RAPIDITY DISTRIBUTION AND DECAY ANGULAR DISTRIBUTION OF CLUSTERS IN THE GIOVANNINI--VAN HOVE MODEL

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The data on particle production in $p\bar{p}$ collisions at 540 GeV c.m. energy are fitted using Giovannini-Van Hove model with identical clusters. Rapidity distribution of clusters as well as angular distribution in their decay are determined. The corresponding widths (in (pseudo)rapidity) are 5.2 and 2.6, respectively.

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It was recently discovered [1] that in $p\bar{p}$ collisions at 540 GeV c.m. energy the multiplicity of charged particles produced in (pseudo)rapidity intervals

$$|\eta| \leqslant \eta_c \tag{1}$$

follow the negative binomial distribution:

$$P(n, \langle n\rangle, k) = \frac{\Gamma(n+k)}{\Gamma(k)\Gamma(n)}\left(\frac{\langle n\rangle/k}{1+\langle n\rangle/k}\right)^n (1+\langle n\rangle/k)^{-k}, \tag{2}$$

where $\langle n\rangle$ is the average multiplicity and k is a parameter. Both $\langle n\rangle$ and k increase with increasing size of the intervals η_c.

Giovannini and Van Hove proposed interpretation of these data in terms of a new cluster model [2]. They observed that independent production of clusters of particles leads

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to negative binomial distribution (2), provided the distribution in cluster decay is of the form:

$$W(n) = \frac{-1}{\ln(1-b)} \frac{b^n}{n}, \qquad 0 < b < 1, \tag{3}$$

where the parameter b is related to the average multiplicity n_c in cluster decay by:

$$n_c = \frac{-b}{(1-b)\ln(1-b)}. \tag{4}$$

The parameters of the resulting negative binomial distribution are given in terms of n_c and the average number of clusters $\bar{N}$ by:

$$\langle n \rangle = \bar{N} n_c, \tag{5}$$

$$k = -\frac{\bar{N}}{\ln(1-b)}. \tag{6}$$

From these formulae both $\bar{N}$ and n_c were determined from experimental data in each rapidity interval considered [2]. Both $\bar{N}$ and n_c increase with increasing η_c and show a tendency to saturation at η_c close to the kinematic limit. This effect was qualitatively explained in Ref. [2]: with increasing η_c more clusters contribute particles into interval (1) (and thus $\bar{N}$ grows) and more particles from a given cluster fall into it (and thus n_c grows).

The purpose of the present note is to develop these qualitative observations into quantitative statements about the distributions of clusters in rapidity and the angular distribution in cluster decay. To this end we consider a simple extension of the model of Ref. [2] by assuming that all clusters decay identically with the angular distribution given by:

$$\chi(\eta) = (2\omega \cosh^2 \eta/\omega)^{-1}, \qquad \int \chi(\eta) d\eta = 1, \tag{7}$$

where η is the (pseudo)rapidity and ω is a free parameter determining the width d of the distribution, $d = 1.64$ ($\omega = 1$ for isotropic decay).

For the (pseudo)rapidity distribution of clusters we take the shape suggested by bremsstrahlung analogy and longitudinal phase-space [3, 4]:

$$\frac{dN}{dy} = \lambda(1-x^+)^\lambda (1-x^-)^\lambda \qquad \text{with} \qquad x^\pm = \frac{m}{\sqrt{s}} e^{\pm y}, \tag{8}$$

where λ is a parameter (plateau height), m is the (transverse) mass of the cluster, y is the cluster (pseudo)rapidity and $\sqrt{s}$ is the total c.m. energy.

Our task is (i) to check if the data can be reproduced with these new assumptions and (ii) to determine the parameters of the model from the data and thus to determine quantitative properties of the clusters. We proceed by calculating the generating function $\phi(z, \eta_c)$ of the probability distribution for n particles falling into (pseudo)rapidity interval (1). This is done as follows [5]:

We observe that probability that in a given event we have N clusters with i-th located in the (pseudo)rapidity interval $[y_i, y_i+dy_i]$ and decaying into n_i particles is equal to:

$$\frac{1}{N!}e^{-\bar{N}}\prod_{i=1}^{N}\frac{dN}{dy_i}dy_i W(n_i). \tag{9}$$

Denoting by $p(\eta_c, y)$ the probability that a particle from a cluster produced at (pseudo) rapidity y falls into interval (1) we obtain:

$$\frac{e^{-\bar{N}}}{N!}\prod_{i=1}^{N}\frac{dN}{dy_i}dy_i W(n_i)\binom{n_i}{k_i}[p(\eta_c, y_i)]^{k_i}[1-p(\eta_c, y_i)]^{n_i-k_i} \tag{10}$$

for the probability that a given collision produces N clusters with i-th one located in the interval $[y_i, y_i+dy_i]$ decaying into n_i particles with exactly k_i ones falling into the interval (1). Multiplying the last formula by $z^n = z^{\Sigma k_i}$, integrating over y_i and summing over all possible $N > 0$, $n_i \geqslant k_i \geqslant 0$ one obtains the generating function:

$$\phi(z, \eta_c) = \sum_{n=0}^{\infty} P(n, \eta_c)z^n$$

$$= \exp\left\{\frac{1}{\ln(1-b)}\int\frac{dN}{dy}\ln\left[1-\frac{bp(\eta_c, y)(z-1)}{1-b}\right]dy\right\}. \tag{11}$$

Using (7) we obtain the following formula for $p(\eta_c, y)$:

$$p(\eta_c, y) = \int_{-\eta_c}^{\eta_c}\chi(\eta-y)d\eta = \frac{\sin(2\eta_c/\omega)}{\cosh(2\eta_c/\omega)+\cosh(2y/\omega)}. \tag{12}$$

Eq. (11) expresses the measured quantities in terms of the parameters of the model. We have calculated the resulting probability distributions $P(n, \eta_c)$ using the relation:

$$P(n, \eta_c) = n!\left.\frac{d^n\phi(z, \eta_c)}{dz^n}\right|_{z=0} \tag{13}$$

and employing the Cauchy formula for numerical estimate of derivatives of $\phi(z, \eta_c)$. The results are described below.

First, we have noticed that although the model does not give exactly the negative binomial distribution for $P(n, \eta_c)$, the departures are significant only for small n ($n \lesssim 3$) provided that both distributions have the same average and dispersion. This is illustrated in Fig. 1 where results of our calculations are compared with experimental data and with the results of the negative binomial fit of Ref. [1].

The experimental data are best described by the following set of parameters:

$$\lambda = 0.855, \quad m = 3.15 \text{ GeV}, \quad \omega = 1.45, \quad b = 0.90. \tag{14}$$

This gives $n_c = 3.94$ and $\bar{N} = 7.25$. In Fig. 2 $\langle n \rangle$ and in Fig. 3 the parameter k calculated from the formula:

$$k = \frac{\langle n \rangle^2}{\langle n^2 \rangle - \langle n \rangle^2 - \langle n \rangle} \tag{15}$$

are compared with the data of Ref. [1]. One sees good agreement, except perhaps at the end of phase space in Fig. 3. Finally, in Fig. 4 the cluster decay distribution (7) is plotted versus (pseudo)rapidity. One sees that the clusters turn out to be fairly broad, as already anticipated in Ref. [2].

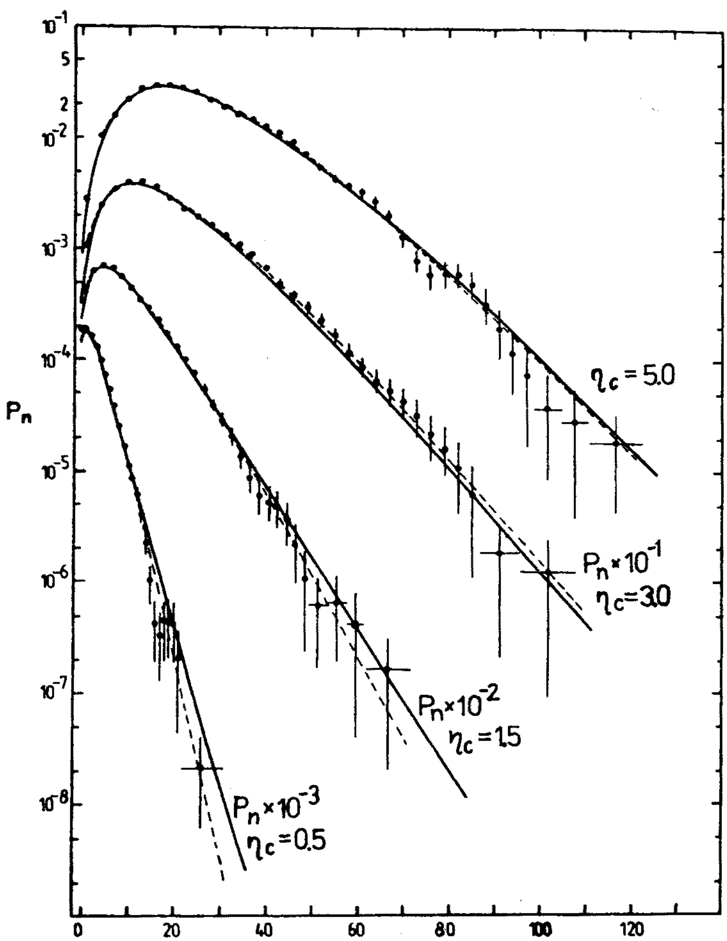

Fig. 1. Multiplicity distribution following from Eq. (11) compared with the data and fit of Ref. [2]

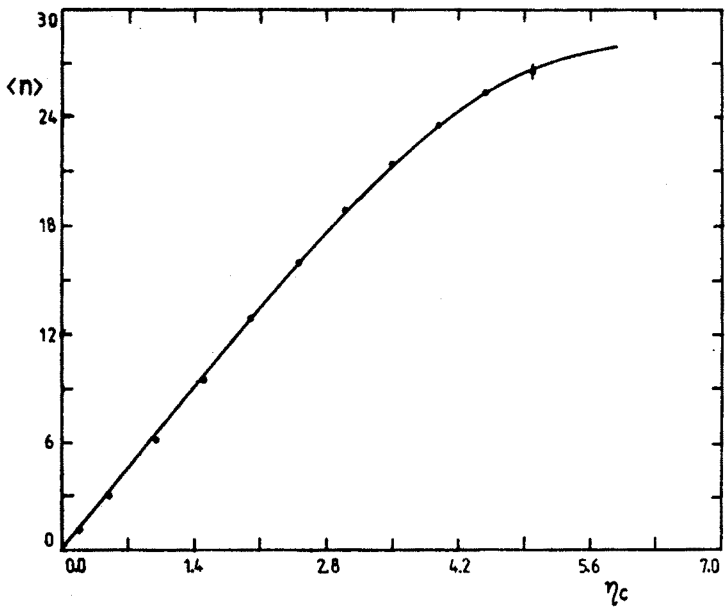

Fig. 2. $\langle n \rangle$ versus η_c compared with the data of Ref. [2]

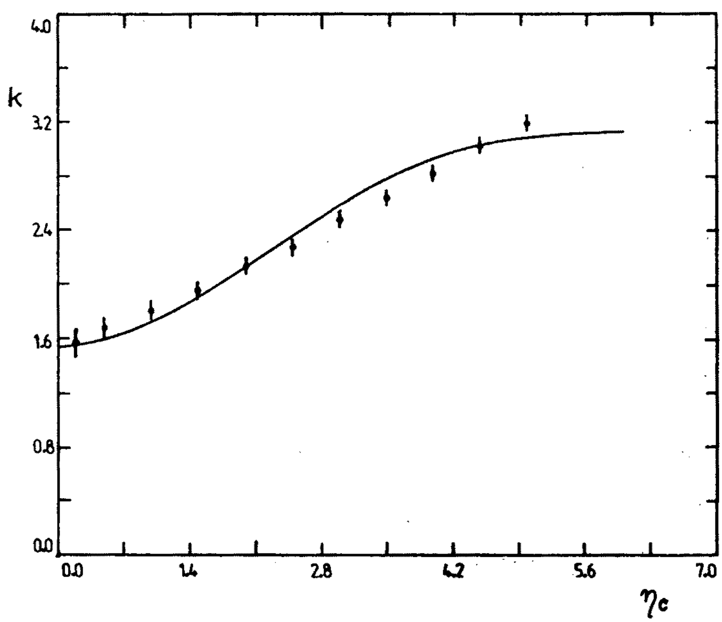

Fig. 3. Parameter k calculated from Eq. (15) compared with the data of Ref. [2]

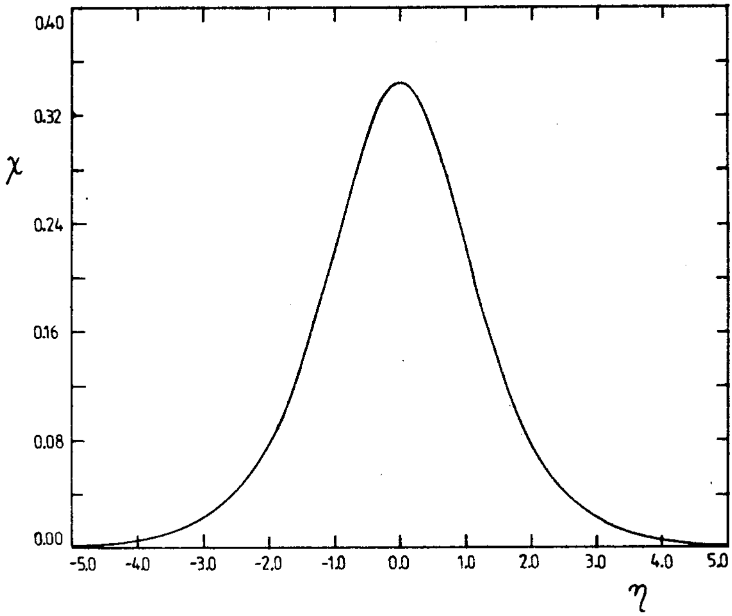

Fig. 4. Pseudorapidity cluster decay distribution

In conclusion, we have shown that a simple version of the Giovannini-Van Hove model with identical clusters produced according to longitudinal phase-space does explain quantitatively the data of Ref. [1]. As a bonus we obtain determination of cluster parameters which confirm the qualitative expectations of Ref. [2].

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