THE SHARP RADIUS, EFFECTIVE INTERACTION RANGE AND THE VOLUME INTEGRAL OF THE SHELL MODEL POTENTIAL

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An analysis of the central part of the shell model potential in terms of the sharp radius, effective interaction range and the volume integral, for nuclei with $36 \leqslant A \leqslant 65$ is given. A comparison of the A-behaviour of these quantities obtained in this analysis with that characterizing the real part of the optical model potential is made. For nuclei investigated the linear dependence of the volume integral per nucleon on the binding energy of the last proton is observed.

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The intuitive assumption that the real part of the optical model potential (OMP) can be represented by the shell model potential (SMP), modified properly to include the energy dependence, could be a good approximation for not very light nuclei for which the introduction of an additional level in the continuum (elastically scattered nucleon) does not influence very much the average nuclear potential. This way of obtaining the real part of the OMP is very appealing since the bound state problem is much easier to solve than the elastic scattering problem. The SMP obtained by reproducing energies of the single particle states near the Fermi surface could be regarded as the real part of the OMP at zero energy (in this region the depth of the single-particle potential is energy independent [1]). If so, the energy dependence of the OMP should be deduced [2]. In this paper we present the results of an analysis of the shell model potentials (SMPs) of the $1f_{7/2}$ shell nuclei obtained by Malaguti [3] performed in the way followed usually for the optical model potentials (OMPs).

It has been shown that from an elastic scattering experiment two quantities of the real part of the OMPs are determined accurately, namely: the volume integral ($J_V = \int V(r)d^3r$) and the effective sharp radius $\left(R_V = \left\{\frac{1}{V_0} J_V/\frac{4}{3}\pi\right\}^{1/3}\right)$ of the potential.

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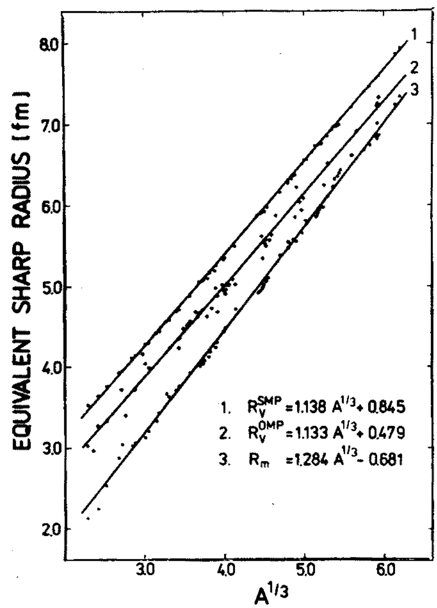

Fig. 1. The mass dependence of the equivalent sharp radius of the shell model potential (*1*), the real part of the optical model potential (*2*) and the mass distribution in nuclei (*3*)

The effective sharp radii of the SMPs, R_V^{SMP}, following from the formula proposed by Malaguti: $R_V^{SMP} = C(1+x^2-x^4)$, where $C = (1.1\, A^{1/3}+0.75)\left(\frac{A}{2Z}\right)^{0.137}$ and $x^2 = \frac{\pi^2}{3}\frac{0.52^2}{C^2}$ [3], are shown in Fig. 1 as a function of $A^{1/3}$. These radii have been calculated for the nuclei investigated by Malaguti, and additionally for those for which the root mean square (rms) charge radii have been summarized by Brown et al. [4]. We found the following linear dependence to occur:

$$R_V^{SMP} = 1.138(3)A^{1/3}+0.845(13). \quad (1)$$

The dependence found for the nuclei investigated by Malaguti [3] does not differ significantly from the one above. In Fig. 1 the effective sharp radii of the real part of the OMP are also shown for comparison. These radii were calculated for the best fit potentials obtained for each nucleus contained in the Perey and Perey [5] work and taken at the energy closest to 40 MeV. In this case the dependence has the form

$$R_V^{OMP} = 1.133(17)A^{1/3}+0.479(76), \quad (2)$$

which is obviously very close to the forms found by other authors (e.g. Myers [6]: $R_V^{OMP} = 1.16\, A^{1/3}+0.45$, Srivastava et al. [7]: $R_V^{OMP} = 1.13\, A^{1/3}+0.55$). By comparing (1) and (2) we see that the factors at $A^{1/3}$ are almost the same and it is only the second terms that are different. This difference may be due to an energy dependence of the sharp radius of the potential. Let the sharp radius of the potential be

$$R_V = 1.138(3)A^{1/3}+0.845(13)-f(E). \tag{3}$$

The form of the $f(E)$ term is unknown. All what we know is that the sharp radius decreases by 0.366(77) fm between $E = 0$ MeV and $E = 40$ MeV and that for $E > 25$ MeV its energy dependence is expected to be very weak [7].

In the frame of the folding model (Greenlees et al. [8], Srivastava et al. [9]) the equivalent sharp radius of the potential is written as

$$R_V = R_m+\Delta. \tag{4}$$

In Fig. 1 the calculated equivalent sharp radii of matter distributions, R_m, in the nuclei are also shown. In these calculations we started from the experimental values of the rms radii of the charge distribution, $\langle r^2\rangle_{ch}^{1/2}$ and obtained the rms radii of the point proton $\langle r^2\rangle_p^{1/2} = \left(\langle r^2\rangle_{ch}+0.1128\frac{N}{Z}-0.6893\right)^{1/2}$ and point neutron $\langle r^2\rangle_n^{1/2} = \langle r^2\rangle_p^{1/2}+\Delta\mathrm{RMS}$ distributions, where ΔRMS have been calculated according to Myers and Świątecki [10] with $Q = 24$ MeV. Next, these two distributions were added $\langle r^2\rangle_m = \frac{Z}{A}\langle r^2\rangle_p+\frac{N}{A}\langle r^2\rangle_n$ and from the resulting rms radii of point matter distribution the sharp radii were found: $\langle r^2\rangle_m = \frac{3}{5}Q_x^2$, $Q_x = R(1+\frac{5}{2}x^2-\frac{21}{8}x^4)$, $x = \frac{b}{R}$, $b = 1$ fm. The gross feature of the dependence of sharp radii on $A^{1/3}$ can be expressed as

$$R_m = 1.284(8)A^{1/3}-0.681(40). \tag{5}$$

As should be expected the simple proportionality between R_m and $A^{1/3}$, usually assumed in this kind of analyses, does not hold. The same is true for two different procedures of calculating rms radii of matter distribution: $\langle r^2\rangle_m = \langle r^2\rangle_{ch}$, $R_m = 1.200\ (9)\ A^{1/3} -0.295\ (46)$; $\langle r^2\rangle_m = \langle r^2\rangle_{ch}-0.64$, $R_m = 1.228\ (10)\ A^{1/3}-0.536\ (47)$.

Combining (5) and (3) we get

$$R_V = R_m+1.526(42)-0.146(9)A^{1/3}-f(E). \tag{6}$$

The quantity Δ (Eq. (4)) which represents the effective sharp radius of the density dependent effective interaction is not constant and depends on the mass number and energy. The range of this effective interaction as a function of the mass number for $E = 0$ is shown in Fig. 2. This dependence (obtained for $b_v = b_m = 1$ fm)

$$\begin{aligned}\langle r^2\rangle_v &\equiv \langle r^2\rangle_V-\langle r^2\rangle_m\\ &= 0.15+2.203A^{1/3}-0.212A^{2/3}+0.099A^{-2/3}-1.074A^{-1}\end{aligned} \tag{7}$$

is close to the one obtained by us earlier for the OMPs [11].

The volume integrals per nucleon, $A^{-1}J_V$, of the SMPs obtained by Malaguti [3] are shown in Fig. 3a as a function of the mass number. In this limited region of A we observe

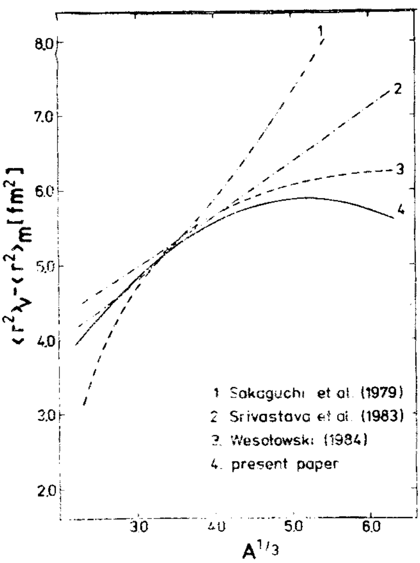

Fig. 2. The mass dependence of the nucleon-nucleon effective interaction range obtained from the analyses of the optical model potentials by Sakaguchi et al. [19], Srivastava et al. [9], Wesołowski [11] and from the analysis of the shell model potentials (present paper)

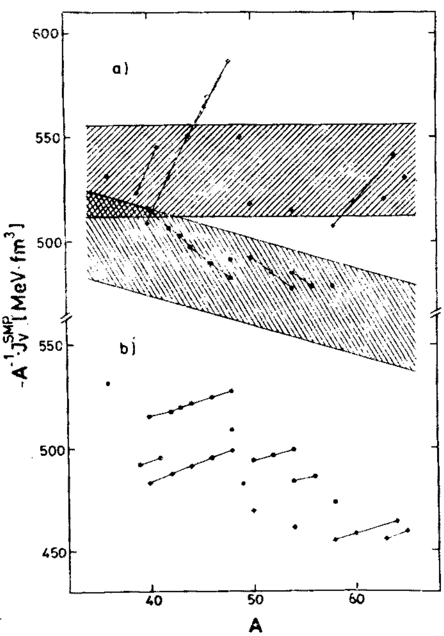

Fig. 3. The mass dependence of the volume integral per nucleon of the shell model potential of Malaguti [3] (a) and of its isoscalar part only (b). The shaded areas represent the results of the JLM model calculations [14], [15] with the error bars ±2%. The crosses and dots represent the proton and neutron potentials, respectively

a picture similar to that obtained by Fabrici et al. [12] and Rapaport [13] for the OMPs. The volume integral per nucleon for the proton SMPs does not depend on A for nuclei close to the β-stability valley with the mean value of about 530 ± 20 MeV fm^3. This value should be compared with that of 500 ± 15 MeV fm^3 obtained by Rapaport [13] or $510 \pm$ ± 20 MeV fm^3 derived from Fabrici et al. data [12]: $410 \pm 20 + 2.9\,E$ (= 35 MeV). The volume integral per nucleon calculated for the neutron SMPs shows a decreasing tendency similar to that presented by Rapaport for the neutron OMPs. The isoscalar part of the volume integral per nucleon of the proton and neutron SMPs (Fig. 3b) is a decreasing function of A with the slope close to that calculated for the proton OMPs by Jeukenne et al. [14] (we used the parametrization given by Gupta and Murthy [15]). The absolute values of the isoscalar part obtained for neutrons are larger than those for protons in this region of masses but this difference seems to decrease with A. These phenomena can be traced back to the negative neutron skin (neutrons are bound stronger than protons) observed for nuclei from this region.

Exploiting the inverse methods Hefter showed in his paper on the mass dependence of the real OMP [16] that the volume integral per nucleon is proportional to the square root of the mean binding energy of the nucleon in a nucleus. We could not observe such an "average" behaviour because of the limited A-region investigated. Instead, we observe a fine structure — the linear dependence of the proton and neutron volume integral per nucleon vs. the square root of the binding energy of the "last" proton (not neutron), $S_p^{1/2}$, (Fig. 4). This is true also for the isoscalar part of the volume integral. A similar linear

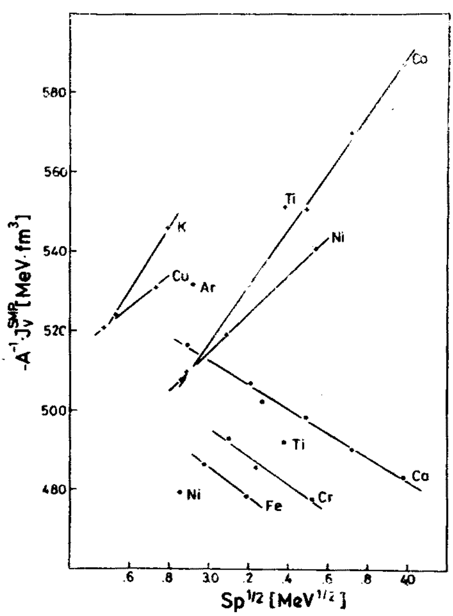

Fig. 4. The dependence of the volume integral per nucleon of the shell model potential of Malaguti [3] on the square-root of the binding energy of the last proton in the nucleus

dependence is observed for those quantities as functions of the nuclear asymmetry parameter $\varepsilon = \frac{N-Z}{A}$ (Fig. 5). Therefore, the linear dependence of $A^{-1}J_V^{SMP}$ on $S_p^{1/2}$ can be traced to the dependence of the binding energy of the single particle states on ε [17] or on the nuclear isospin $T_0 = \frac{1}{2}(N-Z)$ [18].

In conclusion, the quantities such as the equivalent sharp radius, the effective interaction range and the volume integral per nucleon calculated for the shell model potentials, which describe the proton and neutron single particle states near the Fermi surface in nuclei

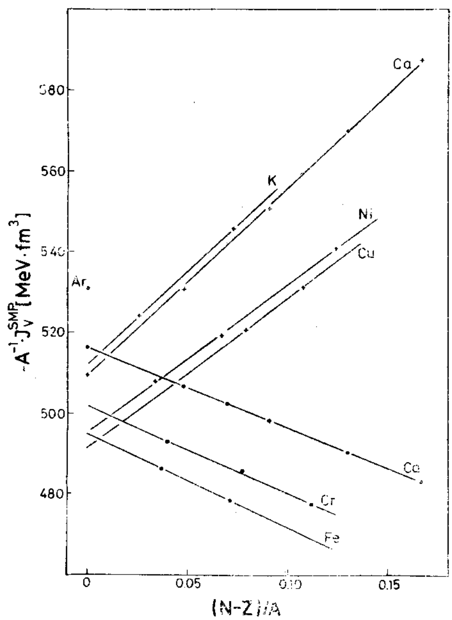

Fig. 5. The dependence of the volume integral per nucleon of the shell model potential of Malaguti [3] on the nuclear asymmetry parameter

with $36 < A < 65$, show a very similar (practically the same) behaviour as functions of the mass number as those calculated for the real part of the optical model potentials obtained for protons and neutrons of 10–50 MeV energy. The equivalent sharp radii of the shell model potentials are larger than those of the optical model potentials, obtained at energy of ca. 40 MeV, by about 0.36 fm what can reflect their energy dependence. The absolute values of the volume integral of the SMPs are close to those expected from an extrapolation of the optical model potential values to zero energy with the energy dependence found for energies of 15–60 MeV. Also the linear dependence of these values on the asymmetry parameter and on the square root of the last proton binding energy is observed. It is desirable to check these conclusions by an analysis based on the SMPs valid in the whole range of mass number.

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