GENERATION OF COMPOSITE OPERATORS IN SUPERGRAVITY

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I discuss the generation of quantum composite operators in two and higher dimensions. In two dimensions the problem is discussed in detail, and the supergravity fields, trivial at the beginning, acquire the status of independent fields, non trivial features being obtained as a consequence. In higher dimensions we are led to non compact symmetry groups when dealing with supergravity. The symmetry SU(p, q) is discussed; quantization presents several problems. In one case, $p = q$, it is possible to obtain a prescription leading to finite results, with a quantization procedure breaking the symmetry to SU(p) ⊗ SU(q).

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1. Introduction

Supergravity has been very much studied in the last few years, and it is believed to accommodate in a natural way a quantum theory of gravitation [1] — albeit not renormalizable. In order to achieve renormalizability it will be possibly necessary to search for extended objects, like superstrings [2]. However $N = 8$ supergravity is quantum mechanically well defined (possibly finite up to 7 loops), being consequently worthwhile to be studied as a relevant theory for a broad variety of physical processes. Nevertheless, it is well known [3] that the SO(8) symmetry present in supergravity does not accommodate the known spectrum, described at low energy by SU(3)⊗SU(2)⊗U(1). To further extend the group SO(8), we must introduce undesirable higher spin fields, and enlarge the number of gravitons. One possible solution to this problem is the preonic picture, namely, the observed particles are bound states of fundamental preons [9]. In this case composite fields acquire the status of independent fields, as in the case of 2 dimensional CP^{N-1} models [4], where the gauge field is originally a composite operator, but in the end acquires an independent dynamics.

In four dimensions this mechanism is spoiled by ultraviolet divergences, and new counterterms should be added. Those counterterms are kinetic terms for the composite

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field. Consequently, the composite fields should have to be independent from the beginning, i.e. elementary, in order to the theory be defined at all! However, there is some hope to define these bound states in the case of supergravity, due to the non compact character of the symmetry group [5].

In order to get some insight into the problem, two dimensional supergravity is considered as a toy model. Notice that classically this is a trivial theory. This part of the work was done in collaboration with R. S. Jasinschi [6].

Afterwards a model with indefinite metric fields is considered, and the quantization procedure discussed [7, 8, 111].

2. *Generation of dynamics for non dynamical fields*[1]

We consider now a real multiplet interacting with supergravity fields. Although these are trivial in 2 dimensions, i.e. their kinetic part is either a total divergence (the Gauss-Bonnet theorem) or identically zero, we shall derive some interesting results in the quantum theory.

We consider the following Lagrangian of an O(N) supersymmetric multiplet (ϕ_i, ψ_i) interacting via σ-model like interaction, and also with G_μ and e^a_μ [6]

$$\mathscr{L} = \frac{N}{2f}\int d^2xe\left\{\tfrac{1}{2}\, e^\mu_a e^{a\nu}\partial_\mu\phi_i\partial_\nu\phi_i + \frac{i}{2}\,\bar{\psi}_i\gamma^\mu\partial_\mu\psi_i\right.$$

$$\left.+\bar{\psi}_i\gamma^\mu\gamma^\nu\partial_\nu\phi_i G_\mu - \tfrac{1}{4}\,\bar{\psi}_i\psi_i\bar{G}_\mu\gamma^\nu\gamma^\mu G_\nu + \tfrac{1}{8}\,(\bar{\psi}_i\psi_i)^2\right\}, \tag{2.1}$$

$$i = 1, \dots, N, \quad \phi_i^2 \equiv \sum_i \phi_i^2 = 1, \quad \psi_i\phi_i = 0, \quad e = \det e^a_\mu.$$

The above Lagrangian has the following symmetries:

1) Local supersymmetry ($\varepsilon = \varepsilon_{(x)}$)

$$\delta\phi_i = \varepsilon\psi_i, \tag{2.2a}$$

$$\delta\psi_i = -i[\partial_\mu\phi_i + \bar{G}_\mu\psi_i]\gamma^\mu\varepsilon + \tfrac{1}{2}\,\phi_i\bar{\psi}\psi, \tag{2.2b}$$

$$\delta e^\mu_a = 2i\bar{G}^\mu\gamma_a\varepsilon, \tag{2.2c}$$

$$\delta G_\mu = -D_\mu\varepsilon. \tag{2.d}$$

2) (Super) Weyl symmetry

$$\psi_i \to \Lambda^{-1/2}\psi_i, \tag{2.3a}$$

$$\phi_i \to \phi_i, \tag{2.3b}$$

[1] This part of the present work was done in collaboration with R. S. Jasinschi.

$$G_\mu \to \Lambda^{1/2} G_\mu, \tag{2.3c}$$

$$e^a_\mu \to \Lambda e^a_\mu, \tag{2.3d}$$

$$G_\mu \to G_\mu + \gamma_\mu \xi. \tag{2.4}$$

The classical invariances (2.2, 3, 4) are enough to eliminate completely the fields e^a_μ and G^a_μ. The equations of motion give rise to a "string-like" picture.

However, quantum corrections generate a mass for the fields ϕ_i and ψ_i, breaking the Weyl symmetry. This implies that e^a_μ and G_μ can no longer be eliminated from the theory! In fact, the quantum corrections with a finite mass for ϕ_i and ψ_i generate non trivial propagators for G_μ and e^a_μ, through the graphs shown in Fig. 1. All these properties

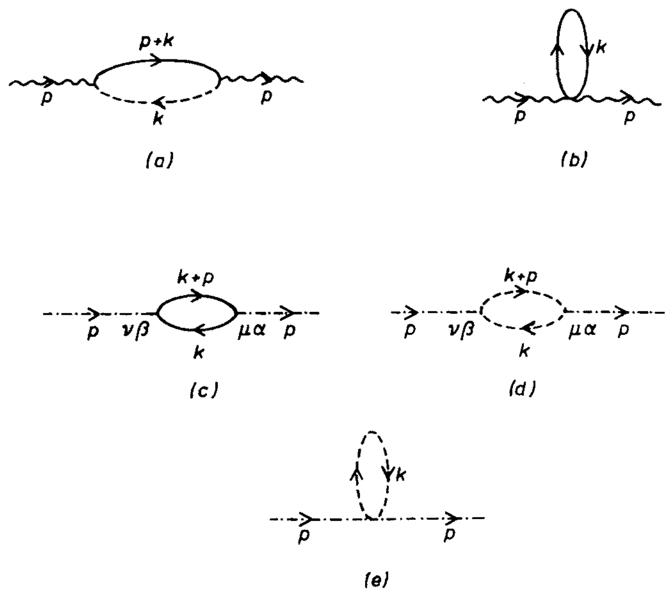

Fig. 1. Graphs with external wavy lines (a and b) correspond to the gravitino 2 point function (2.8a), the others (c, d and e) contribute to the graviton 2 point function (2.8b)

can be verified using a path integration of the problem, introducing Lagrange multipliers, and the Gross-Neveu trick to have a Lagrangian quadratic in the fields ϕ_i and ψ_i, integrating them afterwards. The path integral just before integrating these fields is

$$\mathscr{Z}(J, \ldots) = N \int \mathscr{D}\phi \mathscr{D}\psi \mathscr{D}\alpha \mathscr{D}\beta \mathscr{D}\bar{c} \mathscr{D}c \mathscr{D}G_\mu \mathscr{D}e^a_\mu \exp \int d^2x e$$

$$\times \left\{ -\tfrac{1}{2}\, \phi \Delta_B \phi - \tfrac{1}{2}\, \bar{\psi} \Delta_F \psi + \frac{i}{2\sqrt{N}}\, \bar{\psi}_i [c + \gamma^\mu \gamma^\nu G_\mu \partial_\nu] \phi_i \right.$$

$$\left. + \frac{1}{2\sqrt{N}}\, \phi_i [-G_\mu \gamma^\nu \gamma^\mu \partial_\nu - c] \psi_i - \frac{i\sqrt{N}\,\alpha}{2f} - \frac{i\beta^2}{N} + \text{source terms} \right\}, \tag{2.5}$$

where

$$\Delta_{\mathrm{B}} = \frac{i}{e}\,\partial_\mu(e^\mu_a\, e^{av} e\partial_v) - \frac{2i\alpha}{\sqrt{N}}, \tag{2.6a}$$

$$\Delta_{\mathrm{F}} = \gamma^\mu\partial_\mu + \frac{2i\sqrt{f}}{N}\beta + \frac{i}{2N}\bar{G}_\mu\gamma^\nu\gamma^\mu G_\nu. \tag{2.6b}$$

Integration of ϕ_i and ψ_i gives rise to an effective action for the other fields. Most important is the fact that auxiliary fields β and α acquire a nonzero vacuum expectation value, generating a mass for ϕ_i and ψ_i. In terms of the cut-off Λ, the generated mass is given by:

$$\frac{1}{2f} = \frac{1}{4\pi}\ln\frac{\Lambda^2}{m^2}. \tag{2.7}$$

This is the most important feature of the model, since the graviton and gravitino multipoint functions are only non zero due to the mass m. The Feynman rules can be read straightforwardly from (2.5) and (2.6), and the graphs in Fig. 1 readily calculated from the lowest order expansion of (2.5/6) in powers of $1/N$ for the gravitino two graviton/gravitino two point functions.

The results are

$$\Gamma^G_{\mu\nu} = i\pi m\left\{\sqrt{\frac{p^2-4m^2}{p^2}}\ln\frac{\sqrt{p^2}+\sqrt{p^2-4m^2}}{\sqrt{p^2}-\sqrt{p^2-4m^2}} - 2\right\}\eta_{\mu\nu}$$

$$+2\pi i m\left\{1+\frac{2m^2}{\sqrt{p^2(p^2-4m^2)}}\ln\frac{\sqrt{p^2}+\sqrt{p^2-4m^2}}{\sqrt{p^2}-\sqrt{p^2-4m^2}}\right\}\frac{p_\mu p_\nu}{p^2}, \tag{2.8a}$$

$$\Gamma^{e_\mu{}^\alpha e_\nu{}^\beta} = \frac{i\pi m^2\eta^\alpha_\mu\eta^\beta_\nu}{8\sqrt{p^2(p^2-4m^2)}}\ln\frac{\sqrt{p^2}+\sqrt{p^2-4m^2}}{\sqrt{p^2}-\sqrt{p^2-4m^2}}. \tag{2.8b}$$

We note that one propagator has a pole at zero momentum. In two dimensions, massless gauge fields confine [9], and in the present case this possibly means that all degrees of freedom are confined, and banished from the physical spectrum. Whether this is related to the spontaneous compactification of Kaluza-Klein theories [10], or superstrings [2], is an open question that should rather be answered in the context of $O(D-1, 1)$ symmetry, instead of $O(N)$. Notice also in the above expressions, that they would be zero if the generated mass were zero.

3. *Non compact symmetries and higher dimensions*

Generalization of the methods shown in the last section for higher dimensions is barred by difficulties. The determinant of the Laplacian, respectively Dirac operator is ambiguous, since several new non trivial divergences occur. In the case of the CP^{N-1}

model in 4 dimensions, a counterterm Lagrangian

$$\mathscr{L}_{ct} = AF^2_{\mu\nu} \tag{3.1}$$

is required. This is a reminiscent from the non-renormalizability, since such a term did not exist in the oirginal Lagrangian. If we simply sum it to the original Lagrangián, this means giving the gauge field A_μ the status of an independent field from the beginning spoiling its interpretation as a bound state of the partons, i.e. a generated gauge field.

Recently there has been some discussion whether the above illness of those nonlinear theories could be cured in the case of fields with non compact symmetry [7, 8, 11] assigning a suitable quantization condition for the negative metric field, in such a way that the counterterm Lagrangian (3.1) has a finite coefficient, being not necessary to be taken into account. Such procedure is achieved by performing path integration with negative metric fields being treated as ghost fields; the determinant coming from Gaussian integration appears to the positive (negative) instead of negative (positive) power for boson (fermion) fields. Namely, if ϕ_+, ψ_+ are the usual boson, fermion fields, and ϕ_-, ψ_- the negative metric counterparts

$$\int \mathscr{D}\phi_+ \exp(-\phi_+ D\phi_+) = (\det D)^{-1/2}, \tag{3.2a}$$

$$\int \mathscr{D}\phi_- \exp(+\phi_- D\phi_-) = (\det D)^{+1/2}, \tag{3.2b}$$

$$\int \mathscr{D}\psi_+ \exp(-\psi_+ D\psi_+) = (\det D)^{1/2}, \tag{3.3a}$$

$$\int \mathscr{D}\psi_- \exp(+\psi_- D\psi_-) = (\det D)^{-1/2}. \tag{3.3b}$$

This can be achieved assigning opposite $i\varepsilon$ prescription to the negative metric fields. It can be argued that this is the way to have a well defined path integral in the Minkowski space time.

If we trace this procedure back to the canonical quantization procedure, this means that the role of creation and anihilation operators for the negative metric field is interchanged. Defining

$$\phi_-(x) = \int \frac{d^3k}{\sqrt{(2\pi)^3 2\omega_k}} \left[a_-(k)e^{-ikx} + a_-^+(k)e^{ikx}\right] \tag{3.4}$$

we define the vacuum by

$$a_-^+(k)\,|0\rangle = 0. \tag{3.5}$$

The Fock space is generated by applying polynomials of $a_-(k)$ to the above vacuum. With the above procedure we can calculate the Feynman propagator, obtaining

$$i\Delta_{F-}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{i}{k^2 - m^2 - i\varepsilon} e^{ikx} \tag{3.6}$$

instead of

$$i\Delta_{F+}(x) = \int \frac{d^4k}{(2\pi)^4} \frac{-i}{k^2-m^2+i\varepsilon} e^{ikx}. \tag{3.7}$$

It can be shown also that [7]:
1) The Hamiltonian is positive definite, in the sense that its eigenvalues are all non-negative.
2) The negative metric fields have indeed a negative norm.

If the fields are in a (fundamental) representation of, say SU(p, q), then we can calculate the corresponding charges, and verify that some of them do not annihilate the vacuum, and that the actual symmetry is SU(p)⊗SU(q) instead.

4. *Discussion*

The preceding discussion shows that two possible ways of quantizing the negative metric fields exist. In one of them we have to deal with negative metric fields. On the other the non compact symmetry is broken to the maximal compact subgroup. In our opinion there is no way to distinguish both theories, and both can be equally used to describe the theory.

The first case has been used to quantize the 2 dimensional O(n, 1) nonlinear σ model [12]. Choosing the sign of the coupling constant it is possible to generate a mass to the fundamental fields [12]; the quantum non local charge can be defined [13], and is anomaly free [14]. Since the fields are massive, asymptotic states are well defined objects, and the in and out non local charges are defined, and equal. Their equality implies that the quantum S matrix is well defined [15]. It was also discussed recently. The negative metric fields must be somehow interpreted in order that the theory has a meaning.

In 2 dimensions the above sign of the coupling constant is possibly the only physically relevant region for this problem, since infrared divergences prevent us to have a well defined theory out of massless boson fields. However, in higher dimensions the situation allows more freedom, and we can choose one of the quantization procedures without further problems.

The best property of the symmetry (8) breaking quantization prescription is the possibility that divergences can be cancelled in the case, where the number of positive and negative metric fields is the same as in Grassmanian models SU(p, p)/S(U(p)⊗U(p)).

In that case, with a Lagrangian density

$$\mathscr{L} = \mathrm{tr}\,\{\overline{D_\mu Z} D_\mu Z + \alpha(Z\bar{Z}-1)\},$$

$$D_\mu Z = \partial_\mu Z - ZA_\mu$$

we can integrate the Z fields obtaining an effective action for the A_μ and α fields, which is finite, provided [8]:
1) we use the symmetry breaking quantization procedure, and
2) we assign different bare masses to the positive and negative metric fields.

In a low energy approximation the effective action for the A_μ field, after integration over Z's, reads [8]:

$$\mathscr{L} = -\frac{1}{4e^2} F_{\mu\nu}^2,$$

where $e = \frac{1}{4\pi} \ln \frac{m_+^2}{m_-^2}$, finite!

As an output one obtains a gauge theory in the low energy approximation.

If we insist in the symmetric phase, we have to deal with a non-renormalizable theory. In particular one needs a counterterm of the form $\mathscr{L}_{ct} = C(\Lambda)F_{\mu\nu}^2$ in order to cancel the divergences. This spoils the physical picture of having a generated gauge boson A_μ, since now we have to include a free Lagrangian for it.

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REFERENCES

[1] P. van Nieuwenhuizen, *Phys. Rep.* **68**, 189 (1981).
[2] J. H. Schwarz, *Phys. Rep.* **89**, 223 (1982).
[3] E. Cremmer, B. Julia, *Nucl. Phys.* **B159**, 141 (1979); E. Cremmer, Trieste School of supersymmetry and supergravity, 1982.
[4] A. D'Adda, P. Di Vecchia, M. Lüscher, *Nucl. Phys.* **B146**, 63 (1978).
[5] J. Ellis, C. Kounnas, D. V. Nanopoulos, CERN TH-3768-1983; J. Ellis, M. C. Gaillard, M. Günaydin, B. Zumino, *Nucl. Phys.* **B224**, 427 (1983).
[6] E. Abdalla, R. S. Jasinschi, *Nucl. Phys.* **B232**, 426 (1984).
[7] E. Abdalla, *On the quantization procedure for indefinite metric fields and non linear σ models*, NBI--HE-84-12.
[8] J. H. van Holten, *Phys. Lett.* **B135**, 427 (1984); J. H. van Holten, Wuppertal preprint.
[9] J. Lowenstein, J. A. Swieca, *Ann. Phys.* **68**, 172 (1971).
[10] M. J. Duff, *Nucl. Phys.* **B219**, 389 (1983).
[11] N. Ohta, *Phys. Lett.* **B134**, 75 (1984); A. C. Davis, A. J. Mc Farlane, J. H. van Holten, *Phys. Lett.* **B125**, 151 (1983); D. J. Smit, A. C. Davis, *Nucl. Phys.* **B225**, 221 (1983).
[12] M. Gomes, Yu. K. Ha, NYU preprint 1984.
[13] H. Eichenherr, M. Forger, *Nucl. Phys.* **B155**, 381 (1979).
[14] E. Abdalla, M. Forger, M. Gomes, *Nucl. Phys.* **B210**, 181 (1982).
[15] M. Lüscher, *Nucl. Phys.* **B135**, 1 (1978); E. Abdalla, M. C. B. Abdalla, M. Gomes, *Phys. Rev.* **D27**, 825 (1983).