GENERALISED FIELD THEORY AND KASNER UNIVERSE

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It is shown that the only Kasner-like solution of the GFT field equations with a nonzero electromagnetic field corresponds to an empty field geometry of the space-time. In this case, the electromagnetic field tensors of the theory coincide as could be expected from general considerations.

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1. Introduction

Generalised Field Theory (GFT; Refs [1, 2]) has so far yielded a unique model of the universe in the case of the static, spherically symmetric solution of its field equations and a reformulation of the theory of the electromagnetic field (Ref. [3]). Although both of the above conclusions are different from the results of standard theories in some respects and could, in principle, lead to empirically verifiable predictions, their immediate usefulness is questionable. Observational data in cosmology are, more often than not, theory--dependent and hence hardly useful as its test. Similarly, macrophysical aspects of the electromagnetic theory do not lend themselves readily to experimental investigation while the implications of the impact of the current reformulation on the microphysical domain remains an open problem. Barely first steps have been taken towards its possible solution (Ref. [4]). In other words, GFT remains a speculative theory however compelling the reasons, logical, mathematical and philosophical, for proposing it as the natural (and, hopefully, physically valid) extension of General Relativity.

These and other problems arising from investigations carried out since the publication of Ref. [1] will be further discussed elsewhere (Ref. [5]). We shall confine ourselves in this article to seeking for more general solutions of the GFT field equations. In particular, we shall consider the case when the metric of the background, Riemannian space is of Kasner type:

$$ds^2 = dt^2 - a_1^2 dx^2 - a_2^2 dy^2 - a_3^2 dz^2, \tag{1}$$

where a_i's are functions of t only. We shall find that again, as in the case of static spherical symmetry, we are led to a unique field structure. This may not be very realistic from a physical point of view (after all, it is doubtful whether Kasner universes are realistic either). However, and quite surprisingly, application to our result of the arguments outlined in Ref. [3] will lead to a further conceptual strengthening of GFT itself.

Now, the field equations of the latter are those of the nonsymmetric unified field theory of Einstein and Straus:

$$g_{\mu\nu,\lambda}-\tilde{\Gamma}^{\sigma}_{\mu\lambda}g_{\sigma\nu}-\tilde{\Gamma}^{\sigma}_{\lambda\nu}g_{\mu\sigma} = 0, \tag{2}$$

$$R_{(\mu\nu)}(\tilde{\Gamma}) = 0, \tag{3}$$

$$R_{[\mu\nu]}(\tilde{\Gamma}) = \tfrac{2}{3}(\Gamma_{\mu,\nu}-\Gamma_{\nu,\mu}), \tag{4}$$

and

$$\tilde{\Gamma}_{\mu} \equiv 0\,(= \mathfrak{g}^{\mu\nu}{}_{,\nu},\ \mathfrak{g}^{\mu\nu} = \sqrt{-g}g^{\mu\nu},\ \tilde{\Gamma}_{\mu} = \tilde{\Gamma}^{\sigma}_{[\mu\sigma]}) \tag{5}$$

(the last one is an equation in the Einstein-Straus theory and not an identity which here arises from the relation between the two "connections" of GFT

$$\tilde{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu}+\tfrac{2}{3}\,\delta^{\lambda}_{\mu}\Gamma_{\nu};$$

for reasons explained in the first two references, I call Γ the physical connection and $\tilde{\Gamma}$ the geometrical connection). (Greek indices go from 0 to 3 and Latin, when used, from 1 to 3.) In addition to the noted difference in the relation (5), we have also the definition of the space-time metric $a_{\mu\nu}$ $(= a_{\mu\nu})$ ("the metric hypothesis" or theorem), in requiring the symmetric part of the geometrical connection $\tilde{\Gamma}$ to be a Christoffel bracket constructed from the tensor $\boldsymbol{a}$:

$$\tilde{\Gamma}^{\lambda}_{(\mu\nu)} = \left\{ {\lambda \atop \mu\nu} \right\}_{a}. \tag{6}$$

An even more comprehensive theory than GFT can be obtained by explicitly including the definition (6) in the Lagrangian. It has been pointed out (Ref. [6]), however, that the resulting system of field equations is virtually unsolvable. The reason is the difficulty of deciding whether symmetry restrictions (and consequent simplifications) should be imposed on the metric $\boldsymbol{a}$ or on the field $\boldsymbol{g}$. It seems that only in the Kasner case, which we are going to consider, does the imposition of such simplifying conditions on the metric alone (with the assumption that the components of $\boldsymbol{g}$ are likewise only functions of t, which seems eminently reasonable) lead unambiguously to a complete solution of the problem.

We shall start with a general solution of the equation (2) in the Generalised Field Theory.

2. *Solution of equation (2) in GFT*

It has been shown by Mme Tonnelat (e.g. Ref. [7]) that, except when

$$\bar{g}(\bar{g}-2) = 0, \quad \bar{g} = \frac{\det g_{\mu\nu}}{\det g_{(\mu\nu)}},$$

equation (2) is algebraically solvable for the connection $\tilde{\Gamma}$ in terms of the tensor $\boldsymbol{g}$ and its first derivatives. The Tonnelat solution is considerably simplified in the case of GFT on account of the metric hypothesis (6) since the symmetric part of $\tilde{\Gamma}$ can now be regarded as known.

Let, then, a stroke denote covariant differentiation with respect to $\tilde{\Gamma}^{\lambda}_{(\mu\nu)}$ as affine connection. If we denote by $h_{\mu\nu}$ and $k_{\mu\nu}$ the symmetric and the skew-symmetric parts of $g_{\mu\nu}$ respectively,

$$g_{\mu\nu} = h_{\mu\nu}+k_{\mu\nu}, \quad h_{\mu\nu} = h_{\nu\mu}, \quad k_{\mu\nu} = -k_{\nu\mu}, \tag{7}$$

then

$$h_{\mu\nu|\lambda} = h_{\mu\nu,\lambda}-\tilde{\Gamma}^{\sigma}_{(\sigma\lambda)}h_{\sigma\nu}-\tilde{\Gamma}^{\sigma}_{(\nu\lambda)}h_{\mu\sigma} = \tilde{\Gamma}^{\sigma}_{[\mu\lambda]}k_{\sigma\nu}+\tilde{\Gamma}^{\sigma}_{[\lambda\nu]}k_{\mu\sigma}. \tag{8}$$

We easily see that permuting the indices cyclically and adding

$$h_{[\mu\nu|\lambda]} = 0. \tag{9}$$

Similarly,

$$k_{\mu\nu|\lambda} = k_{\mu\nu,\lambda}-\tilde{\Gamma}^{\sigma}_{(\mu\lambda)}k_{\sigma\nu}-\tilde{\Gamma}^{\sigma}_{(\nu\lambda)}k_{\mu\sigma} = \tilde{\Gamma}^{\sigma}_{[\mu\lambda]}h_{\sigma\nu}+\tilde{\Gamma}^{\sigma}_{[\lambda\nu]}h_{\mu\sigma}, \tag{10}$$

and

$$k_{\nu\lambda|\mu} = \tilde{\Gamma}^{\sigma}_{[\nu\mu]}h_{\sigma\lambda}+\tilde{\Gamma}^{\sigma}_{[\mu\lambda]}h_{\nu\sigma}; \quad k_{\lambda\mu|\nu} = \tilde{\Gamma}^{\sigma}_{[\lambda\nu]}h_{\sigma\mu}+\tilde{\Gamma}^{\sigma}_{[\nu\mu]}h_{\lambda\sigma}. \tag{11}$$

Hence, adding the last two equations and subtracting (10) as usual,

$$2\tilde{\Gamma}^{\sigma}_{[\nu\mu]}h_{\sigma\lambda} = k_{\nu\lambda|\mu}+k_{\lambda\mu|\nu}-k_{\mu\nu}|_{\lambda},$$

or, since $h_{\mu\nu}$ is necessarily nonsingular,

$$\tilde{\Gamma}^{\alpha}_{[\nu\mu]} = \tfrac{1}{2}\, h^{\alpha\lambda}(k_{\lambda\mu|\nu}+k_{\nu\lambda|\mu}-k_{\mu\nu|\lambda}).$$

Interchanging μ and ν and using skewsymmetry of $\boldsymbol{k}$

$$\tilde{\Gamma}^{\lambda}_{[\mu\nu]} = \tfrac{1}{2}\, h^{\lambda\sigma}(k_{\sigma\nu|\mu}+k_{\mu\sigma|\nu}+k_{\mu\nu|\sigma}).$$

The same relations, of course, hold also in the Tonnelat solution but now, because

$$\tilde{\Gamma}^{\lambda}_{(\mu\nu)} = \begin{Bmatrix} \lambda \\ \mu\nu \end{Bmatrix}_{a} \tag{12}$$

is known for given tensors $\boldsymbol{a}$ and $\boldsymbol{g}$, our task is completed. We may notice that the same procedure applied to equation (8) gives

$$\tilde{\Gamma}^{\lambda}_{(\mu\nu)} = \begin{Bmatrix} \lambda \\ \mu\nu \end{Bmatrix}_{h} + h^{\lambda\sigma}(\tilde{\Gamma}^{\alpha}_{[\mu\lambda]}k_{\alpha\nu}+\tilde{\Gamma}^{\alpha}_{[\lambda\nu]}k_{\mu\alpha})$$

because

$$\tilde{\Gamma}_{\mu} = 0 = h^{\lambda\sigma}k_{\mu\lambda|\sigma},$$

whence

$$\left\{ {\sigma \atop \mu\sigma} \right\}_a = \left\{ {\sigma \atop \mu\sigma} \right\}_h, \tag{13}$$

or

$$\sqrt{-h} \propto \sqrt{-a}, \tag{14}$$

where $h = \det h_{\mu\nu}$, $a = \det a_{\mu\nu}$. We cannot, in particular, choose $h_{\mu\nu} = a_{\mu\nu}$ without imposing an intolerable restriction $\tilde{\Gamma}^{\alpha}_{[\mu\lambda]}k_{\alpha\nu} + \tilde{\Gamma}^{\alpha}_{[\lambda\nu]}k_{\mu\alpha} = 0$ on the nonsymmetric affine connection.

Finally, we may also note that the skewsymmetric part of the Ricci tensor constructed from the connection $\tilde{\Gamma}^{\lambda}_{\mu\nu}$ and given by equation (4) as proportional to the curl of the vector Γ_μ (contracted skew part of the "physical" connection Γ) is also given by

$$R_{[\mu\nu]}(\tilde{\Gamma}) = -\tilde{\Gamma}^{\sigma}_{[\mu\nu],\sigma} + \tilde{\Gamma}^{\sigma}_{(\mu\varrho)}\tilde{\Gamma}^{\varrho}_{[\sigma\nu]} + \tilde{\Gamma}^{\sigma}_{(\varrho\nu)}\tilde{\Gamma}^{\varrho}_{[\mu\sigma]} - \tilde{\Gamma}^{\sigma}_{(\sigma\varrho)}\tilde{\Gamma}^{\varrho}_{[\mu\nu]} = -\tilde{\Gamma}^{\sigma}_{[\mu\nu]|\sigma}. \tag{15}$$

In an n-dimensional space Schrödinger's relation between Γ and $\tilde{\Gamma}$ must be replaced by

$$\tilde{\Gamma}^{\lambda}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} + \frac{1}{n-1}\,\delta^{\lambda}_{\mu}\Gamma_\nu \tag{16}$$

when, again,

$$\tilde{\Gamma}_\mu \equiv 0.$$

3. Solution of equation (2) in the Kasner case

Let us now return to the case when the metric of the space-time is given by equation (1) and the components of the field g are also functions of time only. Then, of course, the only nonzero Christoffel brackets are (no summation)

$$\left\{ {0 \atop ii} \right\} = a_i \dot{a}_i, \qquad \left\{ {i \atop 0i} \right\} = \frac{\dot{a}_i}{a_i}, \tag{17}$$

with the dot denoting differentiation with respect to time.

If we now write out in full (given in the Appendix) equations (8), we immediately find by inspection that

$$h_{01} = h_{02} = h_{03} = 0 \quad \text{and} \quad h_{00} = \text{constant} \tag{18}$$

unless the background space-time is flat. Then, however, we easily obtain equations

$$\dot{h}_{ii} - 4\frac{\dot{a}_i}{a_i}h_{ii} - 2a_i\dot{a}_i h_{00} = 0, \tag{19}$$

and

$$\dot{h}_{ij} - 2\left(\frac{\dot{a}_i}{a_i} + \frac{\dot{a}_j}{a_j}\right)h_{ij} = 0, \tag{20}$$

again without summation. Hence the components $h_{\mu\nu}$ of the tensor $\boldsymbol{h}$ are fully determined in the form

$$h_{ii} = -h_{00}a_i^2 + c_i a_i^4 \tag{21}$$

and

$$h_{ij} = k_k a_i^2 a_j^2, \quad (i, j, k \text{ cyclic } 1, 2, 3) \tag{22}$$

where c_1, c_2, c_3; k_1, k_2, k_3 are constants of integration. Moreover, we obtain additional equations

$$\tilde{\Gamma}^{\sigma}_{[0i]} k_{\sigma i} = -c_i a_i^3 \dot{a}_i, \tag{23}$$

$$\tilde{\Gamma}^{\sigma}_{[0i]} k_{\sigma 0} = 0, \quad \tilde{\Gamma}^{\sigma}_{[ij]} k_{\sigma i} = \tilde{\Gamma}^{\sigma}_{[ij]} k_{\sigma j} = 0 \quad (i \neq j), \tag{24}$$

where only a summation over σ is understood.

If we similarly write out equations (10) we find that the only nonzero components of the skewsymmetric part of the affine connection are

$$\tilde{\Gamma}^3_{[01]} = \frac{k_3 a_2^2}{k_{23}} a_1 \dot{a}_1, \quad \tilde{\Gamma}^2_{[01]} = -\frac{k_2 a_3^2}{k_{23}} a_1 \dot{a}_1,$$

$$\tilde{\Gamma}^3_{[02]} = \frac{k_3 a_1^2}{k_{13}} a_2 \dot{a}_2, \quad \tilde{\Gamma}^1_{[02]} = -\frac{k_1 a_3^2}{k_{13}} a_2 \dot{a}_2,$$

$$\tilde{\Gamma}^2_{[03]} = \frac{k_2 a_1^2}{k_{12}} a_3 \dot{a}_3, \quad \tilde{\Gamma}^1_{[03]} = -\frac{k_1 a_2^2}{k_{12}} a_3 \dot{a}_3, \tag{25}$$

when

$$\tilde{\Gamma}^0_{[12]} = -\frac{\dot{a}_1}{a_1} k_{12} + \frac{a_1 a_2^2 a_3^2 \dot{a}_1}{k_{23}} P_2,$$

$$\tilde{\Gamma}^0_{[13]} = -\frac{\dot{a}_1}{a_1} k_{13} + \frac{a_1 a_2^2 a_3^2 \dot{a}_1}{k_{23}} P_3,$$

$$\tilde{\Gamma}^0_{[23]} = -\frac{\dot{a}_2}{a_2} k_{23} + \frac{a_1^2 a_2 a_3^2 \dot{a}_2}{k_{13}} P_3, \tag{26}$$

where

$$P_i = h_{00} k_i + (k_j k_k - k_i c_i) a_i^2, \tag{27}$$

and

$$k_{01} = k_{02} = k_{03} = 0. \tag{28}$$

(Notice that the P_i notation appears somewhat clumsy in equation (26); this, however, is immaterial since complete symmetry of the notation will be restored below so that there is no point in trying to restore it at this stage.) The solution now proceeds by finding conditions for the equations

$$R_{(ij)}\tilde{\Gamma} = 0,$$

or

$$\tilde{\Gamma}^{\sigma}_{[i\varrho]}\tilde{\Gamma}^{\varrho}_{[\sigma j]} = 0 \quad i \neq j \tag{29}$$

to be identically satisfied. Somewhat tedious but elementary algebra shows that this is going to be the case providing

$$k_{31}k_{23} = -a_1^2a_2^2a_3^2P_3 \tag{30}$$

and

$$\frac{k_{31}}{k_{12}} = \frac{P_3}{P_2}, \tag{31}$$

so that we can equivalently have (30) and

$$k_{12}k_{23} = -a_1^2a_2^2a_3^2P_2. \tag{31'}$$

However, there remain still three differential equations for k_{12}, k_{31} and k_{23} amongst equations (10) which must be satisfied under these conditions. These now acquire the symmetric form

$$\frac{k_{31}}{a_1^2a_3^2}\left(\dot{k}_{12}-2\frac{\dot{a}_1}{a_1}k_{12}\right) = -2a_2\dot{a}_2P_1,$$

$$\frac{k_{23}}{a_2^2a_3^2}\left(\dot{k}_{31}-2\frac{\dot{a}_3}{a_3}k_{31}\right) = -2a_1\dot{a}_1P_3,$$

$$\frac{k_{12}}{a_1^2a_2^2}\left(\dot{k}_{23}-2\frac{\dot{a}_2}{a_2}k_{23}\right) = -2a_3\dot{a}_3P_2. \tag{32}$$

In the next Section, we shall obtain a complete solution of thesee quations and, a fortiori, of the field equations (2) through (6) as well. Before we do so, however, let us observe that if one of the P factors vanishes, say $P_i = 0$, then the corresponding $k_i = 0$ so that $k_j = 0$ and therefore $P_j = 0$ as well. In other words, if one of the P's vanishes, then another must vanish as well.

4. *Solution of the field equations*

Let us suppose first that

$$P_i \neq 0. \tag{33}$$

Then the last two of equations (32) immediately give (because of (30) and (31'))

$$k_{31} = \gamma_2a_1^2a_3^2, \quad k_{23} = \gamma_1a_2^2a_3^2, \quad \gamma_1, \gamma_2 \text{ const.},$$

so that equation (30) requires

$$\gamma_1\gamma_2 a_1^2 a_2^2 a_3^4 = -a_1^2 a_2^2 a_3^2[h_{00}k_3+(k_1k_2-k_3c_3)a_3^2],$$

where either $k_3 = 0$ and $\gamma_1\gamma_2 = -k_1k_2$, or $a_3 =$ constant.
(*i*) If $a_3 =$ constant, then either $k_{12} = 0$ or $k_{23} = \bar{\gamma}_1 a_2^2$, $\bar{\gamma}_1 = \gamma_1 a_3^2$.
a) If $a_3 =$ constant, $k_{12} = 0$, then $P_2 = 0$ which is a contradiction.
b) If $a_3 =$ constant, $k_{23} = \bar{\gamma}_1 a_2^2$, $k_{31} = \bar{\gamma}_2 a_1^2 = \gamma_2 a_1^2 a_3^2$, then

$$k_{12} = -\frac{h_{00}k_2}{\gamma_1}a_1^2 - \frac{(k_1k_3-k_2c_2)}{\gamma_1}a_1^2a_2^2 = \alpha_1 a_1^2+\alpha_2 a_1^2 a_2^2, \text{ say.}$$

The first of equations (32) now gives $a_2 =$ constant, $k_{23} =$ constant and so, as we can easily verify, only two components $\tilde{\Gamma}^{\lambda}_{[\mu\nu]}$ survive, namely

$$\tilde{\Gamma}^3_{[01]} = \frac{k_3 a_2^2}{k_{23}}a_1\dot{a}_1 \quad \text{and} \quad \tilde{\Gamma}^2_{[01]} = -\frac{k_2 a_3^2}{k_{23}}a_1\dot{a}_1.$$

Moreover, the equation $R_{00} = 0$ now gives $a_1 = \alpha+\beta t$ (α, β constant) and, by equation (15), there is no electromagnetic field.

In the same way, we can verify that the case $k_3 = 0$, $\gamma_1\gamma_2 = -k_1k_2$ leads to vanishing of the electromagnetic field as well (in both these conclusions we may recall that, in GFT, the electromagnetic intensity field is identified as proportional to $R_{[\mu\nu]}(\tilde{\Gamma})$).
(*ii*) Similarly, if $k_3 = 0$, $\gamma_1\gamma_2 = -k_1k_2$, we easily show that either a_1 or a_2 is necessarily constant and again we get a zero electromagnetic field.

It now follows that at least two of the factors P_i must vanish. Let us suppose that

$$P_1 = P_2 = k_1 = k_2 = 0. \tag{34}$$

Then $k_{12} = 0$ and $k_{31} = \gamma_2 a_1^2 a_3^2$, $\gamma_2 k_{23} = -k_3 a_2^2(h_{00}-c_3a_3^2)$. Therefore

$$\tilde{\Gamma}^3_{[01]} = -\frac{\gamma_2 a_1\dot{a}_1}{h_{00}-c_3a_3^2}, \quad \tilde{\Gamma}^3_{[02]} = -\frac{k_3}{\gamma_2}\frac{a_2\dot{a}_2}{a_3^2},$$

but

$$h_{00}\tilde{\Gamma}^0_{[13]} = 0 = \frac{\dot{a}_1}{a_1}\gamma_2 a_1^2 a_3^2+\gamma_2\frac{a_1\dot{a}_1}{h_{00}-c_3a_3^2}(-h_{00}a_3^2+c_3a_3^4) = \gamma_2 a_1^2 a_3\dot{a}_3. \tag{35}$$

Hence, either

$$\gamma_2 = 0 \tag{36}$$

or a_3 is a constant. We have already seen that the latter possibility leaves us with no electromagnetic field. Therefore equation (36) must be assumed to hold but then $P_3 = 0$ implying $k_3 = 0$ and, with

$$a_2 = a_3 = a, \tag{37}$$

the only nonzero component of the skewsymmetric part of the affine connection is

$$\tilde{\Gamma}^0_{[23]} = -\frac{1}{h_{00}}\frac{\dot{a}}{a}k_{23}. \tag{38}$$

The functions a_1 and a are now determined by equations (3) which become

$$\frac{\ddot{a}_1}{a_1}+2\frac{\ddot{a}}{a} = 0,$$

$$a_1\ddot{a}_1+2\frac{a_1}{a}\dot{a}_1\dot{a} = 0,$$

$$(a\dot{a})^{\cdot}+\frac{a}{a_1}\dot{a}\dot{a}_1 = 0. \tag{39}$$

We easily find that these are satisfied by

$$a_1 = c_1(t-t_0)^{-1/3}, \quad a = a_2 = a_3 = c(t-t_0)^{2/3}, \tag{40}$$

where c_1, c and t_0 are constants, the first two of which can be absorbed into a scaling of space-like coordinates.

Furthermore, only one of the components of the skew part of the Ricci tensor (equation (4)) now survives in the form

$$R_{[23]} = A, \quad \text{a constant.} \tag{41}$$

This equation, finally, determines $g_{[23]} = k_{23}$, which, with B being another constant, becomes

$$k_{23} = \tfrac{3}{2}A(t-t_0)^2+B(t-t_0)^{4/3}. \tag{42}$$

A straightforward calculation shows that the Russell-Klotz tensor

$$w_{\mu\nu} = a^{\alpha\beta}k_{\mu\nu|\alpha\beta} \tag{43}$$

is then given by

$$w_{23} = -2A. \tag{44}$$

5. *Conclusion*

We have shown that the only Kasner-like solution of the GFT field equations which allows a nonzero electromagnetic field corresponds to an empty field geometry of space-time, that is to the solution of the general relativistic field equations

$$R_{\mu\nu} = 0.$$

The standard formula for the energy-momentum tensor (Ref. [1])

$$T_{\mu\nu} = -\frac{1}{k} G_{(\mu\nu)}\left(\left\{{\alpha \atop \beta\gamma}\right\}\right)$$

fails in this case since it gives

$$T_{\mu\nu} = 0,$$

while we know that there is a nonzero energy of the electromagnetic field. Of course, the reason for this is that, in the highly pathological situation of the current model, gravity and electromagnetism become separated from each other. Energy of the latter must therefore be calculated from the Maxwell energy-stress-momentum tensor.

On the other hand, the empty field structure of geometry implies that the two electromagnetic field tensors, namely the Russell-Klotz and the GFT intensity tensors, should coincide. That this, indeed, is the case (constant proportionality or units factors apart) can therefore be regarded as a very satisfactory feature of the theory.

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APPENDIX

$$h_{\mu\nu,\lambda} - \tilde{\Gamma}^{\sigma}_{(\mu\lambda)}h_{\sigma\nu} - \tilde{\Gamma}^{\sigma}_{(\lambda\nu)}h_{\mu\sigma} = \tilde{\Gamma}^{\sigma}_{[\mu\lambda]}k_{\sigma\nu} + \tilde{\Gamma}^{\sigma}_{[\lambda\nu]}k_{\mu\sigma}.$$

$$h_{00,0} = 0,$$

$$h_{00,1} - 2\tilde{\Gamma}^{1}_{(01)}h_{01} = 2\tilde{\Gamma}^{\sigma}_{[01]}k_{\sigma 0},$$

$$h_{00,2} - 2\tilde{\Gamma}^{2}_{(02)}h_{02} = 2\tilde{\Gamma}^{\sigma}_{[02]}k_{\sigma 0},$$

$$h_{00,3} - 2\tilde{\Gamma}^{3}_{(03)}h_{03} = 2\tilde{\Gamma}^{\sigma}_{[03]}k_{\sigma 0};$$

$$h_{01,0} - \tilde{\Gamma}^{1}_{(01)}h_{01} = \tilde{\Gamma}^{\sigma}_{[01]}k_{0\sigma},$$

$$h_{01,1} - \tilde{\Gamma}^{1}_{(01)}h_{11} - \tilde{\Gamma}^{0}_{11}h_{00} = \tilde{\Gamma}^{\sigma}_{[01]}k_{\sigma 1},$$

$$h_{01,2} - \tilde{\Gamma}^{2}_{(02)}h_{21} = \tilde{\Gamma}^{\sigma}_{[02]}k_{\sigma 1} + \tilde{\Gamma}^{\sigma}_{[21]}k_{0\sigma},$$

$$h_{01,3} - \tilde{\Gamma}^{3}_{(03)}h_{31} = \tilde{\Gamma}^{\sigma}_{[03]}k_{\sigma 1} + \tilde{\Gamma}^{\sigma}_{[31]}k_{0\sigma};$$

$$h_{02,0} - \tilde{\Gamma}^{2}_{(02)}h_{02} = \tilde{\Gamma}^{\sigma}_{[02]}k_{0},$$

$$h_{02,1} - \tilde{\Gamma}^{1}_{(01)}h_{12} = \tilde{\Gamma}^{\sigma}_{[01]}k_{\sigma 2} + \tilde{\Gamma}^{\sigma}_{[12]}k_{0\sigma},$$

$$h_{02,2} - \tilde{\Gamma}^{2}_{(02)}h_{22} - \tilde{\Gamma}^{0}_{22}h_{00} = \tilde{\Gamma}_{[02]}k_{\sigma 2},$$

$$h_{02,3}-\tilde{\Gamma}^3_{(03)}h_{32} = \tilde{\Gamma}^\sigma_{[03]}k_{\sigma 2}+\tilde{\Gamma}^\sigma_{[32]}k_{0\sigma};$$

$$h_{03,0}-\tilde{\Gamma}^3_{(03)}h_{03} = \tilde{\Gamma}^\sigma_{[03]}k_{0\sigma},$$

$$h_{03,1}-\tilde{\Gamma}^1_{(03)}h_{13} = \tilde{\Gamma}^\sigma_{[01]}k_{\sigma 3}+\tilde{\Gamma}^\sigma_{[13]}k_{0\sigma},$$

$$h_{03,2}-\tilde{\Gamma}^2_{(02)}h_{23} = \tilde{\Gamma}^\sigma_{[02]}k_{\sigma 3}+\tilde{\Gamma}^\sigma_{[23]}k_{0\sigma},$$

$$h_{03,3}-\tilde{\Gamma}^3_{(03)}h_{33}-\tilde{\Gamma}^0_{33}h_{00} = \tilde{\Gamma}^\sigma_{[03]}k_{\sigma 3};$$

$$h_{11,0}-2\tilde{\Gamma}^1_{(01)}h_{11} = 2\tilde{\Gamma}^\sigma_{[10]}k_{\sigma 1},$$

$$h_{11,1}-2\tilde{\Gamma}^0_{11}h_{01} = 0 = h_{01},$$

$$h_{11,2} = 2\tilde{\Gamma}^\sigma_{[12]}k_{\sigma 1},$$

$$h_{11,3} = 2\tilde{\Gamma}^\sigma_{[13]}k_{\sigma 1};$$

$$h_{12,0}-\tilde{\Gamma}^1_{(10)}h_{12}-\tilde{\Gamma}^2_{(02)}h_{12} = \tilde{\Gamma}^\sigma_{[10]}k_{\sigma 2}+\tilde{\Gamma}^\sigma_{[02]}k_{1\sigma},$$

$$h_{12,1}-\tilde{\Gamma}^0_{11}h_{02} = \tilde{\Gamma}^\sigma_{[12]}k_{1\sigma},$$

$$h_{12,2}-\tilde{\Gamma}^0_{22}h_{10} = \tilde{\Gamma}^\sigma_{[12]}k_{\sigma 2},$$

$$h_{12,3} = \tilde{\Gamma}^\sigma_{[13]}k_{\sigma 2}+\tilde{\Gamma}^\sigma_{[32]}k_{1\sigma};$$

$$h_{13,0}-\tilde{\Gamma}^1_{(10)}h_{13}-\tilde{\Gamma}^3_{(03)}h_{13} = \tilde{\Gamma}^\sigma_{[10]}k_{\sigma 3}+\tilde{\Gamma}^\sigma_{[03]}k_{1\sigma},$$

$$h_{13,1}-\tilde{\Gamma}^0_{11}h_{03} = \tilde{\Gamma}^\sigma_{[13]}k_{1\sigma},$$

$$h_{13,2} = \tilde{\Gamma}^\sigma_{[12]}k_{\sigma 3}+\tilde{\Gamma}^\sigma_{[23]}k_{1\sigma},$$

$$h_{13,3}-\tilde{\Gamma}^0_{33}h_{10} = \tilde{\Gamma}^\sigma_{[13]}k_{\sigma 1};$$

$$h_{22,0}-2\tilde{\Gamma}^2_{(02)}h_{22} = 2\tilde{\Gamma}^\sigma_{[20]}k_{\sigma 2},$$

$$h_{22,1} = 2\tilde{\Gamma}^\sigma_{[21]}k_{\sigma 2},$$

$$h_{22,2}-2\tilde{\Gamma}^0_{22}h_{02} = 0 = h_{02},$$

$$h_{22,3} = 2\tilde{\Gamma}^\sigma_{[23]}k_{\sigma 2};$$

$$h_{23,0}-\tilde{\Gamma}^2_{(20)}h_{23}-\tilde{\Gamma}^3_{(03)}h_{23} = \tilde{\Gamma}^\sigma_{[20]}k_{\sigma 3}+\tilde{\Gamma}^\sigma_{[03]}k_{2\sigma},$$

$$h_{23,1} = \tilde{\Gamma}^\sigma_{[21]}k_{\sigma 3}+\tilde{\Gamma}^\sigma_{[13]}k_{2\sigma},$$

$$h_{23,2}-\tilde{\Gamma}^0_{22}h_{03} = \tilde{\Gamma}^\sigma_{[23]}k_{2\sigma},$$

$$h_{23,3}-\tilde{\Gamma}^0_{33}h_{20} = \tilde{\Gamma}^\sigma_{[23]}k_{\sigma 3};$$

$$h_{33,0}-2\tilde{\Gamma}^3_{(03)}h_{33} = 2\tilde{\Gamma}^\sigma_{[30]}k_{\sigma 3},$$

$$h_{33,1} = 2\tilde{\Gamma}^\sigma_{[31]}k_{\sigma 3},$$

$$h_{33,2} = 2\tilde{\Gamma}^{\sigma}_{[32]}k_{\sigma 3},$$

$$h_{33,3}-2\tilde{\Gamma}^{0}_{33}h_{03} = 0 = h_{03}.$$

$$h_{01} = h_{02} = h_{03} = 0, \quad h_{00} = \text{const.}$$

$$k_{01,0}-\tilde{\Gamma}^{1}_{(01)}k_{01} = \tilde{\Gamma}^{\sigma}_{[01]}h_{0\sigma} = \tilde{\Gamma}^{0}_{[01]}h_{00},$$

$$k_{01,1} = \tilde{\Gamma}^{\sigma}_{[01]}h_{\sigma 1},$$

$$k_{01,2}-\tilde{\Gamma}^{2}_{(02)}k_{21} = \tilde{\Gamma}^{\sigma}_{[02]}h_{\sigma 1}+\tilde{\Gamma}^{0}_{[21]}h_{00},$$

$$k_{01,3}-\tilde{\Gamma}^{3}_{(03)}k_{31} = \tilde{\Gamma}^{\sigma}_{[03]}h_{\sigma 1}+\tilde{\Gamma}^{0}_{[31]}h_{00};$$

$$k_{02,0}-\tilde{\Gamma}^{2}_{(02)}k_{02} = \tilde{\Gamma}^{\sigma}_{[02]}h_{0\sigma} = \tilde{\Gamma}^{0}_{[02]}h_{00},$$

$$k_{02,1}-\tilde{\Gamma}^{1}_{(01)}k_{12} = \tilde{\Gamma}^{\sigma}_{[01]}h_{\sigma 2}+\tilde{\Gamma}^{0}_{[12]}h_{00},$$

$$k_{02,2} = \tilde{\Gamma}^{\sigma}_{[02]}h_{\sigma 2},$$

$$k_{02,3}-\tilde{\Gamma}^{3}_{(03)}k_{32} = \tilde{\Gamma}^{\sigma}_{[03]}h_{\sigma 2}+\tilde{\Gamma}^{0}_{[32]}h_{00};$$

$$k_{03,0}-\tilde{\Gamma}^{3}_{(03)}k_{03} = \tilde{\Gamma}^{0}_{[03]}h_{00},$$

$$k_{03,1}-\tilde{\Gamma}^{1}_{(01)}k_{13} = \tilde{\Gamma}^{\sigma}_{[01]}h_{\sigma 3}+\tilde{\Gamma}^{0}_{[13]}h_{00},$$

$$k_{03,2}-\tilde{\Gamma}^{2}_{(02)}k_{23} = \tilde{\Gamma}^{\sigma}_{[02]}h_{\sigma 3}+\tilde{\Gamma}^{0}_{[22]}h_{00},$$

$$k_{03,3} = \tilde{\Gamma}^{\sigma}_{[03]}h_{\sigma 3};$$

$$k_{12,0}-\tilde{\Gamma}^{1}_{(10)}k_{12}-\tilde{\Gamma}^{2}_{(02)}k_{12} = \tilde{\Gamma}^{\sigma}_{[10]}h_{\sigma 2}+\tilde{\Gamma}^{\sigma}_{[02]}h_{1\sigma},$$

$$k_{12,1}-\tilde{\Gamma}^{0}_{11}k_{02} = \tilde{\Gamma}^{\sigma}_{[12]}h_{1\sigma},$$

$$k_{12,2}-\tilde{\Gamma}^{0}_{22}k_{10} = \tilde{\Gamma}^{\sigma}_{[12]}h_{\sigma 2},$$

$$k_{12,3} = \tilde{\Gamma}^{\sigma}_{[13]}h_{\sigma 2}+\tilde{\Gamma}^{\sigma}_{[32]}h_{1\sigma};$$

$$k_{13,0}-\tilde{\Gamma}^{1}_{(10)}k_{13}-\tilde{\Gamma}^{3}_{(03)}k_{13} = \tilde{\Gamma}^{\sigma}_{[10]}h_{\sigma 3}+\tilde{\Gamma}^{\sigma}_{[03]}h_{1\sigma},$$

$$k_{13,1}-\tilde{\Gamma}^{0}_{11}k_{03} = \tilde{\Gamma}^{\sigma}_{[13]}h_{1\sigma},$$

$$k_{13,2} = \tilde{\Gamma}^{\sigma}_{[12]}h_{\sigma 3}+\tilde{\Gamma}^{\sigma}_{[23]}h_{1\sigma},$$

$$k_{13,3}-\tilde{\Gamma}^{0}_{33}k_{10} = \tilde{\Gamma}^{\sigma}_{[13]}h_{\sigma 3};$$

$$k_{23,0}-\tilde{\Gamma}^{2}_{(20)}k_{23}-\tilde{\Gamma}^{3}_{(03)}k_{23} = \tilde{\Gamma}^{\sigma}_{[20]}h_{\sigma 3}+\tilde{\Gamma}^{\sigma}_{[03]}h_{2\sigma},$$

$$k_{23,1} = \tilde{\Gamma}^{\sigma}_{[21]}h_{\sigma 3}+\tilde{\Gamma}^{\sigma}_{[13]}h_{2\sigma},$$

$$k_{23,2}-\tilde{\Gamma}^{0}_{22}k_{03} = \tilde{\Gamma}^{\sigma}_{[23]}h_{2\sigma},$$

$$k_{23,3}-\tilde{\Gamma}^{0}_{33}k_{20} = \tilde{\Gamma}^{\sigma}_{[23]}h_{\sigma 3}.$$