HEAVY ION REACTIONS WITH ALPHA PARTICLE TRANSFER

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Nuclear reactions between heavy ions with alpha particle transfer are considered. Theoretical studies for these heavy ion reactions are introduced. Theoretical expressions for the differential cross section are developed using pure Coulomb interaction between the interacting particles in the initial and final channels. Numerical calculations are carried out for the reaction $^{12}C(^{13}C, ^{9}Be)^{16}O$ using the Coulomb Wave Born approximation and also by the DWBA. The agreement between the theoretical calculations and the experimental data is good. The extracted spectroscopic factors are reasonable.

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1. *Introduction*

Recently, quantitative studies for heavy ion reactions are introduced by several authors [1–3] on the basis of several models. Elastic and inelastic scattering of the heavy ion collisions between the heavy ions ^{13}C and ^{12}C were analyzed by Chua et al. [4] and Gobbi [5]. Computer codes for distorted wave Born approximation calculations of heavy ion collisions were introduced [6, 7]. Osman [8] considered the ^{6}Li induced reactions as a direct stripping nuclear reaction mechanism with alpha particle transfer. A theoretical study of heavy ion reactions as a three-body problem has been introduced by Osman [9, 10] using projection operators [11].

Heavy ion reactions with alpha particle transfer have been studied as a direct nuclear reaction. Westfall and Zaidi [12] used cluster-model to study the alpha particle transfer reaction for the heavy ion reaction with ^{13}C projectile incident on ^{12}C target. They tried to investigate the experimental results by a theoretical approach. Two theoretical approaches were used to fit the experimental data. To compare the theoretical calculations with the experimental data they used both of the exact finite-range DWBA calculations and also they used $|P_l(\cos\theta)|^2$, where l is the classical grazing trajectory of ^{13}C incident on ^{12}C. The behaviour of the angular distributions is shown in the experimental graph as strong oscillations superimposed upon a gradual decrease of the cross section with angle. Exact finite-range DWBA based on the cluster model form factor are used to describe the cluster bound state of ^{16}O nucleus as composed of an alpha particle bound to a ^{12}C

nucleus with a number of nodes $N = 2$ (excluding the one at the origin) and a relative angular momentum $L = 2$. Also the alpha cluster model is used to describe the ^{13}C nucleus as composed of an alpha particle bound to ^{9}Be nucleus with a node $N = 1$ and a relative angular momentum $L = 2$. In the theoretical calculations, a Woods-Saxon potential is used with parameters of $r_0 = 1.0$ fm and $a = 0.65$ fm, and with well depths of 47.93 MeV for $^{12}C+\alpha$ system and of 56.49 MeV for $^{9}Be+\alpha$ system, where the radius of the well is taken as $R = r_0. (A_1^{1/3}+A_2^{1/3})$. The calculated cross sections indicate little agreement with the experimental data. The calculations were repeated for different values of r_0 with 20% changes, but in these cases they did not agree with the measured cross sections. This disagreement between the theoretical calculations and the experimental data is due to the values of the parameters of the optical potential. To improve these calculations, the parameters of the optical potentials should be obtained by fitting the theoretical to the experimental values for the corresponding scattering process between these heavy ions. The measured cross sections are well described by the function $|P_l(\cos\theta)|^2$, with $l = 10$, where l is the orbital angular momentum of the trajectory of ^{13}C projectile incident on ^{12}C target nucleus. $[J_0((l_0+\frac{1}{2})\theta)]^2$ is a good approximation of $[P_{l_0}(\cos\theta)]^2$ where l_0 is given by $l_0+\frac{1}{2} = \eta \cot(\frac{1}{2}\theta_0)$ and θ_0 is calculated from the relation $\sigma_{el}(\theta_0)/\sigma_R(\theta_0) = 1/4$ and η is the Sommerfeld parameter.

In the present work, a theoretical investigation of the heavy ion reactions with alpha particle transfer is presented. We consider a special case according to the energies of the incident and outgoing particles. If the energy of the heavy ion is below or close to the Coulomb barrier of the interacting particles, then the interaction can be approximated by considering only the Coulomb interaction. Thus, the interaction wave functions are Coulomb distorted wave functions. Taking Coulomb wave functions with hypergeometric functions for the interaction wave functions in the initial and final channels, an expression for the differential cross section is developed for heavy ion reactions with alpha particle transfer. The angular distribution of the reaction $^{12}C(^{13}C, ^{9}Be)^{16}O$ is considered at ^{13}C incident energy of 36 MeV. The differential cross section of this reaction is also calculated using the DWBA approximation. The spectroscopic amplitudes are extracted in both cases.

In Section 2, an expression for the differential cross section is developed by using Coulomb wave functions. Calculations and results are given in Section 3. Discussion and conclusions are introduced in Section 4.

2. *Differential cross section with Coulomb wave function*

The heavy ion reactions with alpha particle is considered. In these reactions the projectile A will be considered as composed of a bound state of a core C bound with an alpha particle α. This projectile A interacts with the target nucleus T in the initial channel, with the result of giving an outgoing particle C and the residual nucleus R. The residual nucleus R is the bound state of the target nucleus T and the alpha particle α. This transfer reaction is expressed as,

$$A(\alpha+C)+T \rightarrow C+R(\alpha+T), \tag{1}$$

where α and C are bound with relative angular momentum l, while α and T are bound with relative angular momentum L. Let us consider the alpha particle as a single structureless entity. Thus the transition amplitude for this direct transfer reaction can be given by [13],

$$T_{\mathrm{fi}} = \langle \Psi_{\mathrm{f}}^{(-)} | V_{\alpha\mathrm{C}} | \Psi_{i}^{(+)} \rangle, \tag{2}$$

where $V_{\alpha\mathrm{C}}$ is the nuclear interaction potential between the alpha particle and the particle C. $\Psi_{\mathrm{i}}^{(+)}$ and $\Psi_{\mathrm{f}}^{(-)}$ are the ingoing and outgoing wave functions describing the corresponding states in the initial and the final channels, respectively. These wave functions can be expressed as [14],

$$\Psi_{\mathrm{i}}^{(+)} = \phi_{\mathrm{i}} \chi^{(+)} \left(\boldsymbol{r} + \frac{m_\alpha}{m_{\mathrm{A}}} \boldsymbol{\varrho} \right), \tag{3}$$

$$\Psi_{\mathrm{f}}^{(-)} = \phi_{\mathrm{f}} \chi^{(-)} \left(\frac{m_{\mathrm{T}}}{m_{\mathrm{R}}} \boldsymbol{r} - \frac{m_\alpha}{m_{\mathrm{R}}} \boldsymbol{\varrho} \right), \tag{4}$$

where ϕ_{i} and ϕ_{f} are the initial and final non-interacting states of the system, and given as,

$$\phi_{\mathrm{i}} = e^{i\boldsymbol{k}_{\mathrm{i}} \cdot \boldsymbol{R}} \sum \theta(l, j) \, [[\phi_{I_\alpha}^{\mu_\alpha}(\xi) \phi_{lm}(\varrho)]_j^{\mu} \phi_{I_{\mathrm{C}}}^{\mu_{\mathrm{C}}}(\tau)]_{I_{\mathrm{A}}}^{\mu_{\mathrm{A}}} \phi_{I_{\mathrm{T}}}^{\mu_{\mathrm{T}}}(\zeta), \tag{5}$$

$$\phi_{\mathrm{f}} = e^{i\boldsymbol{k}_{\mathrm{f}} \cdot \boldsymbol{R}'} \sum \theta(L, J) \, [[\phi_{I_\alpha}^{\mu_\alpha}(\xi) \Phi_{LM}(\boldsymbol{r}_\alpha)]_J^{\mu} \phi_{I_{\mathrm{T}}}^{\mu_{\mathrm{T}}}(\zeta)]_{I_{\mathrm{R}}}^{\mu_{\mathrm{R}}} \phi_{I_{\mathrm{C}}}^{\mu_{\mathrm{C}}}(\tau). \tag{6}$$

$\chi^{(+)}$ and $\chi^{(-)}$ contain all the distortion effects resulting from the interaction of the heavy ion nuclei.

Introducing equations (3)–(6) into equation (2), the transition amplitude can be given by,

$$T_{\mathrm{fi}} = \sum_l \sum \theta(l, j) \theta^*(L, J) \, (I_\alpha \mu_\alpha l m \,|\, j\mu) \, (j\mu I_{\mathrm{C}} \mu_{\mathrm{C}} | I_{\mathrm{A}} \mu_{\mathrm{A}})$$
$$\times (I_\alpha \mu_\alpha L M \,|\, J\mu) \, (J\mu I_{\mathrm{T}} \mu_{\mathrm{T}} | I_{\mathrm{R}} \mu_{\mathrm{R}}) I_{lL}^{mM} \tag{7}$$

with,

$$I_{lL}^{mM} = \int d\boldsymbol{\varrho} e^{i\boldsymbol{Q} \cdot \boldsymbol{\varrho}} V_{\alpha\mathrm{C}}(\varrho) \phi_{lm}(\boldsymbol{\varrho}) \int d\boldsymbol{r} e^{i\boldsymbol{q} \cdot \boldsymbol{r}} \Phi_{LM}(\boldsymbol{r} + \boldsymbol{\varrho})^*$$
$$\times \chi^{(-)} \left(\frac{m_{\mathrm{T}}}{m_{\mathrm{R}}} \boldsymbol{r} - \frac{m_\alpha}{m_{\mathrm{R}}} \boldsymbol{\varrho} \right) \chi^{(+)} \left(\boldsymbol{r} + \frac{m_\alpha}{m_{\mathrm{A}}} \boldsymbol{\varrho} \right), \tag{8}$$

where $\boldsymbol{r}$ and $\boldsymbol{\varrho}$ are the relative coordinates between (T and C) and (C and α), respectively, $\boldsymbol{q}$ and $\boldsymbol{Q}$ also are given by,

$$\boldsymbol{q} = \boldsymbol{k}_i - \frac{m_{\mathrm{T}}}{m_{\mathrm{R}}} \boldsymbol{k}_{\mathrm{f}} \tag{9}$$

and,

$$\boldsymbol{Q} = \frac{m_\alpha}{m_{\mathrm{A}}} \boldsymbol{k}_{\mathrm{i}} + \frac{m_\alpha}{m_{\mathrm{R}}} \boldsymbol{k}_{\mathrm{f}} \tag{10}$$

m_i is the mass of the particle i. $\theta(l,j)$ and $\theta^*(L,J)$ are the spectroscopic amplitudes. $\phi_{lm}(\varrho)$ and $\Phi_{LM}(\boldsymbol{r}+\varrho)$ are the bound state wave functions describing the bound state of the alpha particle to the core C in the projectile and to the nucleus T in the residual nucleus R with relative momenta l and L, respectively.

$\chi^{(+)}\left(\boldsymbol{r}+\frac{m_\alpha}{m_A}\varrho\right)$ and $\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}-\frac{m_\alpha}{m_R}\varrho\right)$ which appeared in equations (3), (4) and (8) are the wave functions describing the relative motions between the interacting particles in the initial and final channels, respectively. These wave functions include the distortion effects in these interactions. The distortion between the interacting particles is due to both, the nuclear and Coulomb forces. If $\chi^{(+)}$ and $\chi^{(-)}$ are the solutions of the relative motions of the interacting particles considering the nuclear and Coulomb forces, then $\chi^{(+)}$ and $\chi^{(-)}$ are referred to and called the distorted wave functions. If $\chi^{(+)}$ and $\chi^{(-)}$ are obtained by considering only pure Coulomb forces and neglecting the nuclear forces, then $\chi^{(+)}$ and $\chi^{(-)}$ are referred to and called the Coulomb distorted wave functions. If $\chi^{(+)}$ and $\chi^{(-)}$ are the solutions of the interacting particles by neglecting both the nuclear and Coulomb forces, then $\chi^{(+)}$ and $\chi^{(-)}$ are referred to and called the plane wave functions.

Since the relative coordinate ϱ between the alpha particle and the core in the projectile is much smaller than the relative coordinate $\boldsymbol{r}$ between the core and the target nucleus, then the bound state wave function $\Phi_{LM}(\boldsymbol{r}+\varrho)$ and the interaction wave functions $\chi^{(+)}\left(\boldsymbol{r}+\frac{m_\alpha}{m_A}\varrho\right)$ and $\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}-\frac{m_\alpha}{m_R}\varrho\right)$ can be expanded by using the Taylor expansion as,

$$\chi^{(+)}\left(\boldsymbol{r}+\frac{m_\alpha}{m_A}\varrho\right) = e^{\frac{m_\alpha}{m_A}\varrho\cdot\nabla\chi^{(+)}}\chi^{(+)}(\boldsymbol{r}),$$

$$\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}-\frac{m_\alpha}{m_R}\varrho\right) = e^{\frac{m_\alpha}{m_R}\varrho\cdot\nabla\chi^{(-)}}\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}\right),$$

and,

$$\Phi_{LM}(\boldsymbol{r}+\varrho) = e^{\varrho\cdot\nabla\Phi}\Phi_{LM}(\boldsymbol{r}).$$

These expansions are also justified since m_α/m_A and m_α/m_R, which are multiplied by the relative coordinate ϱ, are less than unity because the mass of the alpha particle is smaller than the mass of the projectile A and smaller than the mass of the residual nucleus R. Thus, we see that $(m_\alpha/m_A)\varrho \ll \boldsymbol{r}$ and $(m_\alpha/m_R)\varrho \ll (m_T/m_R)\boldsymbol{r}$. Thus, the use of the Taylor expansion is justified.

The operators $\nabla\chi^{(+)}$, $\nabla\chi^{(-)}$ and $\nabla\Phi$ act on the wave functions $\chi^{(+)}$, $\chi^{(-)}$ and Φ_{LM}. Since these wave functions are the solutions of Schrödinger equations, then the result of letting these operators to operate on the corresponding wave functions will lead to the appearance of the corresponding interaction potentials.

$\nabla\chi^{(+)}$ and $\nabla\chi^{(-)}$ will lead to the appearance of the optical potentials V_{opt}^{AT} and V_{opt}^{CR} in the initial and final channels, respectively. Also, $\nabla\Phi$ will lead to the appearance of the bound state potential $V_{\alpha T}$ representing the bound state interaction of the bound state between the alpha particle and the target nucleus forming the residual nucleus.

The bound state wave function $\Phi_{LM}(\boldsymbol{r}+\boldsymbol{\varrho})$, after using the Taylor expansion, can be given with the Morinigo form as,

$$\Phi^{*}_{LM}(\boldsymbol{r}) = N(L, \ldots)e^{-\beta r}r^{L-1}Y^{*M}_{L}(\hat{\boldsymbol{r}}), \tag{11}$$

where $\beta^2 = \left(\frac{2m^{*}_{\alpha C}}{\hbar^2}\right)|E^{\alpha C}_{\text{bin}}|$ with the reduced mass $m^{*}_{\alpha C}$ and the binding energy $E^{\alpha C}_{\text{bin}}$ of the alpha particle in the residual nucleus R, and

$$N(L, \ldots) = [(2\beta)^{2L+1}/(2L)!]^{1/2}. \tag{12}$$

In the case when the projectile A is incident with energy which is well below the Coulomb barrier in the initial channel, then the Coulomb interaction is the dominant one. Thus, we can say that $\chi^{(+)}\left(\boldsymbol{r}+\frac{m_\alpha}{m_A}\boldsymbol{\varrho}\right)$ and $\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}-\frac{m_\alpha}{m_A}\boldsymbol{\varrho}\right)$ include all the distortions for the interacting particles in the initial and final channels, respectively. But in the case that the projectile incident energy is well below the Coulomb barrier between the projectile and the target nucleus in the incident channel, then $\chi^{(+)}$ is distorted due to the Coulomb potential only. This happens since the Coulomb potential is the dominant interaction in the initial channel. Also, in the final channel, if the energy of the outgoing particle is well below the Coulomb barrier between the outgoing particle and the residual nucleus, then the dominant interaction is the Coulomb potential and $\chi^{(-)}$ is distorted due to the Coulomb potential only. Using the Taylor expansion for $\chi^{(+)}\left(\boldsymbol{r}+\frac{m_\alpha}{m_A}\boldsymbol{\varrho}\right)$ and $\chi^{(-)}\left(\frac{m_T}{m_A}\boldsymbol{r}-\frac{m_\alpha}{m_A}\boldsymbol{\varrho}\right)$ leads to $\chi^{(+)}(\boldsymbol{r})$ and $\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}\right)$, respectively. Thus $\chi^{(+)}(\boldsymbol{r})$ and $\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}\right)$ describe the wave functions for the interacting particles as functions of the relative coordinate $\boldsymbol{r}$ between the core nucleus C and the target nucleus T. So, as we have described, the interaction potential between these two particles C and T is dominated by the Coulomb potential. From all these descriptions, the wave functions $\chi^{(+)}(\boldsymbol{r})$ and $\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}\right)$ are distorted due to the Coulomb potential only. Then, $\chi^{(+)}(\boldsymbol{r})$ and $\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}\right)$ are referred to and called the distorted Coulomb wave functions.

Then, they have the Coulomb distorted forms,

$$\chi^{(+)}(\boldsymbol{r}) = e^{-\frac{1}{2}\pi\eta_i}\Gamma(1+i\eta_i)\,{}_1F_1(-i\eta_i; 1; i(k_i r-\boldsymbol{k}_i\cdot\boldsymbol{r})), \tag{13}$$

$$\chi^{(-)}\left(\frac{m_T}{m_R}\boldsymbol{r}\right) = e^{-\frac{1}{2}\pi\eta_f}\Gamma(1-i\eta_f)\,{}_1F_1\left(i\eta_f; 1; -i\frac{m_T}{m_R}(k_f r-\boldsymbol{k}_f\cdot\boldsymbol{r})\right). \tag{14}$$

Introducing equations (11)–(14) into equation (8), we get,

$$I^{mM}_{lL} = N(L, \ldots)e^{-\frac{1}{2}\pi(\eta_i+\eta_f)}\Gamma(1+i\eta_i)\Gamma(1+i\eta_f)\int F_{lm}(\boldsymbol{r})e^{i\boldsymbol{q}\cdot\boldsymbol{r}}e^{-\beta r}e^{L-1}$$
$$\times Y^{*M}_{L}(\boldsymbol{r})\,{}_1F_1(-i\eta_i; 1; i(k_i r-\boldsymbol{k}_i\cdot\boldsymbol{r}))\,{}_1F_1\left(-i\eta_f; 1; i\frac{m_T}{m_R}(k_f r+\boldsymbol{k}_f\cdot\boldsymbol{r})\right)d\boldsymbol{r}, \tag{15}$$

where

$$F_{lm}(r) = \int d\varrho\, e^{i\left[Q - i\nabla\Phi - i\frac{m_\alpha}{m_A}\nabla\chi^{(+)} + i\frac{m_\alpha}{m_R}\nabla\chi^{(-)}\right]\cdot\varrho} V_{\alpha C}(\varrho)\phi_{lm}(\varrho). \tag{16}$$

We use a Morinigo wave function for the bound state of the alpha particle in the projectile $\phi_{lm}(\varrho)$, and also a Yukawa potential for $V_{\alpha C}(\varrho)$ as,

$$V_{\alpha C}(\varrho) = V^0_{\alpha C}\frac{R_{\alpha C}}{\varrho}\, e^{-\alpha_{\alpha C}(\varrho - R_{\alpha C})}. \tag{17}$$

The exponential present in the integral given by equation (16) is used in an expansion form as,

$$e^{i\left[Q - i\nabla\Phi - i\frac{m_\alpha}{m_A}\nabla\chi^{(+)} + i\frac{m_\alpha}{m_R}\nabla\chi^{(-)}\right]\cdot\varrho} = 4\pi\sum_{\lambda=0}^{\infty} i^\lambda$$

$$\times j_\lambda\left[\left|Q - i\nabla\Phi - i\frac{m_\alpha}{m_A}\nabla\chi^{(+)} + i\frac{m_\alpha}{m_R}\nabla\chi^{(-)}\right|\varrho\right]$$

$$\times Y^{m_\lambda}_\lambda(\hat{\varrho})Y^{m_\lambda}_\lambda\left[\left(Q - i\nabla\Phi - i\frac{m_\alpha}{m_A}\hat{\nabla}\chi^{(+)} + i\frac{m_\alpha}{m_R}\nabla\chi^{(-)}\right)\right]. \tag{18}$$

Also, let us use the local WKB-approximations by letting $\nabla\Phi$, $\nabla\chi^{(+)}$ and $\nabla\chi^{(-)}$ operate on the functions $\Phi_{LM}(r)$, $\chi^{(+)}$ and $\chi^{(-)}$, respectively, giving,

$$(\nabla^2)^n\Psi = (-1)^n\left(\frac{2m^*}{\hbar^2}\right)^n [V(r) - E]^n\Psi. \tag{19}$$

Then, we get for the integral given by equation (16), the expression,

$$F_{lm}(r) = 4\pi^{1/2}N(l, \ldots)V^0_{\alpha C}R_{\alpha C}e^{\alpha_{\alpha C}R_{\alpha C}}(2i)^l\Gamma(l+\tfrac{1}{2})$$

$$\times Y^m_l[Q - \hat{P}(r)]\frac{[Q - P(r)]^l}{\{(\alpha_{\alpha C}+\beta)^2 + [Q - P(r)]^2\}^{l+1/2}}, \tag{20}$$

where

$$P(r) = \sqrt{\frac{2m^*_{\alpha T}}{\hbar^2}[E^{\alpha T}_{\text{bin}} - V_{\alpha T}(r)]} + \frac{m_\alpha}{m_A}\sqrt{\frac{2m^*_{AT}}{\hbar^2}[E_i - V^{AT}_{\text{opt}}(r)]}$$

$$-\frac{m_\alpha}{m_R}\sqrt{\frac{2m^*_{CR}}{\hbar^2}[E_f - V^{CR}_{\text{opt}}(r)]}. \tag{21}$$

$m^*_{\alpha T}$ and $E^{\alpha T}_{\text{bin}}$ are the reduced mass and the binding energy for the bound state between the alpha particle and the particle T forming the residual nucleus R. E_i and E_f are the energies of ingoing and outgoing particles. $V^{AT}_{\text{opt}}(r)$ and $V^{CR}_{\text{opt}}(r)$ are the optical potentials in the initial and final channels.

Introducing equations (15) and (20) into equation (7), the transition amplitude is determined. Then the differential cross section can be given by the expression,

$$\frac{d\sigma}{d\Omega} = \frac{m^*_{AT} m^*_{CR}}{(2\pi\hbar^2)^2} \frac{1}{(2I_A+1)(2I_T+1)} \sum_{\substack{\mu_A \mu_T \\ \mu_C \mu_R}} |T_{fi}|^2. \tag{22}$$

3. Calculations and results

In the present work, we developed an expression for the differential cross section for these transfer reactions using Coulomb wave functions in both, the initial and final channels. The heavy ion reaction with alpha particle transfer using ^{13}C projectile incident on the target nucleus ^{12}C has been studied. The incident energy of the ^{13}C projectile is 36 MeV which is close to the Coulomb barrier in the initial channel. The quantum numbers describing the bound state of alpha particle and ^{12}C to produce the ^{16}O residual nucleus are $N = 2$ and $L = 0$. The depth of binding potential of the ^{16}O residual nucleus as a cluster structure of alpha particle and ^{12}C nucleus is 47.93 MeV. For the projectile heavy ion ^{13}C which is considered as a cluster structure of alpha particle and ^{9}Be nucleus with the depth of 56.49 MeV and quantum numbers $N = 1$ and $L = 2$.

The optical model potentials are used for describing the interactions in the initial and final channels. A Woods-Saxon potential forms are used for the optical model potentials. The potential is given by,

$$V_{opt}(r) = V(1+e^{\frac{r-r_{0R}}{a_0}})^{-1} + iW(1+e^{\frac{r-r_{0I}}{a_I}})^{-1}. \tag{23}$$

The parameters are changed till the calculated elastic scattering cross section fit the experimentally measured values [12]. The obtained potential parameters for the entrance channel of the scattering of ^{13}C on ^{12}C are: $V = -86.602$ MeV, $r_{0R} = 1.084$ fm, $a_0 = 0.586$ fm and $W = -9.563$ MeV, $r_{0I} = 1.263$ fm, and $a_I = 0.329$ fm.

For the exit channel, the parameters are the same as those of the entrance channel by scaling the radii with $(A_1^{1/3}+A_2^{1/3})$. The radius parameter R is related to the mass number A_1 and A_2 by the relation $R = r_0(A_1^{1/3}+A_2^{1/3})$.

Calculations for the differential cross sections are made in case the $\chi^{(+)}$ and $\chi^{(-)}$ include only Coulomb distortions. These calculations are referred to as CWBA, and are shown by dashed curve in Fig. 1. Also, the differential cross section is calculated using the distorted wave born approximation referred to as EFR — DWBA. The computer program code used in the present calculations is LAJOLLA, where the numerical calculations are performed on the CDC 6600 computer. The results are shown in figure 1 by the solid curve. The experimental measurements are also shown by the solid circles. From the fitting, the spectroscopic factors are extracted.

The obtained values of the spectroscopic factors in the cases of the CWBA and DWBA calculations are 0.5236 and 0.7741 respectively. The obtained values of the normalization amplitudes seem reasonable, but unfortunately, there are no values obtained by the other authors that we could compare our values with.

From the present calculations of the differential cross sections for heavy ion reactions with alpha particle transfer using CWBA and DWBA we can conclude the following:

(*i*) The CWBA calculations give nearly the same qualitative shape of the whole angular distribution as that done by the DWBA approximation. Although the CWBA angular

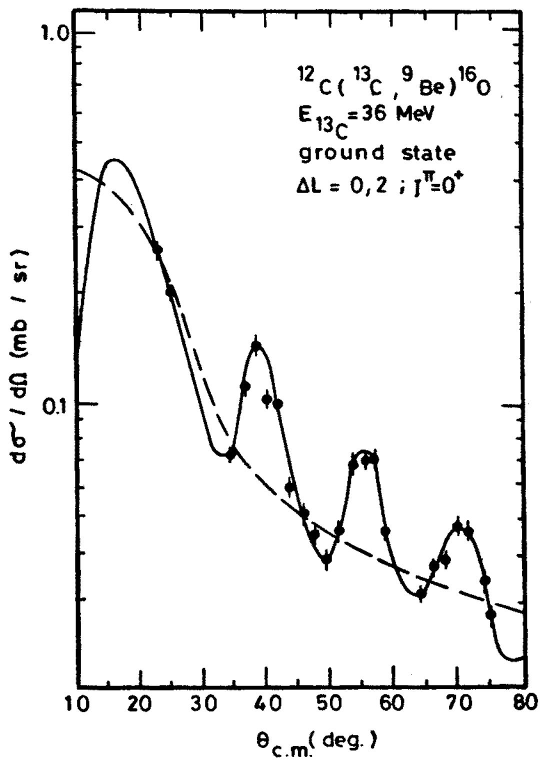

Fig. 1. Differential cross sections of the alpha transfer reaction $^{12}C(^{13}C, ^{9}Be)^{16}O$ at ^{13}C, incident energy 36.0 MeV, leaving the ^{16}O residual nucleus in its ground state. The solid curve is our present distorted wave Born approximation calculation. The dashed curve is our present Coulomb wave Born approximation calculation. The points are experimental data and are taken from reference [12]

distribution is a smooth curve and the DWBA differential cross section calculations give oscillatory patterns, as a whole, both of the angular distributions are nearly the same qualitatively.

(*ii*) Thus, calculations using CWBA have the merit of being easier in analytical work, and they are given in the more closed analytical mathematical expressions than those which use DWBA.

(*iii*) Numerical calculations using the CWBA expressions are easier and need a much shorter time and much less effort on the computer than those which use the DWBA expressions.

4. *Discussion and conclusions*

In the present work, the alpha particle transfer heavy ion reactions are studied. The theoretical investigations given here are introduced for the special case of using Coulomb wave functions to describe the initial and final channels wave functions in the case when the projectile and outgoing ions energies are close to the corresponding Coulomb barriers. Also, for the purpose of comparison, distorted wave Born approximation calculations are carried out. From Fig. 1, we see that the distorted wave Born approximation (DWBA) calculations are in good agreement with the experimental data of the angular distributions. Also, the extracted spectroscopic factors from both calculations (CWBA and DWBA) are 0.5236 and 0.7741 respectively, and are reasonably good values. The CWBA differential cross sections are given in the form of hypergeometric functions which are smooth functions. That is why the CWBA calculations for the differential cross sections are represented by a smooth curve as shown by the dashed curve in Fig. 1. Since in the present calculations the energy of the incident heavy projectile is above the Coulomb barrier between the interacting ions, and because of the lack of the experimental data the presently calculated differential cross sections have a forward peak. This is expected and understandable. Beside, the present calculations give reasonable values for the cross sections in the backward direction. On the other hand, the DWBA calculations for the differential cross sections show up and fit the oscillatory pattern of the angular distributions. This is also expected and understandable since in the DWBA calculations the nuclear distortions from the nuclear potentials are included.

From Fig. 1 and from the obtained values for the spectroscopic factors, we can draw the following conclusions. Both, the CWBA and DWBA calculations give the shape of the angular distributions of heavy ion reactions with alpha particle transfer. The DWBA calculations show the exact peaks of the differential cross section. The CWBA calculations give higher values for the differential cross sections in the backward direction, which is due to strong Coulomb repulsion in the backward direction. Both calculations closely reproduce the magnitude of the cross sections, with better results in the case of the DWBA calculations. Thus, the present theoretical calculations well reproduce the shape and magnitude of the differential cross sections of the heavy ion reactions with alpha particle transfer.

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