THE NEW DIAGONALIZATION PROCEDURE IN THE INTERACTING BOSON MODEL AND ITS APPLICATION*

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(Received April 3, 1986)

The recently constructed complete basis of states in the Interacting Boson Model has been adopted to the calculation of matrix elements of relevant physical operators. The new numerical program has been written for the IBM analysis. The program has been tested by the calculating energy levels and boson eigenstates of Gd-Xe isotopes. The boson states have been used to calculate transition probabilities $E2$ for the same isotopes. Only one-parameter calculations fairly well reproduce experimental data.

PACS numbers: 21.60.Gx

1. Introduction

Interacting Boson Model (IBM) had been introduced by Iachello [1] and then developed by Arima and Iachello [2–5] in the field of nuclear low-energy phenomena. The model has already gained a significant success in both single-particle and collective behaviour of nuclei.

In special, although important, cases like rotational and vibrational limits of the IBM, the model provided final analytical formula for energies and transitional probabilities. Some other approximate methods to calculate the boson operator matrix elements are also known [7–8]. However, general application of the IBM needs a large computer code to deal with physical observables in a specially suited basis and such program developed by Scholten [6] is referred to as PHINT.

The aim of the paper is to adopt the alternative scheme recently considered [9–10] for the diagonalization of the boson physical operators including the general boson Hamiltonian (IBM-1)

$$H = H_1 + H_2 + H_3, \tag{1}$$

* Supported in part by the contract CPBP 01.06.

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with

$$H_1 = H_0 + \varepsilon_s s^+ s + \sum_m d_m^+ d_m + \frac{u_0}{2} s^+ s^+ ss$$

$$+ \frac{u_2}{\sqrt{5}} s^+ s (d^+ \tilde{d})^0 + \sum_{L=0,2,4} c_L \sqrt{(2L+1)}\, [(d^+ d^+)^L (\tilde{d}\tilde{d})^L]^0,$$

$$H_2 = \frac{v_0}{2} (d^+ d^+)^0 ss + s^+ s^+ (\tilde{d}\tilde{d})^0,$$

$$H_3 = \frac{v_2}{\sqrt{2}} \{[(d^+ d^+)^2 (\tilde{d}s)^2]^0 + [(d^+ s^+)^2 (\tilde{d}\tilde{d})^2]^0\},$$

where $s_{l=0}^+ \equiv s^+$ and $d_{l=2,m}^+ \equiv d_m^+$ are the creation operators of monopole and quadrupole bosons with their annihilation operators s and $\tilde{d}_m = (-1)^m d_{-m}$. H_0 is a constant part of a nuclear core.

We have taken into consideration only one kind of bosons but the method presented here can be easily extended to the IBM-2 version of neutron and proton bosons. The method can be regarded as an alternative way, in a way simpler, for boson numerical calculations.

The presented procedure is based on the results of the works [9–10] whose formulas are slightly extended for numerical application. We will not repeat the proofs of the results given in [9–10].

In the following three sections we present the construction of the boson complete and orthonormal vector-basis essentially equivalent to the results given in [9–10], the results of calculation of matrix elements of boson physical operators, and we give the test of a new numerical scheme for the IBM calculations.

2. The basis for a numerical boson program

The 36 boson second-order operators

$$b_{lm}^+ b_{lm}; \quad l = 0, 2; \quad m = -l, \ldots, l \tag{2}$$

are known [2] to form generators for the transformation group U(6). We adopt here the chain of subgroups of the vibrational limit of the IBM [2]

$$\underset{[N]}{\mathrm{SU}(6)} \supset \underset{[n]}{\mathrm{SO}(5)} \times \underset{[v]}{\mathrm{SU}(1,1)} \supset \underset{[L]}{\mathrm{SO}(3)} \tag{3}$$

with the physical numbers which distinguish irreducible representations of the relevant subgroups: N — being the total number of bosons; n — the number of quadrupole bosons; v — the seniority number of quadrupole bosons; L — the total angular quantum number.

The complete orthonormal vector basis reads

$$|NnvxLM\rangle = \frac{1}{\sqrt{(N-n)!}}(s^+)^{N-n}|nvxLM\rangle_0, \tag{4}$$

where $|nvxLM\rangle_0$ is the complete orthonormal quadrupole-boson basis and x — is the additional quantum number for a complete classification which is taken here as a maximal number of scalar quadrupole-boson triplets $(d^+d^+d^+)^{L=0}$ in the state (4).

For a given symmetric irreducible representation $[N]$ of the group SU(6) the rest of quantum numbers follow the changes

$$\begin{gathered} n = 0, 1, \ldots, N \\ v = n, n-2, \ldots, 0 \text{ or } 1 \\ x = 0, 1, \ldots, v/3 \\ (v-3x) \leqslant L \leqslant 2(v-3x) \quad \text{but} \quad L \neq 2(v-3x)-1. \end{gathered} \tag{5}$$

The construction of the basis $|nvxLM\rangle_0$ for quadrupole bosons has been done in several steps [10].

(*i*) First, we take $n = v$ and construct the complete but non-orthogonal vectors $|vvxLM\rangle$

$$|vvxLM\rangle = \sum_{n_{-2},\ldots,n_2} (n_{-2} \ldots n_2|vxLM)\,(d^+_{-2})^{n_{-2}} \ldots (d^+_2)^{n_2}|0\rangle. \tag{6}$$

We adopt in (6) the non-normalized initial basis $(d^+_{-2})^{n_{-2}} \ldots (d^+_2)^{n_2}|0\rangle$ contrary to the formulas (15)–(16) of [10] and hence the transformation coefficients $(n_{-2}, \ldots, n_2|vxLM)$ are changed by a numerical factor $(n_{-2}! \ldots n_2!)^{1/2}$. We use also the recurrence formula (27) of [10] to get the analytical expression for the general transformation coefficients in (6) which reads

$$\begin{aligned} (n_{-2} \ldots n_2|vxLM) &= (2L+1)\,(\sqrt{6})^{n_0}[(L+M)!(L-M)!]^{1/2} \\ &\times [(L-v+3x)!(L+v-3x)!]^{1/2} \sum_{k,p_1,\ldots,p_8} \binom{v-x}{p_1 \ldots p_8} x! \\ &\times (-1)^{k+p_1-p_4-p_6+p_7+n_{-1}+n_1} 2^{p_1+p_3-p_4-p_5+p_6+p_8+n_{-1}+n_1} \\ &\times \frac{(k+p_2+p_3+2p_5+p_7+3p_8)!}{(n_{-2}-p_1)!(n_{-1}-p_3-p_4)!(n_0-p_2-p_7)\,(n_1-p_5-p_6)!(n_2-p_8)!} \\ &\times \frac{(L-M+k+p_2-p_3-p_4-3p_5-2p_6-p_7-3p_8+n_{-1}+2n_0+3n_1+4n_2)!}{k!(L-M-k)!(L+3x-v-k)!(M+v-3x+k)!} \\ &\times \frac{1}{(L+2v-p_4-p_5+1)!}, \end{aligned} \tag{7}$$

where

$$\sum_i n_i = v; \quad \sum_i i n_i = M; \quad \sum_k p_k = v-x.$$

It was also proved [14] that the basis (6) can be alternatively expressed by the Hill-Wheeler type of an integral [12], namely

$$|vvxLM\rangle = (2L+1) \int_{SO(3)} D^{(L)}_{M3x-v}(\Omega)\hat{R}(\Omega)\,|vx\rangle, \tag{8}$$

where $\hat{R}(\Omega)$ is the rotation operator with Euler angles $\Omega \equiv (\alpha\beta\gamma)$, $D^{(L)}_{M,k}(\Omega)$ is the usual Wigner spherical function and [11]

$$\int_{SO(3)} \equiv \frac{1}{8\pi^2} \int_0^{2\pi} d\alpha \int_0^{\pi} \sin\beta d\beta \int_0^{\pi} d\gamma, \tag{9}$$

while

$$|vx\rangle = (d^+_{-1})^{v-x}(d^+_2)^x|0\rangle. \tag{10}$$

(*ii*) The basis (6) is complete but non-orthogonal. Hence, the numerical standard procedure has been used to get the orthonormal basis of state-vectors

$$|vvxLM\rangle_0 = \sum_{x'=\left[\frac{v-L+2}{3}\right]}^{x} C^{vL}_{xx'}|vvx'LM\rangle, \tag{11}$$

where the transformation coefficients $C^{vL}_{xx'}$ are numerically calculated.
(*iii*) To get the vector state with a given number of quadrupole bosons $n > v$, one needs only to apply the quasi-spin boson operator to the state (11)

$$\hat{\mathscr{K}}^+ = \frac{\sqrt{5}}{2}(d^+d^+)^{L=0} \tag{12}$$

what changes a number of quadrupole bosons. Then, final normalizing factor $\mathscr{N}$ is also needed. Hence,

$$|nvxLM\rangle_0 = \mathscr{N}(n, v)\,(\hat{\mathscr{K}}^+)^{\frac{n-v}{2}}\,|vvxLM\rangle. \tag{13}$$

Formula (13) completes the construction of the orthonormal boson basis (4) where

$$\mathscr{N}(n, v) = 2^{-\frac{n-v}{2}} \left(\frac{n-v}{2}\right)! \frac{(n+v+3)!!}{(2v+3)!!}.$$

3. *Matrix elements of the IBM Hamiltonian and an electromagnetic quadrupole transition operator*

In the Hamiltonian (1)

$$H = H_1 + H_2 + H_3.$$

H_1 is already diagonal in the basis $|NnvxLM\rangle$, H_2 has non-vanishing matrix elements only for $n' = n \pm 2$, and H_3 — only for $n' = n \pm 1$.

The diagonal part of H reads:

$$\langle NnvxLM|H|NnvxLM\rangle = \langle NnvxLM|H_1|NnvxLM\rangle$$

$$= \varepsilon_s(N-n)+\varepsilon_d n+\frac{u_0}{2}(N-n)(N-n-1)$$

$$+\frac{u_2}{\sqrt{5}}n(N-n)+\frac{3c_4-10c_2+7c_0}{70}(n-v)(n+v-1)$$

$$+\frac{6c_4+8c_2}{14}n(n-1)+\frac{c_4-c_2}{14}[L(L+1)-6n]. \tag{14}$$

The non-diagonal parts of H reduce to H_2 and H_3 only:

$$\langle Nn\pm 2vx'LM|H|NnvxLM\rangle = \langle Nn\pm 2vx'LM|H_2|NnvxLM\rangle \tag{15}$$

and

$$\langle Nn\pm 1v'x'LM|H|NnvxLM\rangle = \langle Nn\pm 1v'x'LM|H_3|NnvxLM\rangle. \tag{16}$$

We proceed to sketch the calculation of the matrix elements (15)–(16). We notice in (15) that the operators $(d^+d^+)^0$ and $(dd)^0$ are the generators of the quasi-spin boson group SU(1, 1) and by the Wigner theorem we get:

$$\langle Nn' = n\pm 2vxLM|H|NvxLM\rangle = \frac{v_0}{2\sqrt{5}}[(N-n\mp 1)(N-n+2\delta_{n',n-2})]^{1/2}$$

$$\times[(n-v+1\pm 1)(n+v+4\pm 1)]^{1/2}. \tag{17}$$

The s-part of the matrix element (17) has been simply fixed because $s^+(s)$ boson operators are the phonon creation (annihilation) operators of one-dimensional spherical harmonic oscillator.

In the matrix element (16) the quadrupole operators are scalars in the rotational group SO(3) and tensors of the rank 3/2 in the quasi-spin group SU(1, 1)

$$[d^+(\tilde{d}\tilde{d})^2]^0 = \frac{1}{\sqrt{3}}T^{(3/2,0)}_{-1/2},$$

$$[(d^+d^+)^2\tilde{d}]^0 = \frac{1}{\sqrt{3}}T^{(3/2,0)}_{1/2}. \tag{18}$$

Hence, in the first step, we apply the Wigner-Eckart theorem to operators (18) and replace the s-operators in H_3 by their simple matrix elements, which gives

$$\langle Nn' = n\pm 1v'x'LM|H|NnvxLM\rangle$$

$$= \frac{v_2}{\sqrt{6}}(N-n-\delta_{n,n-1})^{1/2}(KK_0\,\tfrac{3}{2},\pm\tfrac{1}{2}|K'K'_0)$$

$$\times\langle v'x'LM\parallel T^{(3/2,0)}\parallel vxLM\rangle_{SU(1,1)}, \tag{19}$$

where

$$K = \tfrac{1}{2}(v+5/2), \quad K_0 = \tfrac{1}{2}(n+5/2) \tag{20}$$

are the quasi-spin quantum numbers in the Clebsch-Gordan coefficient of the group SU(1, 1) [13].

In the second step, using the formula (8), we get:

$$\langle nv'x'LM|T^{(q,0)}_{q0}|vvxLM\rangle = \langle nv'x'L3x-v|T^{(q,0)}_{q0}|vx\rangle \tag{21}$$

and after straightforward calculation we obtain the final analytical matrix elements reduced in the SU(1, 1):

$$\langle v+3x'LM \parallel T^{(3/2,0)} \parallel vxLM\rangle_{SU(1,1)}$$

$$= 3\left[\frac{3(2v+3)}{35(2v+9)}(v-x'+1)!\,(x-1)!\right]^{1/2} x'(v-x+3)$$

$$\times(v-x'+1)\,(0, v-x'+1, 0, 0, x'-1|vxL3x'-v-3) \tag{22a}$$

$$\langle v+1x'LM \parallel T^{(3/2,0)} \parallel vxLM\rangle_{SU(1,1)}$$

$$= 3(v-x'+1)\left[\frac{(2v+1)\,(2v+3)}{35(2v+5)\,(2v+7)}\,x'!(v-x')!\right]^{1/2}[2\sqrt{x'}$$

$$\times(0, v-x', 0, 1, x-1|vxL3x'-v-1)-\sqrt{v-x'}$$

$$\times(1, v-x'-1, 0, 0, x'|vxL3x'-v-1)] \tag{22b}$$

$$\langle v-1xLM \parallel T^{(3/2,0)} \parallel vxLM\rangle_{SU(1,1)}$$

$$= -3(v-x)\left[\frac{2v-1}{35(2v+5)}\,x!(v-x-1)!\right]^{1/2}[2\sqrt{x}$$

$$\times(0, v-x-1, 0, 1, x-1|v-1x'L3x-v)-\sqrt{v-x-1}$$

$$\times(1, v-x-2, 0, 0, x|v-1x'L3x-v)] \tag{22c}$$

$$\langle v-3x'LM \parallel T^{(3/2,0)} \parallel vxLM\rangle_{SU(1,1)}$$

$$= -3[3(v-x-2)!(x-1)!/35]^{1/2}x(v-x)\,(v-x-1)$$

$$\times(0, v-x-2, 0, 0, x-1|v-3xL3x'-v). \tag{22d}$$

The formulae complete all non-vanishing matrix elements which can be used to solve any eigenvalue problem within the IBM model for given even-nuclei. The number of bosons (N) is the half of the valence nucleons and the set of Hamiltonian-parameters are chosen to fit the experimental excited levels. Hence, the vector-eigenstates of an energy ε are

obtained in the form:

$$|N\varepsilon LM\rangle = \sum_{\alpha} c_{\alpha}^{(\varepsilon L)} |N\alpha LM\rangle, \tag{23}$$

where α stands for (nvx) and $C_{\alpha}^{(\varepsilon L)}$ are numerically fixed.

We proceed now to evaluate the matrix elements of the electromagnetic quadrupole operator which in the IBM approximation reads:

$$T(E2)_m = \beta_1(s^+\tilde{d}_m + d_m^+ s) + \beta_2(d^+\tilde{d})_m^2. \tag{24}$$

The reduced transition probability between the states (23) is usually defined as

$$B(E2: L\varepsilon \to L'\varepsilon') = (2L+1)^{-1} |\langle N\varepsilon' L' \| T(E2) \| N\varepsilon L\rangle|^2. \tag{25}$$

To calculate the reduced matrix elements in (25), we transform the quadrupole transition operator (24) to an explicit tensor form in both groups SU(1, 1) and SO(3)

$$T(E2)_m = \beta_1(s^+ T_{-1/2,m}^{(1/2,2)} + s T_{1/2,m}^{(1/2,2)}) + \beta_2 \frac{1}{\sqrt{2}} T_{0,m}^{(1,2)}. \tag{26}$$

The reduced transition probability is then calculated in the basis (23) in which the further reduction in the group SU(1, 1) is performed:

$$\begin{aligned} B(E2: L\varepsilon \to L'\varepsilon') = (2L+1)^{-1} \Bigg\{ & \sum_{\alpha\alpha'} C_{\alpha}^{(\varepsilon L)} C_{\alpha'}^{(\varepsilon' L')} \\ & \times [\beta_1 \delta_{q,1/2}(\delta_{q_0,1/2}\sqrt{N-n} + \delta_{q_0,-1/2}\sqrt{N-n'}) \\ & + \beta_2 \frac{1}{\sqrt{2}} \delta_{q,1}\delta_{q_0,0}] (KK_0, q, q_0 | K'K_0')_{SU(1,1)} \\ & \times \langle v'x'L' \| T^{(q,2)} \| vxL\rangle_{SU(1,1)\otimes SO(3)} \Bigg\}^2, \end{aligned} \tag{27}$$

where

$$\begin{aligned} q &= \tfrac{1}{2}, 1; \quad q_0 = \pm\tfrac{1}{2}, 0, \\ \alpha &= (n, v, x); \quad \alpha' = (n', v', x'), \\ K &= \tfrac{1}{2}(v+\tfrac{5}{2}); \quad K' = \tfrac{1}{2}(v'+\tfrac{5}{2}), \\ K_0 &= \tfrac{1}{2}(n+\tfrac{5}{2}); \quad K_0' = \tfrac{1}{2}(n'+\tfrac{5}{2}). \end{aligned}$$

The reduced matrix element in (27) is then evaluated in two steps. First, with the help of (8), the matrix element of the transition operator is transformed:

$$\begin{aligned} \langle nv'x'L'M' | T_{q_0,m}^{(q,2)} | vvxLM\rangle = (LM2m|L'M') \left(\frac{2L+1}{2L'+1}\right)^{1/2} \sum_{\mu} (-1)^{\mu} \\ \times (2, -\mu, L'3x-v+\mu | L3x-v) \langle nv'x'L'3x-v+\mu | T_{q_0,\mu}^{(q,2)} | vx\rangle. \end{aligned} \tag{28}$$

Then, on the left-hand side of (28) we perform the reduction in two groups SO(3) and SU(1, 1) and on the right-hand side we explicitly evaluate the simplified matrix element of the operator $T^{(q,2)}_{q_0,2}$. The sum over μ in (28) is then taken and all the needed reduced matrix elements of the operator $T^{(q,2)}$ can be put in the explicit analytical form with the known transformation coefficients:

$$\langle v-1xL \,\|T^{(1/2,0)}\| \, vxL\rangle_{SU(1,1)\otimes SO(3)}$$

$$= [(2L+1)\,(v-x)!x!]^{1/2}[\sqrt{v-x}\,(2,\ -1L'3x-v+1|L3x-v)$$

$$\times(0,\ v-x-1,\ 0,\ 0,\ x|v-1,\ x'L'3x-v+1)+\sqrt{x}\,(22L'3x-v-2|L3x-v)$$

$$\times(0,\ v-x,\ 0,\ 0,\ x-1|v-1x'L'3x-v-2)]. \tag{29a}$$

$$\langle v+1x'L' \,\|\, T^{(1/2,0)} \,\|\, vxL\rangle_{SU(1,1)\otimes SO(3)}$$

$$= (-1)^{L+L'+1}[(2v+3)\,(2L'+1)\,(v-x'+1)!x'!/(2v+5)]^{1/2}$$

$$\times[\sqrt{v-x'+1}\,(2,\ -1L3x'-v|L'3x'-v-1)\,(0,\ v-x',\ 0,\ 0,\ x'|vxL3x'-v)$$

$$+\sqrt{x'}\,(22L3x'-v-3|L'3x'-v-1)\,(0,\ v-x'+1,\ 0,\ 0,\ x-1|vxL3x'-v-3)]. \tag{29b}$$

$$\langle v-2,\ x'L' \,\|\, T^{(1,2)} \,\|\, vxL\rangle_{SU(1,1)\otimes SO(3)}$$

$$= (v-x)\,[3(2L+1)\,(v-x-1)!x!/7]^{1/2}[2\sqrt{x}$$

$$\times(21L'3x-v-1|L3x-v)\,(0,\ v-x-1,\ 0,\ 0,\ x-1|v-2x'L'3x-v-1)$$

$$-\sqrt{v-x-1}\,(2,\ -2,\ L'3x-v+2|L3x-v)$$

$$\times(0,\ v-x-2,\ 0,\ 0,\ x|v-2,\ x'L'3x-v+2)]. \tag{29c}$$

$$\langle v+2,\ x'L' \,\|\, T^{(1,2)} \,\|\, vxL\rangle_{SU(1,1)\otimes SO(3)}$$

$$= (-1)^{L+L'}\{3(2v+3)\,(2L'+1)\,(v-x'+1)!x'!/[7(2v+7)]^{1/2}\}$$

$$\times[2\sqrt{x'}\,(21L3x'-v-3|L'3x-v-2)$$

$$\times(0,\ v-x'-1,\ 0,\ 0,\ x'-1|vxL3x'-v-3)-\sqrt{v-x'+1}$$

$$\times(2,\ -2,\ L3x'-v|L'3x'-v-2)\,(0,\ v-x',\ 0,\ 0,\ x|vxL3x'-v)]\,(v-x'+2). \tag{29d}$$

$$\langle vx'L' \,\|\, T^{(1,2)} \,\|\, vxL\rangle_{SU(1,1)\otimes SO(3)}$$

$$= -\{2(2v+1)\,(2L+1)\,(v-x)!x!/[7(2v+5)]\}^{1/2}$$

$$\times[\sqrt{2x}\,(22L'3x-v-2|L3x-v)\,(0,\ v-x,\ 1,\ 0,\ x-1|vx'L'3x-v-2)$$

$$+\sqrt{3(v-x)}\,(21L'3x-v-1|L3x-v)\,(1,\ v-x-1,\ 0,\ 0,\ x|vx'L'3x-v-1)$$

$$\cdot\sqrt{3x}\,(21L'3x-v-1|L3x-v)\,(0, v-x0, 1, x-1|vx'L'3x-v-1)$$

$$+\frac{3x-v}{\sqrt{2}}\,(20L'3x-v|L3x-v)\,(0, v-x, 0, 0, x|vx'L'3x-v)$$

$$-\sqrt{(v-x)/2}\,(2, -1L'3x-v+1|L3x-v)\,(0, x-v-1, 1, 0, x|vx'L'3x-v+1)$$

$$+\sqrt{3(v-x)}\,(2, -2L3x-v+2|L3x-2)\,(0, v-x-1, 0, 1, x|vx'L'3x-v+2)]. \quad (29e)$$

The evaluation of the matrix elements of physical operators under consideration is now completed.

4. *Applications*

At first, we want to test the invented new numerical program for the IBM analysis. We have chosen the examples of Gd and Xe isotopes for which the IBM energy calculations were already done [15]. We adopt the Hamiltonian parameters as in [15], i.e. $H_0 = 0$, $\varepsilon_s = 0$, $u_0 = 0.112$ MeV, $\varepsilon_d = 0.818$ MeV, $c_0 = -0.055$ MeV, $c_2 = -0.325$ MeV, $c_4 = -0.059$ MeV, $u_2 = -0.0109$ MeV, $v_0 = 0.472$ MeV, $v_2 = 0.414$ MeV for Gd isotopes and $H_0 = 0$, $\varepsilon_s = 0$, $u_0 = -0.009$ MeV, $\varepsilon_d = 2.599$ MeV, $c_0 = -0.668$ MeV, $c_2 = -0.456$ MeV, $c_4 = -0.437$ MeV, $u_2 = -0.578$ MeV, $v_0 = -0.074$ MeV, $v_2 = -0.102$ MeV for Xe isotopes. With the above parameters, within our numerical program, the energy levels with exactly the same accuracy as in [15] have been obtained.

The tested boson-eigenstates of the Gd-Xe isotopes have been then used to calculate the transition probabilities E2. We face the problem of fixing the two free parameters

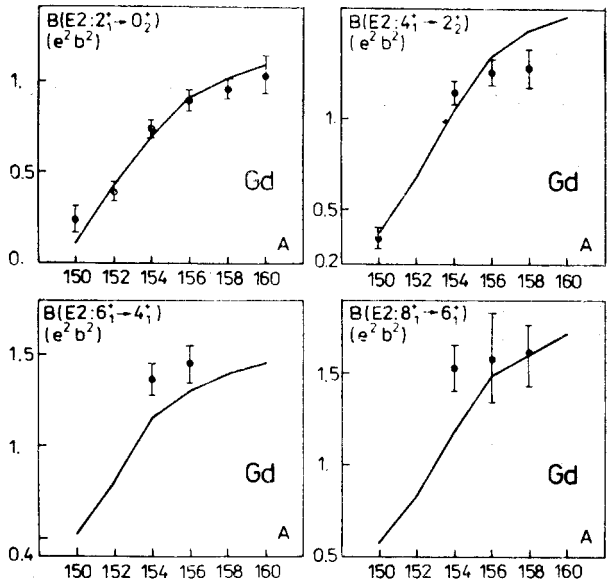

Fig. 1. Reduced theoretical (IBM-1) and experimental [16–18] E2 transitional probabilities in Gd isotopes

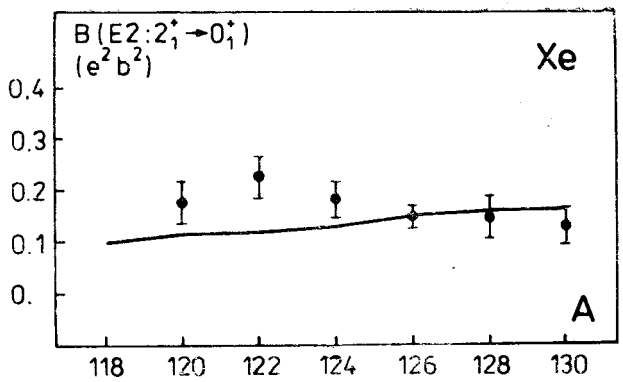

Fig. 2. Calculated and experimental [18–20] $B(E_2 : 2_1^+ \to 0_{g.s.}^+)$ in Xe isotopes

β_1 and β_2 of (26). The parameter β_2 can be related to β_1 by symmetry consideration. Namely, in a so-called quadrupole approximation we want the transition operator (26) to be the generator of the group SU(3). It is the case, if:

$$\beta_2 = -\frac{\sqrt{7}}{2}\beta_1. \tag{30}$$

Hence, we deal only with the one-parameter (β_1) transition model. The parameter β_1 has been then chosen to well reproduce the transition $B(E2 : 2_1^+ \to 0_{g.s.}^+)$ for ^{156}Gd, and hence

$$\beta_1 = 0.0149\ e^2b^2. \tag{31}$$

The results are given in Fig. 1 for Gd isotopes and in Fig. 2 for Xe isotopes. It is seen that the one-parameter model quite well reproduces the transition probabilities for Gd isotopes as well as for Xe isotopes.

In such a way we have completed the introduction and testing of the new basis for the IBM calculations[1]

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