A CONFORMALLY INVARIANT ZERO-MASS SCALAR FIELD WITH A TRACELESS ENERGY-MOMENTUM TENSOR IN STATIC SPACETIME

By A. Banerjee, S. B. Duttachoudhury and N. Banerjee

Department of Physics, Jadavpur University, Calcutta-700032, India

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A conformally invariant zero-mass scalar field with a traceless energy momentum tensor is discussed in static space-time. Exact solutions are obtained for a diagonal metric representing planar, cylindrical and toroidal symmetries in special cases and the calculations are extended to the case of a spherically symmetric spacetime including electromagnetic field. The scalar field may be shown to be formally the same as in Nordtvedt's general scalar tensor theory for a particular choice of the parameter ω as a function of the scalar field ψ.

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1. Introduction

The important property of the action integral for the conformally invariant scalar field θ is that the action integral remains invariant under the conformal transformation

$$g'_{\mu\nu} \to \xi(x) g_{\mu\nu}, \quad g'^{\mu\nu} \to \xi^{-1}(x) g^{\mu\nu} \quad \text{and} \quad \theta'(x) \to \xi^{1/2}(x)\theta(x).$$

Apart from the conformal invariance of the scalar field equation, another characteristic of such a field is that the energy momentum tensor is tracefree. For a zero mass conformally invariant scalar field, the field equations can be derived from the appropriate action integral

$$I = \int d^4x \sqrt{-g}\left[\frac{(1-\theta^2/6)R}{2k} + \tfrac{1}{2}\,\theta_\mu\theta^\mu\right],$$

where R is the curvature scalar and $k = \dfrac{8\pi G}{c^4}$.

Such fields have been considered earlier by Penrose [1] and Callan, Coleman and Jackiw [2]. Recently, this conformally invariant scalar field has attracted interest of many workers. Fryland [3] obtained exact spherically symmetric solutions for matterfree space in curvature coordinates and Vaidya and Som [4] extended his procedure to planar symmetry.

Accioly, Vaidya and Som [5] discussed the behaviour of a Bianchi type I cosmological model with this scalar field.

In the present paper, we make an extensive study of this scalar field in static spacetime including electromagnetic field in some special cases. It can be shown that the conformally invariant scalar field discussed here is formally equivalent to that in Nordtvedt's [6] scalar tensor theory with a particular choice of the parameter $\omega(\psi)$, namely $\omega(\psi) = \frac{3\psi}{2(1-\psi)}$, where ψ is the scalar field.

In Section 2 we write the field equations and the equation for the scalar field and derive some general relationships between the metric and the scalar field for a general static spacetime. These calculations are extended to the presence of electromagnetic field as well. In Section 3, static spacetime with a diagonal metric, describing cylindrical, planar or toroidal symmetry, is discussed and corresponding solutions obtained. In Section 4, spherically symmetric metric in curvature coordinates is considered only to show that conformally flat solution in this case does not exist. In Section 5 exact solutions are obtained in isotropic form of the spherically symmetric metric for the static spacetime surrounding a charged particle in the scalar tensor theory under consideration. Finally it is shown how one can pass over to the present theory from Nordtvedt's scalar tensor theory and vice-versa. Thus the solutions obtained in Section 5 are noted to correspond to a class of solutions obtained previously by the authors (Banerjee, Duttachoudhury and Banerjee [7]) in Nordtvedt's scalar tensor theory.

2. *Field equations and some general results connecting g_{00} and the scalar field in static spacetime*

The standard technique of variation of equation (1.1) with respect to $g_{\mu\nu}$ and θ yields the following equations for the metric and the scalar field respectively,

$$(1-k\theta^2/6)(R_{\mu\nu}-\tfrac{1}{2}g_{\mu\nu}R) = k(\tfrac{1}{2}g_{\mu\nu}\theta_\alpha\theta^\alpha-\theta_\mu\theta_\nu) + \frac{k}{6}[(\theta^2)_{;\mu\nu}-g_{\mu\nu}(\theta^2)^{;\alpha}_{;\alpha}] \tag{2.1}$$

$$\Box\theta+\tfrac{1}{6}R\theta = 0. \tag{2.2}$$

With (2.2), the trace of the equation (2.1) yields

$$R = 0. \tag{2.3}$$

In view of (2.3), one may express the equations (2.1) and (2.2) in the following forms

$$(1-u^2)R^\mu_\nu = (1-u^2)G^\mu_\nu = \delta^\mu_\nu u_\alpha u^\alpha-4u^\mu u_\nu+2uu^\mu_{;\nu} \tag{2.4}$$

$$\Box u = u^\alpha_{;\alpha} = 0, \tag{2.5}$$

where we have introduced u defined by

$$\left(\frac{k}{6}\right)^{1/2}\theta = u. \tag{2.6}$$

Now we consider a static spacetime described by the line element

$$ds^2 = g_{00}dt^2 + g_{ij}dx^i dx^j, \tag{2.7}$$

where i, j run from 1 to 3. If we define

$$f = (1-u^2), \tag{2.8}$$

then from (2.4) and (2.5) one can write

$$fR^0_0 = 3u_\alpha u^\alpha + f^{;\alpha}{}_{;\alpha} - f^{;0}{}_{;0}, \tag{2.9}$$

and

$$f^{;\alpha}{}_{;\alpha} = -2u_\alpha u^\alpha. \tag{2.10}$$

For a static spacetime, time derivative of any variable is zero and from the definition of $R_{\mu\nu}$ one can write

$$R^0_0 = \frac{1}{2\sqrt{-g}}(g^{00}g^{ij}\sqrt{-g}\, g_{00,j})_{,i}. \tag{2.11}$$

From the last three equations we obtain

$$f(g^{00}g^{ij}\sqrt{-g}\, g_{00,j})_{,i} = -[g^{ij}\sqrt{-g}f_{,j}]_{,i} - \sqrt{-g}\, g^{00}g^{ij}g_{00,j}f_{,i}.$$

With $g^{00}g_{00} = 1$ this equation can be written as

$$(g^{00}g^{ij}\sqrt{-g}\, g_{00j}f)_{,i} = -(g^{00}g_{00}g^{ij}\sqrt{-g}f_{j})_{,i} \tag{2.12}$$

or

$$[g^{00}g^{ij}\sqrt{-g}\,(g_{00}f)_{,j}]_{,i} = 0.$$

Assuming that g_{00} is functionally related to the scalar field u, equation (2.12) can be written as

$$[g^{00}g^{ij}\sqrt{-g}\, u_{j}(g_{00}f)^{\dagger}]_{,i} = 0, \tag{2.13}$$

where "†" represents differentiation with respect to u. So we get

$$(g^{ij}\sqrt{-g}\, u_j)_{,i}g^{00}(g_{00}f)^{\dagger} + g^{ij}\sqrt{-g}\, u_j u_i[g^{00}(g_{00}f)^{\dagger}]^{\dagger} = 0$$

which in view of the wave equation (2.5) yields

$$g^{ij}\sqrt{-g}\, u_j u_i[g^{00}(g_{00}f)^{\dagger}]^{\dagger} = 0. \tag{2.14}$$

For non-vanishing scalar field we have $g^{ij}u_i u_j \neq 0$ and (2.14) gives

$$[g^{00}(g_{00}f)^{\dagger}]^{\dagger} = 0$$

or

$$g^{00}g_{00}f^{\dagger}+g^{00}(g_{00})^{\dagger}f = p,$$

where p is a constant of integration. With $g^{00}g_{00} = 1$, we obtain

$$\frac{f^{\dagger}}{f}+\frac{(g_{00})^{\dagger}}{(g_{00})} = \frac{p}{f},$$

or,

$$\ln g_{00}f = p\int\frac{du}{(1-u^2)} = \ln k\left(\frac{1+u}{1-u}\right)^p$$

i.e.

$$g_{00} = \frac{k}{f}\left(\frac{1+u}{1-u}\right)^p,$$

k being a constant of integration which can be absorbed in g_{00} by a scale transformation. So with $f = (1-u^2)$, we finally obtain

$$g_{00} = \frac{1}{f}\left(\frac{1+u}{1-u}\right)^p = \frac{(1+u)^{p-1}}{(1-u)^{p+1}}. \tag{2.15}$$

If we include an electrostatic field given by $F_{0i} = F_{i0} = \phi_i$, $F_{00} = F_{ik} = 0$ where ϕ is the electric potential, the wave equation will remain unchanged as the energy momentum tensor for an electromagnetic field is traceless. Equation (2.4) will become

$$(1-u^2)R^{\mu}_{\nu} = \delta^{\mu}_{\nu}u_{\alpha}u^{\alpha}-4u^{\mu}u_{\nu}+2uu^{\mu}_{;\nu}-\tfrac{1}{2}T^{\mu}_{\nu}. \tag{2.16}$$

The electric field will also satisfy Maxwell's equation

$$(g^{00}g^{ij}\sqrt{-g}\,\phi_j)_{,i} = 0. \tag{2.17}$$

From (2.16) we can write

$$fR^0_0 = g^{00}g^{ij}\phi_i\phi_j+3g^{ij}u_iu_j+f^{;\alpha}_{;\alpha}-f^{;0}_{,0}. \tag{2.18}$$

Assuming that both g_{00} and u are functionally related to the electric potential ϕ, the equations (2.17), (2.18), (2.5) and (2.11) will lead, after a straightforward calculation, to the result

$$g_{00}f = \phi^2+a\phi+b, \tag{2.19}$$

where a and b are constants of integration.

With the same assumption that u is a function of ϕ, equations (2.5) and (2.17) will lead to another general result

$$g_{00}u' = c, \tag{2.20}$$

where c is a constant of integration and the prime represents differential with respect to ϕ. The equation (2.19) and (2.20) may be used to find the scalar field u in terms of the electric potential ϕ alone. All the general relations are verified in the explicit solutions given subsequently.

3. Static solutions with planar, cylindrical and toroidal symmetry

We now consider a static spacetime described by the metric

$$as^2 = e^{2\nu(x)}dt^2 - e^{2\lambda(x)}dx^2 - e^{2\mu(x)}d\eta^2 - e^{2\beta(x)}d\xi^2. \tag{3.1}$$

This metric has been considered by Bronnikov and Kovalchuk [8], and Banerjee and Santos [9]. It corresponds to cylindrical symmetry if ξ and η represent azimuthal and longitudinal coordinates respectively so that $0 \leqslant \xi \leqslant 2\pi$ and $-\infty \leqslant \eta \leqslant +\infty$. If both ξ and η are angular coordinates, it represents toroidal symmetry whereas if both ξ and η represent longitudinal coordinates, the metric represents pseudo-planar or planar symmetry. For the static spacetime under consideration, one can use without loss of generality the following coordinate condition,

$$\lambda = \nu + \mu + \beta. \tag{3.2}$$

The field equations (2.4) for this metric are

$$(1-u^2)U = 3u_1^2 + 2\lambda_1 uu_1 - uu_{11}, \tag{3.3}$$

$$(1-u^2)(\beta_{11}+\nu_{11}-U) = -u_1^2 - 2\mu_1 uu_1, \tag{3.4}$$

$$(1-u^2)(\nu_{11}+\mu_{11}-U) = -u_1^2 - 2\beta_1 uu_1, \tag{3.5}$$

$$(1-u^2)(\beta_{11}+\mu_{11}-U) = -u_1^2 - 2\nu_1 uu_1, \tag{3.6}$$

and the wave equation (2.5) becomes

$$u_{11} = 0, \tag{3.7}$$

where

$$U = \beta_1\nu_1 + \nu_1\mu_1 + \mu_1\beta_1, \tag{3.8}$$

and the subscript "1" indicates differentiation with respect to x. Adding equations (3.3) to (3.6) together and then using (3.2) and (3.7) one easily finds that for $(1-u^2) \neq 0$

$$\nu_{11} + \beta_{11} + \mu_{11} = U, \tag{3.9}$$

so that the field equations (3.3) to (3.6) reduce to the following forms

$$(1-u^2)(\nu_{11}+\beta_{11}+\mu_{11}) = 3u_1^2 + 2(\nu_1+\beta_1+u_1)uu_1, \tag{3.10}$$

$$(1-u^2)u_{11} = u_1^2 + 2\mu_1 uu_1, \tag{3.11}$$

$$(1-u^2)\beta_{11} = u_1^2 + 2\beta_1 uu_1, \tag{3.12}$$

$$(1-u^2)\nu_{11} = u_1^2 + 2\nu_1 uu_1. \tag{3.13}$$

Evidently, equation (3.10) is a consequence of the remaining three field equations. Further, the latter set of equations are structurally identical. When $u_1 \neq 0$, equation (3.13) can be written, in view of (3.7), as

$$\left[(1-u^2)\frac{\nu_1}{u_1}\right]_{,1} = u_1. \tag{3.14}$$

Equation (3.14) yields on integration

$$v_1 = \frac{u+k_1}{(1-u^2)} u_1, \tag{3.15}$$

where k_1 is an arbitrary constant. Equation (3.15), in turn, yields

$$e^{2v} = \frac{(1+u)^{k_1-1}}{(1-u)^{k_1+1}}, \tag{3.16}$$

where the second integration constant is absorbed, without loss of generality, by a scale transformation of the 't' coordinate. Similar procedure to integrate (3.11) and (3.12) yields respectively

$$e^{2\mu} = \frac{(1+u)^{k_2-1}}{(1-u)^{k_2+1}}, \tag{3.17}$$

and

$$e^{2\beta} = \frac{(1+u)^{k_3-1}}{(1-u)^{k_3+1}}, \tag{3.18}$$

where k_2 and k_3 are arbitrary constants. Here also the remaining integration constants are absorbed by suitable scale transformations of η and ξ coordinates. Equations (3.2), (3.8), (3.9) along with (3.16) to (3.18) yield

$$e^{2\lambda} = \frac{(1+u)^{k_4-3}}{(1-u)^{k_4+3}}, \tag{3.19}$$

where

$$k_4 = k_1+k_2+k_3, \tag{3.20}$$

and

$$k_1k_2+k_2k_3+k_3k_4 = 3. \tag{3.21}$$

The wave equation (3.7) readily gives

$$u = n_1x+n_2, \tag{3.22}$$

with n_1 and n_2 as constants of integration. Then (3.16) to (3.22) constitutes the complete solutions of the field equations.

In the special case when $k_2 = k_3 = k$ (say), a transformation of coordinates according as

$$x = \frac{1}{n_1}\left[\frac{(2k)^{1/k}(n_1\bar{x}+\bar{n}_2)^{1/k}-1}{(2k)^{1/k}(n_1\bar{x}+\bar{n}_2)^{1/k}+1} - n_2\right], \tag{3.23}$$

reduces the above solution to the following form:

$$u = \frac{(2k)^{1/k}(n_1\bar{x}+\bar{n}_2)-1}{(2k)^{1/k}(n_1\bar{x}+\bar{n}_2)+1}, \tag{3.24}$$

$$e^{2\nu} = e^{2\lambda} = \frac{[2k(n_1\bar{x}+\bar{n}_2)]^{(\frac{3}{2}k^2-\frac{1}{2})}}{(1-u^2)}, \tag{3.25}$$

$$e^{2\mu} = e^{2\beta} = \frac{2k(n_1\bar{x}+\bar{n}_2)}{(1-u^2)}, \tag{3.26}$$

which can be readily recognized as the plane symmetric solutions given by Vaidya and Som [4].

4. Non-existence of the conformally flat spherically symmetric solutions

We consider the spherically symmetric line element in the following form

$$ds^2 = e^{\nu}dt^2 - e^{\lambda}dr^2 - r^2d\theta^2 - r^2\sin^2\theta d\Phi^2, \tag{4.1}$$

where ν and λ are functions of the radial coordinate r alone. The field equations (2.4) and the wave equation (2.5) for this metric are

$$(1-u^2)\left[e^{-\lambda}\left(\frac{\nu'}{r}+\frac{1}{r^2}\right)-\frac{1}{r^2}\right] = e^{-\lambda}(3u'^2-2uu''+\lambda uu'), \tag{4.2}$$

$$(1-u^2)\left[e^{-\lambda}\left(\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\nu'\lambda'}{4}+\frac{\nu'-\lambda'}{2r}\right)\right] = e^{-\lambda}\left(-u'^2-\frac{2}{r}uu'\right), \tag{4.3}$$

$$(1-u^2)\left[e^{-\lambda}\left(\frac{1}{r^2}-\frac{\lambda'}{r}\right)-\frac{1}{r^2}\right] = e^{-\lambda}(-u'^2-\nu' uu'), \tag{4.4}$$

$$\frac{(\nu'-\lambda')u'}{2}+\frac{2}{r}u'+u'' = 0, \tag{4.5}$$

where a prime indicates differentiation with respect to r. In view of the wave equation (4.5), the operation: Equation (4.2) $+\ 2\times$ Equation (4.3) + Equation (4.4) yields

$$\frac{\nu''}{2}+\frac{\nu'^2}{4}-\frac{\nu'\lambda'}{4}-\frac{(\nu'-\lambda')}{r}+\frac{1}{r^2}-\frac{e^{\lambda}}{r^2} = 0. \tag{4.6}$$

If the spacetime described by the metric (4.1) is conformally flat, the Weyl tensor $C_{\alpha\beta\gamma\delta}$ vanishes, which in turn gives:

$$-\frac{\nu''}{4}-\frac{\nu'^2}{4}+\frac{\nu'\lambda'}{4}+\frac{(\nu'-\lambda')}{2r}-\frac{1}{r^2}+\frac{e^{\lambda}}{r^2} = 0. \tag{4.7}$$

Combining (4.6) and (4.7) one obtains

$$\nu' = \lambda', \tag{4.8}$$

so that by a simple scale transformation of time coordinate 't' one can set, without loss of generality,

$$\nu = \lambda. \tag{4.9}$$

In view of (4.9), equation (4.5) integrates to give

$$u = a/r + b, \tag{4.10}$$

where a, b are constants of integration. Adding (4.2) and (4.4) and using (4.9) and (4.10), one obtains

$$e^{\lambda} = e^{\nu} = \frac{(1-b^2)}{[1-(a/r+b)^2]}. \tag{4.11}$$

Now it is not difficult to see that the solutions (4.10) and (4.11) can satisfy the field equation (4.3) if and only if $a = 0$, which in turn implies that u is constant everywhere. It is proved, therefore, that non trivial conformally flat static solution does not exist in the spherically symmetric case. The solution mentioned to be conformally flat by Frøyland [3] in his paper is obtained, in fact, for a chosen constant magnitude for u (viz. $u = 1$) and thus does not yield a nontrivial field solution.

5. *Static spherically symmetric solution for a charged particle*

Let us take the line element in the isotropic form

$$ds^2 = e^{\nu}dt^2 - e^{\mu}(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\Phi^2), \tag{5.1}$$

where ν and μ are functions of the radial coordinate 'r' alone. The coupled Einstein-Maxwell field equations will be given by (2.16). In the covariant form, it will become

$$(1-u^2)R_{\mu\nu} = g_{\mu\nu}u_{\alpha}u^{\alpha} - 4u_{\mu}u_{\nu} + 2uu_{\mu;\nu} - \tfrac{1}{2}T_{\mu\nu}. \tag{5.2}$$

Here also $R_{\mu\nu}$ will be equivalent to $G_{\mu\nu}$ as $T_{\mu\nu}$ for an electromagnetic field is traceless. The only non-vanishing components of the electromagnetic field tensor for a radial electric field is given by $F_{01} = -F_{10} = \phi'$, where ϕ is the electric potential and a prime indicates differentiation with respect to r. The field equations will become

$$f\left(\tfrac{1}{4}\mu'\nu' + \tfrac{1}{2}\nu'' + \frac{\nu'^2}{4} + \frac{\nu'}{r}\right) = u'^2 + uu'\nu' + e^{-\nu}\phi'^2, \tag{5.3}$$

$$f\left(\frac{\nu''}{2} + \frac{3\mu''}{2} + \tfrac{1}{4}\nu'^2 + \frac{\mu'}{r} - \frac{\mu'\nu'}{4}\right) = -3u'^2 + 2uu'' - uu'\mu' + e^{-\nu}\phi'^2, \tag{5.4}$$

$$f\left(\frac{\mu''}{2} + \frac{\mu'^2}{4} + \frac{\mu'\nu'}{2} + \tfrac{3}{2}\frac{\mu'}{r} + \tfrac{1}{2}\frac{\nu'}{r}\right) = u'^2 + uu'\mu' - e^{-\nu}\phi'^2. \tag{5.5}$$

The three equations are not independent as they should satisfy $R = 0$. The wave equation for the scalar field will be

$$\frac{(\mu'+\nu')}{2}u'+\frac{2u'}{r}+u''=0. \tag{5.6}$$

The electric field will satisfy Maxwell's equation

$$[r^2 e^{(\mu-\nu)/2}\phi']'=0. \tag{5.7}$$

Equations (5.6) and (5.7) readily integrate to

$$r^2 e^{(\mu+\nu)/2}u' = A, \tag{5.8}$$

$$r^2 e^{(\mu-\nu)/2}\phi' = q, \tag{5.9}$$

respectively, where A and q are constants of integration. This q is related to the total charge of the source.

Now multiplying equation (5.3) with $r^2 e^{(\mu+\nu)/2}$ and using (5.8) and (5.9), one can obtain after integration

$$e^{\nu}f = \phi^2+a\phi+b, \tag{5.10}$$

where a and b are constants of integration. Then we add (5.3) with (5.5) and multiply the result by $r^2 e^{(\mu+\nu)/2}$, this can be integrated, using the wave equation and Maxwell's equation to yield

$$r^2 e^{(\mu+\nu)/2}f = mr^2+n, \tag{5.11}$$

m and n being arbitrary constants. From (5.9), (5.10) and (5.11), one finds that

$$\frac{\phi'}{(\phi^2+a\phi+b)} = q^{(mr^2+n)^{-1}}. \tag{5.12}$$

From equations (5.8), (5.9), (5.10) and (5.12) one obtains

$$\frac{u'}{f} = A(mr^2+n)^{-1},$$

or,

$$\frac{f'}{2f\sqrt{1-f}} = -A(mr^2+n)^{-1}, \tag{5.13}$$

as $f = (1-u^2)$. Equations (5.12) and (5.13) can be integrated to yield solutions for ϕ and f respectively, and with ϕ and f being known, ν and μ can be obtained from (5.10) and (5.11).

These solutions appear to be identical with those previously obtained by the authors [7] in Nordtvedt's theory for a particular choice of ω, namely $\omega = \frac{3\psi}{2(1-\psi)}$, where ψ is the scalar field. This is to be expected as we shall see now. Writing the action integral for the conformally invariant scalar field (1.1) in terms of f where $f = (1-u^2)$ and $u = \left(\frac{k}{6}\right)^{1/2} \theta$, we obtain

$$I = \tfrac{1}{2} \int \left[fR + \tfrac{3}{2} \frac{f^\mu f_\mu}{(1-f)} \right] \sqrt{-g}\, d^4x. \tag{5.14}$$

Now interpreting f as the scalar field ψ in Nordtvedt's theory along with a function $\omega(\psi) = \frac{3\psi}{2(1-\psi)}$, we obtain for the above action integral

$$I = \tfrac{1}{2} \int \left[\psi R + \frac{\omega(\psi)}{\psi} \psi^\mu \psi_\mu \right] \sqrt{-g}\, d^4x \tag{5.15}$$

which is the usual Nordtvedt action integral [6] except for the factor 1/2.

The spherically symmetric solutions in isotropic form for an uncharged particle as well as the solutions for plane symmetry can also be obtained in the same manner and can be shown to correspond to those previously obtained by the authors [7, 10] in Nordtvedt theory for a particular choice of $\omega = \frac{3\psi}{2(1-\psi)}$. Spherically symmetric solutions in the absence of electromagnetic field are given by Frøyland [1].

Unlike the uncharged case, one can have a conformally flat solution for a charged particle in the presence of conformally invariant scalar field. If $a^2 > 4b$, $m < 0$ and $n > 0$, we get from (5.12) the following solution

$$\phi = (a^2/4-b)^{1/2} \left[\frac{1 + a_1 \left(\frac{\sqrt{n} + \sqrt{-m}\, r}{\sqrt{n} - \sqrt{-m}\, r} \right)^{q \left(\frac{b-a^2/4}{mn} \right)^{1/2}}}{1 - a_1 \left(\frac{\sqrt{n} + \sqrt{-m}\, r}{\sqrt{n} - \sqrt{-m}\, r} \right)^{q \left(\frac{b-a^2/4}{mn} \right)^{1/2}}} \right] - \frac{a}{2}, \tag{5.16}$$

where a_1 is a constant of integration. With $n = q^2$, $a^2-4b = -4m$ and $a_1 = 1$, one obtains

$$\phi = -\left(\frac{q}{r} + \frac{a}{2} \right), \tag{5.17}$$

$$\phi' = \frac{q}{r^2}. \tag{5.18}$$

(5.9) and (5.18) yield

$$e^{(\nu-\mu)/2} = 1 \quad \text{(constant)},$$

which is sufficient to ensure that the Weyl tensor vanishes [11]. Thus the metric corresponding to this particular solution of ϕ will be conformally flat.

Very recently Accioly, Vaidya and Som [12] considered static spherically symmetric spacetime with a conformally invariant scalar field. They obtained the solution for the metric in terms of the scalar field which in turn had been solved explicitly. The solution for the electric potential, however, had not been given by them.

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