ELECTROWEAK PLASMA IN A MAGNETIC FIELD

By H. Pérez Rojas

Grupo de Física Teórica, Instituto de Matemática, Cibernética y Computación, Academia de Ciencias de Cuba*

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The leptonic sector of an electroweak plasma in an external magnetic field is studied within the Weinberg-Salam model. Nonvanishing chemical potentials μ_1, μ_2 related to electric charge and leptonic number are introduced and used to establish the set of equations describing the chemical equilibrium of the system, which will be taken as electrically neutral. The partition functional is expressed in terms of an effective Lagrangian dependent on μ_1, μ_2 as background fields. Gauge conditions dependent on μ_1, μ_2 are imposed. By using them and expanding the charged fields in magnetic eigenfunctions, the functional integration is performed in the one-loop approximation. The thermodynamic potential is written in terms of the spectra of the physical particles, and the equilibrium conditions are discussed. No Bose condensation in the usual sense occurs at $T \neq 0$ in the presence of the magnetic field, but it is produced in the $T = 0$ case. By calculating the magnetization it is shown that an increase in the lepton density may provoke an increase in the effective magnetic field.

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1. Introduction

This paper is devoted to the study of an electroweak plasma in an external magnetic field at finite temperature and nonvanishing chemical potentials. It is concerned with the leptonic electrowaek sector only, i.e. hadrons or quarks may be considered as part of the heat bath, in thermal equilibrium with our system. There are at least two reasons for considering the present study as interesting: first, the recent discovery of $W^{\pm}$ and Z mesons, confirming the predictions of the Weinberg-Salam (WS) model, makes electroweak processes very important in the high energy astrophysical context. In second place, from a methodological point of view it may be desirable to have a systematic quantum statistical study of electrowaek systems in magnetic fields — the natural generalization to electroweak theory of previous works [1]–[4] in quantum statistical electrodynamics.

The first part of the paper is devoted to obtain the quantum statistical generating functional Z (or partition function) of an electroweak plasma by considering as non-

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* Address: Instituto de Matemática, Cibernética y Computación, Calle o nr 8, Vedado, Habana 4, Cuba.

vanishing only the chemical potentials μ_1, μ_2 related to electric charge and lepton charge. The conditions imposed on the chemical potentials by the requirement of equilibrium under the elementary interactions between the particles are established. The second part of the paper deals with the case of an external magnetic field. A set of gauge conditions dependent on the chemical potential μ_1, is chosen. This particular gauge, together with the use of magnetic eigenfunctions for the charged fields, makes the functional integration easily calculable in the one loop approximation. The thermodynamics potential is written in this approximation in terms of the spectra of the charged particles in the magnetic field and also in terms of the spectra of the neutral particles. The energy of the vacuum due to the external field is calculated. Expressions are given for the electric charge and leptonic number in terms of the mean number of particles and the conditions for equilibrium are discussed. The problem of the Bose condensation at $T = 0$ is studied, the magnetization of the system is obtained and its relation with neutrino density is discussed. Finally, in the appendix the explicit form of the propagators of charged particles in the magnetic fields is written.

No attempt is made in the present paper to discuss the problem of symmetry restoration. The effect of magnetic field alone is discussed by Linde in [5] (see also Salam and Strathdee [22]). The simultaneous effect of temperature and magnetic field was later studied by Chakrabarti [18], who excluded the unstable mode, and more recently by Reuter and Dittrich [21] by using the fact that instability is cured when higher approximations are taken.

2. *The electroweak partition functional*

a) The WS Lagrangian, currents and charges

We start from the Lagrangian for the WS model [6]

$$\mathcal{L} = -\tfrac{1}{4} G^i_{\mu\nu}G^i_{\mu\nu} - \tfrac{1}{4} F_{\mu\nu}F_{\mu\nu} - \bar{\psi}_L\gamma_\mu\left(\partial_\mu - i\frac{g}{2}\tau^i W^i_\mu + i\frac{g'}{2}B_\mu\right)\psi_L$$
$$-\bar{e}_R\gamma_\mu(\partial_\mu + ig'B_\mu)e_R - \left|\left(\partial_\mu - i\frac{g}{2}\tau^i W^i_\mu - i\frac{g'}{2}B_\mu\right)\phi\right|^2$$
$$-\lambda_1(\bar{\psi}_L\phi e_R + \bar{e}_R\phi^\dagger\psi_L) - \frac{\lambda_2}{4}(\phi^\dagger\phi - a^2)^2, \tag{1}$$

where

$$G^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu + g\varepsilon^{ijk}W^j_\mu W^k_\nu \tag{2}$$

is the field tensor of the SU(2) non-abelian field and

$$F_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu \tag{3}$$

is the abelian gauge field tensor.

The left-handed spinor doublet is[1]

$$\psi_L = \tfrac{1}{2}(1-\gamma^5)\begin{pmatrix} \nu \\ e \end{pmatrix} \tag{4}$$

and the singlet right-handed spinor

$$e_R = \tfrac{1}{2}(1+\gamma^5)e. \tag{5}$$

In (1) Euclidean metric is used, $g_{\mu\nu} = (1, 1, 1, 1)$. This means that $\partial_4 = -i\partial_0$, $W_4 = -iW_0$, $B_4 = -iB_0$ and $\gamma_\mu\gamma_\nu+\gamma_\nu\gamma_\mu = 2\delta_{\mu\nu}$. The scalar field is defined as

$$\phi = \frac{1}{\sqrt{2}}\left\{\begin{pmatrix} 0 \\ \xi \end{pmatrix} + \begin{pmatrix} ih^1+h^2 \\ \sigma - ih^3 \end{pmatrix}\right\}, \tag{6}$$

$1/\sqrt{2}\begin{pmatrix} 0 \\ \xi \end{pmatrix}$ being the vacuum expectation value of ϕ in the tree approximation, where $\xi \neq 0$ is the symmetry breakdown parameter. By taking ξ as the minimum value of the potential scalar term $-\lambda_2(\phi^\dagger\phi - a^2)^2/A$, i.e. $\xi^2 = 2a^2$, we get the σ field as massive ($m_\sigma = \sqrt{\lambda_2}a$), and as a consequence of Higgs mechanism the fields h^1, h^2, h^3 become massless and non-physical, whereas the charged vector meson fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \tag{7}$$

and the neutral field

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(-gW_\mu^3 + g'B_\mu) \tag{8}$$

become massive, and the electromagnetic field

$$A_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'W_\mu^3 + gB_\mu) \tag{9}$$

remains massless.

The Lagrangian (1) is invariant under the set of infinitesimal gauge transformations

$$\begin{aligned} W_\mu^i &\to W_\mu^i + (\partial_\mu\delta^{ij} + g\varepsilon^{ikj}W_\mu^k)\lambda^j, \\ \psi_L &\to \left(1 + i\frac{g}{2}\tau^i\lambda^i - i\frac{g'}{2}\lambda^0\right)\psi_L, \\ \varphi &\to \left(1 + i\frac{g}{2}\tau^i\lambda^i + i\frac{g'}{2}\lambda^0\right)\varphi, \\ B_\mu &\to B_\mu + \partial_\mu\lambda^0, \quad e_R \to (1-ig'\lambda^0)e_R, \end{aligned} \tag{10}$$

[1] Other lepton doublets may be included. We omit them for simplicity.

where τ^i are the Pauli spin matrices and λ^i, $i = 1, 2, 3$, λ^0 are infinitesimal parameters of the SU(2) and U(1) gauge transformations, $S = \exp(ig\tau^i\lambda^i/2)$ and $U = \exp(-ig'\lambda^0/2)$, respectively. The Lagrangian (1) is invariant also under the independent U(1) global transformation

$$\psi_L \to e^{i\alpha}\psi_L, \qquad e_R \to e^{i\alpha}e_R. \tag{11}$$

Due to the invariance of (1) under the global version of the set (10) we get the conserved Noether currents

$$j^k_\mu = -gG^i_{\mu\nu}\varepsilon^{ijk}W^j_\nu - ig\bar{\psi}_L\gamma_\mu\frac{\tau^k}{2}\psi_L + \left(\partial_\mu - i\frac{g}{2}\tau^iW^i_\mu - i\frac{g'}{2}B_\mu\right)\phi\left(i\frac{g}{2}\tau^k\right)\phi^\dagger$$
$$-\left(i\frac{g}{2}\tau^k\right)\phi\left(\partial_\mu + i\frac{g}{2}\tau^iW^i_\mu + i\frac{g'}{2}B_\mu\right)\phi^\dagger, \tag{12}$$

$$j^0_\mu = \bar{\psi}_L\gamma_\mu\left(i\frac{g'}{2}\right)\psi_L + \bar{e}_R\gamma_\mu(ig')e_R + \left(\partial_\mu - i\frac{g}{2}\tau^iW^i_\mu - i\frac{g'}{2}B_\mu\right)\phi\left(i\frac{g'}{2}\right)\phi^\dagger$$
$$-\left(i\frac{g'}{2}\right)\phi\left(\partial_\mu + i\frac{g}{2}\tau^iW^i_\mu + i\frac{g'}{2}B_\mu\right)\phi^\dagger. \tag{13}$$

The electromagnetic current is obtained from the linear combination

$$j^{\text{em}}_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g'j^3_\mu + gj^0_\mu). \tag{14}$$

From the invariance of (1) under (11) we get also the conserved leptonic current

$$j^1_\mu = i\bar{\psi}_L\gamma_\mu\psi_L + i\bar{e}_R\gamma_\mu e_R. \tag{15}$$

The fourth components of (14) and (15) are the electric charge density Q^e (the electromagnetic coupling constant is $e = gg'/(g^2+g'^2)^{1/2}$) and the lepton charge density Q^l. From (12) and (13) we may get also the charged and neutral weak currents. However, in the present paper we do not consider their fourth components charge densities in order to write the partition functional. In other words, their associated chemical potentials will be taken as zero.

b) The Hamiltonian

In order to write the Hamiltonian of our system, we obtain at first the momenta conjugated to the set of fields

$$p^i_\mu = iG^i_{4\mu}, \qquad p^B_\mu = iF_{4\mu}, \tag{16}$$

$$p_{\psi_L} = i\bar{\psi}_L\gamma_0, \qquad p_{\bar{\psi}_L} = 0, \qquad p_{e_R} = ie_R\gamma_0, \qquad p_{\bar{e}_R} = 0, \tag{17}$$

$$p_\phi = i\left(\partial_4 + i\frac{g}{2}\tau^i W_4^i + i\frac{g'}{2} B_4\right)\phi^\dagger, \tag{18}$$

$$p_{\phi^\dagger} = i\left(\partial_4 - i\frac{g}{2}\tau^i W_4^i - i\frac{g'}{2} B_4\right)\phi. \tag{19}$$

In terms of these momenta we give the explicit expressions for the electric and lepton charges

$$Q^e = e\Bigg[p_\nu^i \varepsilon^{i3k} W_\nu^k + \frac{i}{2} p_{\psi_L}(\tau_3 - I)\psi_L - i p_{e_R} e_R$$

$$- i p_\phi + [\tfrac{1}{2}(\tau^3 + I)\phi^\dagger] + [\tfrac{1}{2}(\tau^3 + I)\phi]\,(i p_\phi)\Bigg], \tag{20}$$

$$Q^l = p_{\psi_L}\psi_L + p_{e_R} e_R. \tag{21}$$

Note that as soon as x_4 is taken as a real variable, i.e. $\partial_4 = \partial/\partial x_4$ and W_4^i, B_4 as real fields, the momenta ip_ϕ and $ip_{\phi^\dagger}$ are Hermitean conjugated. From (16)–(19) we get easily the Hamiltonian

$$\mathscr{H} = \tfrac{1}{2} p_\nu^i p_\nu^i + \tfrac{1}{4} G_{jk}^i G_{jk}^i - i p_\nu^i(\partial_\nu W_4^i - g\varepsilon^{ijk} W_4^j W_\nu^k) + \tfrac{1}{2} p_\nu^B p_\nu^B + \tfrac{1}{4} F_{jk} F_{jk}$$

$$- i p_\nu^B(\partial_\nu B_4) - p_{\psi_L}\left(\frac{g}{2}\tau^i W_4^i - \frac{g'}{2} B_4\right)\psi_L + \bar{\psi}_L \gamma_k\left(\partial_k - i\frac{g}{2}\tau^i W_k^i + i\frac{g'}{2} B_k\right)\psi_L$$

$$+ p_{e_R}(igB_4)e_R + \bar{e}_R \gamma_k(\partial_k + ig' B_k)e_R + p_\phi p_{\phi^\dagger}$$

$$+ \left(\partial_k + i\frac{g}{2}\tau^i W_k^i + i\frac{g'}{2} B_k\right)\phi^\dagger\left(\partial_k - i\frac{g}{2}\tau^i W_k^i - i\frac{g'}{2} B_k\right)\phi$$

$$- i p_\phi\left(-i\frac{g}{2}\tau^i W_4^i - i\frac{g'}{2} B_4\right)\phi - i p_{\phi^\dagger}\left(i\frac{g}{2}\tau^i W_4^i + i\frac{g'}{2} B_4\right)\phi^\dagger$$

$$+ \lambda_1(\bar{\psi}_L \phi e_R + \bar{e}_R \phi^\dagger \psi_L) + \frac{\lambda_2}{4}(\phi^\dagger\phi - a^2)^2. \tag{22}$$

Due to the constraints (17), $\mathscr{H}$ is independent of the momenta $p_{\psi_L}, p_{\bar{\psi}_L}, p_{e_R}, p_{\bar{e}_R}$. If adequate four divergencies were added to $\mathscr{H}$ we would get the terms

$$iW_4(\partial_k p_k^j - g\varepsilon^{ijk} p_l^j W_l^k) + iB_4(\partial_k p_k^B) \tag{23}$$

instead of the third and sixth term in (22), the quantities in parenthesis being the constraints of the gauge vector fields, and W_4, B_4 playing the role of Lagrange multipliers. We shall use the Hamiltonian in terms of the Euclidean variables x_4, W_4^i, B_4 in order to write the partition functional.

c) The partition functional and the equilibrium conditions

Following Bernard [7], Kapusta [8] and Tyutin [9] we may write

$$Z = N(\beta) \int Dp^i_\mu Dp^B_\mu Dp_{\bar{\psi}_L} Dp_{\psi_L} Dp_{\bar{e}_R} Dp_{e_R} Dp_\phi Dp_{\phi^\dagger}$$
$$\times \int DW^i_\mu DB_\mu D\bar{\psi}_L D\psi_L D\bar{e}_R De_R D\phi D\phi^\dagger \delta(p^i_4)\delta(p^B_4)$$
$$\times \delta(p_{\psi_L})\delta(p_{\psi_L} - i\bar{\psi}_L\gamma_0)\delta(p_{\bar{e}_R})\delta(p_{e_R} - i\bar{e}_R\gamma_0) \prod_{j=0}^{3} \delta(G_j) \,\mathrm{Det}\,\mathcal{M}$$
$$\exp\Big[\int_0^\beta dx_4 \int d^3x\{i(p^i_\mu \dot{W}^i_\mu + p^B_\mu \dot{B}_\mu + p_{\psi_L}\dot{\psi}_L + p_{\bar{\psi}_L}\dot{\bar{\psi}}_L + p_{\bar{e}_R}\dot{\bar{e}}_R + p_\phi\dot{\phi} + p_{\phi^\dagger}\dot{\phi}^\dagger)$$
$$-\mathcal{H} + \mu_1 N_1 + \mu_2 N_2\}\Big], \tag{24}$$

$N(\beta)$ being a temperature dependent constant. In (24) the momenta-dependent delta functions account for the constraints (17) and also $p^i_4 = 0$, $p^B_4 = 0$ among (16), $G^j = 0$ stands for the set of gauge conditions and

$$\mathrm{Det}\,\mathcal{M} = \mathrm{Det}\,\frac{\delta G^j}{\delta\lambda^k}, \quad j, k = 0, 1, 2, 3 \tag{25}$$

is the Fadeev-Popov determinant. In (24) dots mean x_4 derivatives (not time derivatives), $N_1 = Q^e/e$ is the density of charged particles and $N_2 = Q^l$ the density of leptons. The boson fields are periodic in the variable x_4 with period β whereas the fermion fields are antiperiodic. This is reflected in the p_4 Fourier components of these fields, which are respectively $2\pi n/\beta$ and $(2n+1)\pi/\beta$, n being any arbitrary positive or negative integer. It must be pointed out also that when performing the functional integration the Fermi fields must be taken as Grassman functions. If products of the fields by adequate external currents were added to the exponential in (24), we would obtain a functional Z dependent on these currents from which the Green functions and the set of coupled equations which they satisfy may be obtained by the standard methods of functional differentiation. We do not proceed in this way and shall concentrate later in the calculation of the thermodynamic potential. Before doing that we shall establish some relations among the chemical potentials. By calling N_W, N_e, N_ν and N_h the number of particles densities (particles minus antiparticles) for the W mesons, electrons[2], neutrinos and charged Goldstone boson fields, the term $\mu_1 N_1 + \mu_2 N_2$ in the exponential in (24) can be written as

$$\mu_1 N_1 + \mu_2 N_2 = \mu_1 N_W + (\mu_1 + \mu_2) N_e + \mu_2 N_\nu + \mu_1 N_h \tag{26}$$

and calling μ_e the chemical potential for the electrons we get from (26) that it is equal to the sum of the electric and leptonic chemical potentials

$$\mu_e = \mu_1 + \mu_2. \tag{27}$$

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[2] Due to the fact that the chemical potential for neutral weak charge was put equal to zero, the chemical potential for left and right electrons is the same.

Equation (27) may be better understood from the following reasoning: thermodynamical equilibrium requires that the Gibbs free energy G should be a minimum. We may write G as a sum formed by the products of the number of particles of each kind by their chemical potential, and we shall consider the chemical potentials of particles and antiparticles as different in principle (the chemical potentials of the neutral fields A, Z, h_3 and σ will be taken as zero)

$$G = \mu_{e^-}N_{e^-} + \mu_{e^+}N_{e^+} + \mu_{W^-}N_{W^-} + \mu_{W^+}N_{W^+} + \mu_\nu N_\nu + \mu_{\bar\nu} N_{\bar\nu}$$
$$+ \mu_{h^-}N_{h^-} + \mu_{h^+}N_{h^+},$$
$$(h^{\mp} = (h^1 \pm ih^2)/\sqrt{2}). \tag{28}$$

By equating the variation of G to zero, under the condition that the number of particles changes independently under each of the processes depicted in Fig. 1, we get a set of relations among the chemical potentials. In particular, for variations in the number of particles under the diagrams a, b, c, d as the number of particles and antiparticles must be equal, we get

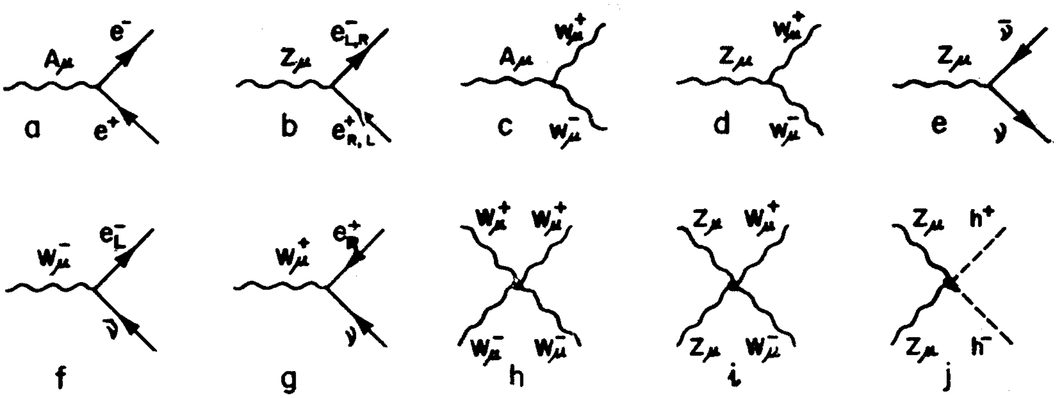

Fig. 1. Some of the elementary processes in the electroweak plasma are illustrated here. An equilibrium equation among the chemical potentials may be associated to each of them. As the chemical potential related to weak neutral charge was taken as zero, the same chemical equilibrium equation is valid for b in the cases of left and right electrons

$$\mu_{e^+} + \mu_{e^-} = 0, \quad \mu_{W^+} + \mu_{W^-} = 0, \quad \mu_\nu + \mu_{\bar\nu} = 0$$
$$(\mu_{e^-} = \mu_e, \quad \mu_{W^-} = \mu_1, \quad \mu_\nu = \mu_2) \tag{29}$$

i.e., the chemical potentials of each particle-antiparticle pair are equal and opposite in sign. Under processes like f, taking $\delta N_{e^-} = \delta N_{\bar\nu} = -\delta N_{W^-}$, we get

$$\mu_{e^-} + \mu_{\bar\nu} = \mu_{W^-} \tag{30}$$

in a similar way we get

$$\mu_{e^+} + \mu_\nu = \mu_{W^+}, \qquad \mu_{h^-} = \mu_{W^-}. \tag{31}$$

The set of chemical equilibrium equations (29)–(31) are nothing but expressions of the general laws of electric and leptonic charge conservation. It is seen in particular that (27) is a consequence of (30) and (29), and it expresses the equilibrium under beta disintegration (and recombination) of W mesons.

By integrating (24) over the momenta we get finally

$$Z = N(\beta) \int \exp \left[- \int_0^\beta dx_4 \int d^3 x \mathscr{L}_{\text{eff}} \right] D W_\mu^i D B_\mu D \bar{\psi} D \psi D \bar{e}_R D e_R D\phi D\phi^+$$

$$\times \prod_{j=0}^{3} \delta(G^j) \operatorname{Det} \mathscr{M}, \tag{32}$$

where

$$\mathscr{L}_{\text{eff}} = \mathscr{L}\Big|_{\substack{W_4^i \to W_4^i + i(\mu_1/g)\delta^{i3} \\ B_4 \to B_4 + i\mu_1/g' \\ \partial_\nu \psi_L \to (\partial_\nu - \mu_2 \delta_{\nu 4})\psi_L \\ \partial_\nu e_R \to (\partial_\nu - \mu_2 \delta_{\nu 4}) e_R}} \tag{33}$$

If we replace $i\mu_1$ by μ_1', and $i\mu_2$ by μ_2' in $\mathscr{L}_{\text{eff}}$ with μ_1' and μ_2' real the generating functional (32) is formally equivalent to that of quantum field theory except for the following facts:

i) Space-time variables have been substituted by Euclidean ones where $0 \leqslant x_4 \leqslant \beta$. Fields four vectors are taken also as Euclidean by taking their four components as real.

ii) Lorentz invariance is replaced by covariance under a real orthogonal transformation.

iii) The W_μ^3 and B_μ fields are shifted by the addition of the background fields $(i\mu_1/g)\delta_{\mu 4}$ and $(i\mu_1/g')\delta_{\mu 4}$ respectively.

iv) Fermion operators have been substituted by their gauge transformed $\psi_L \to \psi_L \exp(i\mu_2' x_4)$, $e_R \to e_R \exp(i\mu_2' x_4)$. For the functional Z we may take as valid all the facts already established for quantum field theory (keeping in mind that in order to obtain good statistical quantities and equations of motion, the fourth components of the fields W_4, B_4 and μ_1', μ_2' must be continued to their imaginary values). In particular, we may accept then as a fact the gauge invariance of (32) under the new transformations for the vector fields[3]

$$W_\nu^3 + i(\mu_1/g)\delta_{\nu 4} \to W_\nu^3 + i(\mu_1/g)\delta_{\nu 4} + \partial_\nu \lambda^3 + ig(W_\nu^+ \lambda^- - W_\nu^- \lambda^+),$$

$$W_\nu^\mp \to W_\nu^\mp + (\partial_\nu \mp \mu_1 \delta_{\nu 4})\lambda^\mp \mp ig\lambda^3 W_\nu^\mp \pm ig\lambda^\mp W_\nu^3,$$

$$\lambda^\mp \to (\lambda^1 \pm i\lambda^2)/\sqrt{2},$$

$$B_\nu + i(\mu_1/g')\delta_{\nu 4} \to B_\mu + i(\mu_1/g')\delta_{\nu 4} + \partial_\mu \lambda^0. \tag{34}$$

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[3] We shall adopt the viewpoint in the following, that the background field is fixed and that the gauge transformations involve only the fields to which it is added. This means that in (34) and (35) W_ν^3, B_ν and σ (and not $(\mu_1/g)\delta_{\nu 4}$, $(\mu_1/g')\delta_{\nu 4}$ or ξ are transformed. (See De Witt [10]).

We also write the infinitesimal gauge transformations for the scalar fields $h^{\pm}$, h^3, $\xi+\sigma$

$$h^{\pm} \to h^{\pm} \pm \frac{i}{2}(g\lambda^3 + g'\lambda^0)h^{\pm} + \frac{g}{2}\lambda^{\pm}(\xi+\sigma\mp ih^3),$$

$$h^3 \to h^3 - i\frac{g}{2}(h^-\lambda^+ + h^+\lambda^-) + \tfrac{1}{2}(g\lambda^3 - g'\lambda^0)(\xi+\sigma),$$

$$\xi+\sigma \to \xi+\sigma - \frac{g}{2}(h^+\lambda^- + h^-\lambda^+) - \tfrac{1}{2}(g\lambda^3 - g'\lambda^0)h^3. \tag{35}$$

It is not difficult to see that the gauge transformations (34) and (35) commute with the abovementioned U(1) gauge transformation associated to lepton charge invariance.

We call the attention of the reader about the fact that although our calculations will be made in the context of quantum statistics (four components of momenta are discrete variables, as pointed before) we have in mind the natural extension of the theory derived from Z to the time dependent quantum kinetical case. In this case analytic continuation $k_4 \to i\omega$ of the four component of momentum must be made according to Fradkin prescriptions [11], and the condition of unitarity must be satisfied by the new time-dependent theory (see [2] for the case of quantum electrodynamics in a medium).

As the functional Z satisfies the requirements of gauge invariance (the fields ξ, μ are to be considered as background fields which satisfy the classical equations of motion), we may choose $h^{+,-,3} = 0$ among the set of gauge conditions, which is the analog of the unitary gauge in quantum field theory. This means that $h^{+,-,3}$ play the role of Goldstone bosons which may be excluded from the theory. Physical quantities like the thermodynamic potential Ω would then be obtained in terms of the spectra of real particles. Nevertheless, the resulting statistical theory would not appear to be renormalizable. For that reason, other gauges are preferable, and according to the gauge invariance of Z, must provide the same results, say, for Ω. We shall confirm it in the first order of perturbation theory in a special gauge which will be used in the next Section.

As a consequence of our reasoning the only permissible direction of the symmetry breakdown parameter is the one already chosen, i.e., along the 3rd isotopic axis. This may be easily seen by writing the classical equations of motion for the effective fields, from the Lagrangian (33). If we start by considering two possible isotopic directions for ξ, i.e. $\begin{pmatrix}\xi\cos\alpha\\ \xi\sin\alpha\end{pmatrix}$ it is easy to see that in our case $\alpha = \pi/2$, contrary to the case discussed by Kapusta [8], where an external compensating charge is considered to be present in the vacuum and consequently the Higgs condensate is charged too, $\alpha = 0$. Our present study is concerned with a neutral vacuum, where the electric charge is carried by the bosons and fermions excited in the medium. (The strict validity of our assumptions requires, nevertheless, that the mean charge of the system $\langle Q\rangle = -e\partial\Omega/\partial\mu_1$ be equal to zero). One of our motivations is in that considering the charged vacuum would require according to Ferrer and de la Incera [12], a drastic rearrangement of quantization and the change in the number of degrees of freedom.

It must be pointed out that the gauge conditions $G^i = 0$ in (32) depend explicitly on the chemical potential μ_1. Once they are fixed, we may use (34) and (35) to obtain Det $\mathscr{M}$. From this we may proceed to evaluate Z. In the next section we shall obtain Z in the one-loop approximation for the case of an external magnetic field. Before it, we shall write the term dependent on the SU(2) gauge field in $\mathscr{L}_{\text{eff}}$, in terms of the fields $W^{\pm}$.

We have

$$\mathscr{L}^{w}_{\text{eff}} = -\tfrac{1}{4}(F^1_{\mu\nu}F^2_{\mu\nu}+F^2_{\mu\nu}F^1_{\mu\nu}+G^3_{\mu\nu}G^3_{\mu\nu}), \tag{36}$$

where

$$F^1_{\mu\nu} = \partial^-_\mu W^+_\nu - \partial^-_\nu W^+_\mu + ig(W^+_\mu W^3_\nu - W^3_\mu W^+_\nu),$$

$$F^2_{\mu\nu} = \partial^+_\mu W^-_\nu - \partial^-_\nu W^-_\mu - ig(W^-_\mu W^3_\nu - W^3_\mu W^-_\nu),$$

$$\partial^\pm_\nu = \partial_\nu \pm \mu_1 \delta_{\nu 4},$$

$$G^3_{\mu\nu} = \partial_\mu W^3_\nu - \partial_\nu W^3_\mu - ig(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu). \tag{37}$$

3. *The case of a constant external magnetic field*

a) The Lagrangian

We shall specialize now in the case of a constant external magnetic field by taking

$$W^3_\mu = \frac{e}{g} A_\mu + w^3_\mu, \quad B_\mu = \frac{e}{g'} A_\mu + b_\mu, \tag{38}$$

where $A_\mu = Bx_1\delta_{\mu 2}$, B being the magnetic field intensity. The form (38) excludes the possibility of an external neutral quasimagnetic field Z_μ i.e., only the classical magnetic field would result. This means that our system is far from the condition of restoration of the broken symmetry (see Linde [5]).

The gauge transformations (34) may be written now in the form (the background field $(ieA_\nu - \mu_1\delta_{\nu 4})/g$ is considered as fixed, see footnote 2):

$$W^\mp_\nu \to W^\mp + (\partial_\nu \mp \mu_1 \delta_{\nu 4} \pm ieA_\nu)\lambda^\mp \pm ig\lambda^3 W^\mp_\nu \pm igw^3\lambda^\mp,$$

$$w^3_\nu \to w^3_\nu + \partial_\nu \lambda^3 + ig(W^+_\nu \lambda^- - W^-_\nu \lambda^+). \tag{39}$$

If the effective Lagrangian (33) is expanded up to terms of second order in the fields $W^\pm_\mu$, w^3_μ, we may eliminate the terms linear in these fields which appear in the expansion, once their coefficients are the classical equations of motion for the fields. We shall take then the terms linear in these fields in (37)

$$F^1_{\mu\nu} = (\partial^-_\mu + ieA_\mu)W^+_\nu - (\partial^-_\nu + ieA_\nu)W^+_\nu,$$

$$F^2_{\mu\nu} = (\partial^+_\mu - ieA_\mu)W^-_\nu - (\partial^-_\nu - ieA_\nu)W^-_\nu. \tag{40}$$

We also have

$$G^3_{\mu\nu} = \partial_\mu w^3_\nu - \partial_\nu w^3_\mu + ig(W^+_\mu W^-_\nu - W^-_\mu W^+_\nu) + G^0_{\mu\nu}, \tag{41}$$

where $G^0_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$; we shall use also

$$g^3_{\mu\nu} = \partial_\mu w^3_\nu - \partial_\nu w^3_\mu . \tag{42}$$

By calling $\nabla^\pm = \partial^\pm_\mu \mp ieA_\mu$ we get finally for the first term of the effective Lagrangian in the quadratic approximation

$$\mathscr{L}^W_{\text{eff}} = -\tfrac{1}{4}\frac{e^2}{g^2} G^0_{\mu\nu}G^0_{\mu\nu} + \tfrac{1}{2} W^+_\mu(\nabla^{(+)2}_\alpha \delta_{\mu\nu} - \nabla^+_\mu \nabla^+_\nu - 2ieG^0_{\mu\nu})W^-_\nu$$
$$+ \tfrac{1}{2} W^-_\mu(\nabla^{(-)2}_\alpha \delta_{\mu\nu} - \nabla^-_\mu \nabla^-_\nu + 2ieG^0_{\mu\nu})W^+_\nu - \tfrac{1}{2} g_{\mu\nu}g_{\mu\nu}. \tag{43}$$

In the same way we get for the second term of the Lagrangian in the same approximation

$$\mathscr{L}^B_{\text{eff}} = -\tfrac{1}{4}\frac{e^2}{g'^2} G^0_{\mu\nu}G^0_{\mu\nu} - \tfrac{1}{4} f_{\mu\nu}f_{\mu\nu}, \tag{44}$$

where $f_{\mu\nu} = \partial_\mu b_\nu - \partial_\nu b_\mu$. From (44) and the last term in (43) we get the term

$$-\tfrac{1}{4} g_{\mu\nu}g_{\mu\nu} - \tfrac{1}{4} f_{\mu\nu}f_{\mu\nu} = -\tfrac{1}{4} h_{\mu\nu}h_{\mu\nu} - \tfrac{1}{4} z_{\mu\nu}z_{\mu\nu} \tag{45}$$

here $h_{\mu\nu} = \partial_\mu a_\nu - \partial_\nu a_\mu$ is the linear part of the electromagnetic radiation field tensor and $Z_{\mu\nu} = \partial_\mu Z_\nu - \partial_\nu Z_\mu$ is the linear part of the neutral Z_μ field tensor, where

$$a_\mu = \frac{1}{\sqrt{g^2+g'^2}}(g' w^3_\mu + g b_\mu),$$

$$Z_\mu = \frac{1}{\sqrt{g^2+g'^2}}(-g' w^3_\mu + g' b_\mu). \tag{46}$$

The third and fourth terms in $\mathscr{L}_{\text{eff}}$ contribute with the three bilinear-in-the-fields terms

$$\mathscr{L}^f_{\text{eff}} = -\bar{\nu}_L \gamma_\nu(\partial_\nu - \mu_2\delta_{\nu 4})\nu_L - \bar{e}_L\gamma_\nu(\partial_\nu - \mu_e\delta_{\nu 4} + ieA_\nu)e_L$$
$$- \bar{e}_R\gamma_\nu(\partial_\nu - \mu_e\delta_{\nu 4} + ieA_\nu)e_R. \tag{47}$$

The fifth and seventh scalar terms can be approximated to the same order as

$$\mathscr{L}^\phi_{\text{eff}} = -[\nabla^+_\nu h^+ \nabla^-_\nu h^- - M_W(\nabla^+_\nu h^+)W^-_\nu - M_W(\nabla^-_\nu h^-)W^+_\nu + M^2_W W^+_\nu W^-_\nu$$
$$+ \tfrac{1}{2} m^2_\sigma \sigma^2 + \tfrac{1}{2}\partial_\nu\sigma\partial_\nu\sigma + \tfrac{1}{2}\partial_\nu h^3 \partial_\nu h^3 + M_Z Z_\nu \partial_\nu h^3 + \tfrac{1}{2} M^2_Z Z^2_\nu], \tag{48}$$

where $M_W = g\xi/2$, $m_\sigma = \sqrt{\lambda_2/2}\,\xi$, $M_Z = \tfrac{1}{2}\sqrt{g^2+g'^2}\,\xi$.

Due to the Higgs mechanism the fields $h^{+,-,3}$ are massless Goldstone bosons whereas the vector fields Z_μ, $W_\mu^\pm$ acquire mass. The scalar field also becomes massive, $m_\sigma = \sqrt{\lambda_2/2}\xi$. Finally, we have the contribution from the interaction Yukawa term

$$\mathscr{L}_{\text{eff}}^{\text{Y}} = -m(\bar{e}_{\text{L}}e_{\text{R}}+\bar{e}_{\text{R}}e_{\text{L}}) \tag{49}$$

which supplies the mass $m = \lambda_1\xi$ to left and right electrons.

b) The gauge conditions

We shall adopt the following set of gauge conditions, dependent on the chemical potential μ_1 and of the masses of the vector fields[4]

$$\begin{aligned} G^1 &= \nabla_\nu^+ W_\nu^+ - M_{\text{W}}h^+ = 0, \\ G^2 &= \nabla_\nu^- W_\nu^- - M_{\text{W}}h^- = 0, \\ G^3 &= \partial_\nu Z_\nu + M_{\text{Z}}h^3 = 0, \\ G^4 &= \partial_\nu a_\nu = 0. \end{aligned} \tag{50}$$

These gauge conditions may be incorporated to the Lagrangian in the form of the additive terms.

$$\begin{aligned} -(\nabla_\nu^- W_\nu^- - M_{\text{W}}h^-)(\nabla_\mu^+ W_\nu^+ - M_{\text{W}}h^+) - \tfrac{1}{2}(\partial_\nu Z_\nu + M_{\text{Z}}h^3)(\partial_\nu Z_\mu + M_{\text{Z}}h^3) \\ -\tfrac{1}{2}(\partial_\nu a_\nu)(\partial_\mu a_\mu). \end{aligned} \tag{51}$$

This may be accomplished by following the usual trick. The gauge conditions are substituted by $\delta(G^i - a^i(x))$ where $a^i(x)$ are a set of arbitrary functions. Then Z is multiplied by an adequate $a^i(x)$ dependent Gaussian functional and integration over these arbitrary functions is finally performed.

One may wonder to what extent the choice of the μ_1-dependent gauge conditions introduces, through $\delta(G^j)$ in (50) an additional dependence of Z on μ_1 (apart from the one coming from the term $-\mu_1 N_1$ in (24)), which fact would ban the use of the chemical potential for producing the average charge density through differentiation $\partial \ln Z/\partial\mu_1 = \langle N_Q\rangle$. The answer is negative and comes from the fact that Z does not depend on the choice of the gauge conditions (50) nor on the $a^i(x)$ dependent gaussian functional which is used to obtain the gauge conditions in the form (51). A change δG^i caused by a variation of μ_1 in Eq. (50) is equivalent to a gauge change in the field functional integration variables W_μ^i, B_μ, $\phi^\dagger$, ϕ which do not change the functional integral due to the gauge invariance of $\mathscr{L}_{\text{eff}}$ (cf. De Witt [10]).

After simple transformations it will be seen that the mixed scalar and vector terms in (48) as well as the mixed derivative terms $\nabla_\mu^\pm \nabla_\nu^\pm$ and $\partial_\mu\partial_\nu$ in (43) and (45) disappear due to the gauge terms (51). Such simplification means to play the price of giving back formally

[4] The first two are the analog of the ones used by Mc Keon in quantum field theory to obtain the radiative corrections to the neutrino propagator in a magnetic field [13].

some mass to $h^\pm$ and h^3, once we have now from (51) the terms

$$-M_W^2 h^+ h^- - \tfrac{1}{2} M_Z^2 (h^3)^2. \tag{52}$$

However, the contribution of these fictitious massive fields $h^\pm$ and h^3 will be completely cancelled by the Fadeev-Popov determinant. For the latter we have, in the zero field limit

$$\mathrm{Det}\,\mathcal{M} = \mathrm{Det}\begin{bmatrix} \nabla_\mu^{(+)2} - M_W^2 & 0 & 0 & 0 \\ 0 & \nabla_\mu^{(-)2} - M_W^2 & 0 & 0 \\ 0 & 0 & -\frac{g}{\sqrt{g^2+g'^2}}(\partial_\mu^2 - M_Z^2) & \frac{M_Z g'\xi}{2} \\ 0 & 0 & 0 & \partial_\mu^2 \end{bmatrix}$$

$$= \mathrm{Det}\,\frac{g}{\sqrt{g^2+g'^2}}\,[(\nabla_\mu^{(+)2} - M_W^2)(\nabla_\mu^{(-)2} - M_W^2)(\partial_\mu^2 - M_Z^2)\partial_\mu^2]. \tag{53}$$

As will be seen later, each of the terms in (53) cancel superfluous degrees of freedom in the functional Z. The first three contribute with a sign opposite to the one of the fictitious fields $h^\pm$, h^3 plus similar terms coming from extra degrees of freedom of each of the gauge fields $W_\mu^\pm$, Z_μ. The last term ∂_μ^2 cancels the contribution of two of the nonphysical degrees of freedom of the photon field term. It is very important to stress here that when the functional representation of Det $\mathcal{M}$ will be given the fourth components of momentum in (53) must be taken as summed over even values, $p_4 = 2\pi n/\beta$ (see [7] and [9]). This means that the propagators of fictitious particles (ghosts) would contain Bose and not Fermi distributions.

c) Expansion in terms of magnetic eigenfunctions

To obtain Ω it is required to perform the functional integration in Z. To that end we shall expand all the charged fields in terms of magnetic eigenfunctions

$$f^\pm_{n,p_2,p_3,p_4}(x_4, \vec{x}) = e^{\pm i(p_2x_2+p_3x_3+p_4x_4)}\varphi_n(\xi) \tag{54}$$

where $\varphi_n(\xi)$ are the Hermite functions multiplied by $(eB)^{1/4}$

$$\varphi_n(\xi) = [(eB)^{1/4}/\pi^{1/4}2^{n/2}(n!)^{1/2}]e^{-\xi^2/2}H_n(\xi) \tag{55}$$

and

$$\xi = \sqrt{eB}\,(\lambda_1 - p_2/eB), \quad n = 0, 1, 2\,\ldots$$

We have the orthogonality relations

$$\int f^+_{n',p_2',p_3',p_4'}(x_4, \vec{x}) f^-_{n,p_2,p_3,p_4}(x_4, \vec{x})\,dx_4 d^3x = (2\pi)^2\beta\delta_{p_4,p_4'}\delta(p_3-p_3')\,\delta(p_2-p_2')\delta_{n,n'} \tag{56}$$

Calling

$$\nabla_1 = \partial_1, \quad \nabla_2 = \partial_2 - ieBx_1, \quad \nabla_\alpha^{(\pm)2} = \nabla_1^2 + \nabla_2^2 + \partial_3^2 + (\partial_4 \pm \mu_1)^2$$

we have

$$(\nabla_2 - i\nabla_1) f^{\pm}_{n,p_2,p_3,p_4} = -i\sqrt{2eBn}\, f^{\pm}_{n-1,p_2,p_3,p_4}, \tag{57}$$

$$(\nabla_2 + i\nabla_1) f^{\pm}_{n,p_2,p_3,p_4} = -i\sqrt{2eB(n+1)}\, f^{\pm}_{n+1,p_2,p_3,p_4}, \tag{58}$$

$$\nabla^{(\pm)2}_{\alpha} f^{\pm}_{n,p_2,p_3,p_4} = [(p_4 \mp i\mu_1)^2 + p_3^2 + 2eBn] f^{\pm}_{n,p_2,p_3,p_4}. \tag{59}$$

Using now the expansion

$$W^{\pm}_{\mu}(x) = \frac{1}{(2\pi)^2\beta} \sum_{n} \sum_{p_4} \int dp_2 \int dp_3 w^{\pm}_{\mu}(n, p_2, p_3, p_4) f^{\pm}_{n,p_2,p_3,p_4}(x_4, \vec{x})$$

we shall evaluate the contribution of the fields $W^{\pm}$ to the partition functional. The set n, p_2, p_3, p_4 whenever possible in the following will be symbolized by q. We must bear in mind that $p_4 = 2\pi l/\beta$, l being any positive or negative integer.

We have

$$Z_W = \int DW^{+}_{\mu} DW^{-}_{\mu} \exp\left[-\int_0^{\beta} dx_4 \int d^3x \mathscr{L}_{\text{eff}}\right] = \int DW^{+}_{\mu} DW^{-}_{\mu}$$

$$\times \exp\left\{-\frac{1}{(2\pi)^4\beta^2}\left[\sum_{p_4'}\sum_{n'}\sum_{p_4}\sum_{n}\int_0^{\beta} dx_4 \int d^3x \int dp_2' dp_3' dp_2 dp_3 w^{-}_{\mu}(q') f^{-}_{q'}\right.\right.$$

$$\left.\left.\times\, [(\nabla^{(-)2}_{\alpha} - M_W^2)\delta_{\mu\nu} + 2ieG^0_{\mu\nu}] w^{+}_{\nu}(q) f^{+}_{q} + \text{h.c.}\right]\right\}$$

$$= \prod_n \int Dw^{+}_{n} Dw^{-}_{n} \exp\left\{-\frac{1}{2n^2\beta}\sum_{p_4}\int dp_2 \int dp_3 [w^{-}_{\mu}(q) A(q)_{\mu\nu} w^{+}_{\mu}(q)\right.$$

$$\left. + w^{+}_{\mu}(q) A^{+}(q)_{\mu\nu} w^{-}_{\nu}(q)]\right\} = \prod_n [\text{Det}\,(A_{\mu\nu}A^{+}_{\mu\nu})]^{-1/2}, \tag{60}$$

where

$$A_{\mu\nu} = [(p_4 + i\mu_1)^2 + p_3^2 + M_W^2 + 2eB(n+\tfrac{1}{2})]\delta_{\mu\nu} - 2ieG^0_{\mu\nu}.$$

d) The thermodynamic potential

We get easily from (60)

$$\Omega_W = -\frac{1}{\beta}\ln Z_W = -\frac{eB}{8\pi^2\beta^2}\sum_{p_4}\sum_{n}\int_{-\infty}^{\infty} dp_3[\ln \text{Det}\, A_n + \ln \text{Det}\, A_n^{+}]. \tag{61}$$

In (61) we have called A_n to the determinant of $A_{\mu\nu}$ with respect to μ, ν indices

$$A_n = [M_W^2+(p_4+i\mu_1)^2+p_3^2+2eB(n+\tfrac{1}{2})]^2$$

$$\times [M_W^2+(p_4+i\mu_1)^2+p_3^2+2eB(n-\tfrac{1}{2})]\,[M_W^2+(p_4+i\mu_1)^2+p_3^2+2eB(n+\tfrac{3}{2})]. \quad (62)$$

In the Appendix we shall see that the second and third brackets are related to the left and right handed circularly polarized eigenmodes of the $W_\mu^\pm$ propagator in the magnetic field, whereas one of the factors in the first bracket is related to the longitudinal eigenmode (the other factor is cancelled by Det $\mathscr{M}$). The zeros of Det $A_{\mu\nu}$ as a function of the variable $ip_4-\mu$ give the energy spectra corresponding to these states of polarization. The ground state occurs in the left-handed mode, for $n = 0$, giving the energy $\varepsilon_q^0 = \sqrt{p_3^2+M_W^2-eB}$. It corresponds to the case of spin opposite to the magnetic field. For $M_W < eB$ it would give a tachyonic unstable state [14]. Here such situation is avoided whenever $M_W \geqslant \sqrt{eB}$.

We must make an important remark concerning (61). The integral in p_2 is absent due to the fact that Ω is the thermodynamic potential per unit volume. The double integral in p_2, p_3 gives quantities per unit area, once we may consider that integral as the limit (if the system is enclosed in a box $L_1 \times L_2 \times L_3$):

$$\sum_{p_2}\sum_{p_3}(\ldots) = \frac{L_2L_3}{(2\pi)^2}\int dp_2\int dp_3(\ldots) \quad (63)$$

if we use $p_2 = eBx_0$, we have $-L_1/2 \leqslant x_0 \leqslant L_1/2$ and the quantity per unit volume is

$$\lim_{L_1,L_2,L_3\to\infty}\frac{1}{L_1L_2L_3}\sum_{p_2}\sum_{p_3}(\ldots) = \frac{eB}{(2\pi)^2}\int dp_3(\ldots). \quad (64)$$

But this is seen directly by the fact that as A_n is degenerate with respect to p_2 we must take

$$\ln \text{Det}\, Q = \text{Sp} \ln Q = \frac{eB}{(2\pi)^2\beta}\sum_n\sum_{p_4}\int dp_3 \ln Q, \quad (65)$$

where Q is any of the brackets in (62), i.e. the Sp operator acts only on the variables p_4, p_3, n.

The sum over p_4 may be carried out by the usual trick of differentiating at first with respect to charge, summing over p_4 the resulting expression, and then integrating over the charge. To obtain the final expression for Ω_W we must take into account the contributions from the terms $h^+(\nabla_\alpha^{(-)2}-M_W^2)h^-$ due to the Goldstone bosons and the first two terms in the brackets (53), to the total thermodynamics potential (remember that $h^{1,2}$ acquire formally a mass due to the gauge conditions). Their whole effect is to cancel the factor two in the exponent of the first bracket in (62), once $h^\pm$ contribute with the term

$$\ln[\text{Det}(\nabla_\alpha^{(-)2}-M_W^2)]^{-1/2}[\text{Det}(\nabla_\alpha^{(+)2}-M_W^2)]^{-1/2} \quad (66)$$

and Det $\mathscr{M}$ with

$$\ln\left[\mathrm{Det}\,(\nabla_\alpha^{(-)2}-M_\mathrm{W}^2)\right]\left[\mathrm{Det}\,(\nabla_\alpha^{(+)2}-M_\mathrm{W}^2)\right]. \tag{67}$$

We have finally for the charged vector boson sector of Ω_W[5]

$$\Omega_\mathrm{W} = \frac{eB}{4\pi^2\beta}\int_{-\infty}^{\infty} dp_3\left[\ln\left(1-e^{-(\varepsilon_{q0}-\mu_1)\beta}\right)\left(1-e^{-(\varepsilon_{q0}+\mu_1)\beta}\right)+\beta\varepsilon_q^0\right]$$

$$+\frac{eB}{4\pi^2\beta}\sum_n \beta_n\int_{-\infty}^{\infty} dp_3\left[\ln\left(1-e^{-(\varepsilon_q-\mu_1)\beta}\right)\left(1-e^{-(\varepsilon_q+\mu_1)\beta}\right)+\beta\varepsilon_q\right];$$

$$\beta_n = 3-\delta_{0n},\quad \varepsilon_q^0 = \sqrt{p_3^2+M_\mathrm{W}^2-eB},\quad \varepsilon_q = \sqrt{p_3^2+M_\mathrm{W}^2+2eB(n+\tfrac{1}{2})}. \tag{68}$$

The ε_q^0, ε_q terms added to the logarithms in the brackets in (68) account for the vacuum energy due to the external field. Later we shall give their separate expressions together with the analog term due to the electron-positron contribution.

By performing the integration over the fields Z_μ, A_ν we get the thermodynamic potential terms due to the neutral vector boson and the electromagnetic fields

$$\Omega_\mathrm{Z} = \frac{3}{(2n)^3\beta}\int\left[\ln\left(1-e^{-\varepsilon_\mathrm{Z}\beta}\right)+\beta\varepsilon_\mathrm{Z}\right]d^3p,\quad \varepsilon_\mathrm{Z} = \sqrt{p^2+M_\mathrm{Z}^2} \tag{69}$$

and

$$\Omega_\mathrm{A} = \frac{2}{(2\pi)^3\beta}\int\left[\ln\left(1-e^{-\omega\beta}\right)+\beta\omega\right]d^3p,\quad \omega = \sqrt{\vec{p}^{\,2}}. \tag{70}$$

It must be pointed out that the functional integrations are performed in (69) and (70) after the linear transformations (8) and (9) or (46) are made in (32). Its Jacobian is equal to unity. The extra degrees of freedom have been cancelled in the following way: for the Z_μ field, by a procedure similar to the $W^\pm$ fields, as a result of the combination of the third term in the bracket in (53) with the contribution of the Goldstone boson h^3, giving the term

$$\ln\mathrm{Det}\left[\partial_\mu^2-M_\mathrm{Z}^2\right]^{1/2}. \tag{71}$$

For the photon field, the last factor in the brackets (53) gives the term

$$\ln\mathrm{Det}\left[\partial_\mu^2\right]. \tag{72}$$

After the functional integration in (32), the expression (71) cancels one degree of freedom to the Z_μ field and (72) two degrees of freedom to the photon field.

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[5] Expressions close to (68) were obtained by Cabo, Kalashnikov and Shabad for a gluonic plasma in a magnetic field [15] and by Chakrabarti [18] in the $\mu_1 = 0$ case.

By using the expansion of the electron-positron field in terms of the magnetic eigenfunctions (54), and using the standard procedure for the functional integration of Fermi fields [16], we get

$$\Omega_{e_L} = -\frac{eB}{8\pi^2\beta}\sum_{n=1}^{\infty}\alpha_n\int_{-\infty}^{\infty}dp_3[\ln(1-e^{-(E_q-\mu_e)\beta})(1+e^{-(E_q+\mu_e)\beta})+\beta E_q] = \Omega_{e_R} = \tfrac{1}{2}\Omega_e, \tag{73}$$

$$\alpha_n = 2-\delta_{0n}, \quad E_q = \sqrt{p_3^2+m^2+2eBn}.$$

The neutrinos contribution is also easily obtained as

$$\Omega_\nu = -\frac{1}{(2\pi)^3\beta}\int d^3p[\ln(1+e^{-(\varepsilon_\nu-\mu_2)\beta})(1+e^{-(\varepsilon_\nu+\mu_2)\beta})+\beta\varepsilon_\nu]. \tag{74}$$

Finally, the Higgs scalar term is given by

$$\Omega_\sigma = \frac{1}{(2\pi)^3\beta}\int d^3p[\ln(1-e^{-\varepsilon_\sigma\beta})+\beta\varepsilon_\sigma], \tag{75}$$

$$\varepsilon_\sigma = \sqrt{p^2+m_\sigma^2}.$$

The total thermodynamic potential of our system in the one-loop approximation is according to (68), (69), (70) and (73)–(75)

$$\Omega = \Omega_W+\Omega_Z+\Omega_{e_L}+\Omega_{e_R}+\Omega_\nu+\Omega_A+\Omega_\sigma. \tag{76}$$

If the logarithms in (68) are expanded in series and use is made of the integral representations

$$\exp(-\beta\sqrt{s}) = \beta/2\int_0^\infty(\pi t^3)^{-1/2}\exp(-\beta^2/4t-st)dt, \tag{77}$$

$$a^{-1/2} = \pi^{-1/2}\int_0^\infty e^{-ay}y^{-1/2}dy, \tag{77a}$$

we may write the term Ω_W in another form (remember that we suppose $M_W^2 \geqslant eB$ so that no imaginary energies would be implied). We have

$$\Omega_W = \Omega_W^{st}+\Omega_W^0, \tag{78}$$

where

$$\Omega_W^{st} = -\frac{1}{8\pi^2}\sum_{l=1}^{\infty}\cosh\mu_1 l\beta\int_0^\infty e^{-M_W^2t-\frac{l^2\beta^2}{4t}}eBcsheBt(1+2\cosh 2eBt)\frac{dt}{t^2} \tag{78a}$$

is the pure statistical term, which vanishes for $\mu_{1,2} = \beta^{-1} = 0$, and

$$\Omega_W^0 = -\frac{1}{16\pi^2}\int_0^\infty e^{-Mw^2t}\left[eBcsheBt(1+2\cosh 2eBt) - \frac{3}{t} - \tfrac{7}{2}e^2B^2t\right]\frac{dt}{t^2} \tag{78b}$$

is the contribution of the charged vector bosons to the energy of the vacuum in the magnetic field. Expressions close to (78) were obtained by Magpantay, Mukku and Sayed [17] in the case of $\mu_1 = 0$ for the effective potential of the Georgi-Glashow SU(2) model in an external magnetic field.

For the electron-positron contribution

$$\Omega_e = \Omega_e^{st} + \Omega_e^0 \tag{79}$$

we may expand the logarithms of (73) in Taylor series in powers of the exponential only if $\mu_e < m$. In that case we get with the help of (77)

$$\Omega_e^{st} = \frac{1}{4\pi^2}\sum_{l=1}^{\infty}(-)^l \cosh l\mu_e\beta \int_0^\infty e^{-m^2t-\frac{l^2\beta^2}{4t}}(eB\coth eBt)\frac{dt}{t^2} \tag{79a}$$

which also vanishes for the limit $\mu_{1,2} = \beta^{-1} = 0$, and for the pure vacuum term we get

$$\Omega_e^0 = \frac{1}{8\pi^2}\int_0^\infty e^{-m^2t}\left(eB\coth eBt - \frac{1}{t} - \frac{e^2B^2}{3}t\right)\frac{dt}{t^2} \tag{79b}$$

which is the well-known Euler-Heisenberg contribution to vacuum energy.

We must point out here that expressions (78a, b) (as well as (79a, b)) are even with respect to the magnetic field.

4. *Thermodynamic properties*

a) Electric charge and leptonic number

By taking the derivative of (76) with respect to μ_2 we get the mean lepton number

$$\langle N_l\rangle = -\partial\Omega/\partial\mu_2 = N_e + N_\nu, \tag{80}$$

where

$$N_\nu = \frac{1}{(2\pi)^3}\int d^3p(n_\nu - n_{\bar\nu}), \quad n_{\nu,\bar\nu} = [1+\exp(\varepsilon_\nu \mp \mu_2)\beta]^{-1}, \tag{81}$$

$$N_e = \frac{eB}{4\pi^2}\int_{-\infty}^{\infty} dp_3 \sum_{n=0}^{\infty}\alpha_n(n_e - n_p), \quad n_{e,p} = [1+\exp(E_q \mp \mu_e)\beta]^{-1} \tag{82}$$

and from (73), $N_{e_L} = N_{e_R} = N_e/2$.

For the mean electric charge density $\langle Q\rangle = eN_Q$, we may differentiate (76) with respect to μ_1, bearing in mind that $\mu_e = \mu_1+\mu_2$, and according to our previous statement about the neutrality of the system we must put finally $\langle Q\rangle = 0$. We have

$$\langle Q\rangle = e\langle N_Q\rangle = -e\partial\Omega/\partial\mu_1 = e(N_e+N_W) = 0, \tag{83}$$

where

$$N_W = \frac{eB}{4\pi^2}\left[\int_{-\infty}^{\infty} dp_3(n_{0W^-}-n_{0W^+})+\sum_{n=0}^{\infty}\beta_n\int_{-\infty}^{\infty} dp_3(n_{W^-}-n_{W^+})\right] \tag{84}$$

and

$$n_{0W^\pm} = [\exp(\varepsilon_q^0\mp\mu_1)\beta-1]^{-1}, \quad n_{W^\pm} = [\exp(\varepsilon_q\pm\mu_1)\beta-1]^{-1}. \tag{85}$$

It is not difficult to see from (82) and (84) that the neutrality condition (83) requires that μ_1 and μ_e would have opposite signs. The condition $(M_W^2-eB)^{1/2}\geqslant|\mu_1|$ is required lest the mean occupation number of the magnetic ground state of $W^\pm$ mesons should become negative. The first term in (84) is the part of the total density N_W which may be attributed to particles two dimensionally "condensed" in the plane perpendicular to the magnetic field $\vec{B}$, once these particles, although free to move parallel to $\vec{B}$, lie in the magnetic ground state. Nevertheless, it must be stressed here that as distinct from the zero field case, no phase transition to a true condensate occurs at $T\neq 0$ in the presence of the magnetic field. At fixed N_W, if temperature is lowered we may get from (84) that μ_1 increases, (i.e. $\partial\mu_1/\partial T<0$) but for finite N_W and $T\neq 0$ the threshold value $(M_W^2-eB)^{1/2}$ is never reached by μ_1, and $n_{0W^\pm}$ remains finite (the first term in (84) becomes infinite for $\mu_1 = (M_W^2-eB)^{1/2}$. On the other hand, a finite fraction of the vector boson gas lies at any temperature in the magnetic ground state $n = 0$. This fraction increases continuously at fixed N_W when the temperature is decreased.

It is also easy to see (i.e., by calculating $\partial N_W/\partial N_1$) that at fixed $T=\beta^{-1}\neq 0$, N_W is an increasing function of μ_1, so that if the density N_W grows enough, $|\mu_1|$ may approach the threshold value $(M_W^2-eB)^{1/2}$ and the fraction of particles in the ground state $p_3 = 0, n = 0$ increases also, but an infinite density in that state would be achieved only if $N_W\to\infty$ or $T=0$.

We may think in the opposite situation when there is no magnetic field and some part of the gas is in a condensate. If the magnetic field is then switched on, according to our previous reasoning, thermodynamic equilibrium requires that the usual condensate would be destroyed (unless $T=0$), although some sort of two-dimensional condensate (in the plane of momentum space is orthogonal to $\vec{B}$) would remain, having the property that the fraction of the gas in that state will diminish continuously if temperature is raised (or will increase by lowering temperature).

The set (27), (80) and (83) is to be considered as a system of three equations with six parameters $\mu_e, \mu_1, \mu_2, \beta, B$ and $\langle N_l\rangle$. For given values of β, B and $\langle N_l\rangle$, we may obtain μ_e, μ_1, μ_2 in terms of them. If we consider that initially $\mu_1 = \mu_2 = \mu_e = \langle N_l\rangle = 0$, we would have an equal distribution of particles and antiparticles of each kind (hot electroweak vacuum in an external magnetic field). It is interesting to study what happens in that case if keeping fixed both the magnetic field and the temperature, the lepton density is allowed to be increased, say by adding neutrinos to the system, or equivalently, by making $\mu_2 > 0$ (it is not difficult to guess what happens in the case of addition of antineutrinos, where the lepton number decreases). The equilibrium conditions (27) would demand an immediate increase of the electron density, or equivalently, to make, $\mu_e > 0$ also. As $\mu_{\bar{\nu}} = -\mu_2$ is a negative quantity, the neutrality condition (83) demands for $\mu_1 = \mu_{W^-}$ a negative value, to permit an adequate excess of W^+ over W^- mesons. As will be shown in the next section, a net magnetization would appear also.

b) The limit $T \to 0$

The behavior of N_W in the limit $T \to 0$ may be discussed from (84). The second integral may be done arbitrarily large by an appropriate choice of the chemical potential. This term diverges in the infrared region $p \simeq 0$ when $\mu_1 = \sqrt{M_W^2 - eB}$ (this is not the case when the magnetic field is absent, as the $p^2 dp$ factor in the integrand cancels the divergency and keeps the expression for N_W finite). To pick up the divergency of (84) we expand the integrand of the first term (which is the leading term) near $p_3 = 0$

$$N_W \simeq \frac{eBT}{2\pi^2} \int_0^\infty \frac{dp_3}{\sqrt{p_3^2 + M_W^2 - eB} - \mu_1}$$

$$\simeq \frac{eBT}{2\pi^2 m'} \int_0^{p_0} \frac{dp_3}{\dfrac{p_3^2}{2m'} + 1 - \dfrac{\mu_1}{m'}} \simeq \frac{1}{2\pi} \frac{eBT}{\sqrt{2(1-\mu_1/m')}}, \tag{86}$$

where p_0 is some characteristic value and $m'^2 = M_W^3 - eB > 0$. From this, the chemical potential μ_1 may be expressed in terms of N_1 to be $\mu_1/m' = 1 - (1/8\pi^2)(eBT/N_W)^2$.

Now as the temperature decreases the most part of the particles fall to the lowest Landau level and are distributed there as

$$n_{0W^+}(p_3) = \frac{T}{\dfrac{p_3^2}{2m'} + \dfrac{m'}{8\pi^2}\left(\dfrac{eBT}{N_W}\right)^2} \underset{T\to 0}{\to} \frac{N_W}{eB}(2\pi)^2 \delta(p_3), \tag{87}$$

and

$$N_W = \frac{eB}{4\pi^2} \int_{-\infty}^{\infty} n_{0W^+}(p_3) dp_3. \tag{88}$$

We see that for $T = 0$ the condensate appears and it is described by the statistical distribution. The equilibrium equations require then that the condition

$$\mu_\nu = \mu_e + m' \tag{89}$$

will be satisfied. If (89) is expressed in terms of the density of neutrinos, it can be written as

$$(6\pi^2 N_\nu)^{1/3} = \mu_e + m'. \tag{90}$$

The critical neutrino density for condensation is then

$$N_\nu = \frac{1}{6\pi^2}(\mu_e + m')^3. \tag{91}$$

The reader may compare our results with those obtained by Linde [19], who studied the W-boson condensation in the $T = 0$, $B = 0$ case induced by a high lepton density. There is a parallelism between our and his method if we understand μ_1/g as the classical expectation value of the A_0 component of the electromagnetic field (the quantity which would correspond to his Z_0, the chemical potential for neutral charge, has been taken as zero in our present approximation). In Linde's work a net charge is included through the statistical average of the leptonic number at $T = 0$ and this fact results in the nonzero expectation value of the $W^\pm$ meson fields.

Without the charge being included explicitly into the classical equations of motion (either as an external charge or as the statistical average), no solution for the vacuum expectation value of a charged field might be obtained and thus the theory might be treated consistently only for temperatures (densities) above (below) the critical value, i.e. before the charged $W^\pm$ mesons start falling into the condensate [12]. For lower temperatures (higher densities) the Bose distribution is no longer able to include the whole charge of the system and one needs to be able to describe the excess of the charge by a vacuum expectation value $\langle W^\pm \rangle$. This is the reason why the charge must be included into the equations of motion as is the case in Linde's work [19].

The situation is completely different when the magnetic field is present. In this case the Bose distribution is able to hold as large a charge as is wanted. No problem appears to be cured by the introduction of $\langle W^\pm \rangle \neq 0$ because all the charge remains in a gaseous state above the condensate down to zero temperature. Consequently, we may and already did work without the charge in the equations of motion and without shifting to any charged vacuum $\langle W^\pm \rangle \neq 0$, and we have just seen that for $T = 0$, the condensate is inside the statistical average, not the vacuum average.

c) The magnetization

We shall write now the expression for the magnetization. We have

$$\langle \mathcal{M} \rangle = -\partial\Omega/\partial B = \mathcal{M}_{W^\pm} + \mathcal{M}_e \tag{92}$$

where $\mathcal{M}_W$ and $\mathcal{M}_e$ are the contributions from the charged mesons and electron-positron subsystems, respectively, and

$$\mathcal{M}_e = -\frac{\Omega_e}{B} - \frac{e}{4\pi^2}\sum_{n=0}^{\infty} \alpha_n \int_{-\infty}^{\infty} dp_3 \left[\frac{eBn}{\varepsilon_q}(n_e + n_p)\right], \tag{93}$$

$$\mathcal{M}_{W^\pm} = -\frac{\Omega_{W^\pm}}{B} - \frac{e}{4\pi^2}\cdot\sum_{n=0}^{\infty}\beta_n\int_{-\infty}^{\infty}dp_3\left[\frac{eB(n+\frac{1}{2})}{\varepsilon_q}(n_{W^-}+n_{W^+})\right]$$

$$+\frac{e^2B}{4\pi^2}\int_{-\infty}^{\infty}\frac{dp_3}{\varepsilon_q}(n_{0W^+}+n_{0W^-}). \tag{94}$$

Expressions (93) and (94) contain both the paramagnetic and diamagnetic effects, which contribute with opposite signs to the total magnetization.

For lepton densities high enough (or temperatures low enough) that $\mu_2\beta \gg 1$, $\mu_e\beta \gg 1$, the fermion sector may be considered as degenerate, and in such case Ω_e takes the form [20], [1]

$$\Omega_e = -\frac{eB}{2\pi^2}\sum_{n=0}^{n_\mu}\alpha_n\left[\mu_e\sqrt{\mu_e^2-m^2-2eBn}-(m^2+2eBn)\ln\frac{\mu_e+\sqrt{\mu_e^2-m^2-2eBn}}{\sqrt{m^2+2eBn}}\right] \tag{95}$$

where n_μ is an integer given by

$$n_\mu = I\left(\frac{\mu_e^2-m^2}{2eB}\right). \tag{96}$$

For magnetic fields strong enough for the condition

$$2eB > \mu_e^2-m^2 \tag{97}$$

to be fulfilled, we would have all the electron subsystem in the magnetic ground state $n = 0$ and it would behave as paramagnetic

$$\mathcal{M}_e = \frac{e}{2\pi^2}\left(\mu_e\sqrt{\mu_e^2-m^2}-m^2\ln\frac{\mu_e+\sqrt{\mu_e^2-m^2}}{m}\right) > 0. \tag{98}$$

The chemical potential may be related to the electron density as [20]

$$\mu_e = \sqrt{\left(\frac{2\pi^2N_e}{eB}\right)^2+m^2} \tag{99}$$

and we get

$$\mathcal{M}_e = \frac{e}{2\pi^2}\left\{\left(\frac{2\pi^2N_e}{eB}\right)\left[\left(\frac{2\pi^2N_e}{eB}\right)^2+m^2\right]^{1/2} - m^2\ln\left[\frac{2\pi^2N_e}{eBm}+\frac{1}{m}\left(\left(\frac{2\pi^2N_e}{eB}\right)^2+m^2\right)^{1/2}\right]\right\}. \tag{100}$$

In what concerns to (94), it is easy to see that the first and third terms are positive and the second negative. The third term is the magnetic ground-state contribution which for the conditions discussed previously for the $T \to 0$ limit, takes the form

$$\frac{e^2 B}{4\pi^2} \int_{-\infty}^{\infty} \frac{dp_3}{\varepsilon_q} (n_{0W^+} + n_{0W^-}) \simeq \frac{e}{\sqrt{M_W^2 - eB}} N_W \tag{101}$$

i.e., for large densities and $T \to 0$, as all the W^+ system falls in the lowest Landau level $n = 0$, all the spins tend to become aligned along $\vec{B}$. As $\vec{B}$ grows, this magnetization increases, and by comparing (101) and (100) we may get that if the condition

$$e^2 B^2 > 2\pi^2 N_W \sqrt{M_W^2 - eB} \tag{102}$$

is satisfied, then (101) prevails over (100), i.e., the main contribution to the magnetization comes from the W^+ Bose condensate.

We conclude that for strong magnetic fields and high lepton densities (so that (97), (100) and (102) are satisfied) an increase in the neutrino density may provoke an increase of the net magnetization, and consequently, an increase in the effective magnetic field intensity. We must point out also that as eB grows, $m' = \sqrt{M_W^2 - eB}$ diminishes, and the critical neutrino density for the W^+ condensation (91) will be also diminished. (The case of instability $eB > M_W^2$, as stated before, is outside the range of our present work).

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APPENDIX

The propagators for charged particles

From (60) we may easily get for the W^- propagator the expression

$$\langle W^+(x) W^-(x') \rangle = \left\langle x \left| \frac{1}{(\nabla_\alpha^{(-)2} - M_W^2)\delta_{\mu\nu} + 2ieG_{\mu\nu}^2} \right| x' \right\rangle$$

$$= \frac{1}{2(2\pi)^2 \beta} \sum_n \sum_{p_4} \int dp_2 \int dp_3 [(R^- + R^+)\psi_{\mu\nu}^{(1)} + R^+ \psi_{\mu\nu}^{(2)} + i(R^- - R^+)\psi_{\mu\nu}^{(3)}]$$

$$\times \varphi_n(\xi)\varphi_n(\xi') \exp[ip_4(x_4 - x_4') + ip_3(x_3 - x_3') + ip_2(x_2 - x_2')]$$

$$= \langle W^-(x) W^+(x) \rangle, \tag{a}$$

where

$$R^\pm = [M_W^2 + (p_4 - i\mu_1)^2 + p_3^2 + 2eB(n \pm \tfrac{1}{2})]^{-1}$$

and $\psi_{\mu\nu}^{(1)} = B^{-2} G_{\mu\nu}^{02}$, $\psi_{\mu\nu}^{(2)} = \delta_{\mu\nu} - B^{-2} G_{\mu\nu}^{02}$ are symmetric tensors whereas $\psi_{\mu\nu}^{(3)} = B^{-1} G_{\mu\nu}^{0}$ is antisymmetric.

(a) can be easily diagonalized and we obtain the eigenvalues

$$\mathscr{K}^i = \frac{1}{2(2\pi)^2}\sum_n \sum_{p_4} \int dp_2 \int dp_3 R^i, \quad i = 1, 2, 3, 4 \tag{b}$$

where

$$R^1 = [M_W^2+(p_4-i\mu)^2+p_3^2+2eB(n+\tfrac{3}{2})]^{-1},$$
$$R^2 = R^-; \quad R^{3,4} = R^+.$$

Their eigenvectors are

$$a_\mu^{1,2} = e_\mu' \pm ie_\mu^2, \quad a_\mu^3 = e_\mu^3, \quad a_\mu^4 = e_\mu^4 \tag{c}$$

$a_\mu^{1,2}$ are respectively right and left-handed circularly polarized unit vectors ($e^{1,2,3,4}$ are unit vectors along the coordinate axes). We see that the eigenvalues related to the modes 1, 2 (respectively $\mathscr{K}^1$, $\mathscr{K}^2$) contain the quantities R^1, R^2. In particular, for $n = 0$, one of the poles of R^2 give the ground state energy $\varepsilon_q^0 = \sqrt{M_W^2+p_3^2-eB}$ to which corresponds a W⁻ particle with the spin opposite to the external field B. These conclusions become reversed for the charge-conjugated propagator $\langle W^-(x)W^+(x')\rangle$. The longitudinal and time-like eigenvalues are respectively $\mathscr{K}^{3,4}$. In a similar way we may obtain for the electron--positron propagator

$$\left\langle x \left| \frac{1}{\gamma_\mu \nabla_\mu - m} \right| x \right\rangle = -\frac{1}{2\pi^2\beta}\sum_n \sum_{p_4} \int_{-\infty}^{\infty} dp_2 \int_{-\infty}^{\infty} dp_3[(p_4+i\mu_e)^2$$
$$+E_q^2]^{-1}M(p_4, q)\exp[ip_2(x_2-x_2')+ip_3(x_3-x_3')+ip_4(x_4-x_4')], \tag{d}$$

where

$$M(p_4, q) = \begin{bmatrix} H_{n-1,n-1}(-ip_4+\mu_e) & 0 & -D_{n-1,n-1} & E_{n-1,n} \\ 0 & H_{n,n}(-ip_4+\mu_e) & E_{n,n-1} & D_{n,n} \\ D_{n-1,n-1} & E_{n-1,n} & H_{n-1,n-1}(ip_4-\mu_e) & 0 \\ -E_{n,n-1} & -D_{n,n} & 0 & H_{n,n}(ip_4-\mu_e) \end{bmatrix}$$

and

$$H_{KK'}(ip_4-\mu_e) = (m+ip_4-\mu_e)\varphi_K(\xi)\varphi_{K'}(\xi'),$$
$$D_{KK'} = p_3\varphi_K(\xi)\varphi_{K'}(\xi'), \quad E_{KK'} = i(2eBn)^{1/2}\varphi_K(\xi)\varphi_{K'}(\xi').$$

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