THE ON-SHELL RENORMALIZATION SCHEME IN THE GWS-MODEL*

By J. Fleischer

Fakultät für Physik, Universität Bielefeld**

(*Received February 3, 1986*)

A review of the on-shell renormalization scheme is given. We stress the analogy of this renormalization procedure with the "usual" one used in QED and discuss the differences. The calculation of some important parameters in the GWS model is sketched.

PACS numbers: 12.15.-y

1. *Introduction*

In the calculation of radiative corrections within the framework of the Glashow-Weinberg-Salam (GWS) model of electroweak interactions different renormalization schemes have been adopted by various authors [1–8]. The on-shell renormalization scheme (ORS) has been studied in detail [2, 5, 6] and has been employed for the calculation of specific processes by the above authors.

The ORS appears as quite "natural" if one remembers the renormalization of quantum electrodynamics: the counterterms are usually defined by performing subtractions *on the mass-shell* of the physical particles (electron and photon) and the wave function renormalization factors are defined as the residues of the propagators of the *physical particles.* In the GWS-model the same procedure is applicable, except that due to the complicated mass-coupling relations additional counterterms have to be introduced. As a consequence of S-matrix elements being gauge invariant, in general, all counterterms can be fixed by on-shell conditions (therefore ORS) and are thus gauge invariant. The latter is not true, of course, for the wave function renormalization factors, which are defined due to the LSZ-formalism as the residues of the poles of the physical (external) particles [9, 10] and are thus *derivatives* of 2-point functions taken on the mass-shell.

---

* Report on work in collaboration with F. Jegerlehner, presented at the IXth Silesian School of Theoretical Physics, Szczyrk, September 1985.

** Address: Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld 1, Federal Republic of Germany.

Two further comments should be added:

First, occasionally one finds in the literature comments on renormalization schemes "with" or "without" wave function renormalization [2], only S-matrix elements being finite in the latter case. Such statements are in disagreement with the proof of renormalizability given by 't Hooft and Veltman [9], who explicitly state that in order to make the S-matrix both gauge invariant and unitary, wave function renormalization has to be applied and the renormalization factors (Z's) have to be obtained according to the LSZ-formalism for the physical particles, which are clearly not the ones which respect the original symmetry.

Secondly, since it is assumed that all subtractions are performed on-shell — also for the intermediate vector bosons — it is silently assumed that all particle masses are experimentally known. In this sense this scheme is a "high energy scheme" and the relation to the low energy parameters requires some extra consideration.

Since the original works are partially quite involved and mainly concerned with applications, it seems worth summarizing the main ideas, technicalities and formulae. We also find it useful to stress the analogy with QED as far as possible. Thus we study at first in Sect. 2 the renormalization of the electric charge, in Sect. 3 mass- and field-renormalization and in Sect. 4 the inclusion of further counterterms. Finally, in Sect. 5 the relation to other parametrizations is discussed.

2. *Renormalization of the electric charge*

As one parameter in the GWS-model we use the electric charge e. Therefore we firs investigate the electromagnetic vertex (Fig. 1)

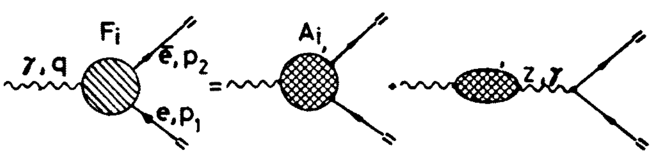
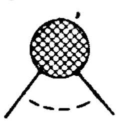


Fig. 1. The irreducible bare γee-vertex function. A_i are the form factors from pure vertex contributions ( = nontrivial one-particle irreducible diagrams) while the F_i's also include the contributions from the self-energies. The equals sign at the end of a line means that this line is on-shell

In QED charge renormalization is performed according to (e_0 = naked, e_r = renormalized charge)

$$e_0 = \frac{Z_1}{Z_2\sqrt{Z_3}}\, e_r, \tag{1}$$

where $Z_2 \equiv Z_e$, $Z_3 \equiv Z_\gamma$ and Z_1 is defined by the bare vertex function ($p = p_2 + p_1$, $q = p_2 - p_1$):

$$\Gamma^\mu_{\gamma ee} = i\Pi^\mu_{\gamma ee},$$

$$\Pi^{\mu}_{\gamma ee} = -e\left(\gamma^{\mu}F^{\gamma ee}_{10} + \gamma^{\mu}\gamma_5 F^{\gamma ee}_{20} + \frac{p^{\mu}}{2m_e}F^{\gamma ee}_{30} + \frac{q^{\mu}}{2m_e}\gamma_5 F^{\gamma ee}_{40}\right),$$

$$Z_1^{-1} - 1 = -eF_{10,\mathrm{OS}} \quad (\mathrm{OS} = \text{on-shell}).$$

Due to the Ward identity

$$q_{\mu}\Pi^{\mu}_{\gamma ee}(p_2, p_1) = -[\Sigma_e(p_2) - \Sigma_e(p_1)](-e), \tag{2}$$

or in differential form (differentiation with respect to p_2 and $q = 0$):

$$\Pi^{\mu}_{\gamma ee}(p, p) = -\frac{\partial}{\partial p_{\mu}}\Sigma_e(p)(-e) \tag{3}$$

one obtains

$$Z_1^{\mathrm{QED}} = Z_2^{\mathrm{QED}} \tag{4}$$

to all orders of perturbation theory: charge is renormalized through vacuum polarization only, yielding a "shift" of the electric charge

$$\delta e = e_0 - e_r = -\tfrac{1}{2}(Z_{\gamma} - 1). \tag{5}$$

How is the situation now in the GWS-model? We must work in a renormalizable gauge, i.e. have to include Faddeev-Popov ghost fields. These also enter the electromagnetic Ward-Takahashi identity (Fig. 2)

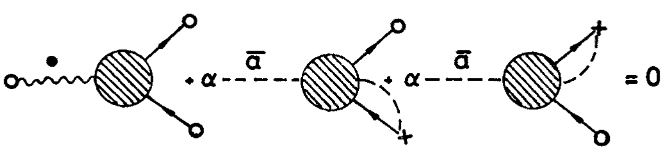


Fig. 2. The electromagnetic Ward identity in the GWS-model. α is the gauge parameter in the 't Hooft gauge (see also [6]). The ● represents differentiation in configuration space. The ○ at the end of a line means that this line is not amputated

where a is the FP-ghost corresponding to the photon field A. x stands for a complicated mixing of ghost fields (including ξ and $\eta^{\pm}$ corresponding to Z and $W^{\pm}$) and fermion fields e^- and ν_e — as obtained from the BRS-transformation.

Amputating the full electron lines in the Slavnov-Taylor (ST)-identity (Fig. 2) yields a relation of similar structure as Eq. (2), more complicated, however, due to the appearance of the ghost loops in the second and third contribution. Thus we cannot obtain Eq. (3) and we now have

$$Z_1^{\mathrm{GWS}} \neq Z_2^{\mathrm{GWS}}. \tag{6}$$

Therefore we rather take the point of view of introducing a shift or "counterterm" for the electric charge like in (5), i.e.

$$\Gamma^{\mu}_{\gamma ee} = i[-(e+\delta e)\gamma^{\mu} + \Pi^{\mu}_{\gamma ee}].$$

The renormalized vertex then reads:

$$\Gamma^{\mu}_{\gamma ee,r} = \sqrt{Z_\gamma}\,\gamma_0\sqrt{Z_e}\,\gamma_0\Gamma^{\mu}_{\gamma ee}\sqrt{Z_e}$$
$$= i\left\{-e\gamma^\mu\left[1+\frac{\delta e}{e}+\tfrac{1}{2}(Z_\gamma-1)+(Z_e-1)\right]+\Pi^{\mu}_{\gamma ee}\right\}.$$

Since the fermion singlets and the doublets are renormalized independently we have

$$Z_f = Z_R\frac{1+\gamma_5}{2}+Z_L\frac{1-\gamma_5}{2} := 1+z_a+z_b\gamma_5.$$

δe is finally fixed by the electron charge form factor ($F_{30} = A_{30}$, see also Fig. 1):

$$F_v = -e\left[1+F_{10}+F_{30}+\tfrac{1}{2}(Z_\gamma-1)+z_a+\frac{\delta e}{e}\right] \equiv -e[1+F_{1r}+A_{30}], \tag{7}$$

namely

$$\lim_{p_1^2,p_2^2\to m_e^2} F_V(p_1^2, p_2^2, q^2=0) = -e,$$

from which we obtain

$$\delta e = -e\{F_{10}+F_{30}+\tfrac{1}{2}(Z_\gamma-1)+z_a\}_{\mathrm{OS}}.$$

This is in fact the same as (1) only that we cannot make use of $Z_1 = Z_2$. We obtain (for details see below) in the dimensional regularization scheme:

$$\delta e = e\frac{1}{16\pi^2 v^2}\{14\sin^2\theta_W[\tfrac{19}{21}M_W^2+A_0(M_W)]-\tfrac{2}{3}(ev)^2\sum_{fs}q_f^2[1+m_f^{-2}A_0(m_f)], \tag{8}$$

with

$$= \frac{1}{(2\pi)^d}\int d^dk\,\frac{1}{k^2-m^2+i0} = \frac{-i}{16\pi^2}A_0(m),$$

where

$$A_0(m) = m^2(\mathrm{Reg}+1-\ln m^2) \quad \text{and}$$

$$\mathrm{Reg} = \frac{2}{\varepsilon}+\ln\mu^2-\gamma+\ln 4\pi \quad (\varepsilon = 4-d \to +0),$$

the sum running over single fermions. We see that δe is gauge invariant and flavour independent (charge universality).

3. Mass- and field-renormalization

As next we choose the physical particle masses (m_f, M_W, M_Z and M_H) as "natural" parameters in the GWS-model. As usual in QED, M^2 and m are the poles of the propagators: we generate counterterms by the shifts

$$M_0^2 = M_r^2+\delta M^2, \quad m_0 = m_r+\delta m.$$

From the bare negative inverse (scalar) propagator (e.g. $\langle THH \rangle$)

$$\Gamma_s^{(2)} = i[p^2 - M_{sr}^2 - \delta M_s^2 + \Pi_s(p^2)]$$

we thus obtain

$$\delta M_s^2 = \Pi_s(M_{sr}^2)$$

and correspondingly for the vector meson and fermion propagators.

The *mass counterterms* are defined for the naked fields (without Z's) and are *gauge invariant* for the following reasons:

(*i*) M_0^2 and M_r^2 are gauge invariant, and so is the difference.

(*ii*) $\Pi(M_r^2)$ is the on-shell two-point function which is gauge invariant.

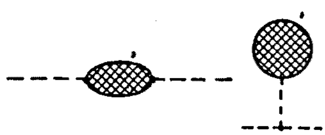
An additive renormalization of v,

$$v_0 = v + \delta v_t$$

is necessary for the physical Higgs field to satisfy the gauge-invariant condition.

$$\langle H \rangle = 0.$$

This condition fixes δv_t. It has been proved [6] that taking into account the proper value of v, this condition amounts to an inclusion of the appropriate *tadpole terms* in the amplitude, e.g.

$i\Pi_H =$ 

Gauge invariance of the mass-counterterms is achieved in fact only after the inclusion of these tadpole terms, a property which has also been verified explicitly in Ref. [6].

Introducing multiplicative renormalizations for the physical fields, we write

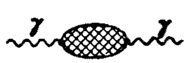
$$A_0^\mu = \sqrt{Z_\gamma}\, A^\mu,\; W_0^\mu = \sqrt{Z}\, W^\mu,\; Z_0^\mu = \sqrt{Z_0}\, Z^\mu,$$

$$H_0 = \sqrt{Z_H}\, H \quad \text{and} \quad \psi_{0f} = \sqrt{Z_f}\, \psi_f.$$

The Z-factors are determined as residues of the corresponding propagator poles, e.g.

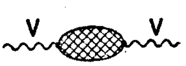
$$Z_s = \left[1 + \frac{\partial \Pi_s(M_{sr}^2)}{\partial p^2}\right]^{-1} \quad \text{or} \quad Z_s - 1 = -\frac{\partial \Pi_s(M_{sr}^2)}{\partial p^2}.$$

Similarly with the definition



$= i(ev)^2\,(g^{\mu\nu} A_{10}^{\gamma\gamma}(q^2) + q^\mu q^\nu A_{20}^{\gamma\gamma}(q^2))$

and



$= iM_W^2(g^{\mu\nu} A_{10}^{VV}(q^2) + q^\mu q^\nu A_{20}^{VV}(q^2))$

we have for the photon

$$Z_\gamma - 1 = (ev)^2 \frac{dA_{10}^{\gamma\gamma}}{dq^2}(0)$$

and for the vector bosons

$$Z_v - 1 = M_v^2 \frac{dA^{vv}}{dq^2}(M_v^2).$$

Special attention requires the γ-Z mixing propagator $\langle TA^\mu Z^\nu \rangle$ ("mixagator")

$$= i(ev)M_Z(g^{\mu\nu}A_{10}^{\gamma Z} + q^\mu q^\nu A_{20}^{\gamma Z})$$

Owing to mixing effects the proper fields A and Z appear rotated relatively to the bare fields A_0 and Z_0:

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \cos\vartheta & -\sin\vartheta \\ \sin\vartheta & \cos\vartheta \end{pmatrix} \begin{pmatrix} A_0 \\ Z_0 \end{pmatrix}.$$

Perturbatively this can be seen as follows [2, 6]: the bare γ-Z Lagrangian

$$L_{0\gamma Z} = -\tfrac{1}{4}(B_{\mu\nu}B^{\mu\nu} + W_{3\mu\nu}W_3^{\mu\nu}) + \tfrac{1}{8}v^2(g'B - gW_3)^2$$

is diagonalized by

$$\begin{pmatrix} B \\ W_3 \end{pmatrix} = \frac{1}{\sqrt{g'^2+g^2}} \begin{pmatrix} g & g' \\ g' & -g \end{pmatrix} \begin{pmatrix} A \\ Z \end{pmatrix},$$

yielding

$$L_{0\gamma Z} = -\tfrac{1}{2}(A_{\mu\nu}A^{\mu\nu} + Z_{\mu\nu}Z^{\mu\nu}) + \tfrac{1}{2}M_Z^2 Z^2.$$

We denote by g_0' and g_0 the couplings and by (A_0, Z_0) the fields to lowest order. With the next order couplings: $g' = g_0' + \delta g'$, $g = g_0 + \delta g$ we obtain

$$g'B - gW_3 = \sqrt{g'^2+g^2}\, Z$$

$$= g_0'B - g_0W_3 + \delta g' \frac{1}{\sqrt{g_0'^2+g_0^2}}(g_0A_0 + g_0'Z_0) - \delta g \frac{1}{\sqrt{g_0'^2+g_0^2}}(g_0'A_0 - g_0Z_0)$$

$$= \sqrt{g_0'^2+g_0^2}\,[(1+c)Z_0 + bA_0], \tag{9}$$

$$\text{where } b = \frac{g_0\delta g' - g_0'\delta g}{g_0'^2+g_0^2} \text{ and } c = \frac{g_0'\delta g' + g_0\delta g}{g_0'^2+g_0^2}.$$

Finally, the fields which diagonalize $L_{0\gamma Z}$ in second order are related to (A_0, Z_0) by the orthogonal transformation

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} 1 & -b \\ b & 1 \end{pmatrix} \begin{pmatrix} A_0 \\ Z_0 \end{pmatrix} + 0(\delta^2),$$

$b = \sin\vartheta$ in this approximation.

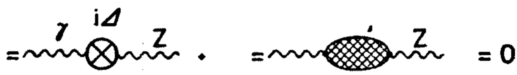
According to (9) the mass term has the form

$$L_{\text{mass},\gamma Z} = \tfrac{1}{2}\left[(M_{0Z}^2+\delta M_Z^2)Z_0^2+2\Delta A_0 Z_0\right]+0(\delta^2),$$

which shows that the photon mass remains zero and the counterterms are

$$\delta M_Z^2 = 2cM_{0Z}^2 \quad \text{and} \quad \Delta = bM_{Z0}^2.$$

The latter can be fixed on the photon mass-shell:



yielding

$$bM_Z^2 = -(ev)M_Z A_{10}^{\gamma Z}(0).$$

This counterterm is gauge dependent. Calculated in the U-gauge (see also [11]), however, we have

$$A_{10}^{\gamma Z} = 0$$

such that no mixing mass counterterm is present.

Due to the mixing effects, the field renormalization must be of the form

$$\begin{pmatrix} A \\ Z \end{pmatrix} = \begin{pmatrix} \sqrt{Z_\gamma} & X \\ 0 & \sqrt{Z_0} \end{pmatrix} \begin{pmatrix} A_r \\ Z_r \end{pmatrix}$$

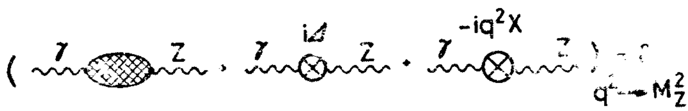
such that the photon mass remains zero:

$$0 \cdot A^2 + M_Z^2 Z^2 = M_Z^2 Z_0 Z_r^2$$

and

$$\overset{0}{F}_{\mu\nu}\overset{0}{F}{}^{\mu\nu}+\overset{0}{Z}_{\mu\nu}\overset{0}{Z}{}^{\mu\nu} = Z_\gamma F_{\mu\nu r}F_r^{\mu\nu}+2XF_{\mu\nu r}Z_r^{\mu\nu}+Z_0 Z_{\mu\nu r}Z_r^{\mu\nu}+ \text{higher order terms}, \tag{10}$$

where the index ^{0} means that only bilinear terms are taken into account. X appears now as further counterterm and to one-loop order can be fixed by the condition



yielding for the wavefunction renormalization of the mixagator

$$X = \frac{ev}{M_Z}\left(A_{10}^{\gamma Z}(M_Z^2)-A_{10}^{\gamma Z}(0)\right).$$

Thus the mixagator is finally diagonalized on the photon- and the Z-mass shell.

Due to the mixing effects, we now write the γ-Z propagator as a 2×2 symmetric matrix $\hat{G}$. In the 't Hooft gauge the free propagator $\hat{G}_0$ is diagonal and the negative inverse propagator $\hat{\Gamma}$ has finally the form

$$\hat{\Gamma} = -\hat{G}^{-1} = -\hat{G}_0^{-1} - i\hat{\Pi} = \begin{pmatrix} -G_{0\gamma}^{-1} - i\Pi_{\gamma\gamma} & -i\Pi_{\gamma Z} \\ -i\Pi_{\gamma Z} & -G_{0Z}^{-1} - i\Pi_{ZZ} \end{pmatrix}.$$

Restricting ourselves to the transversal part, we obtain

$$\hat{G}_1 = \frac{1}{\Gamma_{1\gamma\gamma}\Gamma_{1ZZ} - (\Gamma_{\gamma Z})^2} \begin{pmatrix} -\Gamma_{1ZZ} & \Gamma_{1\gamma Z} \\ \Gamma_{1\gamma Z} & -\Gamma_{1\gamma\gamma} \end{pmatrix}$$

and one easily sees, that $G_{1\gamma\gamma}$ and G_{1ZZ} have the proper poles at $p^2 = 0$ and $p^2 = M_Z^2$, while $G_{1\gamma Z}$ has no pole.

Given now the "renormalized" photon self-energy and mixagator:

$$A_{1r}^{\gamma\gamma} = A_{10}^{\gamma\gamma} - q^2 \frac{dA_{10}^{\gamma\gamma}}{dq^2}(0) \quad \text{and}$$

$$A_{1r}^{\gamma Z} = A_{10}^{\gamma Z} - A_{10}^{\gamma Z}(0) - \frac{q^2}{M_Z^2}(A_{10}^{\gamma Z}(M_Z^2) - A_{10}^{\gamma Z}(0)),$$

the electron charge form factor (7) can be written in terms of renormalized quantities as ($A_{1r}^{\gamma\gamma}(0) = A_{1r}^{\gamma Z}(0) = 0$):

$$F_v = -e(F_1 + F_3),$$

with

$$F_1 = 1 + A_{1r}^{\gamma ee} - 2a \frac{M_Z^2}{q^2 - M_Z^2} A_{1r}^{\gamma Z} - (ev)^2 \frac{1}{q^2} A_{1r}^{\gamma\gamma},$$

where

$$A_{1r}^{\gamma ee} = A_{10}^{\gamma ee} + z_a + \tfrac{1}{2}(Z_\gamma - 1) + \frac{\delta e}{e} + 2aA_{10}^{\gamma Z}(0)$$

is the renormalized γee vertex, and $F_3 = F_{30} = A_{30}$. For $q^2 = 0$ this determines δe as given by (8).

As next we consider the relation of the above scheme to the "symmetric field renormalization". The on-shell wave function renormalizations are *not* compatible with the canonical (bare) form of the ST-identities. The latter would require to choose them according to the unbroken theory which means that not the individual particle fields but the "field multiplets" B_μ and $W_{\mu a}$ become multiplicatively renormalized with as maller number of independent counterterms. The ultraviolet-singular terms of the Z-factors in the different schemes must coincide, however, since the symmetry is broken in such a way that in the high energy limit the theory is $SU(2)\times U(1)$ symmetric.

The kinetic term in the original form reads

$$-\tfrac{1}{4}\, B_{\mu\nu}B^{\mu\nu} - \tfrac{1}{4}\, \mathring{W}_{3\mu\nu}\mathring{W}_3^{\mu\nu} - \tfrac{1}{2}\, W^+_{\mu\nu}W^{-\mu\nu},$$

with $W^\pm_\mu = (W_{1\mu} \mp W_{2\mu})/\sqrt{2}$. Introducing field renormalizations for B_μ and $W_{\mu a}$ we write:

$$B_\mu = \sqrt{Z_\mathrm{B}}\, B_\mathrm{r}, \quad W_{3\mu} = \sqrt{Z}\, W_{3\mu\mathrm{r}} \quad \text{and} \quad W^\pm_\mu = \sqrt{Z}\, W^\pm_{\mu\mathrm{r}}.$$

For the renormalized fields we have

$$\begin{pmatrix} B_\mathrm{r} \\ W_{3\mathrm{r}} \end{pmatrix} = \begin{pmatrix} \cos\theta_\mathrm{W} & -\sin\theta_\mathrm{W} \\ \sin\theta_\mathrm{W} & \cos\theta_\mathrm{W} \end{pmatrix} \begin{pmatrix} A_\mathrm{r} \\ Z_\mathrm{r} \end{pmatrix}$$

and thus

$$B_{\mu\nu\mathrm{r}}B^{\mu\nu\mathrm{r}} = \cos^2\theta_\mathrm{W} F_{\mu\nu\mathrm{r}}F^{\mu\nu\mathrm{r}} + \sin^2\theta_\mathrm{W} \mathring{Z}_{\mu\nu\mathrm{r}}\mathring{Z}^{\mu\nu\mathrm{r}} - 2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W} \mathring{Z}_{\mu\nu\mathrm{r}}F^{\mu\nu}_\mathrm{r},$$

$$W_{3\mu\nu}W_3^{\mu\nu} = \cos^2\theta_\mathrm{W} \mathring{Z}_{\mu\nu\mathrm{r}}\mathring{Z}^{\mu\nu\mathrm{r}} + \sin^2\theta_\mathrm{W} F_{\mu\nu\mathrm{r}}F^{\mu\nu}_\mathrm{r} + 2\sin\theta_\mathrm{W}\cos\theta_\mathrm{W} \mathring{Z}_{\mu\nu\mathrm{r}}F^{\mu\nu}_\mathrm{r},$$

yielding for the kinetic term

$$\begin{aligned} &-\tfrac{1}{4}\,(Z_\mathrm{B}\cos^2\theta_\mathrm{W} + Z\sin^2\theta_\mathrm{W})F_{\mu\nu\mathrm{r}}F^{\mu\nu}_\mathrm{r} - \tfrac{1}{4}\,(Z_\mathrm{B}\sin^2\theta_\mathrm{W} + Z\cos^2\theta_\mathrm{W})\mathring{Z}_{\mu\nu\mathrm{r}}\mathring{Z}^{\mu\nu}_\mathrm{r} \\ &\quad -\tfrac{1}{2}\, ZW^+_{\mu\nu\mathrm{r}}W_\mathrm{r}^{-\mu\nu} + 2(Z - Z_\mathrm{B})\sin\theta_\mathrm{W}\cos\theta_\mathrm{W} F_{\mu\nu\mathrm{r}}\mathring{Z}^{\mu\nu}_\mathrm{r} \\ &\equiv -\tfrac{1}{4}\, Z_\gamma F_{\mu\nu\mathrm{r}}F^{\mu\nu}_\mathrm{r} - \tfrac{1}{4}\, Z_0 \mathring{Z}_{\mu\nu\mathrm{r}}\mathring{Z}^{\mu\nu}_\mathrm{r} - \tfrac{1}{2}\, ZW^+_{\mu\nu\mathrm{r}}W_\mathrm{r}^{-\mu\nu} + 2XF_{\mu\nu\mathrm{r}}\mathring{Z}^{\mu\nu}_\mathrm{r}, \end{aligned}$$

the last relation according to (10). Eliminating Z_B yields *for the ultraviolet singular parts* of the "physical" field renormalization factors the two relations

$$(Z)_\mathrm{UV} = \frac{(\cos^2\theta_\mathrm{W} Z_0 - \sin^2\theta_\mathrm{W} Z_\gamma)_\mathrm{UV}}{\cos^2\theta_\mathrm{W} - \sin^2\theta_\mathrm{W}}$$

and

$$(X)_\mathrm{UV} = \frac{\sin\theta_\mathrm{W}\cos\theta_\mathrm{W}}{\cos^2\theta_\mathrm{W} - \sin^2\theta_\mathrm{W}}\,(Z_0 - Z_\gamma)_\mathrm{UV},$$

which have been verified explicitly.

Even since by the use of the on-shell wave function renormalization factors, the bare form of the ST-identities is changed, in practice there is no problem because all Z-factors associated with *internal* lines drop out from the amplitudes, which on a formal level may be seen using the functional integral representation for the Green functions.

It is important to stress, however, that for the *external* lines we have to use the wave function renormalization factors according to the LSZ formalism, i.e. for the physical fields in order to make the S-matrix both gauge invariant and unitary. This has been proven by 't Hooft and Veltman [9] and earlier by Białynicki-Birula for QED [10].

As a final remark we want to point out that for the calculation of S-matrix elements the renormalization of the gauge parameter is superfluous since S-matrix elements are gauge invariant and furthermore ghost amplitudes drop out due to ST-identities as shown in a special example in Ref. [7].

4. *Further counterterms*

All other counterterms are obtained using the mass-coupling relations

$$g = \frac{2M_W}{v}, \quad g' = \frac{2(M_Z^2 - M_W^2)^{1/2}}{v}, \quad \lambda = \frac{m_H^2}{2v^2} \quad \text{and} \quad G_f = \frac{\sqrt{2}m_f}{v}, \tag{11}$$

where g and g' are the $SU(2)_L \otimes U(1)_Y$ gauge couplings, λ the Higgs self coupling and G the Higgs-fermion coupling.

The weak mixing angle θ_W and the electric charge are given by

$$\sin^2\theta_W = \frac{g'^2}{g'^2 + g^2} = 1 - \frac{M_W^2}{M_Z^2} \quad \text{and} \quad e^2 = \frac{g'^2 g^2}{g'^2 + g^2} = 4M_W^2 \sin^2\theta_W \frac{1}{v^2}. \tag{12}$$

We consider the latter relations as the definitions of the *dependent* parameters $\sin^2\theta_W$ and v^{-2}, the latter being the loop expansion parameter for fixed values of the masses [6]. According to (11) this is equivalent to an expansion in α.

From the mass-coupling relations we obtain the dependent counterterms

$$\frac{\delta\cos^2\theta_W}{\cos^2\theta_W} = -\mathrm{tg}^2\,\theta_W \frac{\delta\sin^2\theta_W}{\sin^2\theta_W} = -\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right), \tag{13}$$

$$\frac{\delta v^{-1}}{v^{-1}} = \frac{\delta e}{e} - \frac{1}{2}\frac{\delta M_W^2}{M_W^2} - \frac{1}{2}\frac{\delta\sin^2\theta_W}{\sin^2\theta_W}.$$

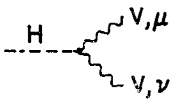
For the dimensionless couplings we have

$$\frac{\delta g'}{g'} = \frac{\delta e}{e} - \frac{1}{2}\frac{\delta\cos^2\theta_W}{\cos^2\theta_W}, \quad \frac{\delta g}{g} = \frac{\delta e}{e} - \frac{1}{2}\frac{\delta\sin^2\theta_W}{\sin^2\theta_W},$$

$$\frac{\delta\lambda}{\lambda} = 2\frac{\delta v^{-1}}{v^{-1}} + \frac{\delta m_H^2}{m_H^2}, \quad \frac{\delta G_f}{G_f} = \frac{\delta v^{-1}}{v^{-1}} + \frac{\delta m_f}{m_f},$$

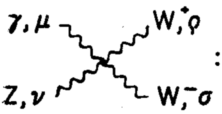
which must exhibit the same UV-singularities as in the unbroken theory.

From these we obtain the counterterms for trilinear and quadrilinear vertices, e.g. [6].



$$: \frac{2M_V^2}{v} g^{\mu\nu} \left(1+\tfrac{1}{2}\,\delta Z_H+\delta Z_V+\frac{\delta M_V^2}{M_V^2}+\frac{\delta v^{-1}}{v^{-1}}\right)$$

$$V = W, Z$$



$$: 2e\,\frac{M_W^2}{vM_Z}\,T^{\mu\nu,\varrho\sigma}\left(1+\tfrac{1}{2}\,\delta Z_\gamma+\tfrac{1}{2}\,\delta Z_0+\delta Z+\frac{\delta M_W^2}{M_W^2}-\tfrac{1}{2}\,\frac{\delta M_Z^2}{M_Z^2}+\frac{\delta e}{e}+\frac{\delta v^{-1}}{v^{-1}}\right)$$

with $\delta Z = Z-1$, $T^{\mu\nu,\varrho\sigma} = (2g^{\mu\nu}g^{\varrho\sigma}-g^{\mu\varrho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\varrho})$.

5. Relation to other parametrizations

In the above scheme the parameters (α, M_W, M_Z) were chosen in a "natural" way, i.e. in close analogy to QED. The main features of this scheme are the gauge invariant determination of the mass-counterterms and the wave function renormalization for the physical (external) fields in the LSZ sense.

If we use different parametrizations of the GWS-model we maintain the above characteristic features except that the additional counterterms (see Sect. 4) will in general be chosen in a different manner. Of particular interest are the "low-energy" parametrization (α, G_μ, $\sin^2\theta$), the "high-energy" parametrization (G_μ, M_W, M_Z) and finally also (α, G_μ, M_Z).

The low-energy parametrization is of particular interest since it makes use of the parametrization of the effective Fermi current-current interaction:

$$L_{\mathrm{eff,int}} = -\frac{4}{\sqrt{2}}(G_c J_\mu^+ J^{-\mu}+G_n J_{\mu Z}J_Z^\mu)+ej_{\mu\,\mathrm{em}}A^\mu,$$

which is obtained as low-energy limit of

$$L_{\mathrm{int}} = \frac{g}{\sqrt{2}}(J_\mu^+ W^{\mu-}+\mathrm{h.c.})+\frac{g}{\cos\theta_W}\,J_{\mu Z}Z^\mu+ej_{\mu\mathrm{em}}\,A^\mu,$$

with

$$\cos^2\theta_W = \frac{g^2}{g'^2+g^2}, \quad e = g\sin\theta_W \text{ and } \sqrt{2}\,G_c = \frac{g^2}{4M_W^2} = v^{-2},$$

$$\sqrt{2}\,G_n = \frac{g'^2+g^2}{4M_Z^2} = \varrho_G v^{-2}.$$

The ϱ-parameter is

$$\varrho_G = \frac{G_n}{G_c} = 1+0(\alpha)$$

to lowest order in the standard model.

The low-energy parameters are thus defined as (they are characterized by an index 0):

1. The fine structure constant $\alpha = \frac{e_0^2}{4\pi} = 1/137.035963(5)$, obtained e.g. from ($\mu e$)-Compton scattering at $q^2 = 0$.
2. The Fermi constant $G_\mu = G_{c0} = \frac{1}{\sqrt{2}\, v_0^2} = (1.16638 \pm 0.00002) \cdot 10^{-5}\ \mathrm{GeV}^{-2}$ from μ-decay.
3. The neutral current parameter $\sin^2 \theta_{W,0}$ is for the time being predominantly obtained from deep inelastic neutrino-hadron scattering. The cleanest determination of $\sin^2 \theta_W$ would come, of course, from neutrino-electron scattering, but here the error is still of the order of 15%. The presently accepted value is [12] $\sin^2 \theta_W = 0.217 \pm 0.014$. Due to the unsatisfactory experimental situation [13], however, we choose for the purpose of this presentation $\sin^2 \theta_{W,0} = 0.22$.

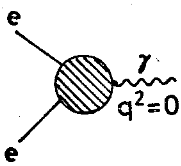
From the mass-coupling relations (11), (12) one obtains as low-energy effective parameters

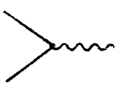
$$v_0 = (\sqrt{2}\, G_\mu)^{-1/2} \cong 246.224 \pm 0.003\ \mathrm{GeV},$$

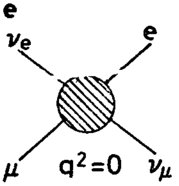
$$M_{W_0} = \frac{e_0 v_0}{2 \sin \theta_0} \quad \text{and} \quad M_{Z_0} = \frac{M_{W_0}}{\sqrt{\varrho_{G_0}} \cos \theta_0}.$$

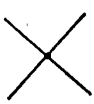
For numerical values of the latter see Table I.

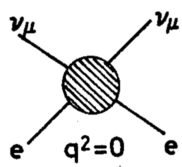
In perturbation theory the low-energy parameters are defined by the following on-shell conditions:

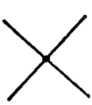


$=$

$$= -ie_0(\bar{e}\gamma^\mu e) \tag{14}$$



$=$

$$+\text{em.} = -\frac{G_{c0}}{\sqrt{2}} (\bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\mu)(\bar{e}\gamma_\alpha(1-\gamma_5)\nu_e) + \text{em.} \tag{15}$$



$=$

$$+\text{em.} = -\sqrt{2}\, G_{n0}(\bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\gamma_\mu)(\bar{e}\gamma_\alpha(a_0 + b_0\gamma_5)e) + \text{em.} \tag{16}$$

with $(a_0, b_0) = (\sin^2 \theta_{W,0} - \frac{1}{4}, \frac{1}{4})$.

By definition the Born diagrams are exact at $q^2 = 0$ while the evaluation of the one--loop diagrams on the l.h.s. at $q^2 = 0$ yields the low-energy counterterms in this scheme:

$$\delta e_0 \equiv \delta e, \quad \delta G_{c_0} = \sqrt{2}\, \frac{\delta v_0^{-1}}{v_0} \quad \text{and} \quad \delta \sin^2 \theta_0.$$

The first important result we obtain, is that the neutral coupling G_{n_0} is renormalized relatively to G_μ by a finite and gauge invariant correction (in the notation of Marciano and Sirlin [3]):

$$G_{n_0} = \varrho_{0NC} G_\mu = G_\mu(1+\varepsilon_{NC}) \quad \text{with} \quad \varepsilon_{NC} = A^{WW}(0) - A^{ZZ}(0) + R_G,$$

where R_G includes the contribution from form factors and box diagrams.

On the other hand it is possible to evaluate the relations (14)–(16) on the l.h.s. by the use of the (α, M_W, M_Z)-parametrization (including, of course, the corresponding counter-terms), yielding thus relations between the low energy parameters in the two schemes:

$$G_\mu = \frac{1}{\sqrt{2}\, v_0^2} = \frac{1}{\sqrt{2}\, v^2}(1-\Delta r)^{-1}, \quad \sin^2\theta_{W,0} = \sin^2\theta_W(1-\Delta) \tag{17}$$

again in the notation of Ref. [3].

From the parameter relations (12) we finally obtain the mass-shifts:

$$M_{W_0}^2 = M_W^2(1+\delta X_W), \quad M_{Z_0}^2 = M_Z^2(1+\delta X_Z),$$

with $\delta X_W = -\Delta r + \Delta$ and $\delta X_Z = \delta X_W - \mathrm{tg}^2\,\theta_W \Delta - \varepsilon_{NC}$.

Given the low-energy parameters α, G_μ and $\sin^2\theta_{W,0}$, the physical vector-boson masses are finally predicted to be

$$M_W = \left(\frac{\pi\alpha}{\sqrt{2}\, G_\mu(1+\delta X_W)}\right)^{1/2} \frac{1}{\sin\theta_{W,0}}$$

and

$$M_Z = \frac{M_W}{(1+\Delta)^{1/2}} \frac{1}{\cos\theta_{W,0}}.$$

Finally we present in Table I some numerical results for the most relevant quantities in the GWS-model. These were calculated in the above mentioned four schemes keeping $\sin^2\theta_{W,0} = 0.22$. The errors in Table I correspond to hadronic uncertainties [14] and the scheme dependence. The errors from the different schemes were added up quadratically. The computations were done for $m_t = 35$ GeV and $M_H = 100$ GeV.

TABLE I

Numerical results obtained for the parameters ϱ_{0NC}, Δr and the physical vector-meson masses

ϱ_{0NC}	Δr	M_W	M_Z	$\sin^2\theta_W$	M_{W_0}	M_{Z_0}
1.00535 (±0.00041)	0.07120 (±0.00145)	82.3596 (±0.0283)	93.2900 (±0.0384)	0.22060 (±0.00035)	79.4831 (±0.0007)	89.7567 (±0.0181)

The experimental values of the vector-boson masses are [15]

	UA_1	UA_2
$M_Z =$	$93.0 \pm 1.4 \pm 3.2$	$92.5 \pm 1.3 \pm 1.5$
$M_W =$	$83.5^{+1.1}_{-1.0} \pm 2.8$	$81.2 \pm 1.1 \pm 1.3$

and there is agreement of the theoretical predictions and these values within the experimental uncertainties — much more precise data are needed, however, and are to be expected from LEP and SLC.

REFERENCES

[1] G. Passarino, M. Veltman, *Nucl. Phys.* **B160**, 151 (1979).
[2] A. Sirlin, *Phys. Rev.* **D22**, 971 (1980).
[3] W. J. Marciano, A. Sirlin, *Phys. Rev.* **D22**, 2695 (1980); *Phys. Rev. Lett.* **46**, 163 (1981).
[4] K. Aoki et al., *Prog. Theor. Phys.* **64**, 707 (1980); **65**, 1001 (1981).
[5] K. Aoki et al., *Prog. Theor. Phys. Suppl.* **73**, 1 (1982).
[6] J. Fleischer, F. Jegerlehner, *Phys. Rev.* **D23**, 2001 (1981).
[7] J. Fleischer, F. Jegerlehner, *Nucl. Phys.* **B216**, 469 (1983); *Nucl. Phys.* **B228**, 1 (1983); F. Jegerlehner, in Workshop on Radiative Corrections in $SU(2)_L \times U(1)_Y$, Miramare, Trieste, June 1983, B. W. Lynn and J. F. Wheater, editors, World Scientific 1984.
[8] S. Sakakibara, *Phys. Rev.* **D24**, 1149 (1981); for further references see also S. Sakakibara, in: Workshop on Radiative Corrections in $SU(2)_L \times U(1)_Y$, Miramare, Trieste, June 1983, B. W. Lynn and J. F. Wheater, editors, World Scientific 1984
[9] G. 't Hooft, M. Veltman, *Nucl. Phys.* **B50**, 318 (1972).
[10] I. Białynicki-Birula, *Phys. Rev.* **D2**, 2877 (1970).
[11] F. Jegerlehner, J. Fleischer, *Acta Phys. Pol.* **B17**, 709 (1986).
[12] A. Sirlin, W. J. Marciano, *Nucl. Phys.* **B189**, 442 (1981).
[13] J. Panman, CERN-preprint EP/85-35.
[14] F. Jegerlehner, *Z. Phyc. C* (1986).
[15] L. DiLella, CERN-preprint EP/85-184.