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MATRIX FORMULATION OF $N=1$ SUPERGRAVITY BASED ON UNIVERSAL NONLINEAR FIRST-ORDER EQUATIONS

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Lagrangian and equations of $N = 1$ supergravity are obtained in terms of universal matrix nonlinear first-order equations with quadratic nonlinearities, which have a number of advantages. The quadratic and cubic matrices are derived and their properties are investigated. It is shown that tetrad-formalism matrices have much simpler minimum polynomial compared with metric one. The field function structure is obtained containing tetrads, Ricci coefficients and Riemann tensor and also both spin-vector and components lowering nonlinearities.

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1. *Introduction*

Gauge fields are presently the basis for the unified theory of fundamental interactions. It was supergravity (SG) where the idea was first realized that fermion fields, usually associated with the matter, could be the true gauge fields and that they unite with the fields of integer spin in the irreducible supermultiplet. Because of the great mathematical difficulties there arises a problem of adequate mathematical formulation of SG. One of the possible approaches to the solution of this problem is based on the universal nonlinear first-order equations (UNE) first suggested in [1] and developed in [2–11] for a number of field theory models including fields with different transformation properties and statistics.

A distinguishing feature of this approach, convenient for supersymmetrical theories is that, by definition, the unified field satisfying the UNE includes interchangeably the fields of fermions and bosons which are the elements of Grassmann algebra. The first order of UNE is also adequate for SG, since Ricci coefficients which actually give transition to the first order of gravity equations, are the gauge fields components. Due to the first-order equations and quadratic nonlinearity of UNE, Ψ contains tetrads, Ricci coefficients and Riemann tensor as components of Bose sector. Using also the 1.5-order formalism

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[12], we can circumvent difficulties connected with transformations of auxiliary components.

In this work we consider formulation of $N = 1$ SG on the basis of such approach. In the UNE formalism all interacting fields, both bosons and fermions, are introduced as components of the self-interacting unified field $\Psi = \{\Psi_A(x)\}$, $(A = 1, 2, ..., n)$ which satisfies the following UNE:

$$(\alpha^\mu \partial_\mu + \alpha^0)\Psi + \tfrac{1}{2} M\Psi\Psi = 0, \tag{1}$$

with Lagrangian of the form:

$$\mathscr{L} = \tfrac{1}{2} \Psi(\alpha^\mu \partial_\mu + \alpha^0)\Psi + \tfrac{1}{3}! M\Psi\Psi\Psi. \tag{2}$$

Here, $\alpha^\mu = (\alpha^\mu_{AB})$, $\alpha^0 = (\alpha^0_{AB})$ are quadratic and $M = (M_{ABC})$ cubic matrices, which completely characterize the system (1), and Ψ are the elements of Grassmann algebra:

$$\Psi_A(x)\Psi_B(x') - (-1)^{ab}\Psi_B(x')\Psi_A(x) = 0, \tag{3}$$

where a, b are Grassmann parity of components Ψ_A, Ψ_B ($a, b = 0$ for Bose fields and $a, b = 1$ for Fermi fields).

From (1) and (2), we have the following symmetry properties of α^μ, α^0, M [2, 4]:

$$\alpha^\mu_{AB} = -(-1)^{ab}\alpha^\mu_{BA}, \quad \alpha^0_{AB} = (-1)^{ab}\alpha^0_{BA},$$

$$M_{ABC} = (-1)^{ab}M_{BAC} = (-1)^{a(b+c)}M_{BCA}, \tag{4}$$

For the UNE formulation of $N = 1$ SG we must find the corresponding component structure of Ψ and explicit form of matrices α^μ, α^0, and M.

2. *N = 1 supergravity*

The action of the minimal $N = 1$ SG is expressed as follows [13–15]:

$$S = \int d^4x[\mathscr{L}_G(x) + \mathscr{L}_{3/2}(x)], \tag{5}$$

$$\mathscr{L}_G = \tfrac{1}{8} k^{-2}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mncd}e^m_\mu e^n_\nu R_{\varrho\sigma}{}^{cd}(\omega), \tag{6}$$

$$\mathscr{L}_{3/2} = -\tfrac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu D_\varrho\psi_\sigma, \tag{7}$$

where

$$R_{\varrho\sigma}{}^{cd} = \partial_\varrho\omega_\sigma{}^{cd} - \partial_\sigma\omega_\varrho{}^{cd} + \omega_\varrho{}^c{}_k\omega_\sigma{}^{kd} - \omega_\sigma{}^c{}_k\omega_\varrho{}^{kd} \tag{8}$$

is the curvature tensor, $\omega_\mu{}^{ab}$ are Ricci coefficients or spin connection, e^a_μ are tetrad ($e^a_\mu e_{\nu a} = g_{\mu\nu}$) components, $\psi_\mu(\bar{\psi}_\mu = -\psi^T_\mu C^{-1})$ is a Majorana spin 3/2 field, C is a charge conjugate matrix, $\gamma_\mu = e^a_\mu\gamma_a$, γ_a are Dirac matrices, $\gamma_5 = \gamma_1\gamma_2\gamma_3\gamma_4$,

$$D_\varrho\psi_\mu = (\partial_\varrho - \tfrac{1}{2}\omega_\varrho{}^{ln}\sigma_{ln})\psi_\mu, \tag{9}$$

where

$$\sigma_{ln} = \frac{1}{2}\gamma_{[l}\gamma_{n]} = \frac{1}{4}(\gamma_l\gamma_n - \gamma_n\gamma_l).$$

Field equations are found by independent varying e^n_μ, ψ_μ, and $\omega_\mu{}^{kn}$ (Palatini formalism)

$$\frac{1}{4}k^{-2}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mncd}e^m_\mu R_{\varrho\sigma}{}^{cd} - \frac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}\bar{\psi}_\mu\gamma_5\gamma_n D_\varrho\psi_\sigma = 0, \tag{10}$$

$$-\frac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}\partial_\varrho(\bar{\psi}_\mu\gamma_5\gamma_\nu) + \frac{1}{4}\varepsilon^{\mu\nu\varrho\sigma}\bar{\psi}_\mu\gamma_5\gamma_\nu\sigma_{kl}\omega_\varrho{}^{kl}$$
$$-\frac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}(D_\varrho\psi_\mu)C^{-1}\gamma_5\gamma_\nu = 0, \tag{11}$$

$$-\frac{1}{2}k^{-2}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mncd}e^m_\mu\partial_\varrho e^n_\nu + \frac{1}{2}k^{-2}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mnck}$$
$$\times e^m_\mu e^n_\nu\omega_{\varrho d}{}^k + \frac{1}{4}\varepsilon^{\mu\nu\sigma\varrho}\bar{\psi}_\mu\gamma_5\gamma_\nu\sigma_{cd}\psi_\varrho = 0. \tag{12}$$

If we postulate that tetrad is covariantly constant:

$$\partial_\varrho e^n_\nu + \omega_\varrho{}^n{}_k e_\nu{}^k - \Gamma^\sigma_{\nu\varrho}e^n_\sigma = 0, \tag{13}$$

where,

$$\Gamma^\varrho_{\mu\nu} = \left\{{\varrho \atop \mu\nu}\right\} + \Gamma^\varrho_{[\mu\nu]} = \left\{{\varrho \atop \mu\nu}\right\} + C^\varrho_{\mu\nu},$$

$$\left\{{\varrho \atop \mu\nu}\right\} = \frac{1}{2}g^{\varrho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}) \tag{14}$$

are the Christoffel symbols, then from (12) and (13) we have the torsion given by

$$C^\varrho_{\mu\nu} = -\frac{k^2}{4}\bar{\psi}_\mu\gamma^\varrho\psi_\nu. \tag{15}$$

3. *Matrix approach in supergravity*

Now, let us construct the appropriate system of first-order equations of the type (1) in the supergravity theory. Equations (10)–(12) contain nonlinearities of higher degrees than quadratic. To make nonlinearities quadratic, we need to introduce as auxiliary components of the field function $\Psi_A(x)$, Riemann tensor $R_{\mu\nu}{}^{ab}$ and antisymmetrical covariant derivative $D_{[\varrho}\psi_{\mu]}$. Then, we must add to the system (10)–(12) an auxiliary equation (8) and an equation for defining components $\varphi_{\varrho\nu} = D_{[\varrho}\psi_{\nu]}$:

$$\varphi_{\varrho\nu} = \partial_{[\varrho}\psi_{\nu]} - \frac{1}{2}\sigma_{kl}\psi_{[\varrho}\omega_{\nu]}{}^{kl}. \tag{16}$$

Let us consider equation (12) and write it in the form:

$$\frac{1}{4}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mncd}\partial_\varrho(k^{-2}e^m_\mu e^n_\nu) - \frac{1}{4}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mncd}$$
$$\times(k^{-2}e^m_\mu e^n_\nu)\omega_{\varrho d}{}^k + \varepsilon^{\mu\nu\varrho\sigma}(\psi_\mu\gamma_5\gamma_\nu)\sigma_{cd}\psi_\varrho = 0. \tag{17}$$

Introducing new variables $\kappa^{mn}{}_{\mu\nu}$ and $\bar{\chi}_{\mu\nu}$ by expressions:

$$\kappa^{mn}{}_{\mu\nu} = k^{-2}e^m_{[\mu}e^n_{\nu]}, \quad \bar{\chi}_{\mu\nu} = \bar{\psi}_{[\mu}\gamma_5\gamma_{\nu]} \tag{18}$$

we shall have a system only with quadratic nonlinearities on field functions

$$\tfrac{1}{4} k^{-2}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mncd}e^{m}_{\mu}R_{\varrho\sigma}{}^{cd}-\tfrac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}\psi_{\mu}C^{-1}\gamma_5\gamma_n\varphi_{\varrho\sigma} = 0, \tag{19}$$

$$-\tfrac{1}{4}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mncd}\partial_{\varrho}\kappa^{mn}{}_{\mu\nu}+\tfrac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}\varepsilon_{mnck}\kappa^{mn}{}_{\mu\nu}$$

$$\times\,\omega_{\varrho}{}^{ak}\eta_{ad}+\tfrac{1}{4}\varepsilon^{\mu\nu\sigma\varrho}\bar{\chi}_{\mu\nu}\sigma_{cd}\psi_{\varrho} = 0, \tag{20}$$

$$\kappa^{mn}{}_{\mu\nu}-k^{-2}e^{m}_{[\mu}e^{n}_{\nu]} = 0, \tag{21}$$

$$2\partial_{[\varrho}\omega_{\sigma]}{}^{kn}-R_{\varrho\sigma}{}^{kn}+2\eta_{ab}\omega_{[\varrho}{}^{ka}\omega_{\sigma]}{}^{bn} = 0, \tag{22}$$

$$-\tfrac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}\partial_{\varrho}\bar{\chi}_{\mu\nu}+\tfrac{1}{4}\varepsilon^{\mu\nu\varrho\sigma}\bar{\chi}_{\mu\nu}\sigma_{kn}\omega_{\varrho}{}^{kn}-\tfrac{1}{2}\varepsilon^{\mu\nu\varrho\sigma}\varphi_{\varrho\mu}C^{-1}\gamma_5\gamma_n e^{n}_{\nu} = 0, \tag{23}$$

$$\bar{\chi}_{\mu\nu}-e_{[\mu}\psi_{\nu]}C^{-1}\gamma_5\gamma_n = 0, \tag{24}$$

$$\partial_{[\mu}\psi_{\nu]}-\varphi_{\mu\nu}+\tfrac{1}{2}\sigma_{kn}\psi_{[\mu}\omega_{\nu]}{}^{kn} = 0, \tag{25}$$

where $\eta = \mathrm{diag}(-1, -1, -1, +1)$ is the Minkowski tensor.

If we introduce as unified field Ψ a multicomponent function

$$\Psi(x) = \{\Psi_A(x)\} = \{\Psi_{\mu k}, \Psi_{\mu[kn]}, \Psi_{[\mu\nu][kn]},$$

$$\Psi_{[kn][\mu\nu]}, \Psi_{\alpha\mu}, \Psi_{\alpha[\mu\nu]}, \Psi_{[\mu\nu]\alpha}\}^{\mathrm{T}} = \{e^{k}_{\mu},$$

$$\omega_{\mu}{}^{kn}, R_{\mu\nu}{}^{kn}, \kappa^{kn}{}_{\mu\nu}, \psi^{\alpha}_{\mu}, \varphi^{\alpha}_{\mu\nu}, \bar{\chi}_{\mu\nu\alpha}\}^{\mathrm{T}} \tag{26}$$

we can write the system (20)–(25) in the universal matrix form (1).

For finding the explicit form of the matrices of (1) it is convenient to use elements e^{AB} of the basis of quadratic and e^{ABC} of cubic matrices in the space $\Psi(x)$, which are expressed in terms of elements of vector basis:

$$e^{AB} = e^{A}\cdot e^{B}, \quad e^{ABC} = e^{A}\cdot e^{B}\cdot e^{C},$$

$$(e^{A})_B = \delta_{AB}, \quad e^{A}e^{B} = \delta_{AB} \tag{27}$$

(the point denotes direct multiplication of basis vectors). Here, δ_{AB} are generalized Kronecker symbols in the form:

$$\delta_{\mu\alpha,\mu'\alpha'} = \delta_{\mu\mu'}\delta_{\alpha\alpha'},$$

$$\delta_{\mu[kn],\mu'[k'n']} = \delta_{\mu\mu'}\delta_{[kn],[k'n']},$$

$$\delta_{[kn],[k'n']} = \tfrac{1}{2}(\delta_{kk'}\delta_{nn'}-\delta_{kn'}\delta_{nk'}),$$

$$\delta_{[\mu\nu][kn],[\mu'\nu'][k'n']} = \delta_{[\mu\nu],[\mu'\nu']}\delta_{[kn],[k'n']}. \tag{28}$$

Using relations (26–28), we get the following expressions for the main matrices

$$\beta^{\mu} = -\tfrac{1}{4}\varepsilon^{\nu\varrho\mu\sigma}\varepsilon_{mncd}e^{\sigma[cd],[mn][\nu\varrho]}$$

$$+2e^{[\mu\nu][kn],\nu[kn]}-\tfrac{1}{2}\varepsilon^{\nu\varrho\mu\sigma}e^{\alpha\sigma,[\nu\varrho]\alpha}+e^{[\mu\nu]\alpha,\alpha\nu}, \tag{29}$$

$$\beta^0 = e^{[\mu\nu][mn],[mn][\mu\nu]} - e^{[mn][\mu\nu],[\mu\nu][mn]} + e^{\alpha[\mu\nu],[\mu\nu]\alpha} - e^{[\mu\nu]\alpha,\alpha[\mu\nu]}, \tag{30}$$

$$\begin{aligned} \Lambda = {} & \tfrac{1}{4} k^{-2} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{mncd} e^{\nu n,\mu m,[\varrho\sigma][cd]} + \tfrac{1}{4} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{mnck} \\ & \times \eta_{ad} e^{\sigma[cd],[mn][\mu\nu],\varrho[\alpha k]} + 2\eta_{ab} e^{[kn][\varrho\sigma],\varrho[ka],\sigma[bn]} \\ & + \tfrac{1}{4} \varepsilon^{\mu\nu\varrho\sigma} (\sigma_{kn})^{\alpha}_{\beta} \left[e^{\sigma[kn],\alpha[\mu\nu],\beta\varrho} + e^{\beta\sigma,\alpha[\mu\nu],\varrho[kn]} \right] \\ & + \tfrac{1}{2} \varepsilon^{\mu\nu\varrho\sigma} (C^{-1}\gamma_5\gamma_n)_{\alpha\beta} e^{\nu n,\alpha\mu,\beta[\varrho\sigma]} - \tfrac{1}{2} (C^{-1}\gamma_5\gamma_n)_{\alpha\beta} \\ & \times e^{\beta[\mu\nu],\alpha\mu,\nu n} - \tfrac{1}{2} (\sigma_{kn})^{\alpha}_{\beta} e^{[\mu\nu],\beta\nu,\mu[kn]}. \end{aligned} \tag{31}$$

It is seen that these matrices do not satisfy symmetry conditions (4), and hence, the system (20)–(25) cannot be obtained from Lagrangian (2) through the variational principle. But with the use of nondegenerate linear transformation, i.e. multiplying (1) by any nonsingular matrix Q ($\det Q \neq 0$) we can receive the equivalent system of equations with matrices $\alpha^\mu = Q\beta^\mu$, $\alpha^0 = Q\beta^0$, $M = Q\Lambda$ satisfying conditions (4). We can get it if we use Q in the form:

$$\begin{aligned} Q = {} & e^{\nu k,\nu k} + e^{\nu[kn],\nu[kn]} - \tfrac{1}{8} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{mncd} \\ & \times \left[e^{[\mu\nu][mn],[\varrho\sigma][cd]} - e^{[mn][\mu\nu],[cd][\varrho\sigma]} + e^{\alpha\mu,\alpha\mu} \right. \\ & \left. \tfrac{1}{2} \varepsilon^{\mu\nu\varrho\sigma} \left[e^{\alpha[\mu\nu],\alpha[\varrho\sigma]} - e^{[\mu\nu]\alpha,[\varrho\sigma]\alpha} \right] \right]. \end{aligned} \tag{32}$$

And for α^μ, α^0, M we obtain

$$\begin{aligned} \alpha^\mu = {} & \tfrac{1}{4} \varepsilon^{\nu\varrho\mu\sigma} \varepsilon_{mncd} \left[e^{[mn][\nu\varrho],\sigma[cd]} \right. \\ & \left. - e^{\sigma[cd],[mn][\nu\varrho]} \right] - \tfrac{1}{2} \varepsilon^{\nu\varrho\mu\sigma} \left[e^{[\nu\varrho]\alpha,\alpha\sigma} + e^{\alpha\sigma,[\nu\varrho]\alpha} \right], \end{aligned} \tag{33}$$

$$\begin{aligned} \alpha^0 = {} & -\tfrac{1}{8} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{mncd} \left[e^{[\varrho\sigma][cd],[mn][\mu\nu]} \right. \\ & \left. + e^{[mn][\mu\nu],[\varrho\sigma][cd]} \right] - \tfrac{1}{2} \varepsilon^{\mu\nu\varrho\sigma} \left[e^{d[\varrho\sigma],[\mu\nu]\alpha} - e^{[\mu\nu]\alpha,\alpha[\varrho\sigma]} \right], \end{aligned} \tag{34}$$

$$\begin{aligned} M = {} & \sum_{\pi} (-1)^{p(\pi)} \left\{ \tfrac{1}{8} \varepsilon^{\mu\nu\varrho\sigma} \varepsilon_{mncd} \left[k^{-2} e^{\mu m,\nu n,[\varrho\sigma][cd]} \right. \right. \\ & \left. + 2\eta_{ak} e^{[mn][\mu\nu],\varrho[ck],\sigma[ad]} \right] - \tfrac{1}{2} \varepsilon^{\mu\nu\varrho\sigma} \left[(C^{-1}\gamma_5\gamma_n)_{\alpha\beta} \right. \\ & \left. \left. \times e^{\alpha\mu,\nu n,\beta[\varrho\sigma]} - \tfrac{1}{2} (\sigma_{kn})^{\alpha}_{\beta} e^{[\mu\nu]\alpha,\varrho[kn],\beta\sigma} \right] \right\}. \end{aligned} \tag{35}$$

Here, $\sum_\pi$ is the sum on all possible transpositions of indices for e^{ABC}, and $p(\pi)$ is the sum of multiplications of Grassmann parities of indices participating in transpositions.

Minimal polynomial for matrices $\hat{p} = p_\mu d^\mu$, where p_μ is any vector, is given by

$$\hat{p}[\hat{p}^2 + (-1)^{ab} \tfrac{1}{2} p^2] = 0. \tag{36}$$

One obtains minimal polynomial of $\hat{p}$ in SG of the third degree as in the case of matrix tetrad formulation of gravity theory [11]. Note that in metric formulation minimum polynomial is of the seventh degree [3].

4. Discussion

Thus, we have obtained the universal form (2) of SG Lagrangian with matrices satisfying symmetry conditions (4). This Lagrangian differs from SG Lagrangian (5) by the total divergence

$$\Delta\mathscr{L} = -\tfrac{1}{8}\,\partial_\varrho\{\varepsilon^{\mu\nu\varrho\sigma}[\varepsilon_{mncd}k^{-2}e^m_\mu e^n_\nu\omega_\sigma{}^{cd} - 2\bar{\psi}_\mu\gamma_5\gamma_\nu\psi_\sigma]\}. \tag{37}$$

Variation of $\delta\Psi$ under general coordinate, local Lorentz and supersymmetry transformations, can be written as

$$\delta\Psi = \zeta^K N_1^K \Psi + \zeta^K N_2^{K\mu}\partial_\mu\Psi + \partial_\mu\zeta^K N_3^{\mu K}\Psi + \partial_\mu\zeta^K V^{\mu K},$$

$$K = (\mu, [ab], \alpha). \tag{38}$$

Here, N_1^K, $N_2^{K\mu}$, $N_3^{\mu K}$ are constant matrices, $V^{\mu K}$ are vectors:

$$N_1^K = (0, N_1^{[ab]}, N_1^\alpha),$$

$$N_1^{[ab]} = \tfrac{1}{2}(\eta_{bc}e^{a\mu,c\mu} - \eta_{ac}e^{b\mu,c\mu}) + \eta_{bc}e^{\mu[ak],\mu[ck]}$$

$$-\eta_{ac}e^{\mu[bk],\mu[ck]} + \eta_{bc}e^{[\mu\nu][ak],[\mu\nu][ck]}$$

$$-\eta_{ac}e^{[\mu\nu][bk],[\mu\nu][ck]} + \eta_{bc}e^{[ak][\mu\nu],[ck][\mu\nu]} - \eta_{ac}e^{[bk][\mu\nu],[ck][\mu\nu]}$$

$$+\tfrac{1}{2}(\sigma_{ab})^\beta_\alpha e^{\alpha\mu,\beta\mu} - \tfrac{1}{2}(\sigma_{ab})^\beta_\alpha e^{\alpha[\mu\nu],\beta[\mu\nu]} - \tfrac{1}{2}(\sigma_{ab})^\beta_\alpha e^{[\mu\nu]\beta,[\mu\nu]\alpha}, \tag{39}$$

$$N_1^\alpha = -\tfrac{1}{2}k(c^{-1}\gamma^n)_{\alpha\beta}e^{n\mu,\beta\mu} + \tfrac{1}{2}k^{-1}(\sigma_{mn})^\beta_\alpha e^{\beta\mu[mn]}$$

$$N_2^{K\mu} = (N_2^{\nu\mu}, 0, 0), \qquad N_2^{\nu\mu} = -\delta_{\mu\nu}e^{AA}, \tag{40}$$

$$N_3^{\mu K} = (N_3^{\mu\nu}, 0, 0),$$

$$N_3^{\mu\nu} = -e^{\mu k,\nu k} - e^{\mu[ab],\nu[ab]} - 2e^{[\mu\sigma][ab],[\nu\sigma][ab]}$$

$$-2e^{[ab][\mu\sigma],[ab][\nu\sigma]} - e^{\alpha\mu,\alpha\nu} - 2e^{\alpha[\mu\sigma],\alpha[\nu\sigma]} - 2e^{[\mu\sigma]\alpha,[\nu\sigma]\alpha}, \tag{41}$$

$$V^{\mu K} = (0, V^{\mu[ab]}, V^{\mu\alpha}),$$

$$V^{\mu[ab]} = e^{\mu[ab]}, \qquad V^{\mu\alpha} = k^{-1}e^{\alpha\mu}, \tag{42}$$

$\zeta^K(x) = (\zeta^\mu(x), \zeta^{[ab]}(x), \zeta^\alpha(x))$ are localized group parameters, ζ^μ define general coordinate transformations, $(x^{\mu'} = x^\mu + \zeta^\mu(x))$, $\zeta^{[ab]}(x)$ local Lorentz and $\zeta^\alpha(x)$ local supersymmetry transformations. We consider the invariance of Lagrangian (2) under supertransformations in the "1.5-formalism", i.e. we suggest $\dfrac{\delta S}{\delta\Psi_A}\,\delta\Psi_A(\zeta^\alpha) = 0$ at $A = \mu[kn]$, $[\mu\nu][kn]$, $[kn][\mu\nu]$, $\alpha[\mu\nu]$, $[\mu\nu]\alpha$. Lagrangian (2) varies under transformations (37) into a total derivative. Matrices (33)–(35) satisfy the corresponding invariance conditions.

The appearance of curvature tensor $R_{\mu\nu}{}^{ab}$ (22) and $\kappa^{ab}{}_{\mu\nu}$ (25) in the unified Ψ structure (26) is due to a greater degree of nonlinearities of tetrad formulation compared to metric formulation of the gravity theory.

Finally, expressions (34–35) for α^{μ}, α^{0}, M, and (26) for the unified Ψ structure completely determine Lagrangian and equations of the $N = 1$ SG in terms of universal metric equations and make it possible to use the earlier developed method in SG. This formulation implies further generalization on extended SG models.

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