ in reference [1], Eq. (13b), is misprinted; the positive sign should be a negative sign as in Eq. (13a).

which F_V is reduced to, say, half its maximum value. If in addition the absolute magnitude of the reflection force in dynes per gram of material can be estimated (e.g. from the small acceleration produced) this would provide a value for the local neutrino number density. Thus, evaluation of the integral (6) for the distribution (1) and a target thickness $a < a_c$ gives the result [1]:

$$F_V(\text{dyn cm}^{-3}) = K_2 \varrho_\nu m U^2 / p_0, \tag{13}$$

where $K_2 = \pi^{1/2} e^2 / 300^2 \hbar c = 1.44 \cdot 10^{-7}$ (m, U, p_0, in eV).

Putting $U = U_1 \varrho_t$, where ϱ_t (g cm^{-3}) is the target material density and U_1 is the known neutrino interaction potential for a material density 1 g cm^{-3} (depending on the specific material only through the approximately constant factor $1 - Z/A$ for ν_μ, ν_τ or $3Z/A - 1$ for ν_e [1]), we can rewrite (13) as

$$F_M(\text{dyn g}^{-1}) = K_2 \varrho_\nu m U_1^2 \varrho_t / p_0. \tag{14}$$

Thus the force per gram is a measure of the quantity $\varrho_\nu m / p_0 = \varrho_\nu c / v_0$ and hence provides an estimate of the neutrino number density, either in conjunction with an estimate of v_0 from the directional dependence (7) or on the basis of the preceding reasoning that v_0 should in any case be within a factor 2 of v_s. This quantity can also be included with the others on Fig. 2 by combining it with the theoretical phase space inequality (3) to give:

$$p_0^2 m > 3.6 \cdot 10^{-13} (\varrho_\nu c / v_0). \tag{15}$$

The different types of experiment discussed above are summarized in Fig. 2. The intersection of any two measurements would thus be sufficient to define probable values for p_0 and m, and the intersection of a third parameter in the same region — together with compatibility between this solution and the constraints shown in Fig. 1 — would provide confirmation of the internal consistency of the technique and that the observed effects arise from the hypothetical neutrino background.

Finally we note a possible elaboration of the above experiments to distinguish between neutrinos and antineutrinos which, although differing in the sign of U, give essentially the same reflection force for $a < a_c$ owing to the dependence of the reflection coefficient predominantly on U^2. Eqs (11) and (12) give a dependence of C on the sign of U at small values of p_z (when n_z becomes imaginary for positive U) and we have already noted that this affects the integral (6) for a target thickness of order a_c (Fig. 5) but this would be very difficult to observe in practice and would in any case not enable the neutrino and antineutrino fluxes to be separately distinguished.

A more promising approach would appear to be the addition of an array of high density material lens elements (for example on an axis parallel to the direction of the galactic orbit) which, although the diffuse angular distribution precludes any significant level of true focusing, could be designed to provide some degree of concentration or non-uniformity of the neutrino flux at the detector surface. Since the focusing properties of any array of optical elements are proportional to $n-1$ (i.e. first order in U), it is evident from (12) that a system which focuses neutrinos will defocus antineutrinos and vice versa.

Thus measurements F_1, F_2, made alternately with neutrino lens systems of opposite focusing sign, giving flux concentration factors $(1 \pm \delta)$, would provide a measurement of any difference between neutrino and antineutrino fluxes ϕ_ν, $\phi_{\bar{\nu}}$, through the asymmetry parameter:

$$(F_1 - F_2)/(F_1 + F_2) = \delta(\phi_\nu - \phi_{\bar{\nu}})/(\phi_\nu + \phi_{\bar{\nu}}). \tag{16}$$

Note that refracting systems can be designed which can be adjusted mechanically to either focusing mode. For example a two dimensional Fresnel lens can be approximated by prism section elements which could be rotated to produce either a converging or diverging lens. A possible complication arises from the sign difference between ν_e and ν_μ, ν_τ (caused by the additional scattering amplitude from the atomic electrons) but it is generally expected that the neutrinos will have sufficiently dissimilar masses (as with the charged leptons) for one type only to dominate the mass of the dark matter.

It remains to comment on the scale and practicality of neutrino concentration systems, since the quantity $n-1$ is of order 10^{-8} for $p_z = p_0$, rising (as p_z^{-2}) to 10^{-4} for $p_z = 0.01p_0$, requiring at first sight unrealistically long and massive focusing arrays. In fact, however, although the focal length of a single lens of given geometry scales as $(n-1)^{-1}$, the focusing distance for a continuous linear array of lens elements scales only as $(n-1)^{-1/2}$. This can be seen by considering the path of a ray or particle through a continuous optical system, given by equations typically of the form:

$$d^2y/dz^2 = 2(n-1)y/(y_{max})^2 \tag{17}$$

or a system of thin lenses of maximum aperture y_{max}, or

$$d^2y/dz^2 = (c^2/mv^2)dU/dy = (U_1c^2/mv^2)d\varrho_t/dy \tag{18}$$

for a transverse gradient in material density ϱ_t.

It can then be seen that the substitution of the scaled co-ordinate $z' = z(n-1)^{1/2}$ in (17), or equivalently $z' = z(U_1c^2/mv^2)^{1/2}$ in (18), converts these equations to a form independent of $(n-1)$ or U, so that devices or systems for visible light, with $n-1 \approx 1$, can be applied also to neutrinos with the axial scale expanded (for $p_0 = 0.02$ eV, $m = 20$ eV) by a factor $\approx 10^4$. For example, a stack of Fresnel lenses a few mm thick for light would scale to about 50 metres thickness for the neutrinos (the transverse dimensions governing the solid angle acceptance). Such a system, although involving substantial material costs, would clearly be quite feasible to fabricate and assemble, and even more ambitious neutrino focusing or deflection arrangements could be contemplated in an orbital (zero-g) environment, where implementation of many of the preceding experimental ideas can perhaps be most easily visualized. We have also noted previously [1] that neutrino number density gradients produced by such concentrators may allow the creation of significant first order forces, which are directly proportional to grad U and thus normally negligible. This would give rise to a new range of experimental possibilities — including, perhaps, interaction with microscopic targets or quantum fluids — but these alternatives have not yet been studied in any detail.

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